Diallel Cross Design

- Diallel cross is an experiment in which two treatments are crossed each other to have a new treatment which will be consider to be a promising variety of crop. If you are interested to find a new variety of best crop, we will go for an experimentation by crossing the two new cross of different types and then testing the experiment which cross yields best and promising variety of crop.
- Grifind has introduced the concept of diallel cross experiment.

$$\begin{bmatrix} \mathbf{1x1} & \mathbf{1x2} & \mathbf{1x3} & \ldots & \mathbf{1xv} \\ \mathbf{2x1} & \mathbf{2x2} & \mathbf{2x3} & \ldots & \mathbf{2xv} \\ \mathbf{3x1} & \mathbf{3x2} & \mathbf{3x3} & \ldots & \mathbf{3xv} \\ \vdots & \vdots & \vdots & \ldots & \vdots \\ \mathbf{vx1} & \mathbf{vx2} & \mathbf{vx3} & \ldots & \mathbf{vxv} \end{bmatrix}$$

- Here there are v treatments and two treatment will be crossed together which gives total v^2 crosses.

- Here there are v crosses in the diagonal which are called self crossed experiment.

- There are v(v-1)/2 crosses of ixj where i<j

These crosses are called f1 crosses.

- There are also v(v-1)/2 crosses of ixj where i>j

These crosses are called reciprocals of f1 crosses.

So, the total crosses are ,

$$\mathbf{v\ self} + \frac{\mathbf{v(v-1)}}{\mathbf{2}}\,\mathbf{f1} + \frac{\mathbf{v(v-1)}}{\mathbf{2}}\ \textbf{resiprocal of f1}$$

$$= \mathbf{v} + \frac{\mathbf{v(v-1)}}{\mathbf{2}} + \frac{\mathbf{v(v-1)}}{\mathbf{2}}$$

$$= \mathbf{v + v(v-1)}$$

$$= \mathbf{v + v^2 - 2v + v}$$

$$= \mathbf{v^2}$$

- Curnow(1943) discussed the construction of diallel crossed experiment using RBD.
- Since RBD yields a large no. of crosses so later on various other authors develop the construction and use of diallel crossed experiment using BIBD,PBIBD and so on.
- After getting the cross one has to test and verify the best or promising variety of crop.
- There are two types of diallel crosses plan

1)Complete diallel cross plan. (CDC)

2)Partial diallel cross plan. (PDC)

CDC Plan

- A dialler crosses experiment is said to be complete dialled crosses plan if all the crosses occur an equal no. of times in experiments. Further the variances of all the lines of the dialled crosses are same that is all the lines are estimated with the same variances.

- CDC are always balanced design

PDC Plan

- A dialled crossed experiment is said to be PDC plan if some of the crosses occur n_1 times and remaining occur n_2 times.

- Here the variance of the treatment contrast are not same that is lines will be estimated two types of variances.

General combining ability

- GCA effect is a effect of combining i^{th} and j^{th} two lines to give the best yield these effect is estimated using least square method of estimation from linear model.
- Griffing classified the dialled crosses experiment into 4 systems namely,

  system 1,system 2,system 3, system 4.

CDC system 1

1) p self crossess

2) $\frac{p(p-1)}{2}$ f1 crossess

3) $\frac{p(p-1)}{2}$ reciprocal of f1 crossess

$$\text{Total}: p + \frac{p(p-1)}{2} + \frac{p(p-1)}{2}$$

$$= p + p(p-1)$$

$$= p^2$$

CDC system 2

1) p self crossess

2) $\frac{p(p-1)}{2}$ f1 crossess

$$\text{Total} : p + \frac{p(p-1)}{2}$$

$$= \frac{2p + p^2 - p}{2}$$

$$= \frac{p^2 + p}{2}$$

$$= \frac{p(p+1)}{2}$$

CDC system 3

1) $\dfrac{p(p-1)}{2}$ f1 crossess

2) $\dfrac{p(p-1)}{2}$ resiprocal of f1 crossess

Total : $\dfrac{p(p-1)}{2} + \dfrac{p(p-1)}{2}$

$= p(p-1)$

CDC system 4

$$1)\ \frac{p(p-1)}{2}\ f1\ \text{crosses}$$

$$\text{Total}: \frac{p(p-1)}{2}$$

- This is the simplest plan among all plan as it contains only f1 distinct crosses of type i<j.

Construction of CDC Plan

Method 1

Ex. Consider a BIBD with,

- $V=4, b=6, r=3, k=2, \lambda=1$

1x2	**(1)**
1x3	**(2)**
1x4	**(3)**
2x3	**(4)**
2x4	**(5)**
3x4	**(6)**

Consider another BIBD with,

- V1=6,b1=6,r1=k1=5, λ1=4

1	**2**	**3**	**4**	**5**		**1x2**	**1x3**	**1x4**	**2x3**	**2x4**
1	**2**	**3**	**4**	**6**		**1x2**	**1x3**	**1x4**	**2x3**	**3x4**
1	**2**	**3**	**5**	**6**		**1x2**	**1x3**	**1x4**	**2x4**	**3x4**
1	**2**	**4**	**5**	**6**		**1x2**	**1x3**	**2x3**	**2x4**	**3x4**
1	**3**	**4**	**5**	**6**		**1x2**	**1x4**	**2x3**	**2x4**	**3x4**
2	**3**	**4**	**5**	**6**		**1x3**	**1x4**	**2x3**	**2x4**	**3x4**

Tr.

BL.	1	2	3	4	
1	3	3	2	2	10
2	3	2	3	2	10
3	3	2	2	3	10
4	2	3	3	2	10
5	2	3	2	3	10
6	2	2	3	3	10
	15	15	15	15	

$$\sum n_{ij}^2 = 39$$

$$\sum n_{ij} n_{mj} = 37 \quad \forall\, i \neq j$$

- Each line will occur rr_1 times
- Each cross will occur k_1 times
- Total no. of crosses will be b_1k_1.

- Each line will occur rr_1 times
- Each cross will occur k_1 times
- Total no. of crosses will be b_1k_1.

- Each line will occur rr1 times
- Each cross will occur k1 times
- Total no. of crosses will be b1k1.

Method 2

Ex. Consider a BIBD with,

- V=5,b=5,r=4,k=4, λ=3

1	**2**	**3**	**4**	**1x2**	**1x3**	**1x4**	**2x3**	**2x4**	**3x4**
1	**2**	**3**	**5**	**1x2**	**1x3**	**1x5**	**2x3**	**2x5**	**3x5**
1	**2**	**4**	**5**	**1x2**	**1x4**	**1x5**	**2x4**	**2x5**	**4x5**
1	**3**	**4**	**5**	**1x3**	**1x4**	**1x5**	**3x4**	**3x5**	**4x5**
2	**3**	**4**	**5**	**2x3**	**2x4**	**2x5**	**3x4**	**3x5**	**4x5**

- Each line will occur $r(k-1)$ times
- Each cross will occur λ times
- Total no. of crosses will be $bk(k-1)/2$ times

Tr.

	1	2	3	4	5	
1	3	3	3	3	0	12
2	3	3	3	0	3	12
Bl. 3	3	3	0	3	3	12
4	3	0	3	3	3	12
5	0	3	3	3	3	12
	12	12	12	12	12	

$$\sum n_{ij}^2 = 36 \ \forall\ i = 1,2$$

$$\sum n_{ij} n_{mj} = 27 \quad \forall i \neq m = 1,2 \ldots 5$$

Method 3

Ghosh and Biswas(2003)

Ex. Consider a BIBD with,

- **V=b=7,r=k=3, λ=1**

$$3 \text{ is premetive element of } GF(7)$$

3^0	3^2	3^4		3^1	3^3	3^5
1	2	4		3	6	5
2	3	5		4	7	6
3	4	6		5	1	7
4	5	7		6	2	1
5	6	1		7	3	2
6	7	2		1	4	3
7	1	3		2	5	4

1x3	2x6	4x5
2x4	3x7	5x6
3x5	1x4	6x7
4x6	2x5	1x7
5x7	3x6	1x2
1x6	4x7	2x3
2x7	1x5	3x4

$$N = \begin{array}{r|ccccccc|c}
 & & & & \text{Tr.} & & & & \\
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \\
\hline
1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 6 \\
2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 6 \\
3 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 6 \\
\text{BL.}\,4 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 6 \\
5 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 6 \\
6 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 6 \\
7 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 6 \\
\hline
 & 6 & 6 & 6 & 6 & 6 & 6 & 6 &
\end{array}$$

$$\sum n_{ij}^2 = 6$$

$$\sum n_{ij} n_{mj} = 5$$

This is binary design.

- Each line will occur $2r$ times
- Each cross will occur λ times
- Total no. of crosses will be bk times

Analysis of CDC plan

- Consider a complete diallel cross plan involving p lines with b blocks, each of block size k.
- The linear model for the CDC system is defined as,
- $Y_{ij}=\mu+g_i+g_j+s_{ij}+e_{ij}$

  where μ is general effect,

  g_i,g_j are i^{th} and j^{th} lines which are unknown and called general combining ability of i^{th} and j^{th} line

- S_{ij} is called specific combining ability of i^{th},j^{th} cross
- e_{ij} are random error and $e_{ij} \sim N(0,\sigma^2)$

Now, in matrix notation, model can be written as,

$Y= \mu In+\Delta_1 g+ \Delta_2 \beta+\epsilon$ _______(2)

Where Y=nx1 vector of observed responses,

μ= general mean

In=Identity matrix

- g and β are vector of p and gca and b block effect respectively
- Δ_1 ,Δ_2 are corresponding design materials.
- Є denotes vector of independent random error~$N(0, \sigma^2)$
- It can be easily seen that (s,w)th element of Δ_1=1 ;If the cross in the sth experiment unit

  as one parent say, w

  =0; otherwise

  s=1,2…n; w=1,2…p

- Let r_{ij}=No. of replication of the cross ixj$(i<j)$

  in the experiment
- $G=\Delta_1'\Delta_1=G_{ij}$
- $N_d=\Delta_1'\Delta_2$
- Using the definition of the Δ_1 it can be verify that,
- $G_{ij}=r_{ij}$ and $G_{ii}=\Sigma r_{ij}$
- It can also verify that, $N_d=(n_{ij})$

  where n_{ij}=No. of times parent i appear in block j

The c matrix can be obtained,

- $C_d = G_d - \frac{N_{d'} N_d}{k}$

  Where G_d is symmetric matrix of order pxp

$$\text{and } G_d = \begin{bmatrix} w_{di} & g_{ii'} \\ & w_{di} \end{bmatrix}$$

Where w_{di} = No. of times i^{th} line occur in CDC plan.

$g_{ii'}$ = No. of times the cross ixi' appear in the plan.

$\forall i \neq i' = 1, 2 \ldots p$

- $N_d = (n_{dij})_{pxb}$

  where n_{dij} = No. of times i^{th} line occur in the block b.

Ex. Consider a BIBD with,

- V=b=7,r=k=3, λ=1

3 is premetive element of GF(7)

3^0	3^2	3^4
1	2	4
2	3	5
3	4	6
4	5	7
5	6	1
6	7	2
7	1	3

3^1	3^3	3^5
3	6	5
4	7	6
5	1	7
6	2	1
7	3	2
1	4	3
2	5	4

1x3	2x6	4x5
2x4	3x7	5x6
3x5	1x4	6x7
4x6	2x5	1x7
5x7	3x6	1x2
1x6	4x7	2x3
2x7	1x5	3x4

1x3 2x6 4x5
2x4 3x7 5x6
3x5 1x4 6x7
4x6 2x5 1x7
5x7 3x6 1x2
1x6 4x7 2x3
2x7 1x5 3x4

$$N = \begin{array}{c|ccccccc|c} & \multicolumn{7}{c}{\text{Tr.}} & \\ & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \\ \hline 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 6 \\ 2 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 6 \\ 3 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 6 \\ \text{BL.}\,4 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 6 \\ 5 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 6 \\ 6 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 6 \\ 7 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 6 \\ \hline & 6 & 6 & 6 & 6 & 6 & 6 & 6 & \end{array}$$

$$\sum n_{ij}^2 = 6 = v - 1$$

$$\sum n_{ij} n_{mj} = 5 = v - 2$$

This is binary design.

$$C_d = G_{di} - \frac{NN'}{k}$$

where $G_{di} = \begin{bmatrix} W_{di} & g_{ii'} \\ & W_{di'} \end{bmatrix}$

$$W_{di} = (v-1)$$

$$g_{ii} = \lambda$$

$$G_{di} = \begin{bmatrix} v-1 & \lambda & \dots & \lambda \\ \lambda & v-1 & \dots & \lambda \\ : & : & \dots & : \\ \lambda & \lambda & \dots & v-1 \end{bmatrix}$$

$$NN' = \begin{bmatrix} \sum n_{1j}^2 & \sum n_{1j} n_{2j} & \sum n_{1j} n_{pj} \\ \sum n_{2j} n_{1j} & \sum n_{2j}^2 & \sum n_{2j} n_{pj} \\ \sum n_{pj} n_{1j} & \sum n_{pj} n_{2j} & \sum n_{pj}^2 \end{bmatrix}$$

$$\sum n_{ij}^2 = (v-1), \sum n_{ij} n_{mj} = (v-2)$$

$$NN' = \begin{bmatrix} v-1 & v-2 & \ldots & v-2 \\ v-2 & v-1 & \ldots & v-2 \\ \vdots & \vdots & \ldots & \vdots \\ v-2 & v-2 & \ldots & v-1 \end{bmatrix}$$

$$\mathbf{c} = \begin{bmatrix} (\mathbf{v}-\mathbf{1}) & \lambda & \lambda \\ \lambda & (\mathbf{v}-\mathbf{1}) & \lambda \\ \lambda & \lambda & (\mathbf{v}-\mathbf{1}) \end{bmatrix} - \frac{\begin{bmatrix} \mathbf{v}-\mathbf{1}\ \mathbf{v}-\mathbf{2} \ldots \mathbf{v}-\mathbf{2} \\ \mathbf{v}-\mathbf{2}\ \mathbf{v}-\mathbf{1} \ldots \mathbf{v}-\mathbf{2} \\ \vdots \qquad \vdots \quad \ldots \quad \vdots \\ \mathbf{v}-\mathbf{2}\ \mathbf{v}-\mathbf{2} \ldots \mathbf{v}-\mathbf{1} \end{bmatrix}}{\mathbf{k}}$$

$$c=\begin{bmatrix}(v-1) & 0\ldots & 0\\ 0 & (v-1) & 0\\ 0 & 0 & (v-1)\end{bmatrix}+\begin{bmatrix}\lambda & \lambda\ldots\lambda\\ \lambda & \lambda\ldots\lambda\\ \lambda & \lambda\ldots\lambda\end{bmatrix}-\begin{bmatrix}\lambda & 0\ldots 0\\ 0 & \lambda\ldots 0\\ 0 & 0\ldots\lambda\end{bmatrix}$$

$$-\frac{\begin{bmatrix}(v-1) & 0\ldots & 0\\ 0 & (v-1) & 0\\ 0 & 0 & (v-1)\end{bmatrix}+\begin{bmatrix}v-2 & v-2\ldots & v-2\\ v-2 & v-2\ldots & v-2\\ v-2 & v-2\ldots & v-2\end{bmatrix}-\begin{bmatrix}v-2 & 0\ldots & 0\\ 0 & v-2\ldots & 0\\ 0 & 0\ldots & v-2\end{bmatrix}}{k}$$

$$=[(v-1)I_p+\lambda E_{pp}-\lambda I_p]-\frac{[(v-1)I_p+(v-2)E_{pp}-(v-2)I_p]}{k}$$

$$= [(v-1)-\lambda]I_p + \lambda E_{pp}$$

$$-\frac{[(v-1)-(v-2)]I_p + (v-2)E_{pp}}{k}$$

$$= \left[(v-1-\lambda)-\frac{1}{k}\right]I_p + \left[\lambda - \frac{(v-2)}{k}\right]E_{pp}$$

$$= \left[\frac{k(v-1)-k\lambda-1}{k}\right]I_p + \left[\frac{\lambda k - v + 2}{k}\right]E_{pp}$$

In this design each cross occur 1 time

so, $\lambda = 1$

$$= \left[\frac{k(v-1)-k-1}{k}\right] I_p + \left[\frac{k-v+2}{k}\right] E_{pp}$$

$$= \left[\frac{k(v-2)-1}{k}\right] I_p - \left[\frac{v-2-k}{k}\right] E_{pp}$$

$$= \left[\frac{k(v-2)-1}{k}\right] \left[I_p - \frac{1}{p} E_{pp} \right]$$

For any balanced design,

$$C_d = \theta(I_v - \tfrac{1}{v} E_{vv})$$

Then θ is called non zero eigrn value of matrix c

$$\theta = \left[\frac{k(v-2)-1}{k}\right] \text{ with multiplicity } (v-1)$$

The estimate of i^{th} line is obtained from $\dfrac{1}{\theta}$

$$\therefore \hat{g}_i = \frac{1}{\theta} = \left[\frac{k}{k(v-2)-1}\right]$$

The general combined ability can given by,

$$gca = \left[\frac{k}{k(v-2)-1}\right]$$

Variance of gca effect is given by,

$$v(g_i - \hat{g}_i) = \frac{2}{\theta}\sigma^2 = \left[\frac{2k}{k(v-2)-1}\right]\sigma^2$$

Efficiency

- **Singh and Hinkallman(1990) showed that for CDC plan in RBD, Variance of the gca effect is given by,**

$$\mathbf{V(U'g) = \frac{U'U}{q(p-2)}}\sigma^2$$

- **Where U'g=the contrasts among gca.**
- **P=No. of lines**
- **q= No. of block in RBD with $U'I_p$=0**
- **σ^2=Error mean square**

For, CDC plan we know that,

$$V(U'g) = \frac{2}{\theta}\sigma^2$$

where, θ is non – zero eigen value of c – matrix of CDC plan.

- Now, efficiency factor of the CDC plan is define as the ratio of gca effect in RBD. to the variance of gca effect of concerned CDC plan.
- That is efficiency factor is a factor to compare the CDC plan of the new plan with the CDC plan taken in a RBD.

$$E = \frac{V(U'g)_{\text{under RBD}}}{V(U'g)_{\text{New plan}}}$$

$$= \frac{\dfrac{U'U}{q(p-2)}\sigma^2}{\dfrac{2}{\theta}\sigma^2}$$

$$= \frac{2\sigma^2}{q(p-2)} \text{ x } \frac{\theta}{2\sigma^2}$$

$$= \frac{\theta}{q(p-2)} = \frac{k(v-2)-1}{kq(p-2)}$$

$$= \frac{3(7-2)-1}{3*1(7-2)} = \frac{14}{15}$$

- CDC plan with,
- V=b=7,r=k=3, λ=1

1	2	4	1x2	1x4	2x4
2	3	5	2x3	2x5	3x5
3	4	6	3x4	3x6	4x6
4	5	7	4x5	4x7	5x7
5	6	1	5x6	5x1	6x1
6	7	2	6x7	6x2	7x2
7	8	3	7x1	7x3	1x3

Blocks

TREAT.		1	2	3	4	5	6	7	
	1	2	2	0	2	0	0	0	6
	2	0	2	2	0	2	0	0	6
	3	0	0	2	2	0	2	0	6
	4	0	0	0	2	2	0	2	6
	5	2	0	0	0	2	2	0	6
	6	0	2	0	0	0	2	2	6
	7	2	0	2	0	0	0	2	6
		6	6	6	6	6	6	6	

$$\sum n_{ij}{}^{2} = 12 = r(k-1)^2$$

$$\sum n_{ij} n_{mj} = 4 = \lambda (k-1)^2$$

$$C_d = G_{di} - \frac{NN'}{k}$$

$$\text{where} \quad G_{di} = \begin{bmatrix} W_{di} & g_{ii'} \\ & W_{di'} \end{bmatrix}$$

$$W_{di} = W_{di'} = r(k-1)$$

$$g_{ii} = \lambda$$

- Since any block for I th line will occur (k-1) time in ixj th cross. Again there are r lines in a plan so, each line will occur r(k-1) times.

$$NN' = \begin{bmatrix} \sum n_{1j}^2 & \sum n_{1j}n_{2j} & \sum n_{1j}n_{pj} \\ \sum n_{2j}n_{1j} & \sum n_{2j}^2 & \sum n_{2j}n_{pj} \\ \sum n_{pj}n_{1j} & \sum n_{pj}n_{2j} & \sum n_{pj}^2 \end{bmatrix}$$

$$\sum n_{ij}^2 = r(k-1)^2$$

$$\sum n_{ij}n_{mj} = \lambda(k-1)^2$$

$$k = \frac{k(k-1)}{2}$$

$$
c = \begin{bmatrix} r(k-1) & \lambda & \lambda \\ \lambda & r(k-1) & \lambda \\ \lambda & \lambda & r(k-1) \end{bmatrix} - \frac{\begin{bmatrix} r(k-1)^2 & \lambda(k-1) & \lambda(k-1) \\ \lambda(k-1) & r(k-1)^2 & \lambda(k-1) \\ \lambda(k-1) & \lambda(k-1) & r(k-1)^2 \end{bmatrix}}{\frac{k(k-1)}{2}}
$$

$$= [r(k-1)I_p + \lambda E_{pp} - \lambda I_p]$$

$$-\frac{2[r(k-1)^2 I_p + \lambda (k-1)^2 E_{pp} - \lambda (k-1)^2 I_p]}{k(k-1)}$$

$$= [r(k-1) - \lambda]I_p - \frac{2[r(k-1)^2 - \lambda (k-1)^2]I_p}{k(k-1)}$$

$$+\left[\lambda - \frac{2\lambda (k-1)^2]E_{pp}}{k(k-1)}\right]$$

$$= \left[\{r(k-1) - \lambda\} - \frac{2\{r(k-1) - \lambda(k-1)\}}{k} \right] I_p$$

$$+ \left[\lambda - \frac{2\lambda(k-1)}{k} \right] E_{pp}$$

$$= \left[\frac{rk(k-1) - \lambda k - 2r(k-1) + 2\lambda(k-1)\}}{k} \right] I_p$$

$$+ \left[\frac{\lambda k - 2\lambda(k-1)}{k} \right] E_{pp}$$

$$= \left[\frac{r(k-1)(k-2) - \lambda k + 2\lambda k - 2\lambda}{k} \right] I_p$$

$$+ \left[\frac{\lambda(k - 2k + 2)}{k} \right] E_{pp}$$

$$= \left[\frac{r(k-1)(k-2) + \lambda(k-2)}{k} \right] I_p + \left[\frac{\lambda(2-k)}{k} \right] E_{pp}$$

$$= \left[\frac{(k-2)\{r(k-1) + \lambda\}}{k} \right] I_p - \left[\frac{\lambda(k-2)}{k} \right] E_{pp}$$

$$= \left[\frac{(k-2)\{r(k-1)+\lambda\}}{k}\right] I_p - \left[\frac{\lambda(k-2)}{k}\right] E_{pp}$$

$$= \left[\frac{(k-2)r(k-1)}{k}\right] I_p + \left[\frac{\lambda(k-2)}{k}\right] I_p - \left[\frac{\lambda(k-2)}{k}\right] E_{pp}$$

$$= \left[\frac{\lambda(k-2)}{k}\right] [I_p - E_{pp}] + \left[\frac{(k-2)\lambda(v-1)}{k}\right] I_p$$

$$= \left[\frac{\lambda(k-2)}{k}\right][I_p - vI_p - I_p] - \left[\frac{\lambda(k-2)}{k}\right]E_{pp}$$

$$= \left[\frac{\lambda v(k-2)}{k}\right]\left[I_p - \frac{E_{pp}}{v}\right]$$

$$= \left[\frac{\lambda p(k-2)}{k}\right]\left[I_p - \frac{E_{pp}}{p}\right]$$

$$\text{So, } \theta = \left[\frac{\lambda p(k-2)}{k}\right]$$

$$\hat{g}_i = \frac{1}{\theta} Q_i$$

$$= \frac{k}{\lambda v(k-2)} Q_i$$

$$v(g_i - \hat{g}_i) = \frac{2}{\theta}\sigma^2$$

$$= \frac{2}{\frac{\lambda p(k-2)}{k}}\sigma^2$$

$$= \frac{2k}{\lambda p(k-2)}\sigma^2$$

with multiplicity $(v-1)$

Design is balanced

Robustness of CDC Plan

- Consider a CDC plan with line p arrange in b blocks of size k. Let delete s observations from the block where s=1,2…k
- Cd denote C matrix of original CDC plan.
- Cd* denote C matrix of residual CDC plan.
- Robustness criteria against the loss of s observation is A efficiency, defined as ,

$$E = \frac{\text{sum of the resiprocal of the non zero eigen value of } c_d}{\text{sum of the resiprocal of the non zero eigen value of } c_{d*}}$$

- Let delete one block from the CDC plan.
- So, here each crosses will not repeated same times. Some crosses repeated λ1 times and remaining will repeated λ2 times.
- λ1= λ
- λ2 = λ-1
- Which is nothing but $g_{ii'}$

Now, w_{di} is no. of times ith line occur in CDC plan.

i line will occur r_1 times

i' line will occur r_2 times.

$$r = r(k-1)$$

$$r' = (r-1)(k-1)$$

$$\sum n_{ij}^2 = r(k-1)^2$$

$$\sum n_{i'j}^2 = (r-1)(k-1)^2$$

$$C_d = G_{di} - \frac{NN'}{k}$$

$$\text{where } G_{di} = \begin{bmatrix} W_{di} & g_{ii'} \\ & W_{di'} \end{bmatrix}$$

$$NN' = \begin{bmatrix} \sum n_{1'j}^{2} & \sum n_{1'j} n_{2j} & \sum n_{1'j} n_{p'j} \\ \sum n_{2'j} n_{1'j} & \sum n_{2'j}^{2} & \sum n_{2'j} n_{p'j} \\ \sum n_{p'j} n_{1'j} & \sum n_{p'j} n_{2'j} & \sum n_{p'j}^{2} \end{bmatrix}$$

$$c = \begin{bmatrix} (r-1)(k-1) & \lambda_2 & \lambda_1 & \dots & \lambda_1 \\ \lambda_2 & (r-1)(k-1) & \lambda_1 & \dots & \lambda_1 \\ \lambda_1 & \lambda_1 & r(k-1) & \dots & \lambda_1 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \lambda_1 & \lambda_1 & \lambda_1 & \dots & r(k-1) \end{bmatrix}$$

$$- \frac{\begin{bmatrix} (r-1)(k-1)^2 & (\lambda-1)(k-1)^2 & \lambda(k-1)^2 & \dots & \lambda(k-1)^2 \\ (\lambda-1)(k-1)^2 & (r-1)(k-1)^2 & \lambda(k-1)^2 & \dots & \lambda(k-1)^2 \\ \lambda(k-1)^2 & \lambda(k-1)^2 & r(k-1)^2 & \dots & \lambda(k-1)^2 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \lambda(k-1)^2 & \lambda(k-1)^2 & \lambda(k-1)^2 & \dots & r(k-1)^2 \end{bmatrix}}{\frac{k(k-1)}{2}}$$

$$\sim> (r-1)(k-1) - \frac{2(r-1)(k-1)^2}{k(k-1)}$$

$$= (r-1)(k-1) - \frac{2(r-1)(k-1)}{k}$$

$$= \frac{k(r-1)(k-1) - 2(r-1)(k-1)}{k}$$

$$= \frac{(r-1)(k-1)(k-2)}{k}$$

$$\sim> (\lambda - 1) - \frac{2(\lambda - 1)(k - 1)^2}{k(k - 1)}$$

$$= (\lambda - 1) - \frac{2(\lambda - 1)(k - 1)}{k}$$

$$= \frac{k(\lambda - 1) - 2(\lambda - 1)(k - 1)}{k}$$

$$= \frac{(\lambda - 1)(k - 2k + 2)}{k}$$

$$= \frac{-(\lambda - 1)(k - 2)}{k}$$

$$\sim> \lambda - \frac{2\lambda(k-1)^2}{k(k-1)}$$

$$= \lambda - \frac{2\lambda(k-1)}{k}$$

$$= \frac{\lambda k - 2\lambda k + 2\lambda}{k}$$

$$= \frac{2\lambda - \lambda k}{k}$$

$$= \frac{-\lambda(k-2)}{k}$$

$$\sim> r(k-1) - \frac{2r(k-1)^2}{k(k-1)}$$

$$= r(k-1) - \frac{2r(k-1)}{k}$$

$$= \frac{kr(k-1) - 2r(k-1)}{k}$$

$$= \frac{r(k-1)(k-2)}{k}$$

$$\mathbf{C}_{d^*} =$$

$$\left[\begin{array}{cc:ccc}
(r-1)(k-1)(k-2) & -(\lambda-1)(k-2) & -\lambda(k-2) & \ldots & -\lambda(k-2) \\
-(\lambda-1)(k-2) & (r-1)(k-1)(k-2) & -\lambda(k-2) & \ldots & -\lambda(k-2) \\
\hdashline
-(\lambda-1)(k-2) & -\lambda(k-2) & r(k-1)(k-2) & \ldots & -\lambda(k-2) \\
\vdots & \vdots & \vdots & \ldots & \vdots \\
-\lambda(k-2) & -\lambda(k-2) & -\lambda(k-2) & \ldots & r(k-1)(k-2)
\end{array}\right]$$

$$\theta_1 = \frac{(r-1)(k-1)(k-2)+(\lambda-1)(k-2)}{k}$$

$$= \frac{(k-2)[(r-1)(k-1)+(\lambda-1)]}{k}$$

$$= \frac{(k-2)[(r(k-1)-(k-1)+(\lambda-1)]}{k}$$

$$= \frac{(k-2)[(\lambda(v-1)-(k-1)+(\lambda-1)]}{k}$$

$$= \frac{(k-2)[(\lambda v-\lambda-k+1+\lambda-1]}{k}$$

$$= \frac{(k-2)(\lambda v-k)}{k}$$

$$\theta_2 = \frac{r(k-1)(k-2)+\lambda(k-2)}{k}$$

$$= \frac{(k-2)[r(k-1)+\lambda]}{k}$$

$$= \frac{(k-2)[\lambda(v-1)+\lambda]}{k}$$

$$= \frac{(k-2)(\lambda v-\lambda+\lambda)}{k}$$

$$= \frac{(k-2)(\lambda v)}{k}$$

$$\hat{g}_i = \frac{1}{\theta_1} = \frac{k}{(k-2)(\lambda v - k)}$$

Here $\hat{g}_i$ are those lines which are deleted in the blocks.

$$\hat{g}_j = \frac{1}{\theta_2} = \frac{k}{\lambda v(k-2)}$$

Here $\hat{g}_j$ are those lines which are not deleted in the blocks.

Variance is,

$$v(g_i - g_j) = \left[\frac{2k}{(k-2)(\lambda v - k)}\right]\sigma^2$$

$$v(g_i - g_{i'}) = \left[\frac{2k}{\lambda v(k-2)}\right]\sigma^2$$

Efficiency

$$E = \frac{V(U'g)C_d}{V(U'g)C_{d*}}$$

$$= \frac{\text{sum of resiprocal of eigen value of } C_d}{\text{sum of resiprocal of eigen value of } C_{d*}}$$

$$V(U'g)C_d = \frac{2k}{\lambda v(k-2)}(v-1)$$

$$V(U'g)C_{d*} = \frac{k}{(\lambda v - k)(k-2)}(k-1) + \frac{k}{\lambda v(k-2)}(v-k)$$

$$= \frac{\lambda vk(k-1) + k(v-k)(\lambda v - k)}{(\lambda v - k)(k-2)\lambda v}$$

$$= \frac{k[\lambda v(k-1) + (v-k)(\lambda v - k)]}{(\lambda v - k)(k-2)\lambda v}$$

$$E = \frac{\dfrac{k(v-1)}{\lambda v(k-2)}}{\dfrac{k[\lambda v(k-1) + (v-k)(\lambda v - k)]}{(\lambda v - k)(k-2)\lambda v}}$$

$$= \frac{k(v-1)}{\lambda v(k-2)} \ \mathrm{x}\ \frac{(\lambda v - k)(k-2)\lambda v}{k[\lambda v(k-1) + (v-k)(\lambda v - k)]}$$

$$= \frac{(v-1)(\lambda v - k)}{\lambda vk - \lambda v + \lambda v^2 - \lambda vk - kv - k^2}$$

$$E = \frac{(v-1)(\lambda v - k)}{\lambda v(v-1) - k(v-k)}$$

$$\sim> \text{Loss} = 1 - E$$

$$\therefore \text{Loss} = 1 - \frac{(v-1)(\lambda v - k)}{\lambda v(v-1) - k(v-k)}$$

$$=\frac{\lambda v(v-1)-k(v-k)-(v-1)(\lambda v-k)}{\lambda v(v-1)-k(v-k)}$$

$$=\frac{(v-1)[\lambda v-\lambda v+k]-k(v-k)}{\lambda v(v-1)-k(v-k)}$$

$$=\frac{k(v-1)-k(v-k)}{\lambda v(v-1)-k(v-k)}$$

$$\text{Loss} = \frac{k(v - 1 - v + k)}{\lambda v(v - 1) - k(v - k)}$$

We can write,

$$E = 1 - \frac{k(v - 1 - v + k)}{\lambda v(v - 1) - k(v - k)}$$