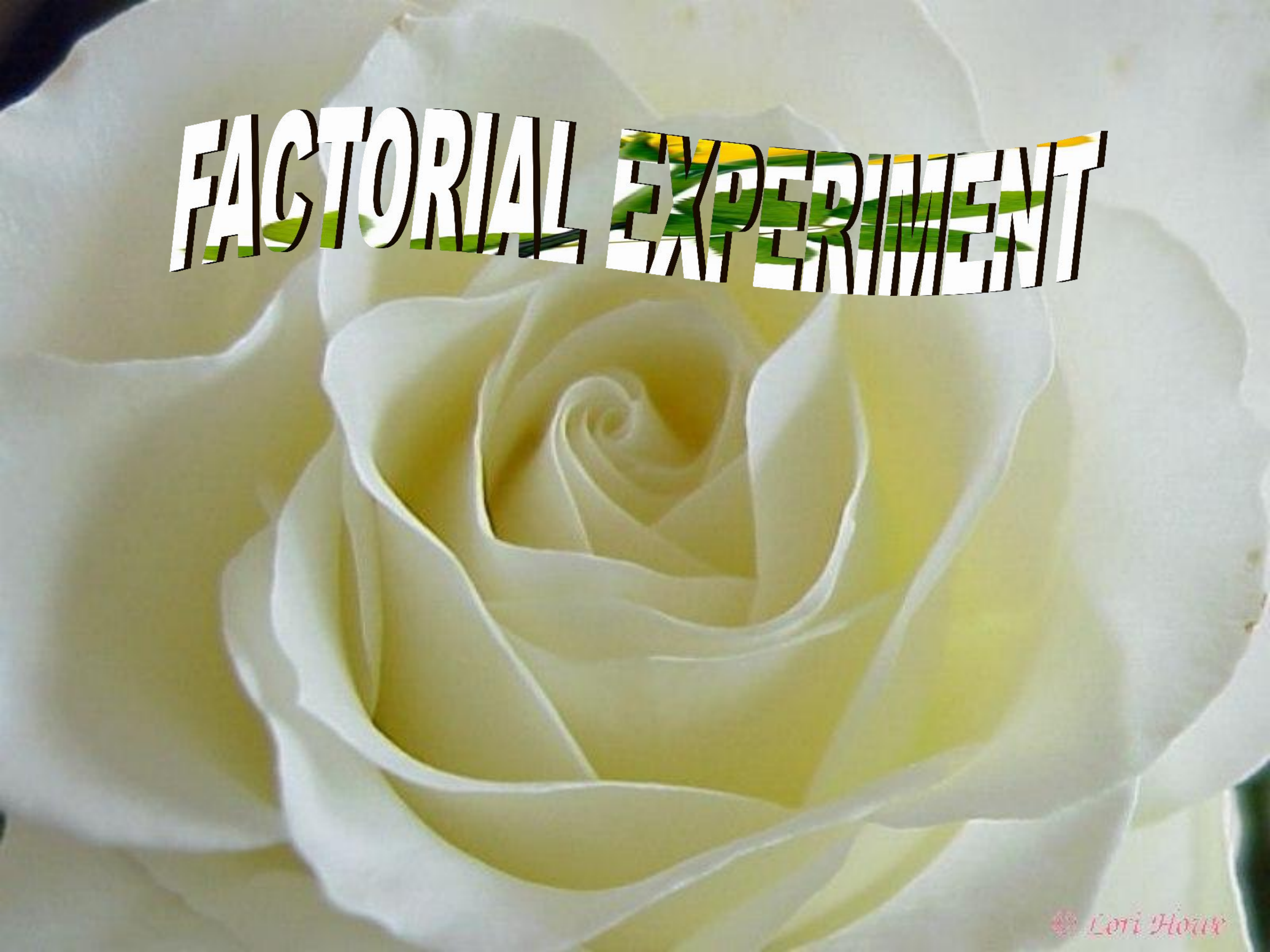
FACTORIAL EXPERIMENT



Factorial Experiment:

Definition:

Let $x_1,x_2,\ldots,x_n$ are n factors at $s_1,s_2,\ldots,s_n$ levels, the combination of $s_1^{\ x1},s_2^{\ x2},\ldots s_n^{\ xn}$ called factorial experiment, where this combinations are called treatment combinations.

Factorial experiment are of two types namely,

1) **Symmetrical factorial experiment** ($2^x*2^y=2^{x+y}$)
2) **Asymmetrical factorial experiment** (2^x*3^y)

2^n factorial experiment:

Definition:

If we have n factors and each are at 2 levels,the treatment combinations in a block randomly, then the experiment is called 2^n symmetrical factorial experiment.

2^2 factorial experiment:

The factors are two namely A and B, each are at two levels namely 0 and 1, so the four possible treatment combinations are,

a_0b_0	1	**A**		**B**	
a_0b_1	b	0	1	0	1
a_1b_0	a				
a_1b_1	ab				

Main effect:

Main effect is defined as average effect of all possible treatment combination at 2 level formed in 2 groups,such that level of 1 factor at each group is same, while level of other factors are different.

4 treatments are,

a_1b_1	a_0b_1
a_1b_0	a_0b_0
-----------------	------------------
$a_1b_1 + a_1b_0$	$a_0b_1 + a_0b_0$

$$A = \frac{[a_1b_1 + a_1b_0]/2 - [a_0b_1 + a_0b_0]/2}{2}$$

$$B = \frac{[a_1b_1 + a_0b_1]/2 - [a_1b_0 + a_0b_0]/2}{2}$$

Interaction effect:

The interaction effect is defined as average of the average of the average effect of all possible treatment combination in 2^n factorial experiment. Such that level of one factor in each group is same while level of other factors are different.

$$AB = \frac{(a_1b_1 - a_1b_0)/2 + (a_0b_0 - a_0b_1)/2}{2}$$

$$= \frac{(a_1b_1 - a_1b_0)/2 - (a_0b_1 - a_0b_0)/2}{2}$$

Other definition of Main effect and Interaction effect:

Main effect:

Main effect is defined as contrast of all possible treatment combinations at two level formed in two groups, such that level of one factor in each group is same,while level of other factors are different.

Interaction effect:

Interaction effect is defined as contrast of the contrast of all possible treatment combination in 2 factorial experiment, such that level of one factor in each group is same,while level of other factors are different.

There are three methods to estimate Main effect and Interaction effect:

1) Yate's method:

combinations	contrasts		
	A	B	AB
a_0b_0	-	-	+
a_0b_1	-	+	-
a_1b_0	+	-	-
a_1b_1	+	+	+

$$A = -a_0b_0 - a_0b_1 + a_1b_0 + a_1b_1 = (a_1b_1 + a_1b_0) - (a_0b_1 + a_0b_0)$$

$$B = -a_0b_0 + a_0b_1 - a_1b_0 + a_1b_1 = (a_0b_1 + a_1b_1) - (a_0b_0 + a_1b_1)$$

$$AB = a_0b_0 - a_0b_1 - a_1b_0 + a_1b_1 = (a_1b_1 - a_0b_1) - (a_1b_0 - a_0b_0)$$

2) Fisher & Yate's method :

$A = (a-1)(b+1) = ab-b+a-1$

OR

$A = (a_1-a_0)(b_1+b_0) = a_1b_1+a_1b_0-a_0b_1-a_0b_0$

$= (a_1b_1+a_1b_0) - (a_0\ b_1+a_0b_0)$

$B = (b_1-b_0)(a_1+a_0) = a_1b_1+a_0b_1-a_1b_0-a_0b_0$

$= (a_1b_1+a_0b_1) - (a_1b_0+a_0b_0)$

$AB = (a_1-a_0)(b_1-b_0) = a_1b_1-a_1b_0-a_0b_1+a_0b_0$

$= (a_1b_1-a_0b_1) - (a_1b_0-a_0b_0)$

3)Ghosh method:

Hadmard matrix of size m, where m=2,4,8,…

i.e.m = 2^n

Take n= 2;

Then

$$H_4= \begin{matrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{matrix}$$

Delete the first row of Hadmard matrix and write the remaining matrix of (m-1) * m and call H.

$$H= \begin{matrix} 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{matrix}$$

Now the treatment combination from right to left,

	a_1b_1	a_0b_1	a_1b_0	a_0b_0
A	1	-1	1	-1
B	1	1	-1	-1
AB	1	-1	-1	1

$$A = a_1b_1 - a_0b_1 + a1b0 - a_0b_0 = (a_1b_1 + a_1b_0) - (a_0b_1 + a_0b_0)$$

$$B = a_1b_1 + a_0b_1 - a_1b_0 - a_0b_0 = (a_1b_1 + a_0b_1) - ((a_1b_0 + a_0b_0)$$

$$AB = a_1b_1 - a_0b_1 - a_1b_0 + a_0b_0 = (a_1b_1 - a_0b_1) - (a_1b_0 - a_0b_0)$$

ANOVA TABLE OF 2^2 FACTORIAL EXPERIMENT:

S.V.	d.f.	s.s.	M.s.s.	F-ratio
Replication	r-1	$\frac{\sum R_i^2}{t}$ – c.f. = **R.S.S.**	σ_R^2	σ_R^2 / σ_e^2
Treatment combination	4-1=3			
A	1	$[A]^2/4r$ = **A S.S.**	$[A]^2/4r=\sigma_A^2$	σ_A^2 / σ_e^2
B	1	$[B]^2/4r$ =**B S.S.**	$[B]^2/4r=\sigma_B^2$	σ_B^2 / σ_e^2
AB	1	$[AB]^2/4r$ = **AB S.S.**	$[AB]^2/4r=\sigma_{AB}^2$	$\sigma_{AB}^2 / \sigma_e^2$
Error	3(r-1)	By subtraction=Se^2 = ***E.S.S.***	σ_e^2	
Total	4(r-1)	$\sum Y_{ij}^2$ – c.f. = **T.S.S.**		

Example:

An agriculture experiment was conducted at Junagadh agriculture university to find the increase of groundnut of the application of 2 fertilizers of Nitrogen and Potash. Nitrogen was applied as 40 gm/hector and 80 gm/h, while Potash was applied at 50 gm/h and 100 gm/h. The groundnut seeds were planted in such way that the distance between 2 plants in a row is same. This experiment is conducted in 3 replication. The collected observations of groundnut are shown below.

Rep-1	Rep-2	Rep-3
a_0b_0(60)	a_1b_1 (79)	a_0b_1(72)
a_0b_1 (70)	a_0b_0 (66)	a_0b_0 (55)
a_1b_0 (75)	a_0b_1 (88)	a_1b_0 (69)
a_1b_1 (80)	a_1b_0 (100)	a_1b_1 (120)

Analyse the data and give your comments.

Calculation:

H_0:The distance between two plants in a row is same.

Table:

	Rep-1	Rep-2	Rep-3	Total
a_0b_0	60	66	55	T_{00}=181
a_0b_1	70	88	72	T_{01}=230
a_1b_0	75	100	69	T_{10}=244
a_1b_1	80	79	120	T_{11}=279
	285	333	316	G=934

C.F.= $G^2/N = (934)^2/12 = 72696.33$

Total S.S = $\sum Y_{ij}^2$ – C.F. =3539.67

Repli.s.s. = $\sum R_i^2/t$ – C.F. =296.17

Now,

A= $-T_{00}-T_{01}+T_{10}+T_{11}$

= -181-230+244+279 = **112**

s.s. due to A= $[A]^2/4r = (112)^2/12 =$ **1045.33**

B= $T_{01} + T_{11} - T_{10} - T_{00}$ = 230+279-181-244=**84**

s.s. due to B= $[B]^2/4r = (84)^2/12 =$ **588**

AB= $T_{11} - T_{10} + T_{00} - T_{01}$= **-14**

s.s. due to AB= $[AB]^2/4r =$**16.333**

ANOVA TABLE:

S.V.	d.f.	s.s.	M.s.s.	F-ratio	F_t	
					5%	1%
Repli.	3-1=2	296.17	148.085	0.5574	5.14	10.92
Treat. combination	4-1=3					
A	1	1045.33	1045.33	3.9351	5.99	13.74
B	1	588	588	2.2135	5.99	13.74
AB	1	16.333	16.333	0.0614	5.99	13.74
Error	6	1593.837	265.639			
Total	12-1=11	3539.67	321.788			

For repli.: **Fc<Ft** (for both level) => test is insignificant & we accept our H_0.

For main effect **A** & **B** and interaction effect **AB**: **Fc<Ft** (for both level) => test is insignificant & we accept our H_0.

2^3 Factorial Experiment:

Let A,B,C are three factors, each are at two levels namely 0 and 1,so 2^3 possible treatment combinations are,

$\mathbf{a_0b_0c_0}$

$\mathbf{a_0b_0c_1}$

$\mathbf{a_0b_1c_0}$

$\mathbf{a_0b_1c_1}$

$\mathbf{a_1b_0c_0}$

$\mathbf{a_1b_0c_1}$

$\mathbf{a_1b_1c_0}$

$\mathbf{a_1b_1c_1}$

A		B		C	
0	1	0	1	0	1

Main effects and Interaction contrasts in 2^3 factorials:

Combinations	Main effects and Interactions						
	A	B	C	AB	AC	BC	ABC
$a_0b_0c_0$	–	–	–	+	+	+	–
$a_0b_0c_1$	–	–	+	+	–	–	+
$a_0b_1c_0$	–	+	–	–	+	–	+
$a_0b_1c_1$	–	+	+	–	–	+	–
$a_1b_0c_0$	+	–	–	–	–	+	+
$a_1b_0c_1$	+	–	+	–	+	–	–
$a_1b_1c_0$	+	+	–	+	–	–	–
$a_1b_1c_1$	+	+	+	+	+	+	+

Here,

$A= -a_0b_0c_0-a_0b_0c_1-a_0b_1c_0-a_0b_1c_1+a_1b_0c_0+a_1b_0c_1+a_1b_1c_0+a_1b_1c_1$

$=(a_1b_0c_0+a_1b_0c_1+a_1b_1c_0+a_1b_1c_1) – (a_0b_0c_0+a_0b_0c_1+a_0b_1c_0+a_0b_1c_1)$

$B= -a_0b_0c_0-a_0b_0c_1+ a_0b_1c_0 + a_0b_1c_1 - a_1b_0c_0-a_1b_0c_1+a_1b_1c_0+a_1b_1c_1$

$= (a_1b_1c_0+a_1b_1c_1 + a_0b_1c_0 + a_0b_1c_1) – (a_0b_0c_0+a_0b_0c_1 + a_1b_0c_0+a_1b_0c_1)$

$C= -a_0b_0c_0+a_0b_0c_1- a_0b_1c_0 + a_0b_1c_1 - a_1b_0c_0+a_1b_0c_1-a_1b_1c_0+a_1b_1c_1$

$= (a_1b_0c_1 +a_1b_1c_1 + a_0b_1c_1 +a_0b_0c_1) –(a_0b_0c_0+ a_0b_1c_0 + a_1b_0c_0 +a_1b_1c_0)$

$AB= a_0b_0c_0+a_0b_0c_1-a_0b_1c_0-a_0b_1c_1-a_1b_0c_0-a_1b_0c_1+a_1b_1c_0+a_1b_1c_1$

$= (a_0b_0c_0+a_0b_0c_1 +a_1b_1c_0+a_1b_1c_1) – (a_0b_1c_0+a_0b_1c_1+a_1b_0c_0+a_1b_0c_1)$

$ABC= -a_0b_0c_0+a_0b_0c_1+a_0b_1c_0-a_0b_1c_1+a_1b_0c_0-a_1b_0c_1-a_1b_1c_0+a_1b_1c_1$

$=(a_0b_0c_1+a_0b_1c_0+a_1b_0c_0+a_1b_1c_1) – (a_0b_0c_0+a_0b_1c_1+a_1b_0c_1+a_1b_1c_0)$

ANOVA TABLE OF 2^3 FACTORIAL EXPERIMENTS:

s.v.	d.f.	s.s.	M.s.s.	F-ratio
Replication	r-1	$\frac{\sum Ri2}{ti}$ – C.F.	σ_R^2	
Treatment combination	2^3-1=7			
A	1	$[A]^2/2^3 r$	σ_A^2	σ_A^2/σ_e^2
B	1	$[B]^2/2^3 r$	σ_B^2	σ_B^2/σ_e^2
AB	1	$[AB]^2/2^3 r$	σ_{AB}^2	σ_{AB}^2/σ_e^2
C	1	$[C]^2/2^3 r$	σ_C^2	σ_C^2/σ_e^2
AC	1	$[AC]^2/2^3 r$	σ_{AC}^2	σ_{AC}^2/σ_e^2
BC	1	$[BC]^2/2^3 r$	σ_{BC}^2	σ_{BC}^2/σ_e^2
ABC	1	$[ABC]^2/2^3 r$	σ_{ABC}^2	$\sigma_{ABC}^2/\sigma_e^2$
Error	7(r-1)	By subtraction	σ_e^2	
Total	2^3 r-1	$\sum Y_{ij}2$-C.F.		

3^n Factorial Experiment:

Definition:

If we have n factors and each are at 3 levels, then the treatment combinations in a block randomly then the experiment is called 3^n symmetrical factorial experiment.

* when factors are taken at 3 levels instead of 2, the scope of the experiment increases.It becomes more informative. Further, when the levels of a factor are quantitative, the pattern of change of response with the increase of the level values of the factor can be studied in a better way. A study to investigate if the change is linear or quadratic is possible when the factors at 3 levels, though from this point of view the more the number of levels the better.

3^2 Factorial Experiment:

Let there are two factors N and P, each are at 3 levels namely 0,1, and 2.

N			P		
0	1	2	0	1	2

All possible combinations are,

n_0p_0	n_1p_0	n_2p_0
n_0p_1	n_1p_1	n_2p_1
n_0p_2	n_1p_2	n_2p_2

Method 1:

In 3^2 factorial experiment, there are two types of Main effect namely,

1) Linear Main effect
2) Quadratic Main effect

The contrast N_2-N_0 gives the linear contrast among the three levels of N, if they are equispaced. It indicates a linear relation between the response and level values of N.

The other contrast, $N_2 - 2N_1 + N_0$. This contrast indicates a quadratic relation between the levels and their responses.

Now,

$N_L = N_2 - N_0$

$= (n_2p_0+n_2p_1+n_2p_2) - (n_0p_0+n_0p_1+n_0p_2)$

$N_Q = N_2 - 2N_1 + N_0$

$= (n_2p_0+n_2p_1+n_2p_2) - 2(n_1p_0+n_1p_1+n_1p_2) + (n_0p_0+n_0p_1+n_0p_2)$

$$P_L = P_2 - P_0 = (p_2n_0+p_2n_1+p_2n_2) - (p_0n_0+p_0n_1+p_0n_2)$$

$$P_Q = P_2 - 2P_1 + P_0$$

$$= (p_2n_0+p_2n_1+p_2n_2) - 2(p_1n_0+p_1n_1+p_1n_2) + (p_0n_0+p_0n_1+p_0n_2)$$

$$N_LP_L = (N_2-N_0)(P_2-P_0)$$

$$= [(n_2p_0+n_2p_1+n_2p_2) - (n_0p_0+n_0p_1+n_0p_2)]$$

$$[(p_2n_0+p_2n_1+p_2n_2) - (p_0n_0+p_0n_1+p_0n_2)]$$

$$N_LP_Q = (N_2-N_0)(P_2 -2P_1+P_0)$$

$$= [(n_2p_0+n_2p_1+n_2p_2) - (n_0p_0+n_0p_1+n_0p_2)]$$

$$[(p_2n_0+p_2n_1+p_2n_2) - 2(p_1n_0+p_1n_1+p_1n_2) + (p_0n_0+p_0n_1+p_0n_2)]$$

$$N_QP_L = (N_2-N1+N_0)(P_2-P_0)$$

$$=[(n_2p_0+n_2p_1+n_2p_2) - 2(n_1p_0+n_1p_1+n_1p_2) + (n_0p_0+n_0p_1+n_0p_2)]$$

$$[(p_2n_0+p_2n_1+p_2n_2) - (p_0n_0+p_0n_1+p_0n_2)]$$

$$N_QP_L = (N_2-N1+N_0)(P_2 -2P_1+P_0)$$

$$=[(n_2p_0+n_2p_1+n_2p_2) - 2(n_1p_0+n_1p_1+n_1p_2) + (n_0p_0+n_0p_1+n_0p_2)]$$

$$[(p_2n_0+p_2n_1+p_2n_2) - (p_1n_0+p_1n_1+p_1n_2) + (p_0n_0+p_0n_1+p_0n_2)]$$

Now,

s.s. of $N_L = (N_L)^2/6r$

s.s. of $N_Q = (N_Q)^2/18r$

s.s. of $P_L = (P_L)^2/6r$

s.s. of $P_Q = (P_Q)^2/18r$

s.s. of $N_L P_L = (N_L P_L)^2/36r$

s.s. of $N_L P_Q = (N_L P_Q)^2/108r$

s.s. of $N_Q P_L = (N_Q P_L)^2/108r$

s.s. of $N_Q P_Q = (N_Q P_Q)^2/324r$

s.s. of N= s.s. of N_L + s.s. of N_Q

s.s. of P = s.s. of P_L + s.s. of P_Q

s.s. of NP= s.s. of $N_L P_L$ + s.s. of $N_L P_Q$ + s.s. of $N_Q P_L$ + s.s. of $N_Q P_Q$

Method-2 (Ghosh method) : ***0 -> 3, 1->2, 2->6***

Treatment combination	Replication total	Col-1	Col-2	divisor	s.s.= $\frac{(\text{col-2})^2}{\text{divisor}}$
00	R_1	$R_1+R_2+R_3=A$	A+B+C=**G**	9r	
01	R_2	$R_4+R_5+R_6=B$	D+E+F=P	6r	$P^2/6r$=s.s.of P_L
02	R_3	$R_7+R_8+R_9=C$	G+H+I=Q	18r	$Q^2/18r$=s.s.of P_Q
10	R_4	$R_3-R_1=D$	C-A=R	6r	$R^2/6r$=s.s.of N_L
11	R_5	$R_6-R_4=E$	F-D=S	4r	$S^2/4r$=s.s.of N_LP_L
12	R_6	$R_9-R_7=F$	I-G=T	12r	$T^2/12r$=s.s.of N_LP_Q
20	R_7	$R_3-2R_2+R_1=G$	C-2B+A=U	18r	$U^2/18r$=s.s.of N_Q
21	R_8	$R_6-2R_5+R_4=H$	F-2E+D=V	12r	$V^2/12r$=s.s.of N_QP_L
22	R_9	$R_9-2R_8+R_7=I$	I-2H+G=W	36r	$W^2/36r$=s.s.of N_QP_Q

ANOVA TABLE OF 3^2 FACTORIAL EXPERIMENT:

s.v.	d.f.	s.s.	M.s.s.(s.s./d.f.)	F-ratio
Replication	r-1	$\sum R_i^2 / t - c.f.$	σ_R^2	σ_R^2 / σ_e^2
Treat.Combi.	3^2-1=8			
N	2	N_L s.s +N_Q s.s	σ_N^2	σ_N^2 / σ_e^2
N_L	1	N_L s.s	σN_L^2	$\sigma N_L^2 / \sigma_e^2$
N_Q	1	N_Q s.s	σN_Q^2	$\sigma N_Q^2 / \sigma_e^2$
P	2	P_L s.s +P_Q s.s	σ_P^2	σ_P^2 / σ_e^2
P_L	1	P_L s.s	σP_L^2	$\sigma P_L^2 / \sigma_e^2$
P_Q	1	P_Q s.s	σP_Q^2	$\sigma P_Q^2 / \sigma_e^2$
NP	4	$N_L P_L$ s.s+…+ $N_Q P_Q$ s.s	σ_{NP}^2	$\sigma_{NP}^2 / \sigma_e^2$
$N_L P_L$	1	$N_L P_L$ s.s	$\sigma N_L P_L^2$	$\sigma N_L P_L^2 / \sigma_e^2$
$N_L P_Q$	1	$N_L P_Q$ s.s	$\sigma N_L P_Q^2$	$\sigma N_L P_Q^2 / \sigma_e^2$
$N_Q P_L$	1	$N_Q P_L$ s.s	$\sigma N_Q P_L^2$	$\sigma N_Q P_L^2 / \sigma_e^2$
$N_Q P_Q$	1	$N_Q P_Q$ s.s	$\sigma N_Q P_Q^2$	$\sigma N_Q P_Q^2 / \sigma_e^2$
Error	8(r-1)	By subtraction	σ_e^2	
Total	3^2r-1	$\sum Y_{ij}^2 - c.f.$		

Method-3:

		N			SUM
		0	1	2	
P	0	n_0p_0	n_1p_0	n_2p_0	$n_0p_0+n_1p_0+n_2p_0$ ----→eq.(1)
	1	n_0p_1	n_1p_1	n_2p_1	$n_0p_1+n_1p_1+n_2p_1$ ----→eq.(2)
	2	n_0p_2	n_1p_2	n_2p_2	$n_0p_1+n_1p_2+n_2p_2$ ----→eq.(3)
SUM		$n_0p_0+n_0p_1+n_0p_2$ ----→eq.(4)	$n_1p_0+n_1p_1+n_1p_2$ ----→eq.(5)	$n_2p_0+n_2p_1+n_2p_2$ ----→eq.(6)	Grand Total

s.s. of P $= \dfrac{[eq.(1)]^2+[eq.(2)]^2+[eq.(3)]^2}{3r} - c.f.$; Where $c.f.=G^2/N$

s.s. of N $= \dfrac{[eq.(4)]^2+[eq.(5)]^2+[eq.(6)]^2}{3r} - c.f.$

Cell s.s. due to NP = $[n_0p_0^2+n_1p_0^2+\ldots+n_2p_2^2]/r$ - c.f.

s.s. of NP = cell s.s. due to NP – s.s. of N – s.s. of P

ANOVA TABLE OF 3^2 FACTORIAL EXPERIMENT:

s.v.	d.f.	s.s.	M.s.s.	F-ratio
Repli.	r-1	$\frac{\sum Ri2}{ti}$ – C.F.	σ_R^2	
Treatment combination	$3^2 - 1=8$			
N	2	N s.s.	σ_N^2	σ_N^2/σ_e^2
P	2	P s.s.	σ_P^2	σ_P^2/σ_e^2
NP	4	NP s.s.	σ_{NP}^2	σ_{NP}^2/σ_e^2
Error	8(r-1)	E s.s.	σ_e^2	
total	9r-1	T s.s.		

Example:

N and P were considered sugarcane in Saurashtra region.Nitrogen are considered at 3 level namely 30 gm/h, 80 gm/h, 120 gm/h. Similarly phospharas was taken at 3 different level namely 60 gm/h, 100 gm/h and 150 gm/h. The experiment is conducted in 3 replications. The collected data is shown below:

Repli.-1	Repli.-2	Repli.-3
n_0p_0 (40)	n_1p_0 (67)	n_2p_2 (76)
n_0p_1 (60)	n_1p_1 (86)	n_2p_0 (89)
n_0p_2 (65)	n_1p_2 (94)	n_0p_0 (44)
n_1p_0 (80)	n_0p_1 (55)	n_1p_1 (99)
n_1p_1 (85)	n_0p_2 (48)	n_1p_2 (105)
n_1p_2 (90)	n_0p_0 (42)	n_0p_1 (49)
n_2p_0 (70)	n_2p_1 (78)	n_0p_2 (50)
n_2p_1 (72)	n_2p_2 (85)	n_1p_0 (106)
n_2p_2 (84)	n_2p_0 (60)	n_2p_1 (87)

Analyse the data and give your comments.

Calculation:

H_0: The experiment is conducted in 3 replications.

	Replications			Total
	R1	R2	R3	
n_0p_0	40	42	44	126
n_1p_1	60	55	49	164
n_0p_2	65	48	50	163
n_1p_0	80	67	106	253
n_1p_1	85	86	99	270
n_1p_2	90	94	105	289
n_2p_0	70	60	89	219
n_2p_1	72	78	87	237
n_2p_2	84	85	76	245
Total	646	615	705	**1966 = G**

$$c.f. = G^2/N = (1966)^2/27 = 143153.9259$$

		N			SUM
		0	1	2	
P	0	$n_0p_0 = 126$	$n_1p_0 = 253$	$n_2p_0 = 219$	$n_0p_0+n_1p_0+n_2p_0 = 598$
	1	$n_0p_1 = 164$	$n_1p_1 = 270$	$n_2p_1 = 237$	$n_0p_1+n_1p_1+n_2p_1 = 671$
	2	$n_0p_2 = 163$	$n_1p_2 = 289$	$n_2p_2 = 245$	$n_0p_1+n_1p_2+n_2p_2 = 697$
SUM		$n_0p_0+n_0p_1+n_0p_2 = 453$	$n_1p_0+n_1p_1+n_1p_2 = 812$	$n_2p_0+n_2p_1+n_2p_2 = 701$	Grand Total = **1966**

$$\text{s.s. of P} = \frac{[(598)^2 + (671)^2 + (697)^2]}{3*3} - c.f.$$

s.s. of P = 144005.5556 – 143153.9259

= **585.4074**

s.s. of N = $\frac{(453)^2 + (812)^2 + (701)^2}{3*3}$ - 143153.9259

= **7507.6296**

Cell s.s. due to NP = $\frac{(126)^2 + \ldots + (245)^2}{3}$ - 143153.9259

= (453926/3) – 143153.9259

= **8154.740767**

s.s. of NP = cell s.s. of NP – s.s. of N – s.s. of P

= 8154.740767-7507.6296-585.4074

= **61.703767**

ANOVA TABLE:

s.v.	d.f.	s.s.	M.s.s.	F-ratio	F_t	
					5%	1%
Repli	3-1=2	464.5185	232.2592	2.6452	3.63	6.23
Treat.comb.	$3^2-1=8$					
N	2	7507.629	3753.814	42.7537	3.63	6.23
P	2	585.4074	292.7037	3.3337	3.63	6.23
NP	4	61.7037	15.4259	0.1756	3.01	4.77
Error	16	1404.814	87.8009			
total	27-1=26	10024.074				

For repli.: **Fc<Ft** (for both level) => test is insignificant & we accept our H_0.

For main effect N: **Fc>Ft** (for both level) => test is significant & we reject our H_0.

For main effect P; **Fc<Ft** (for both level)=> test is insignificant & we accept our H_0

For interaction effect NP; **Fc<Ft** (for both level)=> test is insignificant & we accept our H_0

3^3 Factorial Experiment:

Let there are three factors N and P and K, each are at 3 levels namely 0,1, and 2.

N			P			K		
0	1	2	0	1	2	0	1	2

All possible combinations are,

$n_0p_0k_0$	$n_1p_0k_0$	$n_2p_0k_0$
$n_0p_0k_1$	$n_1p_0k_1$	$n_2p_0k_1$
$n_0p_0k_2$	$n_1p_0k_2$	$n_2p_0k_2$
$n_0p_1k_0$	$n_1p_1k_0$	$n_2p_1k_0$
$n_0p_1k_1$	$n_1p_1k_1$	$n_2p_1k_1$
$n_0p_1k_2$	$n_1p_1k_2$	$n_2p_1k_2$
$n_0p_2k_0$	$n_1p_2k_0$	$n_2p_2k_0$
$n_0p_2k_1$	$n_1p_2k_1$	$n_2p_2k_1$
$n_0p_2k_2$	$n_1p_2k_2$	$n_2p_2k_2$

Method 1:

In 3^2 factorial experiment, there are two types of Main effect namely,

1) Linear Main effect
2) Quadratic Main effect

The contrast N_2-N_0 gives the linear contrast among the three levels of N, if they are equispaced. It indicates a linear relation between the response and level values of N.

The other contrast, $N_2-2N_1+N_0$. This contrast indicates a quadratic relation between the levels and their responses.

Now,

$N_L = N_2-N_0$

$$=(n_2p_0k_0+n_2p_0k_1+n_2p_0k_2+n_2p_1k_0+n_2p_1k_1+n_2p_1k_2+n_2p_2k_0+n_2p_2k_1+n_2p_2k_2)-(n_0p_0k_0+n_0p_0k_1+n_0p_0k_2+n_0p_1k_0+n_0p_1k_1+n_0p_1k_2+n_0p_2k_0+n_0p_2k_1+n_0p_2k_2)$$

$N_Q= N_2 - 2N_1+N_0$

$=(n_2p_0k_0+n_2p_0k_1+n_2p_0k_2+n_2p_1k_0+n_2p_1k_1+n_2p_1k_2+n_2p_2k_0+n_2p_2k_1+n_2p_2k_2)-$

$2(n_1p_0k_0+n_1p_0k_1+n_1p_0k_2+n_1p_1k_0+n_1p_1k_1+n_1p_1k_2+n_1p_2k_0+n_1p_2k_1+n_1p_2k_2)$

$+(n_0p_0k_0+n_0p_0k_1+n_0p_0k_2+n_0p_1k_0+n_0p_1k_1+n_0p_1k_2+n_0p_2k_0+n_0p_2k_1+n_0p_2k_2)$

$P_L= P_2-P_0$

$K_L= K_2-K_0$

$N_QP_LK_L=(N_2-2N_1+N_0)(P_2-P_0)(K_2-K_0)$

$N_LP_LK_L = (N_2-N_0)(P_2-P_0) (K_2-K_0)$

$N_LP_LK_Q = (N_2-N_0)(P_2-P_0) (K_2 -2K_1 +K_0)$

$P_Q= P_2 - 2P_1+P_0$

$K_Q= K_2 - 2K_1+K_0$

$N_QK_Q = (N_2 - 2N_1+N_0) (K_2 - 2K_1 +K_0)$

$N_QP_L = (N_2 - 2N_1+N_0) (P_2 - P_0)$

$N_QP_Q = (N_2 - 2N_1+N_0) (P_2 - 2P_1 +P_0)$

$P_LK_L = (P_2-P_0)(K_2-K_0)$

$P_LK_Q = (P_2-P_0)(K_2 - 2K_1+K_0)$

$N_QP_LK_Q = (N_2 - 2N_1+N_0) (P_2 - P_0)(K_2 - 2K_1 +N_0)$

$N_LP_QK_Q = (N_2-N_0)(P_2 - 2P_1+P_0))(K_2 - 2K_1+K_0)$

$N_LP_LK_Q=(N_2-N_0)(P_2-P_0)(K_2 - 2K_1+K_0)$

$N_QP_QK_L=(N_2 - 2N_1+N_0) (P_2 - 2P_1 +P_0)(K_2-K_0)$

$N_QP_QK_Q = (N_2 - 2N_1+N_0) (P_2 - 2P_1 +P_0)(K_2 - 2K_1 +N_0)$

$P_QK_Q = (P_2 - 2P_1+P_0) (K_2 - 2K_1 +N_0)$

$N_QK_L = (N_2 - 2N_1 +N_0)(K_2-K_0)$

$N_LP_L = (N_2-N_0)(P_2-P_0)$

$N_LK_L = (N_2-N_0)(K_2-K_0)$

$N_LP_Q = (N_2-N_0)(P_2 - 2P_1+P_0)$

$N_LK_Q = (N_2-N_0)(K_2 - 2K_1+K_0)$

Now,

- s.s. of $N_L = (N_L)^2/18r$
- s.s. of $N_Q = (N_Q)^2/54r$
- s.s. of $P_L = (P_L)^2/18r$
- s.s. of $P_Q = (P_Q)^2/54r$
- s.s. of $K_L = (K_L)^2/18r$
- s.s. of $K_Q = (K_Q)^2/54r$

- s.s. of $N_LP_L = (N_LP_L)^2/324r$
- s.s. of $N_LP_Q = (N_LP_Q)^2/972r$
- s.s. of $N_QP_L = (N_QP_L)^2/972r$
- s.s. of $N_QP_Q = (N_QP_Q)^2/2916r$
- s.s. of $P_LK_L = (P_LK_L)^2/324r$
- s.s. of $P_LK_Q = (P_LK_Q)^2/972r$
- s.s. of $P_QK_L = (P_QK_L)^2/972r$
- s.s. of $P_QK_Q = (P_QK_Q)^2/2916r$
- s.s. of $N_LK_L = (N_LK_L)^2/324r$
- s.s. of $N_LK_Q = (N_LK_Q)^2/972r$
- s.s. of $N_QK_L = (N_QK_L)^2/972r$
- s.s. of $N_QK_Q = (N_QK_Q)^2/2916r$

- s.s. of $N_LP_LK_L = (N_LP_LK_L)^2/5832r$
- s.s. of $N_LP_LK_Q = (N_LP_LK_Q)^2/17496r$
- s.s. of $N_LP_QK_L = (N_LP_QK_L)^2/17496r$
- s.s. of $N_LP_QK_Q = (N_LP_QK_Q)^2/52488r$
- s.s. of $N_QP_LK_L = (N_QP_LK_L)^2/17496r$
- s.s. of $N_QP_LK_Q = (N_QP_LK_Q)^2/52488r$
- s.s. of $N_QP_QK_L = (N_QP_QK_L)^2/52488r$
- s.s. of $N_QP_QK_Q = (N_QP_QK_Q)^2/157464r$

Method-3:

			N			SUM
			0	1	2	
		P	0 1 2	0 1 2	0 1 2	
K	0		$n_0p_0k_0+n_0p_1k_0+n_0p_2k_0$	$n_1p_0k_0+n_1p_1k_0+n_1p_2k_0$	$n_2p_0k_0+n_2p_1k_0+n_2p_2k_0$	$n_0p_0k_0+\ldots+n_2p_2k_0$ ----→eq.(1)
	1		$n_0p_0k_1+n_0p_1k_1+n_0p_2k_1$	$n_1p_0k_1+n_1p_1k_1+n_1p_2k_1$	$n_2p_0k_1+n_2p_1k_1+n_2p_2k_1$	$n_0p_0k_1+\ldots n_2p_2k_1$ ----→eq.(2)
	2		$n_0p_0k_2+n_0p_1k_2+n_0p_2k_2$	$n_1p_0k_2+n_1p_1k_2+n_1p_2k_2$	$n_2p_0k_2+n_2p_1k_2+n_2p_2k_2$	$n_0p_0k_2+\ldots n_2p_2k_2$ ----→eq.(3)
			$n_0p_0k_0+\ldots+n_0p_2k_2$ ----→eq.(4)	$n_1p_0k_0+\ldots+n_1p_2k_2$ ----→eq.(5)	$n_2p_0k_0+\ldots+n_2p_2k_2$ ----→eq.(6)	Grand total

s.s. of K= $\dfrac{[eq.(1)]^2+[eq.(2)]^2+[eq.(3)]^2}{9r}$ – c.f. ; Where c.f.= G^2/N

s.s. of N= $\dfrac{[eq.(4)]^2+[eq.(5)]^2+[eq.(6)]^2}{9r}$ – c.f. ; Where c.f.= G^2/N

cell s.s. due to NK={[$n_0p_0k_0+n_0p_1k_0+n_0p_2k_0$]^2+…+[$n_2p_0k_2+n_2p_1k_2+n_2p_2k_2$]^2}/3r–c.f.

s.s. of NK = cell s.s. – Ns.s. – Ks.s.

			N			SUM
			0	1	2	
		K	0 1 2	0 1 2	0 1 2	
P	0		$n_0p_0k_0+n_0p_0k_1+n_0p_0k_2$	$n_1p_0k_0+n_1p_0k_1+n_1p_0k_2$	$n_2p_0k_0+n_2p_0k_1+n_2p_0k_2$	$n_0p_0k_0+\ldots+n_2p_0k_2$ ----→eq.(7)
	1		$n_0p_1k_0+n_0p_1k_1+n_0p_1k_2$	$n_1p_1k_0+n_1p_1k_1+n_1p_1k_2$	$n_2p_1k_0+n_2p_1k_1+n_2p_1k_2$	$n_0p_1k_0+\ldots n_2p_1k_2$ ----→eq.(8)
	2		$n_0p_2k_0+n_0p_2k_1+n_0p_2k_2$	$n_1p_2k_0+n_1p_2k_1+n_1p_2k_2$	$n_2p_2k_0+n_2p_2k_1+n_2p_2k_2$	$n_0p_2k_0+\ldots n_2p_2k_2$ ----→eq.(9)
			$n_0p_0k_0+\ldots+n_0p_2k_2$ ----→eq.(10)	$n_1p_0k_0+\ldots+n_1p_2k_2$ ----→eq.(11)	$n_2p_0k_0+\ldots+n_2p_2k_2$ ----→eq.(12)	Grand total

s.s. of P= $\frac{[eq.(7)]^2+[eq.(8)]^2+[eq.(9)]^2}{9r}$ – c.f. ; Where c.f.=G^2/N

s.s. of N= $\frac{[eq.(10)]^2+[eq.(11)]^2+[eq.(12)]^2}{9r}$ – c.f. ; Where c.f.=G^2/N

cell s.s. due to NP=$\{[n_0p_0k_0+n_0p_0k_1+ n_0p_0k_2]^2+\ldots+[n_2p_2k_0+n_2p_2k_1+ n_2p_2k_2]^2\}/3r$–c.f.

s.s. of NP = cell s.s. – Ns.s. – Ps.s.

			P			SUM
			0	1	2	
		N	0 1 2	0 1 2	0 1 2	
K	0		$n_0p_0k_0+n_1p_0k_0+n_2p_0k_0$	$n_0p_1k_0+n_1p_1k_0+n_2p_1k_0$	$n_0p_2k_0+n_1p_2k_0+n_2p_2k_0$	$n_0p_0k_0+\ldots+n_2p_2k_0$ ----→eq.(13)
	1		$n_0p_0k_1+n_1p_0k_1+n_2p_0k_1$	$n_0p_1k_1+n_1p_1k_1+n_2p_1k_1$	$n_0p_2k_1+n_1p_2k_1+n_2p_2k_1$	$n_0p_0k_1+\ldots n_2p_2k_1$ ----→eq.(14)
	2		$n_0p_0k_2+n_1p_0k_2+n_2p_0k_2$	$n_0p_1k_2+n_1p_1k_2+n_2p_1k_2$	$n_0p_2k_2+n_1p_2k_2+n_2p_2k_2$	$n_0p_0k_2+\ldots n_2p_2k_2$ ----→eq.(15)
			$n_0p_0k_0+\ldots+n_2p_0k_2$ ----→eq.(16)	$n_0p_1k_0+\ldots+n_2p_1k_2$ ----→eq.(17)	$n_0p_2k_0+\ldots+n_2p_2k_2$ ----→eq.(18)	Grand total

$$\text{s.s. of K} = \frac{[\text{eq.}(13)]^2+[\text{eq.}(14)]^2+[\text{eq.}(15)]^2}{9r} - \text{c.f.} \quad ; \quad \text{Where c.f.}=G^2/N$$

$$\text{s.s. of P} = \frac{[\text{eq.}(16)]^2+[\text{eq.}(17)]^2+[\text{eq.}(18)]^2}{9r} - \text{c.f.} \quad ; \quad \text{Where c.f.}=G^2/N$$

cell s.s. due to PK=$\{[n_0p_0k_0+n_1p_0k_0+n_2p_0k_0]^2+\ldots+[n_0p_2k_2+n_1p_2k_2+n_2p_2k_2]^2\}/3r -$ c.f.

s.s. of PK = cell s.s. – Ps.s. – Ks.s.

ANOVA TABLE OF 3^3 FACTORIAL EXPERIMENT:

s.v.	d.f.	s.s.	M.s.s.	F-ratio
Repli.	r-1	$\sum R_i^2/t$ – C.F.	σ_R^2	
Treat.combi.	3^3 -1=26			
N	2	N s.s.	σ_N^2	σ_N^2/σ_e^2
P	2	P s.s.	σ_P^2	σ_P^2/σ_e^2
NP	4	NP s.s.	σ_{NP}^2	σ_{NP}^2/σ_e^2
K	2	K s.s.	σ_K^2	σ_K^2/σ_e^2
NK	4	NK s.s.	σ_{NK}^2	σ_{NK}^2/σ_e^2
PK	4	PK s.s.	σ_{PK}^2	σ_{PK}^2/σ_e^2
NPK	8	NPK s.s.	σ_{NPK}^2	$\sigma_{NPK}^2/\sigma_e^2$
Error	26(r-1)	E s.s.	σ_e^2	
total	3^3r -1	$\sum Y_{ij}^2$		

Importance of Factorial Experiment:

1) The efficiency of factorial experiment is more.
2) Give an information about effects of factors and interactions of factorial experiment.
3) The results of factorial experiment is very optimum.

Limits:

- If the no. of factors or no. of level of factors is very large, then the no. of possible treatment combinations is very large => the variance σ^2 is increase => efficiency is decrease.

Ex.: $2^8 = 256$, $2^9 = 512$, $8^3 = 512$ …