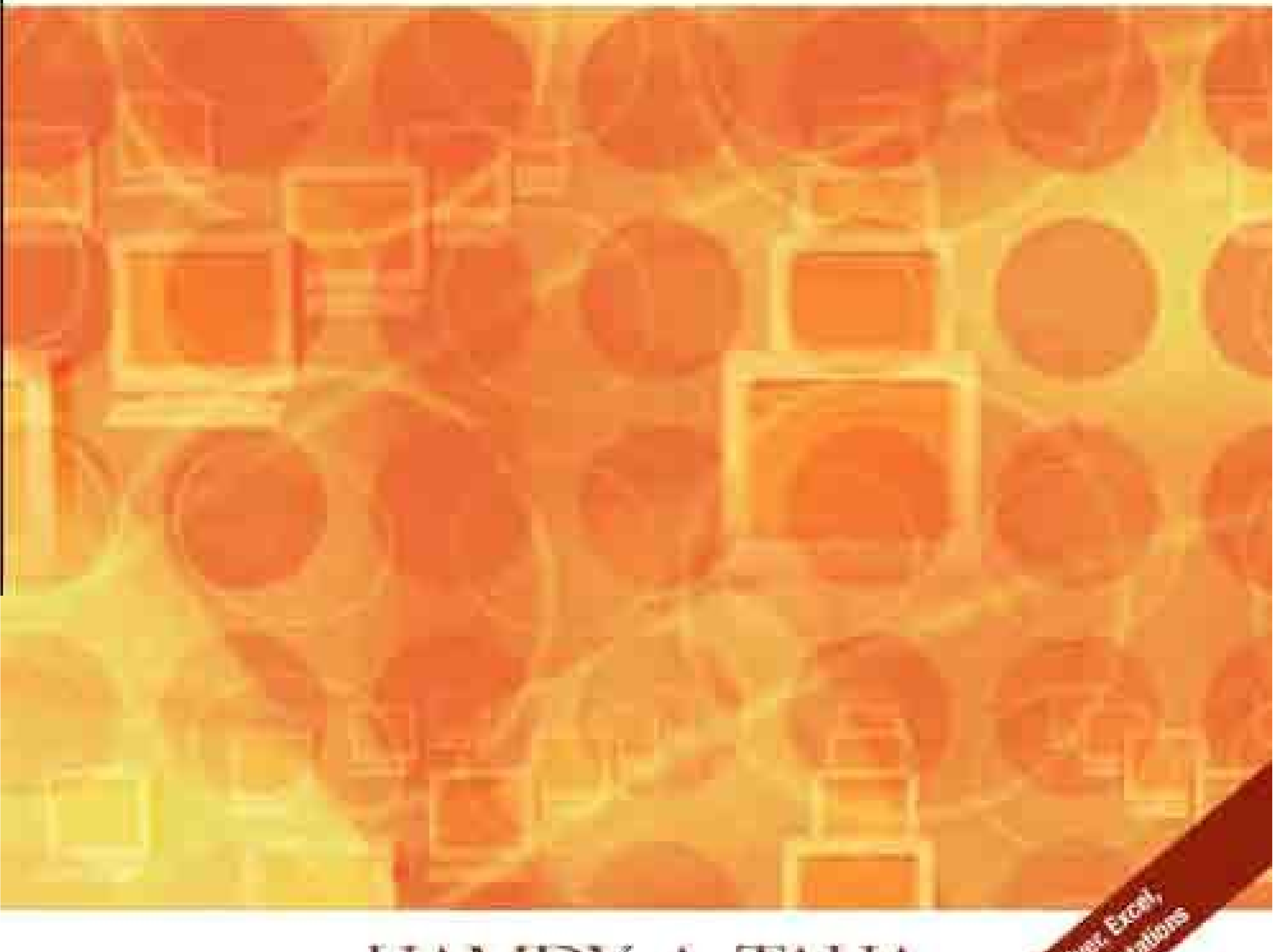
OPERATIONS RESEARCH

AN INTRODUCTION

EIGHTH EDITION



HAMDY A. TAHA

With AMPL®, Solver, Excel, and TORA Implementations

APPENDIX C

Partial Answers to Selected Problems[1]

CHAPTER 1

Set 1.1a

4. 17 minutes
5. (a) Jim's alternatives: Throw curve or fast ball.
   Joe's alternatives: Prepare for curve or fast ball.
   (b) Joe wants to increase his batting average.
   Jim wants to reduce Joe's batting average.

CHAPTER 2

Set 2.1a

1. (a) $-x_1 + x_2 \geq 1$
   (c) $x_1 - x_2 \leq 0$
   (e) $.5x_1 - .5x_2 \geq 0$
3. Unused $M1$ = 4 tons/day

Set 2.2a

1. (a and e) See Figure C.1.
2. (a and d) See Figure C.2.

[1]Solved problems in this appendix are designated by * in the text.

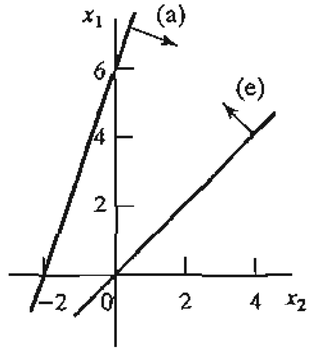


FIGURE C.1

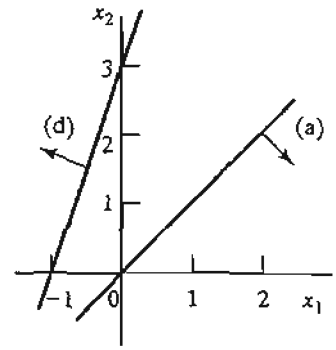


FIGURE C.2

5. Let

x_1 = Number of units of A

x_2 = Number of units of B

Maximize $z = 20x_1 + 50x_2$ subject to

$$-.2x_1 + .8x_2 \leq 0, 2x_1 + 4x_2 \leq 240$$
$$x_1 \leq 100, x_1, x_2 \geq 0$$

Optimum: $(x_1, x_2) = (80, 20), z = \$2,600$

7. Let

x_1 = Dollars invested in A

x_2 = Dollars invested in B

Maximize $z = .05x_1 + .08x_2$ subject to

$$.75x_1 - .25x_2 \geq 0, .5x_1 - .5x_2 \geq 0,$$
$$x_1 - .5x_2 \geq 0, x_1 + x_2 \leq 5000, x_1, x_2 \geq 0$$

Optimum: $(x_1, x_2) = (2500, 2500), z = \$325$

11. Let

x_1 = Play hours per day

x_2 = Work hours per day

Maximize $z = 2x_1 + x_2$ subject to

$$x_1 + x_2 \leq 10, x_1 - x_2 \leq 0$$
$$x_1 \leq 4, x_1, x_2 \geq 0$$

Optimum: $(x_1, x_2) = (4, 6), z = 14$

14. Let

x_1 = Tons of $C1$ per hour

x_2 = Tons of $C2$ per hour

Maximize $z = 12000x_1 + 9000x_2$ subject to

$$-200x_1 + 100x_2 \leq 0, 2.1x_1 + .9x_2 \leq 20, x_1, x_2 \geq 0$$

Optimum: $(x_1, x_2) = (5.13, 10.26)$, $z = 153{,}846$ lb

(a) Optimum ratio $C1 : C2 = .5$.

(b) Optimum ratio is the same, but steam generation will increase by 7692 lb/hr.

18. Let

x_1 = Number of HiFi1 units

x_2 = Number of HiFi2 units

Minimize $z = 1267.2 - (15x_1 + 15x_2)$ subject to

$$6x_1 + 4x_2 \leq 432, 5x_1 + 5x_2 \leq 412.8$$

$$4x_1 + 6x_2 \leq 422.4, x_1, x_2 \geq 0$$

Optimum: $(x_1, x_2) = (50.88, 31, 68)$, $z = 28.8$ idle min.

Set 2.2b

1. (a) See Figure C.3

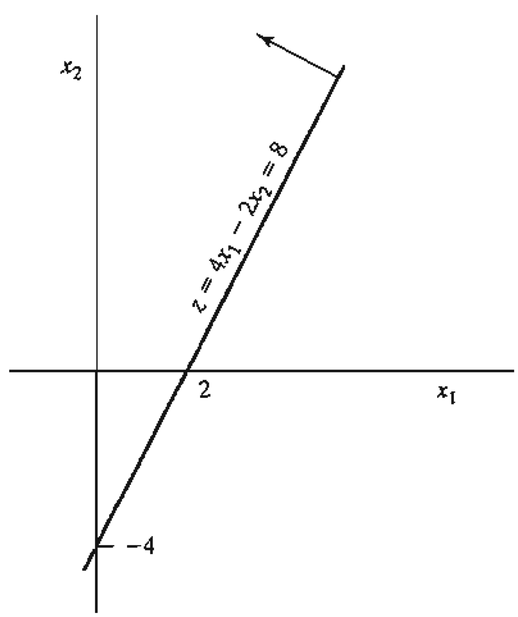


FIGURE C.3

5. Let

x_1 = Thousand bbl/day from Iran

x_2 = Thousand bbl/day from Dubai

Minimize $z = x_1 + x_2$ subject to

$$-.6x_1 + .4x_2 \le 0, .2x_1 + .1x_2 \ge 14$$
$$.25x_1 + .6x_2 \ge 30, .1x_1 + .15x_2 \ge 10$$
$$.15x_1 + .1x_2 \ge 8, x_1, x_2 \ge 0$$

Optimum: $x_1 = 55, x_2 = 30, z = 85$

7. Let

x_1 = Ratio of scrap A alloy

x_2 = Ratio of scrap B alloy

Minimize $z = 100x_1 + 80x_2$ subject to

$$.03 \le .06x_1 + .03x_2 \le .06, .03 \le .03x_1 + .06x_2 \le .05$$
$$.03 \le .04x_1 + .03x_2 \le .07, x_1 + x_2 = 1, x_1, x_2 \ge 0$$

Optimum: $x_1 = .33, x_2 = .67, z = \$86{,}667$

Set 2.3a

3. Let

x_{ij} = Portion of project i completed in year j

$$\text{Maximize } z = .05(4x_{11} + 3x_{12} + 2x_{13}) + .07(3x_{22} + 2x_{23} + x_{24})$$
$$+ .15(4x_{31} + 3x_{32} + 2x_{33} + x_{34}) + .02(2x_{43} + x_{44})$$

subject to

$$x_{11} + x_{12} + x_{13} = 1, x_{43} + x_{44} = 1$$
$$.25 \le x_{22} + x_{23} + x_{24} + x_{25} \le 1$$
$$.25 \le x_{31} + x_{32} + x_{33} + x_{34} + x_{35} \le 1$$
$$5x_{11} + 15x_{31} \le 3, 5x_{12} + 8x_{22} + 15x_{32} \le 6$$
$$5x_{13} + 8x_{23} + 15x_{33} + 1.2x_{43} \le 7$$
$$8x_{24} + 15x_{34} + 1.2x_{44} \le 7, 8x_{25} + 15x_{35} \le 7$$
$$\text{all } x_{ij} \ge 0$$

Optimum: $x_{11} = .6, x_{12} = .4, x_{24} = .255, x_{25} = .025$

$$x_{32} = .267, x_{33} = .387, x_{34} = .346, x_{43} = 1, z = \$523{,}750$$

Set 2.3b

2. The model can be generalized to account for any input currency p and any output currency q. Define x_{ij} as in Example 2.3-2 and r_{ij} as the exchange rate from currency i to currency j. The associated model is

Maximize $z = y$ subject to

$$\text{capacity: } x_{ij} \le c_i, \text{ for all } i \ne j$$

$$\text{Input currency } p\text{: } I + \sum_{j \ne p} r_{jp} x_{jp} = \sum_{j \ne p} x_{pj}$$

$$\text{Output currency } q\text{: } y + \sum_{j \ne q} x_{qj} = \sum_{j \ne q} r_{jq} x_{jq}$$

$$\text{Currency } i \ne p \text{ or } q\text{: } \sum_{j \ne i} r_{ji} x_{ji} = \sum_{j \ne i} x_{ij}$$

$$\text{all } x_{ij} \ge 0$$

Rate of return: 1.8064% for \$ → \$, 1.7966% for \$ → €, 1.8287% for \$ → £, 2.8515% for \$ → ¥, and 1.0471% for \$ → KD. Wide discrepancies in ¥ and KD currencies may be attributed to the fact that the given exchange rates may not be totally consistent with the other rates. Nevertheless, the problem demonstrates the advantage of targeting accumulation in different currencies.
[*Note*: Interactive AMPL (file ampl2.3b-2.txt) or Solver (file solver2.3b-2.xls) is ideal for solving this problem. See Section 2.4.]

Set 2.3c

2. Let

x_i = Dollars invested in project i, $i = 1, 2, 3, 4$

y_j = Dollars invested in bank in year j, $j = 1, 2, 3, 4$

Maximize $z = y_5$ subject to

$$x_1 + x_2 + x_4 + y_1 \le 10{,}000$$

$$.5x_1 + .6x_2 - x_3 + .4x_4 + 1.065y_1 - y_2 = 0$$

$$.3x_1 + .2x_2 + .8x_3 + .6x_4 + 1.065y_2 - y_3 = 0$$

$$1.8x_1 + 1.5x_2 + 1.9x_3 + 1.8x_4 + 1.065y_3 - y_4 = 0$$

$$1.2x_1 + 1.3x_2 + .8x_3 + .95x_4 + 1.065y_4 - y_5 = 0$$

$$x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4, y_5 \ge 0$$

Optimum solution:

$$x_1 = 0,\ x_2 = \$10{,}000,\ x_3 = \$6000,\ x_4 = 0$$

$$y_1 = 0,\ y_2 = 0,\ y_3 = \$6800,\ y_4 = \$33{,}642$$

$$z = \$53{,}628.73 \text{ at the start of year 5}$$

5. Let x_{iA} = amount invested in year i using plan A, $i = 1, 2, 3$
   x_{iB} = amount invested in year i using plan B, $i = 1, 2, 3$
   Maximize $z = 3x_{2B} + 1.7x_{3A}$ subject to

$$x_{1A} + x_{1B} \le 100 \text{ (start of year 1)}$$

$$-1.7x_{1A} + x_{2A} + x_{2B} = 0 \quad \text{(start of year 2)}$$

$$-3x_{1B} - 1.7x_{2A} + x_{3A} = 0 \quad \text{(start of year 3)}$$

$$x_{iA}, x_{iB} \ge 0, i = 1, 2, 3$$

   Optimum solution: Invest \$100,000 in plan A in year 1 and \$170,000 in plan B in year 2. Problem has alternative optima.

Set 2.3d

3. Let x_j = number of units of product j, $j = 1, 2, 3$
   Maximize $z = 30x_1 + 20x_2 + 50x_3$ subject to

$$2x_1 + 3x_2 + 5x_3 \le 4000$$

$$4x_1 + 2x_2 + 7x_3 \le 6000$$

$$x_1 + .5x_2 + .33x_3 \le 1500$$

$$2x_1 - 3x_2 = 0$$

$$5x_2 - 2x_3 = 0$$

$$x_1 \ge 200,\ x_2 \ge 200,\ x_3 \ge 150$$

$$x_1, x_2, x_3 \ge 0$$

   Optimum solution: $x_1 = 324.32$, $x_2 = 216.22$, $x_3 = 540.54$, $z = \$41{,}081.08$

7. Let x_{ij} = Quantity produced by operation i in month j, $i = 1, 2$, $j = 1, 2, 3$
   I_{ij} = Entering inventory of operation i in month j, $i = 1, 2$, $j = 1, 2, 3$
   Minimize $z = \sum_{j=1}^{3}(c_{1j}x_{1j} + c_{2j}x_{2j} + .2I_{1j} + .4I_{2j})$ subject to

$$.6x_{11} \le 800,\ .6x_{12} \le 700,\ .6x_{13} \le 550$$

$$.8x_{21} \le 1000,\ .8x_{22} \le 850,\ .8x_{23} \le 700$$

$$x_{1j} + I_{1,j-1} = x_{2j} + I_{1j}, x_{2j} + I_{2,j-1} = d_j + I_{2j}, j = 1, 2, 3$$

$$I_{1,0} = I_{2,0} = 0, \text{all variables} \geq 0$$

$$d_j = 500, 450, 600 \text{ for } j = 1, 2, 3$$

$$c_{1j} = 10, 12, 11 \text{ for } j = 1, 2, 3$$

$$c_{2j} = 15, 18, 16 \text{ for } j = 1, 2, 3$$

Optimum: $x_{11} = 1333.33$ units, $x_{13} = 216.67$, $x_{21} = 1250$ units, $x_{23} = 300$ units, $z = \$39{,}720$.

Set 2.3e

2. Let x_s = lb of screws/package, x_b = lb of bolts/package, x_n = lb of nuts/package, x_w = lb of washers/package

   Minimize $z = 1.1x_s + 1.5x_b + \left(\frac{70}{80}\right)x_n + \left(\frac{20}{30}\right)x_w$ subject to

$$y = x_s + x_b + x_n + x_w$$

$$y \geq 1, x_s \geq .1y, x_b \geq .25y, x_n \leq .15y, x_w \leq .1y$$

$$\left(\tfrac{1}{10}\right)x_b \leq x_n, \left(\tfrac{1}{50}\right)x_b \leq x_w$$

   All variables are nonnegative

   Solution: $z = \$1.12, y = 1, x_s = .5, x_b = .25, x_n = .15, x_w = .1$

5. Let x_A = bbl of crude A/day, x_B = bbl of crude B/day, x_r = bbl of regular/day x_p = bbl of premium/day, x_j = bbl of jet fuel/day

   Maximize $z = 50(x_r - s_r^+) + 70(x_p - s_p^+) + 120(x_j - s_j^+)$
   $- (10s_r^- + 15s_p^- + 20s_j^- + 2s_r^+ + 3s_p^+ + 4s_j^+)$
   $- (30x_A + 40x_B)$ subject to

$$x_A \leq 2500, x_B \leq 3000, x_r = .2x_A + .25x_B, x_p = .1x_A + .3x_B, x_j = .25x_A + .1x_B$$

$$x_r + s_r^- - s_r^+ = 500, x_p + s_p^- - s_p^+ = 700, x_j + s_j^- - s_j^+ = 400, \text{All variables} \geq 0$$

   Solution:

$$z = \$21{,}852.94, x_A = 1176.47 \text{ bbl/day}, x_B = 1058.82, x_r = 500 \text{ bbl/day}$$

$$x_p = 435.29 \text{ bbl/day}, x_j = 400 \text{ bbl/day}, s_p^- = 264.71$$

Set 2.3f

1. Let $x_i(y_i)$ = Number of 8-hr (12-hr) buses starting in period i

   Minimize $z = 2\sum_{i=1}^{6} x_i + 3.5\sum_{i=1}^{6} y_i$ subject to

$$x_1 + x_6 + y_1 + y_5 + y_6 \geq 4, x_1 + x_2 + y_1 + y_2 + y_6 \geq 8,$$

$$x_2 + x_3 + y_1 + y_2 + y_3 \geq 10, x_3 + x_4 + y_2 + y_3 + y_4 \geq 7,$$

$$x_4 + x_5 + y_3 + y_4 + y_5 \geq 12, x_5 + x_6 + y_4 + y_5 + y_6 \geq 4$$

All variables are nonnegative

Solution: $x_1 = 4, x_2 = 4, x_4 = 2, x_5 = 4, y_3 = 6$, all others $= 0$, $z = 49$.

Total number of buses = 20. For the case of 8-hr shift, number of buses = 26 and comparable $z = 2 \times 26 = 52$. Thus, (8-hr + 12-hr) shift is better.

5. Let x_i = Number of students starting in period i ($i = 1$ for 8:01 A.M., $i = 9$ for 4:01 P.M.)

Minimize $z = x_1 + x_2 + x_3 + x_4 + x_6 + x_7 + x_8 + x_9$ subject to

$$x_1 \geq 2, x_1 + x_2 \geq 2, x_1 + x_2 + x_3 \geq 3,$$

$$x_2 + x_3 + x_4 \geq 4, x_3 + x_4 \geq 4, x_4 + x_6 \geq 3,$$

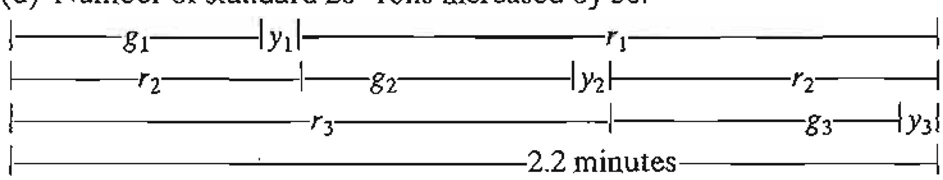
$$x_6 + x_7 \geq 3, x_6 + x_7 + x_8 \geq 3, x_7 + x_8 + x_9 \geq 3$$

$$x_5 = 0, \text{ all other variables are nonnegative}$$

Solution: Hire 2 at 8:01, 1 at 10:01, 3 at 11:01, and 3 at 2:01. Total = 9 students

Set 2.3g

1. (a) $1150L$ ft^2
   (b) (3,0,0), (1,1,0), (1,0,1), and (0,2,0) with respective 0, 3, 1, and 1 trim loss per foot.
   (c) Number of standard 20′-rolls decreased by 30.
   (d) Number of standard 20′-rolls increased by 50.

6. 

Let g_i, y_i, and r_i be the durations of green, yellow, and red lights for cars exiting highway i. All time units are in seconds. No cars move on yellow.

maximize $z = 3(500/3600)g_1 + 4(600/3600)g_2 + 5(400/3600)g_3$ subject to

$$(500/3600)g_1 + (600/3600)g_2 + (400/3600)g_3 \leq (510/3600)(2.2 \times 60 - 3 \times 10)$$

$$g_1 + g_2 + g_3 + 3 \times 10 \leq 2.2 \times 60, g_1 \geq 25, g_2 \geq 25, g_3 \geq 25$$

Solution: $g_1 = 25$ sec., $g_2 = 43.6$ sec., $g_3 = 33.4$ sec. Booth income = \$58.04/hr

Set 2.4a

2. (d) See file solver2.4a-2(d).xls in folder AppenCFiles.

Set 2.4b

2. (c) See file ampl2.4b-2(c).txt in folder AppenCFiles.
   (f) See file ampl2.4b-2(f).txt in folder AppenCFiles.

CHAPTER 3

Set 3.1a

1. 2 tons/day and 1 ton/day for raw materials $M1$ and $M2$, respectively.
4. Let x_{ij} = units of product i produced on machine j.
   Maximize $z = 10(x_{11} + x_{12}) + 15(x_{21} + x_{22})$ subject to

$$x_{11} + x_{21} - x_{12} - x_{22} + s_1 = 5$$
$$-x_{11} - x_{21} + x_{12} + x_{22} + s_2 = 5$$
$$x_{11} + x_{21} + s_3 = 200$$
$$x_{12} + x_{22} + s_4 = 250$$
$$s_i, x_{ij} \geq 0, \text{ for all } i \text{ and } j$$

Set 3.1b

3. Let x_j = units of product j, $j = 1, 2, 3$.
   Maximize $z = 2x_1 + 5x_2 + 3x_3 - 15x_4^+ - 10x_5^+$
   subject to

$$2x_1 + x_2 + 2x_3 + x_4^- - x_4^+ = 80$$
$$x_1 + x_2 + 2x_3 + x_5^- - x_5^+ = 65$$
$$x_1, x_2, x_3, x_4^-, x_4^+, x_5^-, x_5^+ \geq 0$$

   Optimum solution: $x_2 = 65$ units, $x_4^- = 15$ units, all others $= 0$, $z = \$325$.

Set 3.2a

1. (c) $x_1 = \frac{6}{7}, x_2 = \frac{12}{7}, z = \frac{48}{7}$.
   (e) Corner points $(x_1 = 0, x_2 = 3)$ and $(x_1 = 6, x_2 = 0)$ are infeasible.
3. Infeasible basic solutions are:

$$(x_1, x_2) = \left(\tfrac{26}{3}, -\tfrac{4}{3}\right), \quad (x_1, x_3) = (8, -2)$$
$$(x_1, x_4) = (6, -4), \quad (x_2, x_3) = (16, -26)$$
$$(x_2, x_4) = (3, -13), (x_3, x_4) = (6, -16)$$

Set 3.3a

3. (a) Only (A, B) represents successive simplex iterations because corner point A and B are adjacent. In all the remaining pairs the associated corner points are not adjacent.
   (b) (i) Yes. (ii) No, C and I are not adjacent. (iii) No, path returns to a previous corner point, A.
5. (a) x_3 enters at value 1, $z = 3$ at corner point D.

Set 3.3b

3.

New basic variable	x_1	x_2	x_3	x_4
Value	1.5	1	0	.8
Leaving variable	x_7	x_7	x_8	x_5

6. (b) x_2, x_5, and x_6 can increase value of z. If x_2 enters, x_8 leaves and $\Delta z = 5 \times 4 = 20$. If x_5 enters, x_1 leaves and $\Delta z = 0$ because x_5 equals 0 in the new solution. If x_6 enters, no variable leaves because all the constraint coefficients of x_6 are less than or equal to zero. $\Delta z = \infty$ because x_6 can be increased to infinity without causing infeasibility.

9. Second best value of $z = 20$ occurs when s_2 is made basic.

Set 3.4a

3. (a) Minimize $z = (8M - 4)x_1 + (6M - 1)x_2 - Ms_2 - Ms_3 = 10M$

(b) Minimize $z = (3M - 4)x_1 + (M - 1)x_2 = 3M$

6. The starting tableau is

Basic	x_1	x_2	x_3	x_4	Solution
z	−1	−12	0	0	−8
x_3	1	1	1	0	4
x_4	1	4	0	1	8

Set 3.4b

1. Always minimize the sum of artificial variables because the sum represents the amount of infeasibility in the problem.

7. Any nonbasic variable having nonzero objective coefficients at end of Phase I cannot become positive in Phase II because it will mean that the optimal objective value in Phase I will be positive; that is, infeasible Phase I solution.

Set 3.5a

1. (a) $A \rightarrow B \rightarrow C \rightarrow D$.

(b) 1 at A, 1 at B, $C_2^4 = 6$ at C, and 1 at D.

Set 3.5b

1. Alternative basic optima: $\left(0, 0, \frac{10}{3}\right)$, $(0, 5, 0)$, $\left(1, 4, \frac{1}{3}\right)$. Nonbasic alternative optima: $\left(\alpha_3, 5\alpha_2 + 4\alpha_3, \frac{10}{3}\alpha_1 + \frac{1}{3}\alpha_3\right)$, $\alpha_1 + \alpha_2 + \alpha_3 = 1, 0 \leq \alpha_i \leq 1, i = 1, 2, 3$.

Set 3.5c

2. (a) Solution space is unbounded in the direction of x_2.
   (b) Objective value is unbounded because each unit increase in x_2 increases z by 10.

Set 3.5d

1. The most that can be produced is 275 units.

Set 3.6a

2. Let

x_1 = number of Type 1 hats per day

x_2 = number of Type 2 hats per day

Maximize $z = 8x_1 + 5x_2$ subject to

$$2x_1 + x_2 \le 400$$
$$x_1 \le 150, x_2 \le 200$$
$$x_1, x_2 \ge 0$$

   (a) See Figure C.4: $x_1 = 100$, $x_2 = 200$, $z = \$1800$ at point B.
   (b) \$4 per Type 2 hat in the range (200, 500).
   (c) No change because the dual price is \$0 per unit in the range $(100, \infty)$.
   (d) \$1 worth per unit in the range (100, 400). Maximum increase = 200 Type 2.

Set 3.6b

3. (a) $0 \le \frac{c_1}{c_2} \le 2$.
   (b) New $\frac{c_1}{c_2} = 1$. Solution remains unchanged.

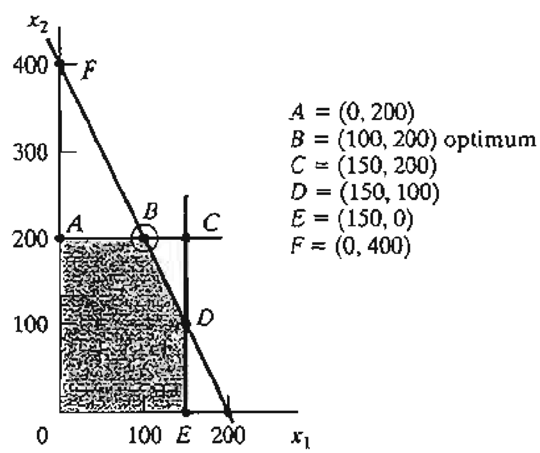


FIGURE C.4

Set 3.6c

2. (a) Yes, because additional revenue per min = \$1 (for up to 10 min of overtime) exceeds additional cost of \$.83/min.
   (b) Additional revenue is \$2/min (for up to 400 min of overtime) = \$240 for 2 hr. Additional cost for 2 hr = \$110. Net revenue = \$130.
   (c) No, its dual price is zero because the resource is already abundant.
   (d) $D_1 = 10$ min. Dual price = \$1/min for $D_1 \leq 10$. $x_1 = 0, x_2 = 105, x_3 = 230$, net revenue = (\$1350 + \$1 × 10 min) − $\left(\frac{\$40}{60} \times 10 \text{ min}\right)$ = \$1353.33.
   (e) $D_2 = -15$. Dual price = \$2/min for $D_2 \geq -20$. Decrease in revenue = \$30. Decrease in cost = \$7.50. Not recommended.
6. Let

$$x_1 = \text{radio minutes}, x_2 = \text{TV minutes}, x_3 = \text{newspaper ads}$$

Maximize $z = x_1 + 50x_2 + 10x_3$ subject to

$$15x_1 + 300x_2 + 50x_3 + s_1 = 10{,}000, x_3 - S_2 = 5,$$
$$x_1 + s_3 = 400, -x_1 + 2x_2 + s_4 = 0, x_1, x_2, x_3 \geq 0,$$
$$s_1, s_2, s_3, s_4 \geq 0$$

   (a) $x_1 = 59.09$ min, $x_2 = 29.55$ min, $x_3 = 5$ ads, $z = 1561.36$
   (b) From TORA, $z + .158s_1 + 2.879S_2 + 0s_3 + 1.364s_4 = 156.364$. Dual prices for the respective constraints are .158, −2.879, 0, and 1.36. Lower limit set on newspaper ads can be decreased because its dual price is negative (= −2.879). There is no advantage in increasing the upper limit on radio minutes because its dual price is zero (the present limit is already abundant).
   (c) From TORA, $x_1 = 59.9091 + .00606D_1 \geq 0, x_3 = 5, s_3 = 340.90909 + .00606D_1 \geq 0, x_2 = 29.54545 + .00303D_1 \geq 0$. Thus, dual price = .158 for the range $-9750 \leq D_1 \leq 56{,}250$. A 50% increase in budget (D_1 = \$5000) is recommended because the dual price is positive.
11. (a) Scarce: resistor and capacitor resource; abundant: chip resource.
    (b) Worths per unit of resistor, capacitor, and chips are \$1.25, \$.25, and \$0.
    (e) Change $D_3 = 350 - 800 = -450$ falls outside the feasibility range $D_3 \geq -400$. Hence problem must be solved anew.
13. (b) Solution $x_1 = x_2 = 2 + \frac{\Delta}{3}$ is feasible for all $\Delta > 0$. For $0 < \Delta \leq 3, r_1 + r_2 = \frac{\Delta}{3} \leq 1 \Rightarrow$ feasibility confirmed. For $3 \leq \Delta < 6, r_1 + r_2 = \frac{\Delta}{3} > 1 \Rightarrow$ feasibility not confirmed. For $\Delta > 6$, the change falls outside the ranges for D_1 and D_2.

Set 3.6d

2. (a) x_1 = Cans of A1, x_2 = Cans of A_2, x_3 = Cans of BK.
   Maximize $z = 80x_1 + 70x_2 + 60x_3$ subject to

$$x_1 + x_2 + x_3 \leq 500, x_1 \geq 100, 4x_1 - 2x_2 - 2x_3 \leq 0$$

Optimum: $x_1 = 166.67, x_2 = 333.33, x_3 = 0, z = 36666.67$.

(b) From TORA, reduced cost per can of BK = 10. Price should be increased by more than 10 cents.

(c) $d_1 = d_2 = d_3 = -5$ cents. From TORA, the reduced costs for the nonbasic variables are

$$x_3: 10 + d_2 - d_3 \geq 0, \text{satisfied}$$

$$s_1: 73.33 + .67d_2 + .33d_1 \geq 0, \text{satisfied}$$

$$s_3: 1.67 - .17d_2 + .17d_1 \geq 0, \text{satisfied}$$

Solution remains the same.

5. (a) x_i = Number of units of motor $i, i = 1, 2, 3, 4$.
Maximize $z = 60x_1 + 40x_2 + 25x_3 + 30x_4$ subject to

$$8x_1 + 5x_2 + 4x_3 + 6x_4 \leq 8000, x_1 \leq 500, x_2 \leq 500,$$

$$x_3 \leq 800, x_4 \leq 750, x_1, x_2, x_3, x_4 \geq 0$$

Optimum: $x_1 = 500, x_2 = 500, x_3 = 375, x_4 = 0, z = \$59{,}375$

(b) From TORA, $8.75 + d_2 \geq 0$. Type 2 motor price can be reduced by up to \$8.75.

(c) $d_1 = -\$15, d_2 = -\$10, d_3 = -\$6.25, d_4 = -\$7.50$. From TORA,

$$x_4: 7.5 + 1.5d_3 - d_4 \geq 0, \text{satisfied}$$

$$s_1: 6.25 + .25d_3 \geq 0, \text{satisfied}$$

$$s_2: 10 - 2d_3 + d_1 \geq 0, \text{satisfied}$$

$$s_3: 8.75 - 1.25d_3 + d_2 \geq 0, \text{satisfied}$$

Solution remains the same, but z will be reduced by 25%.

(d) Reduced cost of $x_4 = 7.5$. Increase price by more than \$7.50.

Set 3.6e

5. The dual price for the investment constraint $x_{1A} + x_{1B} \leq 100$ is \$5.10 per dollar invested for *any* amount of investment.

9. (a) Dual price for raw material A is \$10.27. The cost of \$12.00 per lb exceeds the expected revenue. Hence, purchase of additional raw material A is not recommended.

(b) Dual price for raw material B is \$0. Resource is already abundant and no additional purchase is warranted.

CHAPTER 4

Set 4.1a

2. Let y_1, y_2, and y_3 be the dual variables.
   Maximize $w = 3y_1 + 5y_2 + 4y_3$ subject to

$$y_1 + 2y_2 + 3y_3 \le 15, 2y_1 - 4y_2 + y_3 \le 12$$

$$y_1 \ge 0, y_2 \le 0, y_3 \text{ unrestricted}$$

4. (c) Let y_1 and y_2 be the dual variables.
   Minimize $z = 5y_1 + 6y_2$ subject to

$$2y_1 + 3y_2 = 1, y_1 - y_2 = 1$$

$$y_1, y_2 \text{ unrestricted}$$

5. Dual constraint associated with the artificial variables is $y_2 \ge -M$.
   Mathematically, $M \to \infty \Rightarrow y \ge -\infty$, which is the same as y_2 being unrestricted.

Set 4.2a

1. (a) $\mathbf{AV}_1$ is undefined.
   (e) $\mathbf{V}_2\mathbf{A} = (-14 \quad -32)$

Set 4.2b

1. (a) Inverse $= \begin{pmatrix} \frac{1}{4} & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{8} & \frac{3}{4} & 0 & 0 \\ \frac{3}{8} & -\frac{5}{4} & 1 & 0 \\ \frac{1}{8} & -\frac{3}{4} & 0 & 1 \end{pmatrix}$

Set 4.2c

3. Let y_1 and y_2 be the dual variables.
   Minimize $w = 30y_1 + 40y_2$ subject to

$$y_1 + y_2 \ge 5, 5y_1 - 5y_2 \ge 2, 2y_1 - 6y_2 \ge 3$$

$$y_1 \ge -M(\Rightarrow y_1 \text{ unrestricted}), y_2 \ge 0$$

   Solution: $y_1 = 5, y_2 = 0, w = 150$.

6. Let y_1 and y_2 be the dual variables.
   Minimize $w = 3y_1 + 4y_2$ subject to

$$y_1 + 2y_2 \ge 1, 2y_1 - y_2 \ge 5, y_1 \ge 3$$

$$y_2 \text{ unrestricted}$$

   Solution: $y_1 = 3, y_2 = -1, w = 5$

8. (a) $(x_1, x_2) = (3, 0), z = 15, (y_1, y_2) = (3, 1), w = 14$. Range $= (14, 15)$
9. (a) Dual solution is infeasible, hence cannot be optimal even though $z = w = 17$.

Set 4.2d

2. (a) Feasibility: $(x_2, x_4) = (3, 15) \Rightarrow$ feasible.
   Optimality: Reduced costs of $(x_1, x_3) = (0, 2) \Rightarrow$ optimal.
4.

Basic	x_1	x_2	x_3	x_4	x_5	Solution
z	0	0	$-\frac{2}{5}$	$-\frac{1}{5}$	0	$\frac{12}{5}$
x_1	1	0	$-\frac{3}{5}$	$\frac{1}{5}$	0	$\frac{3}{5}$
x_2	0	1	$\frac{4}{5}$	$-\frac{3}{5}$	0	$\frac{6}{5}$
x_5	0	0	-1	1	1	0

Solution is optimal and feasible.

7. Objective value: From primal, $z = c_1x_1 + c_2x_2$, and from dual, $w = b_1y_1 + b_2y_2 + b_3y_3$. $b_1 = 4, b_2 = 6, b_3 = 8, c_1 = 2, c_2 = 5 \Rightarrow z = w = 34$.

Set 4.3a

2. (a) Let (x_1, x_2, x_3, x_4) = daily units of SC320, SC325, SC340, and SC370
   Maximize $z = 9.4x_1 + 10.8x_2 + 8.75x_3 + 7.8x_4$ subject to

$$10.5x_1 + 9.3x_2 + 11.6x_3 + 8.2x_4 \leq 4800$$
$$20.4x_1 + 24.6x_2 + 17.7x_3 + 26.5x_4 \leq 9600$$
$$3.2x_1 + 2.5x_2 + 3.6x_3 + 5.5x_4 \leq 4700$$
$$5x_1 + 5x_2 + 5x_3 + 5x_4 \leq 4500$$
$$x_1 \geq 100,\ x_2 \geq 100,\ x_3 \geq 100,\ x_4 \geq 100$$

   (b) Only soldering capacity can be increased because it has a positive dual price (= .4944).
   (c) Dual prices for lower bounds are ≤ 0 ($-.6847$, -1.361, 0, and -5.3003), which means that the bounds have an adverse effect on profitability.
   (d) Dual price for soldering is \$.4944/min valid in the range (8920, 10201.72), which corresponds to a maximum capacity increase of 6.26% only.

Set 4.3b

2. New fire truck toy is profitable because its reduced cost $= -2$.
3. Parts PP3 and PP4 are not part of the optimum solution. Current reduced costs are .1429 and 1.1429. Thus, rate of deterioration in revenue per unit is \$.1429 for PP3 and \$1.1429 for PP4.

Set 4.4a

1. (b) No, because point E is feasible and the dual simplex must stay infeasible until optimum is reached.
4. (c) Add the artificial constraint $x_1 \leq M$. Problem has no feasible solution.

Set 4.5a

4. Let Q be the weekly feed in lb (= 5200, 9600, 15000, 20000, 26000, 32000, 38000, 42000, for weeks 1, 2, ..., and 8). Optimum solution: Limestone = .028Q, corn = .649Q, and soybean meal = .323Q. Cost = .81221Q.

Set 4.5b

1. (a) Additional constraint is redundant.

Set 4.5c

2. (a) New dual values = $\left(\frac{1}{2}, 0, 0, 0\right)$. Current solution remains optimal.
   (c) New dual values = $\left(-\frac{1}{8}, \frac{11}{4}, 0, 0\right)$. $z - .125s_1 + 2.75s_2 = 13.5$. New solution: $x_1 = 2, x_2 = 2, x_3 = 4, z = 14$.

Set 4.5d

1. $\frac{p}{100}(y_1 + 3y_2 + y_3) - 3 \geq 0$. For $y_1 = 1, y_2 = 2$, and $y_3 = 0$, $p \geq 42.86\%$.
3. (a) Reduced cost for fire engines $= 3y_1 + 2y_2 + 4y_3 - 5 = 2 > 0$. Fire engines are not profitable.

CHAPTER 5

Set 5.1a

4. Assign a very high cost, M, to the route from Detroit to dummy destination.
6. (a and b) Use $M = 10{,}000$. Solution is shown in bold. Total cost = \$49,710.

	1	2	3	Supply
Plant 1	600	700	400 **25**	**25**
Plant 2	320 **23**	300 **17**	350	**40**
Plant 3	500	480 **25**	450 **5**	**30**
Excess Plant 4	1000 **13**	1000	M	**13**
Demand	**36**	**42**	**30**	

(c) City 1 excess cost = \$13,000.

9. Solution (in million gallons) is shown in bold. Area 2 will be 2 million gallons short. Total cost = $304,000.

	A1	A2	A3	Supply
Refinery 1	12 (**4**)	18 (**2**)	*M*	6
Refinery 2	30	10 (**4**)	8 (**1**)	5
Refinery 3	20	25	12 (**6**)	6
Dummy	*M*	50 (**2**)	50	2
Demand	4	8	7	

Set 5.2a

2. Total cost = $804. Problem has alternative optima.

		Sharpening service			
Day	*New*	*Overnight*	*2-day*	*3-day*	Disposal
Monday	24	0	6	18	0
Tuesday	12	12	0	0	0
Wednesday	2	14	0	0	0
Thursday	0	0	20	0	0
Friday	0	14	0	0	4
Saturday	0	2	0	0	12
Sunday	0	0	0	0	22

5. Total cost = $190,040. Problem has alternative optima.

Period	Capacity	Produced amount	Delivery
1	500	500	400 for (period) 1 and 100 for 2
2	600	600	200 for 2, 220 for 3, and 180 for 4
3	200	200	200 for 3
4	300	200	200 for 4

Set 5.3a

1. (a) Northwest: cost = $42. Least-cost: cost = $37. Vogel: cost = $37.

Set 5.3b

5. (a) Cost = $1475.
   (b) $c_{12} \geq 3, c_{13} \geq 8, c_{23} \geq 13, c_{31} \geq 7$.

Set 5.4a

5. Use the code (city, date) to define the rows and columns of the assignment problem. Example: The assignment (D, 3)-(A,7) means leaving Dallas on Jun 3 and returning from Atlanta June 7 at a cost of $400. Solution is shown in bold. Cost = $1180. Problem has alternative optima.

	(A, 7)	(A, 12)	(A, 21)	(A, 28)
(D, 3)	400	300	300	**280**
(D, 10)	**300**	400	300	300
(D, 17)	300	**300**	400	300
(D, 25)	300	300	**300**	400

6. Optimum assignment: I-*d*, II-*c*, III-*a*, IV-*b*.

Set 5.5a

4. Total cost = $1550. Optimum solution summarized below. Problem has alternative optima.

	Store 1	Store 2	Store 3
Factory 1	50	0	0
Factory 2	50	200	50

CHAPTER 6

Set 6.1a

1. For network (i): (a) 1-3-4-2. (b) 1-5-4-3-1. (c and d) See Figure C.5.
4. Each square is a node. Adjacent squares are connected by arcs. Each of nodes 1 and 8 has the largest number of emanating arcs, and hence must appear in the center. Problem has more than one solution. See Figure C.6.

FIGURE C.5

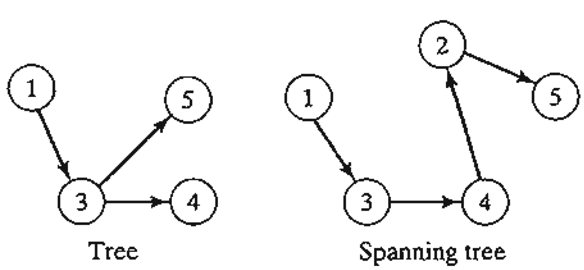
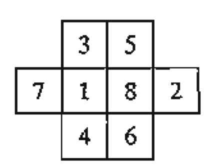


FIGURE C.6



Set 6.2a

2. (a) 1-2, 2-5, 5-6, 6-4, 4-3. Total length = 14 miles.
5. High pressure: 1-2-3-4-6. Low pressure: 1-5-7 and 5-9-8.

Set 6.3a

1. Buy new car in years 1 and 4. Total cost = \$8900. See Figure C.7.
4. For arc (i, v_i)-$(i + 1, v_{i+1})$, define $p(q)$ = value(number of item i). Solution: Select one unit of each of items 1 and 2. Total value = \$80. See Figure C.8.

Set 6.3b

1. (c) Delete all nodes but 4, 5, 6, 7, and 8. Shortest distance = 8 associated with routes 4-5-6-8 and 4-6-8.

Set 6.3c

1. (a) 5-4-2-1, distance = 12.
4. Figure C.9 summarizes the solution. Each arc has unit length. Arrows show one-way routes. Example solution: Bob to Joe: Bob-Kay-Rae-Kim-Joe. Largest number of contacts = 4.

Set 6.3d

1. (a) Right-hand side of equations for nodes 1 and 5 are 1 and −1, respectively, all others = 0. Optimum solution: 1-3-5 or 1-3-4-5, distance = 90.

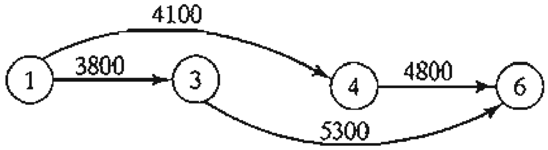


FIGURE C.7

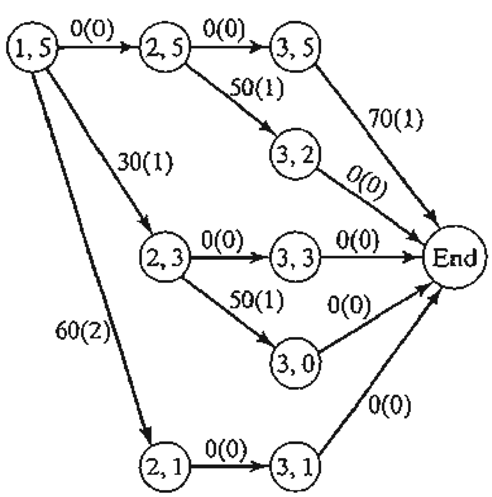
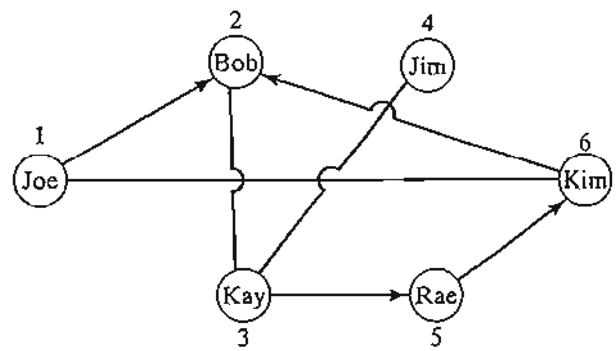


FIGURE C.8

FIGURE C.9



Set 6.4a

1. Cut 1: 1-2, 1-4, 3-4, 3-5, capacity = 60.

Set 6.4b

1. (a) Surplus capacities: arc (2-3) = 40, arc (2-5) = 10, arc (4-3) = 5.
   (b) Node 2: 20 units, node 3: 30 units, node 4: 20 units.
   (c) No, because there is no surplus capacity out of node 1.
7. Maximum number of chores is 4. Rif-3, Mai-1, Ben-2, Kim-5. Ken has no chore.

Set 6.5a

3. See Figure C.10.

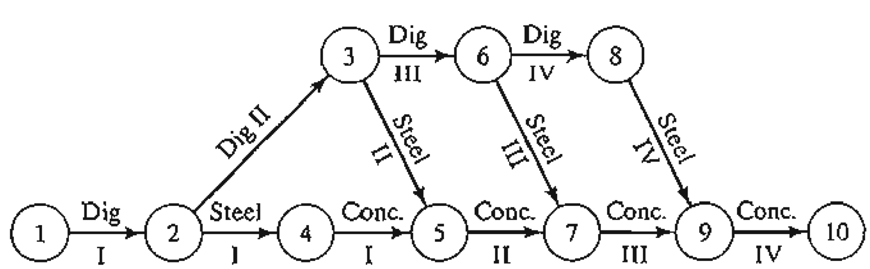
Set 6.5b

1. Critical path: 1-3-4-5-6-7. Duration = 19.

Set 6.5c

3. (a) 10. (b) 5. (c) 0.
5. (a) Critical path: 1-3-6, duration = 45 days.
   (b) A, D, and E.

FIGURE C.10



(c) Each of C, D, and G will be delayed by 5 days. E will not be affected.
(d) Minimum equipment = 2 units.

CHAPTER 7

Set 7.1a

2. (1, 0) and (0, 2) are in Q, but $\lambda(1,0) + (1-\lambda)(0,2) = (\lambda, 2-2\lambda)$ does not lie in Q for $0 < \lambda < 1$.

Set 7.1b

2. (b) Unique solution with $x_1 > 1$ and $0 < x_2 < 1$. See Figure C.11.
   (d) An infinite number of solutions.
   (f) No solution.
3. (a) Basis because det $\mathbf{B} = -4$.
   (d) Not a basis because a basis must include exactly 3 independent vectors.

Set 7.1c

1.

$$\mathbf{B}^{-1} = \begin{pmatrix} .3 & -.2 \\ .1 & .1 \end{pmatrix}$$

Basic	x_1	x_2	x_3	x_4	Solution
z	1.5	−.5	0	0	21.5
x_3	0	.5	1	0	2
x_4	.5	0	0	1	1.5

Solution is feasible but nonoptimal.

4. Optimal $z = 34$.
   Maximize $z = 2x_1 + 5x_2$ subject to $x_1 \leq 4$, $x_2 \leq 6$, $x_1 + x_2 \leq 8$, $x_1, x_2 \geq 0$

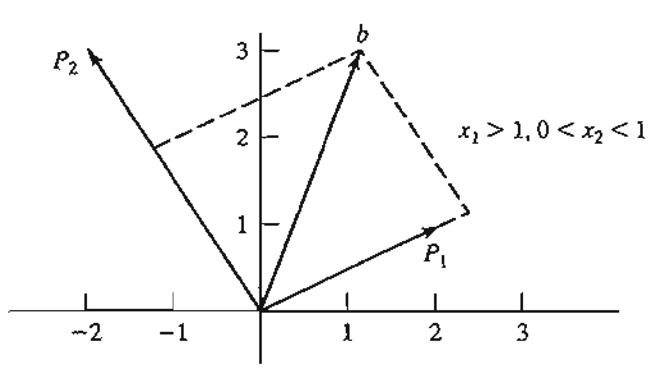


FIGURE C.11

Set 7.2a

1. (a) $\mathbf{P}_1$ must leave.
   (b) $\mathbf{B} = (\mathbf{P}_2, \mathbf{P}_4)$ is a feasible basis.
2. For the basic vector $\mathbf{X}_B$, we have

$$\{z_j - c_j\} = \mathbf{c}_B\mathbf{B}^{-1}\mathbf{B} - \mathbf{c}_B = \mathbf{c}_B\mathbf{I} - \mathbf{c}_B = \mathbf{c}_B - \mathbf{c}_B = \mathbf{0}$$

7. Number of adjacent extreme points is $n - m$, assuming nondegeneracy.
10. In case of degeneracy, number of extreme points is less than the number of basic solutions, else they are equal.
11. (a) new $x_j = \frac{1}{\alpha}$ old x_j.
    (b) new $x_j = \frac{\beta}{\alpha}$ old x_j.

Set 7.2b

2. (b) $(x_1, x_2, x_3) = (1.5, 2, 0), z = 5$.

Set 7.3a

2. (b) $(x_1, x_2, x_3, x_4, x_5, x_6) = (0, 1, .75, 1, 0, 1), z = 22$.

Set 7.4a

2. Maximize $w = \mathbf{Yb}$ subject to $\mathbf{YA} \leq \mathbf{c}, \mathbf{Y} \geq \mathbf{0}$.

Set 7.4b

5. Method 1: $(b_1, b_2, b_3) = (4, 6, 8) \Rightarrow$ dual objective value $= 34$.
   Method 2: $(c_1, c_2) = (2, 5) \Rightarrow$ primal objective value $= 34$.
7. Minimize $w = \mathbf{Yb}$ subject to $\mathbf{YA} = \mathbf{C}, \mathbf{Y}$ unrestricted.

Set 7.5a

1. $-\frac{2}{7} \leq t \leq 1$
2. (a)

Basic solution	Applicable range of t
$(x_2, x_3, x_6) = (5, 30, 10)$	$0 \leq t \leq \frac{1}{3}$
$(x_2, x_3, x_1) = (\frac{25}{4}, \frac{90}{4}, 5)$	$\frac{1}{3} \leq t \leq \frac{5}{2}$
$(x_2, x_4, x_1) = (\frac{5}{2}, 15, 20)$	$\frac{5}{2} \leq t \leq \infty$

5. $\{z_j - c_j\}_{j=1,4,5} = \left(4 - \frac{3t}{2} - \frac{3t^2}{2}, 1 - t^2, 2 - \frac{t}{2} + \frac{t^2}{2}\right)$. Basis remains optimal for $0 \leq t \leq 1$.

Set 7.5b

1. (a) $t_1 = 10, \mathbf{B}_1 = (\mathbf{P}_2, \mathbf{P}_3, \mathbf{P}_4)$
2. At $t = 0, (x_1, x_2, x_4) = (.4, 1.8, 1)$. It remains basic for $0 \le t \le 1.5$. No feasible solution for $t > 1.5$.

CHAPTER 8

Set 8.1a

1. G_5: Minimize $s_5^+, 55x_p + 3.5x_f + 5.5x_s - .0675x_g + s_5^- - s_5^+ = 0$.
3. Let x_1 = No. of in-state freshmen, x_2 = No. of out-of-state freshmen, x_3 = No. of international freshmen.

   G_i: Minimize $s_i^-, i = 1, 2, \ldots, 5$, subject to $x_1 + x_2 + x_3 + s_1^- - s_1^+ = 1200$,

   $$2x_1 + x_2 - 2x_3 + s_2^- - s_2^+ = 0, -.1x_1 - .1x_2 + .9x_3 + s_3^- - s_3^+ = 0,$$

   $$.125x_1 - .05x_2 - .556x_3 + s_4^- - s_4^+ = 0, -.2x_1 + .8x_2 - .2x_3 + s_5^- - s_5^+ = 0$$

   All variables are nonnegative
5. Let x_j = No. of production runs in shift $j, j = 1, 2, 3$.
   Minimize $z = s_1^- + s_1^+$, subject to $-100x_1 + 40x_2 - 80x_3 + s_1^- - s_1^+ = 0$,

   $$4 \le x_1 \le 5, 10 \le x_2 \le 20, 3 \le x_3 \le 20$$

Set 8.2a

1. Objective function: Minimize $z = s_1^- + s_2^- + s_3^- + s_4^+ + s_5^+$
   Solution: $x_p = .0201, x_f = .0457, x_s = .0582, x_g = 2$ cents, $s_5^+ = 1.45$
   Gasoline tax is $1.45 million short of goal.
4. x_1 = lb of limestone/day, x_2 = lb of corn/day, x_3 = lb of soybean meal/day.
   Objective function: Minimize $z = s_1^- + s_2^+ + s_3^- + s_4^- + s_5^+$
   Solution: $x_1 = 166.08$ lb, $x_2 = 2778.56$ lb, $x_3 = 3055.36$ lb, $z = 0$. Problem has alternative optima. All goals are satisfied but goals 3 and 4 are overachieved.
7. x_j = No. of units of product $j, j = 1, 2$.
   Assign a relatively high weight to the quota constraints.
   Objective function: Minimize $z = 100s_1^- + 100s_2^- + s_3^+ + s_4^+$
   Solution: $x_1 = 80, x_2 = 60, s_3^+ = 100$ minutes, $s_4^+ = 120$ minutes.
   Production quota can be met with 100 minutes of overtime for machine 1 and 120 minutes of overtime for machine 2.

Set 8.2b

2. G_1 solution: $x_p = .01745, x_f = .0457, x_s = .0582, x_g = 21.33, s_4^+ = 19.33$, all others = 0. Goals G_1, G_2, and G_3 are satisfied. G_4 is not.

G_4 problem: Same constraints as G_1 plus $s_1^- = 0, s_2^- = 0, s_3^- = 0$.

G_4 solution: $x_p = .0201, x_f = .0457, x_s = .0582, x_g = 2, s_5^+ = 1.45$. All other variables = 0. Goal G_5 is not satisfied.

G_5 problem: Same as G_4 plus $s_4^+ = 0$.

$G5$ solution: Same as G_4, which means that goal 5 cannot be satisfied ($s_5^+ = 1.45$).

CHAPTER 9

Set 9.1a

3. x_{ij} = No. of bottles of type i assigned to individual j, where $i = 1$ (full), 2 (half full), 3 (empty).

   Constraints:

$$x_{11} + x_{12} + x_{13} = 7, x_{21} + x_{22} + x_{23} = 7, x_{31} + x_{32} + x_{33} = 7$$

$$x_{11} + .5x_{21} = 3.5, x_{12} + .5x_{22} = 3.5, x_{13} + .5x_{23} = 3.5$$

$$x_{11} + x_{21} + x_{31} = 7, x_{12} + x_{22} + x_{32} = 7, x_{13} + x_{23} + x_{33} = 7$$

   All x_{ij} are nonnegative integers

   Solution: Use a dummy objective function.

	No. bottles assigned to individual		
Status	*1*	*2*	*3*
Full	1	3	3
Half full	5	1	1
Empty	1	3	3

6. y = Original sum of money. x_j = Amount taken on night $j, j = 1, 2, 3$.

   x_4 = Amount given to each mariner by first officer.

   Minimize $z = y$ subject to $3x_1 - y = 2, x_1 + 3x_2 - y = 2, x_1 + x_2 + 3x_3 - y = 2, y - x_1 - x_2 - x_3 - 3x_4 = 1$. All variables are nonnegative integers.

   Solution: $y = 79 + 81n, n = 0, 1, 2, \ldots$

10. Side 1: 5, 6, and 8 (27 minutes). Side 2: 1, 2, 3, 4, and 7 (28 minutes). Problem has alternative optima.

12. $x_{ij} = 1$ if student i selects course j, and zero otherwise, c_{ij} = associated preference score, C_j = course j capacity. Maximize $z = \sum_{i=1}^{10}\sum_{j=1}^{6} c_{ij}x_{ij}$ subject to

$$\sum_{j=1}^{6} x_{ij} = 2, i = 1, 2, \ldots, 10, \sum_{i=1}^{10} x_{ij} \le C_j, j = 1, 2, \ldots, 6$$

   Solution: Course 1: students (2, 4, 9), 2: (2, 8), 3:(5, 6, 7, 9), 4:(4, 5, 7, 10), 5: (1,3, 8, 10), 6: (1,3). Total score = 1775.

Set 9.1b

1. Let $x_j = 1$ if route j is selected and 0 otherwise. Total distance of route (ABC, 1, 2, 3, 4, ABC) = 10 + 32 + 4 + 15 + 9 = 80 miles.
   Minimize $z = 80x_1 + 50x_2 + 70x_3 + 52x_4 + 60x_5 + 44x_6$ subject to

$$x_1 + x_3 + x_5 + x_6 \geq 1, x_1 + x_3 + x_4 + x_5 \geq 1, x_1 + x_2 + x_4 + x_6 \geq 1,$$
$$x_1 + x_2 + x_5 \geq 1, x_2 + x_3 + x_4 + x_6 \geq 1, x_j = (0, 1), \text{for all } j.$$

   Solution: Select routes (1, 4, 2) and (1, 3, 5), $z = 104$. Customer 1 should be skipped in one of the two routes.

2. Solution: Committee is formed of individuals a, d, and f. Problem has alternative optima.

7. $x_t = 1$ if transmitter t is selected, 0 otherwise. $x_c = 1$ if community c is covered, 0 otherwise. c_t = cost of transmitter t. S_c = set of transmitters covering community c. P_j = population of community j.

   Maximize $z = \sum_{c=1}^{15} P_c x_c$ subject to

$$\sum_{t \in S_c} x_t \geq x_c, c = 1, 2, \ldots, 15, \sum_{t=1}^{7} c_t x_t \leq 15$$

   Solution: Build transmitters 2,4,5,6, and 7. All but community 1 are covered.

Set 9.1c

2. Let x_j = Number of widgets produced on machine j, $j = 1, 2, 3$. $y_j = 1$ if machine j is used and 0 otherwise. Minimize $z = 2x_1 + 10x_2 + 5x_3 + 300y_1 + 100y_2 + 200y_3$ subject to $x_1 + x_2 + x_3 \geq 2000$, $x_1 - 600y_1 \leq 0$, $x_2 - 800y_2 \leq 0$, $x_3 - 1200y_3 \leq 0$, $x_1, x_2, x_3 \geq 500$ and integer, $y_1, y_2, y_3 = (0, 1)$.
   Solution: $x_1 = 600$, $x_2 = 500$, $x_3 = 900$, $z = \$11{,}300$.

3. Solution: Site 1 is assigned to targets 1 and 2, and site 2 is assigned to targets 3 and 4. $z = 18$.

10. x_e = Number of Eastern (one-way) tickets, x_u = Number of US Air tickets, x_c = Number of Continental tickets. e_1, and e_2 binary variables. u and c nonnegative integers. Maximize $z = 1000(x_e + 1.5x_u + 1.8x_c + 5e_1 + 5e_2 + 10u + 7c)$ subject to $e_1 \leq x_e/2$, $e_2 \leq x_e/6$, $u \leq x_u/6$, and $c \leq x_c/5$, $x_e + x_u + x_c = 12$.
    Solution: Buy 2 tickets on Eastern and 10 tickets on Continental. Bonus = 39000 miles.

Set 9.1d

1. Let x_{ij} = Integer amount assigned to square (i, j). Use a dummy objective function with all zero coefficients.

Constraints:

$$\sum_{j=1}^{3} x_{ij} = 15, i = 1, 2, 3, \sum_{i=1}^{3} x_{ij} = 15, j = 1, 2, 3,$$

$$x_{11} + x_{22} + x_{33} = 15, x_{31} + x_{22} + x_{13} = 15,$$

$$(x_{11} \geq x_{12} + 1 \text{ or } x_{11} \leq x_{12} - 1), (x_{11} \geq x_{13} + 1 \text{ or } x_{11} \leq x_{13} - 1),$$

$$(x_{12} \geq x_{13} + 1 \text{ or } x_{12} \leq x_{13} - 1), (x_{11} \geq x_{21} + 1 \text{ or } x_{11} \leq x_{21} - 1),$$

$$(x_{11} \geq x_{31} + 1 \text{ or } x_{11} \leq x_{31} - 1), (x_{21} \geq x_{31} + 1 \text{ or } x_{21} \leq x_{31} - 1),$$

$$x_{ij} = 1, 2, \ldots, 9, \text{ for all i and } j$$

Solution:

2	9	4
7	5	3
6	1	8

Alternative solutions are direct permutations of rows and/or columns.

3. x_j = Daily number of units of product j.
   Maximize $z = 25x_1 + 30x_2 + 22x_3$ subject to

$$\begin{pmatrix} 3x_1 + 4x_2 + 5x_3 \leq 100 \\ 4x_1 + 3x_2 + 6x_3 \leq 100 \end{pmatrix} \text{ or } \begin{pmatrix} 3x_1 + 4x_2 + 5x_3 \leq 90 \\ 4x_1 + 3x_2 + 6x_3 \leq 120 \end{pmatrix}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer}$$

   Solution: Produce 26 units of product 1, 3 of product 2, and none of product 3, and use location 2.

Set 9.2a[2]

2. (a) $z = 6, x_1 = 2, x_2 = 0.$
   (d) $z = 12, x_1 = 0, x_2 = 3.$
3. (a) $z = 7.25, x_1 = 1.75, x_2 = 1.$
   (d) $z = 10.5, x_1 = .5, x_2 = 2.$
9. Equivalent 0-1 ILP:
   Maximize $z = 18y_{11} + 36y_{12} + 14y_{21} + 28y_{22} + 8y_{31} + 16y_{32} + 32y_{33}$
   subject to $15y_{11} + 30y_{12} + 12y_{21} + 24y_{22} + 7y_{31} + 14y_{32} + 28y_{33} \leq 43$
   All variables are binary.
   Solution: $z = 50, y_{12} = 1, y_{21} = 1,$ all others $= 0$. Equivalently, $x_1 = 2, x_2 = 1$.
   The 0-1 version required 41 nodes. The original requires 29.

[2]Use TORA integer programming module to generate the B&B tree.

Set 9.2b

1. (a) Legitimate cut because it passes through an integer point and does not eliminate any feasible integer point. You can verify this result by plotting the cut on the LP solution space.

6. (a) Optimum integer solution: $(x_1, x_2, x_3) = (2, 1, 6)$, $z = 26$.
Rounded solution: $(x_1, x_2, x_3) = (3, 1, 6)$, which is infeasible.

Set 9.3a

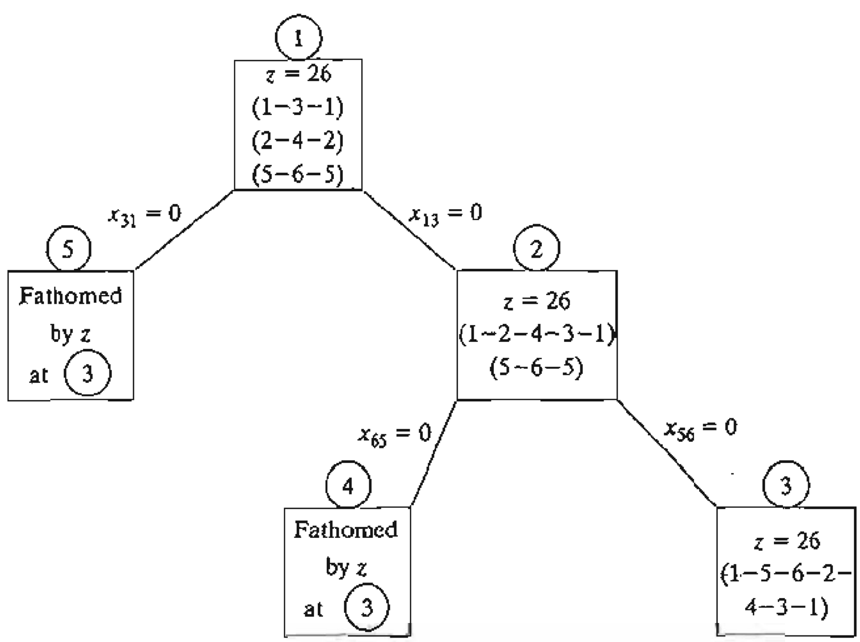
1. The table below gives the number of distinct employees who enter/leave the manager's office when we switch from project i to project j. The objective is to find a "tour" through all projects that will minimize the total traffic.

	1	2	3	4	5	6
1	—	4	4	6	6	5
2	4	—	6	4	6	3
3	4	6	—	4	8	7
4	6	4	4	—	6	5
5	6	6	8	6	—	5
6	5	3	7	5	5	—

Set 9.3c

2. See Figure C.12.

FIGURE C.12



CHAPTER 10

Set 10.1a

1. Solution: Shortest distance = 21 miles. Route: 1-3-5-7.

Set 10.2a

3. Solution: Shortest distance = 17. Route: 1-2-3-5-7.

Set 10.3a

2. (a) Solution: Value = 120. $(m_1, m_2, m_3) = (0,0,3), (0,4,1), (0,2,2)$, or $(0,6,0)$.
5. Solution: Total points = 250. Select 2 courses from I, 3 from II, 4 from III, and 1 from IV,
7. Let $x_j = 1$ if application j is accepted, and 0 otherwise. Equivalent knapsack model is
   Maximize $z = 78x_1 + 64x_2 + 68x_3 + 62x_4 + 85x_5$ subject to
   $$7x_1 + 4x_2 + 6x_3 + 5x_4 + 8x_5 \leq 23, x_j = (0,1), j = 1, 2, \ldots, 5$$
   Solution: Accept all but the first application. Value = 279.

Set 10.3b

1. (a) Solution: Hire 6 for week 1, fire 1 for week 2, fire 2 for week 3, hire 3 for week 4, and hire 2 for week 5.
3. Solution: Rent 7 cars for week 1, return 3 for week 2, rent 4 for week 3, and no action for week 4.

Set 10.3c

2. Decisions for next 4 years: Keep, Keep, Replace, Keep. Total cost = \$458.

Set 10.3d

3. (a) Let x_i and y_i be the number of sheep kept and sold at the end of period i and define $z_i = x_i + y_i$.
   $$f_n(z_n) = \max_{y_n = z_n}\{p_n y_n\}$$
   $$f_i(z_i) = \max_{y_i \leq z_i}\{p_i y_i + f_{i+1}(2z_i - 2y_i)\}, i = 1, 2, \ldots, n-1$$

CHAPTER 11

Set 11.3a

2. (a) Total cost per week = \$51.50.
   (b) Total cost per week = \$50.20, $y^* = 239.05$ lb.

4. (a) Choose policy 1 because its cost per day is \$2.17 as opposed to \$2.50 for policy 2.
   (b) Optimal policy: Order 100 units whenever the inventory level drops to 10 units.

Set 11.3b

2. Optimal policy: Order 500 units whenever level drops to 130 units. Cost per day = \$258.50.
4. No advantage if $TCU_1(y_m) \leq TCU_2(q)$, which translates to no advantage if the discount factor does not exceed .9344%.

Set 11.3c

1. AMPL/Solver solution: $(y_1, y_2, y_3, y_4, y_5) = (4.42, 6.87, 4.12, 7.2, 5.8)$, cost = \$568.12,
4. Constraint: $\sum_{i=1}^{4} \frac{365 D_i}{y_i} \leq 150.$
   Solver/AMPL solution: $(y_1, y_2, y_3, y_4) = (155.3, 118.82, 74.36, 90.09)$, cost = \$54.71.

Set 11.4a

1. (a) 500 units required at the start of periods 1, 4, 7, and 10.

Set 11.4b

3. Produce 173 units in period 1, 180 in period 2, 240 in period 3, 110 in period 4, and 203 in period 5.

Set 11.4c

1. (a) No, because inventory should not be held needlessly at end of horizon.
   (b) (i) $0 \leq z_1 \leq 5, 1 \leq z_2 \leq 5, 0 \leq z_3 \leq 4; x_1 = 4, 1 \leq x_2 \leq 6, 0 \leq x_3 \leq 4.$
   (ii) $5 \leq z_1 \leq 14, 0 \leq z_2 \leq 9, 0 \leq z_3 \leq 5; x_1 = 0, 0 \leq x_2 \leq 9, 0 \leq x_3 \leq 5.$
2. (a) $z_1 = 7, z_2 = 0, z_3 = 6, z_4 = 0$. Total cost = \$33.

Set 11.4d

1. Use initial inventory to satisfy the entire demand of period 1 and 4 units of period 2, thus reducing demand for the four periods to 0, 22, 90, and 67, respectively. Optimal solution: Order 112 units in period 2 and 67 units in period 4. Total cost = \$632.

Set 11.4e

1. Solution: Produce 210 units in January, 255 in April, 210 in July, and 165 in October.

CHAPTER 12

Set 12.1a

1. (a) .15 and .25, respectively. (b) .571. (c) .821.
2. $n \gtrsim 23$.
3. $n > 253$.

Set 12.1b

3. $\frac{5}{32}$.
4. Let p = probability Liz wins. Probability John wins is $3p$, which equals the probability Jim will win. Probability Ann wins is $6p$. Because one of the four wins, $p + 3p + 3p + 3p + 6p = 1$.
   (a) $\frac{3}{13}$.
   (b) $\frac{7}{13}$.
   (c) $\frac{6}{13}$.

Set 12.1c

3. (a) .375. (b) .6.
7. .9545.

Set 12.2a

2. (a) $K = 20$.
3. $P\{\text{Demand} \geq 1100\} = .3$.

Set 12.3a

3. (a) $P\{50 \leq \text{copies sold} \leq 70\} = .6667$.
   (b) Expected number of unsold copies = 2.67
   (c) Expected net profit = \$22.33

Set 12.3b

1. Mean = 3.667, variance = 1.556.

Set 12.3c

1. (a) $P(x_1 = 1) = P(x_2 = 1) = .4, P(x_1 = 2) = P(x_2 = 2) = .2, P(x_1 = 3) = P(x_2 = 3) = .4$.
   (b) No, because $P(x_1, x_2) \neq P(x_1)P(x_2)$.

Set 12.4a

1. $\left(\frac{1}{2}\right)^{10}$.
3. .0547.

Set 12.4b

1. .8646.
3. (a) $P\{n = 0\} = 0$.
   (b) $P\{n \geq 3\}$; 1.

Set 12.4c

1. $\lambda = 12$ arrivals/min. $P\{t \leq 5 \text{ sec}\} = .63$.

Set 12.4d

2. .001435.

CHAPTER 13

Set 13.1a

1. Weights for A, B, and C = (.44214, .25184, .30602).

Set 13.1b

2. CR > .1 for all matrices except **A**. $(w_S, w_J, w_M) = (.331, .292, .377)$. Select Maisa.
4. All matrices are consistent. $(w_H, w_P) = (.502, .498)$. Select H.

Set 13.2a

2. (a) See Figure C.13.
   (b) EV(corn) = −\$8250, EV(soybeans) = \$250. Select soybeans.
6. (a) See Figure C.14.
   (b) EV(game) = −\$.025. Do not play the game.

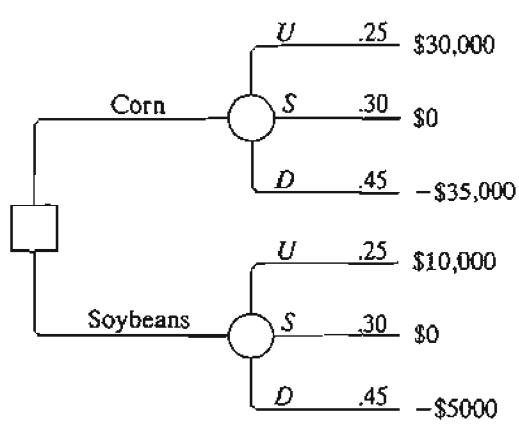


FIGURE C.13

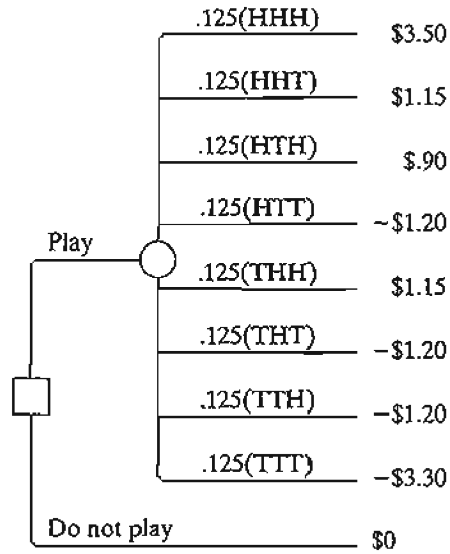


FIGURE C.14

12. Optimum maintenance cycle = 8 years. Cost per year = \$397.50.
15. Optimum production rate = 49 pieces per day.
19. Level must be between 99 and 151 gallons.

Set 13.2b

2. Let z be the event of having one defective item in a sample of size 5. Answer: $P\{A|z\} = .6097$, $P\{B|z\} = .3903$.
4. (a) Expected revenue if you self-publish = \$196,000.
   Expected revenue if you use a publisher = \$163,000.
   (b) If survey predicts success, self-publish, else use a publisher.
7. (b) Ship lot to B if both items are bad, else ship lot to A.

Set 13.2c

1. (a) Expected value = \$5, hence there is no advantage.
   (b) For $0 \leq x < 10$, $U(x) = 0$, and for $x = 10$, $U(x) = 100$.
   (c) Play the game.
2. Lottery: $U(x) = 100 - 100p$, with $U(-\$1{,}250{,}000) = 0$ and $U(\$900{,}000) = 100$.

Set 13.3a

1. (a) All methods: Study all night (action a_1).
   (b) All methods: Select actions a_2 or a_3.

Set 13.4a

2. (a) Saddle-point solution at (2, 3). Value of game = 4.
3. (a) $2 < v < 4$.

Set 13.4b

1. Each player should mix strategies 50-50. Value of game = 0.
2. Police payoff matrix:

	100%A	50%A-50%B	100%B
A	100	50	0
B	0	30	100

Strategy for Police: Mix 50-50 strategies 100%A and 100%B.
Strategy for Robin: Mix 50-50 strategies A and B. Value of game = \$50 (= expected fine paid by Robin).

Set 13.4c

1. (a) Payoff matrix for team 1:

	AB	AC	AD	BC	BD	CD
AB	1	0	0	0	0	−1
AC	0	1	0	0	−1	0
AD	0	0	1	−1	0	0
BC	0	0	−1	1	0	0
BD	0	−1	0	0	1	0
CD	−1	0	0	0	0	1

Optimal strategy for both teams: Mix AB and CD 50-50. Value of the game = 0.

3. (a) (m, n) = (Number of regiments at location 1, No. of regiments at locations 2). Each location has a payoff of 1 if won and -1 if lost. For example, Botto's strategy $(1, 1)$ against the enemy's $(0, 3)$ will win location 1 and lose location 2, with anet payoff of $1 + (-1) = 0$. Payoff matrix for Colonel Blotto:

	3, 0	2, 1	1, 2	0, 3
2, 0	−1	−1	0	0
1, 1	0	−1	−1	0
0, 2	0	0	−1	−1

Optimal strategy for Blotto: Blotto mixes 50-50 strategies (2-0) and (0-2), and the enemy mixes 50-50 strategies (3-0) and (1-2). Value of the game = $-.5$, and Blotto loses. Problem has alternative optima.

CHAPTER 14

Set 14.1a

1. (a) Order 1000 units whenever inventory level drops to 537 units.

Set 14.1b

2. Solution: $y^* = 317.82$ gallons, $R^* = 46.82$ gallons.
3. Solution: $y^* = 316.85$ gallons, $R^* = 58.73$ gallons. In Example 14.1-2, $y^* = 319.44$ gallons, $R^* = 93.61$ gallons. Order quantity remains about the same as in Example 14.1-2, but R^* is smaller because the demand pdf has a smaller variance.

Set 14.2a

3. $.43 \leq p \leq .82$
6. 32 coats.

Set 14.2b

1. Order $9-x$ if $x < 4.53$, else do not order.

Set 14.3a

2. Order $4.61-x$ if $x < 4.61$, else do not order.

CHAPTER 15

Set 15.1a

1. (a) Productivity = 71%.
   (b) The two requirements cannot be met simultaneously.

Set 15.2a

1.

Situation	Customer	Server
(a)	Plane	Runway
(b)	Passenger	Taxi
(h)	Car	Parking space

Set 15.3a

1. (b) (i) $\lambda = 6$ arrivals per hour, average interarrival time $= \frac{1}{6}$hour.
   (c) (i) $\mu = 5$ services per hour, average service time $= .2$ hour.
3. (a) $f(t) = 20e^{-20t}, t > 0$.
   (b) $P\{t > \frac{15}{60}\} = .00674$.
7. Jim's payoff is 2 cents with probability $P\{t \leq 1\} = .4866$ and -2 cents with probability $P\{t \geq 1\} = .5134$. In 8 hours, Jim pays Ann = 17.15 cents.
10. (a) $P\{t \leq 4 \text{ minutes}\} = .4866$.
    (b) Average discount percentage = 6.208.

Set 15.4a

1. $p_{n\geq 5}(1 \text{ hour}) = .55951$.
4. (a) $p_2(t = 7) = .24167$.
6. (a) Combined $\lambda = \frac{1}{10} + \frac{1}{7}$, $p_2(t = 5) = .219$.

Set 15.4b

2. (a) $p_0(t = 3) = .00532$.
   (c) $p_{n\leq 17}(t = 1) = .9502$.
5. $p_0(4) = .37116$.
8. (a) Average order size = 25 − 7.11 = 17.89 items.
   (b) $p_0(t = 4) = .00069$.

Set 15.5a

3. (a) $p_{n\geq 3} = .4445$.
   (b) $p_{n\leq 2} = .5555$.
6. (a) $p_j = .2, j = 0, 1, 2, 3, 4$.
   (b) Expected number in shop = 2 customers.
   (c) $p_4 = .2$.

Set 15.6a

1. (a) $L_q = 1p_6 + 2p_7 + 3p_8 = .1917$ car.
   (c) $\lambda_{lost} = .1263$ car per hour. Average number lost in 8 hr = 1.01 cars.
   (d) No. of empty spaces $= c - (L_s - L_q) = c - \sum_{n=0}^{8} np_n + \sum_{n=c+1}^{8} (n - c)p_n$.

Set 15.6b

2. (a) $p_0 = .2$.
   (b) Average monthly income = $50 $\times \mu t$ = $375.
   (c) Expected payment = $40 $\times L_q$ = $128.

5. (a) $p_0 = .4$.
   (b) $L_q = .9$ car.
   (c) $W_q = 2.25$ min.
   (d) $p_{n\geq 11} = .0036$.
6. (d) No. of spaces is at least 13.

Set 15.6c

1. $P\{\tau > 1\} = .659$.
5. $37.95 per 12-hour day.

Set 15.6d

1. (a) $p_0 = .3654$.
   (b) $W_q = .207$ hour.
   (c) Expected number of empty spaces $= 4 - L_q = 3.212$.
   (d) $p_5 = .04812$.
   (e) 40% reduction lowers W_s to about 9.6 min ($\mu = 10$ cars/hr).
4. (a) $p_8 = .6$.
   (b) $L_q = 6.34$ generators.
   (c) Probability of finding an empty space cannot exceed .4 regardless of belt capacity. This means that the best utilization of the assembly department is 60%.
7. (a) $1 - p_5 = .962$.
   (b) $\lambda_{\text{lost}} = \lambda p_5 = .19$ customer per hour.

Set 15.6e

2. For $c = 2$, $W_q = 3.446$ hour and for $c = 4$, $W_q = 1.681$ hour, an improvement of over 51%.
5. Let K be the number of waiting-room spaces. Using TORA, $p_0 + p_1 + \cdots + p_{K+2} \geq .999$ yields $K \geq 10$.
7. (a) $p_{n\geq 4} = .65772$.
   (e) Average number of idle computers $= .667$ computer.

Set 15.6f

2. (c) Utilization $= 81.8\%$.
   (d) $p_2 + p_3 + p_4 = .545$.
4. (a) $p_{40} = .00014$.
   (b) $p_{30} + p_{31} + \text{L} + p_{39} = .02453$.
   (d) Expected number of occupied spaces $= L_s - L_q = 20.043 - .046 \approx 20$.
   (f) Probability of not finding a parking space $= 1 - p_{n\leq 29} = .02467$. Number of students who cannot park in an 8-hour period is approximately 4.

Set 15.6g

2. (a) Approximately 7 seats.
   (b) $p_{n\geq 8} = .2911$.

Set 15.6h

1. (b) Average number of idle repairpersons = 2.01.
   (d) $P\{2 \text{ or } 3 \text{ idle servers}\} = p_0 + p_1 = .34492$.
4. (a) $L_s = 1.25$ machines.
   (b) $p_0 = .33342$.
   (c) $W_s = .25$ hour.
6. $\lambda = 2$ calls per hour per baby, $\mu = .5$ baby per hour, $R = 5$, $K = 5$.
   (a) Number of awake babies $= 5 - L_s = 1$ baby.
   (b) $p_5 = .32768$.
   (c) $p_{n\leq 2} = .05792$.

Set 15.7a

2. (a) $E\{t\} = 14$ minutes and $\text{var}\{t\} = 12$ minutes^2. $L_s = 7.8672$ cars.
4. $\lambda = .0625$ prescriptions per minute, $E\{t\} = 15$ minutes, $\text{var}\{t\} = 9.33$ minutes^2.
   (a) $p_0 = .0625$.
   (b) $L_q = 7.3$ prescriptions
   (c) $W_s = 132.17$ minutes.

Set 15.9a

2. Use $(M/M/1)$:$(GD/10/10)$. Cost per hour is \$431.50 for repairperson 1 and \$386.50 for repairperson 2.
4. (b) $\mu = \lambda + \sqrt{\dfrac{c_2\lambda}{c_1}}$
   (c) Optimum production rate = 2725 pieces per hour.

Set 15.9b

2. (a) Hourly cost per hour is \$86.4 for two repairpersons and \$94.80 for three.
   (b) Schedule loss per breakdown $= \$30 \times W_s = \$121.11$ for two repairpersons and \$94.62 for three.
4. Rate of breakdowns per machine, $\lambda = .36125$ per hour, $\mu = 10$ per hour. Model $(M/M/3)$:$(GD/20/20)$ yields $L_s = .70529$ machine. Lost revenue = \$36.60 and cost of three repairpersons = \$60.

Set 15.9c

1. (a) Number of repairpersons ≥ 5.
   (b) Number of repairpersons ≥ 4.

CHAPTER 16

Set 16.1a

4. (a) $P\{H\} = P\{T\} = .5$. If $0 \leq R \leq .5$, Jim gets \$10.00. If $.5 < R \leq 1$, Jan gets \$10.00.

7. Lead time sampling: If $0 \leq R \leq .5, L = 1$ day. If $.5 < R \leq 1, L = 2$ days. Demand per day sampling: If $0 \leq R \leq .2$, demand = 0 unit. If $.2 < R \leq .9$, demand = 1 unit. If $.9 < R \leq 1$, demand = 2 units. Use one R to sample L. If $L = 1$, use another R to sample demand for one day, else if $L = 2$, use one R to generate demand for day 1 and then another R to generate demand for day 2.

Set 16.2a

1. (a) Discrete.

Set 16.3a

4. See Figure C.15.

Set 16.3b

1. $t = -\frac{1}{\lambda}\ln(1 - R), \lambda = 4$ customers per hour.

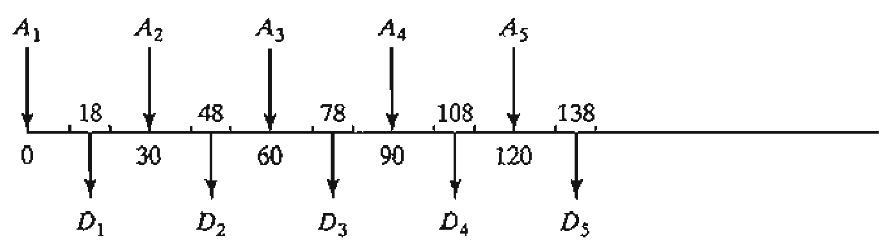
Customer	R	t (hr)	Arrival time
1	—	—	0
2	0.0589	0.015176	0.015176
3	0.6733	0.279678	0.294855
4	0.4799	0.163434	0.458288

2. $t = a + (b - a)R$.

4. (a) $0 \leq R < .2: d = 0, .2 \leq R < .5: d = 1, .5 \leq R < .9: d = 2, .9 \leq R \leq 1$: $d = 3$.

9. If $0 \leq R \leq p$, then $x = 0$, else $x = \left(\text{largest integer} \leq \frac{\ln(1 - R)}{\ln q}\right)$.

FIGURE C.15



Set 16.3c

1. $y = -\frac{1}{5}\ln(.0589 \times .6733 \times .4799 \times .9486) = .803$ hour.
6. $t = x_1 + x_2 + x_3 + x_4$, where $x_i = 10 + 10R_i, i = 1, 2, 3, 4$.

Set 16.4a

1. In Example 16.4-1, cycle length = 4. With the new parameters, cycling was not evident after 50 random numbers were generated. The conclusion is that judicious selection of the parameters is important.

Set 16.5a

2. (a) Observation-based.
   (b) Time-based.
3. (a) 1.48 customers.
   (b) 7.4 hours.

Set 16.6a

2. Confidence interval: $15.07 \leq \mu \leq 23.27$.

CHAPTER 17

Set 17.1a

2. S1: Car on patrol
   S2: Car responding to a call
   S3: Car at call scene
   S4: Apprehension made.
   S5: Transport to police station

	S1	S2	S3	S4	S5
S1	0.4	0.6	0	0	0
S2	0.1	0.3	0.6	0	0
S3	0.1	0	0.5	0.4	0
S4	0.4	0	0	0	0.6
S5	1	0	0	0	0

Set 17.2a

2. Initial probabilities:

S1	S2	S3	S4	S5
0	0	1	0	0

Input Markov chain:

	S1	S2	S3	S4	S5
S1	0.4	0.6	0	0	0
S2	0.1	0.3	0.6	0	0
S3	0.1	0	0.5	0.4	0
S4	0.4	0	0	0	0.6
S5	1	0	0	0	0

Output (2-step or 2 patrols) transition matrix ($\mathbf{P}^2$)

	S1	S2	S3	S4	S5
S1	0.22	0.42	0.36	0	0
S2	0.13	0.15	0.48	0.24	0
S3	0.25	0.06	0.25	0.2	0.24
S4	0.76	0.24	0	0	0
S5	0.4	0.6	0	0	0

Absolute 2-step probabilities = $(0\,0\,1\,0\,0)\mathbf{P}^2$

State	Absolute (2-step)
S1	0.25
S2	0.06
S3	0.25
S4	0.2
S5	0.24

$P\{\text{apprehension, S4, in 2 patrols}\} = .2$

Set 17.3a

1. (a) Using excelMarkovChains.xls, the chain is periodic with period 3.

(b) States 1, 2, and 3 are transient, State 4 is absorbing.

Set 17.4a

1. (a) Input Markov chain:

	S	C	R
S	0.8	0.2	0
C	0.3	0.5	0.2
R	0.1	0.1	0.8

Steady state probabilities:

$(\pi_1, \pi_2, \pi_3) = (\pi_1, \pi_2, \pi_3)\mathbf{P}$

$\pi_1 + \pi_2 + \pi_3 = 1$

Output Results:

State	*Steady state*	*Mean return time*
S	0.50	2.0
C	0.25	4.0
R	0.25	4.0

Expected revenues $= 2 \times .5 + 1.6 \times .25 + .4 \times .25 = \$1{,}500$

(b) Sunny days will return every $\mu_{SS} = 2$ days—meaning two days on no sunshine.

5. (a) Input Markov chain:

	never	some	always
never	0.95	0.04	0.01
some	0.06	0.9	0.04
always	0	0.1	0.9

(b)

Output Results

State	*Steady state*	*Mean return time*
never	0.441175	2.2666728
some	0.367646	2.7200089
always	0.191176	5.2307892

44.12% never, 36.76% sometimes, 19.11% always

(c) Expected uncollected taxes/year $= .12(\$5000 \times .3676 + 12{,}000 \times .1911) \times 70{,}000{,}000 = \$34{,}711{,}641{,}097.07$

14. (a) State = (i, j, k) = (No. in year −2, No. in year −1, No. in current year), i, j, k = (0 or 1)

Example: (1-0-0) this year links to (0-0-1) if a contract is secured next yr.

	0-0-0	1-0-0	0-1-0	0-0-1	1-1-0	1-0-1	0-1-1	1-1-1
0-0-0	0.1	0	0	0.9	0	0	0	0
1-0-0	0.2	0	0	0.8	0	0	0	0
0-1-0	0	0.2	0	0	0	0.8	0	0
0-0-1	0	0	0.2	0	0	0	0.8	0
1-1-0	0	0.3	0	0	0	0.7	0	0
1-0-1	0	0	0.3	0	0	0	0.7	0
0-1-1	0	0	0	0	0.3	0	0	0.7
1-1-1	0	0	0	0	0.5	0	0	0.5

(b)

State	Steady state
0-0-0	0.014859
1-0-0	0.066865
0-1-0	0.066865
0-0-1	0.066865
1-1-0	0.178306
1-0-1	0.178306
0-1-1	0.178306
1-1-1	0.249629

$$\begin{aligned}\text{Expected nbr. of contracts in 3 yrs} &= 1(0.066865 + 0.066865 + 0.066865)\\ &\quad + 2(0.178306 + 0.178306 + 0.178306)\\ &\quad + 3(0.249629) = 2.01932\end{aligned}$$

$$\text{Expected nbr. of contracts/yr} = 2.01932/3 = 0.67311$$

Set 17.5a

1. (a) Initial probabilities:

1	2	3	4	5
1	0	0	0	0

Input Markov chain:

1	2	3	4	5
0	.3333	.3333	.3333	0
.3333	0	.3333	0	.3333
.3333	.3333	0	0	.3333
.5	0	0	0	.5
0	.3333	.3333	.3333	0

State	Absolute (3-step)	Steady state
1	.07407	.214286
2	.2963	.214286
3	.2963	.214286
4	.25926	.142857
5	**.07407**	**.214286**

(b) $a_5 = .07407$
(c) $\pi_5 = .214286$
(d) $\mu_{15} = 4.6666.$

	$(\mathbf{I} - \mathbf{N})^{-1}$				Mu
	1	2	3	5	
1	2	1	1	.6667	4.6666
2	1	1.625	.875	.3333	3.8333
3	1	.875	1.625	.3333	3.8333
4	1	.5	.5	1.3333	3.3333

5. (a) Input Markov chain:

	A	B	C
A	.75	.1	.15
B	.2	.75	.05
C	.125	.125	.75

(b)

State	Steady state
A	.394737
B	.307018
C	.298246

A: 39.5%, B: 30.7%, C: 29.8%

(c)

$(\mathbf{I}-\mathbf{N})^{-1}$

	A	C
A	5.71429	3.42857
C	2.85714	5.71429

Mu

	B
A	**9.14286**
C	8.57143

	1	2
A	5.88235	2.35294
B	4.70588	5.88235

	C
A	**8.23529**
B	1.5882

A → B: 9.14 years
A → C: 8.23 years

Set 17.6a

2. (a) States: 1wk, 2wk, 3wk, Library

Matrix **P**:

	1	2	3	lib
1	0	0.3	0	0.7
2	0	0	0.1	0.9
3	0	0	0	1
lib	0	0	0	1

(b)

$(\mathbf{I}-\mathbf{N})^{-1}$

	1	2	3
1	1	0.3	.03
2	0	1	.01
3	0	0	1

Mu

	lib
1	1.33
2	1.1
3	1

I keep the book 1.33 wks on the average.

8. (a)

Matrix **P**:

	1	2	3	4	F
1	0.2	0.8	0	0	0
2	0	0.22	0.78	0	0
3	0	0	0.25	0.75	0
4	0	0	0	0.3	0.7
F	0	0	0	0	1

(b)

	$(I-N)^{-1}$ 1	2	3	4		Mu F
1	1.25	1.282	1.333	1.429	1	5.29
2	0	1.282	1.333	1.429	2	4.04
3	0	0	1.333	1.429	3	2.76
4	0	0	0	1.429	4	1.43

(c) To be able to take Cal II, the student must finish in 16 weeks (4 transitions) or less. Average number of transitions needed = 5.29. Hence, an average student will not be able to finish Cal I on time.

(d) No, per answer in (c).

10. (a) states:0, 1, 2, 3, D (delete)

Matrix **P**:

	0	1	2	3	D
0	0.5	0.5	0	0	0
1	0.4	0	0.6	0	0
2	0.3	0	0	0.7	0
3	0.2	0	0	0	0.8
D	0	0	0	0	1

(b) A new customer stays 12 years on the list.

	$(I-N)^{-1}$ 0	1	2	3		Mu D
0	5.952	2.976	1.786	1.25	0	12
1	3.952	2.976	1.786	1.25	1	9.96
2	2.619	1.31	1.786	1.25	2	6.96
3	1.19	0.595	0.357	1.25	3	3.39

(c) 6.96 years.

CHAPTER 18

Set 18.1a

1. (a) No stationary points.

(b) Minimum at $x = 0$.

(e) Inflection point at $x = 0$, minimum at $x = .63$, and maximum at $x = -.63$.

4. $(x_1, x_2) = (-1, 1)$ or (2,4).

Set 18.2a

1. (b) $(\partial x_1, \partial x_2) = (2.83, -2.5)\,\partial x_2$

Set 18.2b

3. Necessary conditions: $2\left(x_i - \frac{x_n^2}{x_i}\right) = 0, i = 1, 2, \ldots, n - 1$. Solution is $x_i = \sqrt[n]{C}$, $i = 1, 2, \ldots, n$. $\partial f = 2\delta\sqrt[n]{C^{2-n}}$.
6. (b) Solution $(x_1, x_2, x_3, x_4) = \left(-\frac{5}{74}, -\frac{10}{74}, \frac{155}{74}, \frac{60}{74}\right)$, which is a minimum point.

Set 18.2c

2. Minima points: $(x_1, x_2, x_3) = (-14.4, 4.56, -1.44)$ and $(4.4, .44, .44)$.

CHAPTER 19

Set 19.1a

2. (c) $x = 2.5$, achieved with $\Delta = .000001$.
   (e) $x = 2$, achieved with $\Delta = .000001$.

Set 19.1b

1. By Taylor's expansion, $\nabla f(\mathbf{X}) = \nabla f(\mathbf{X}^0) + \mathbf{H}(\mathbf{X} - \mathbf{X}^0)$. The Hessian $\mathbf{H}$ is independent of $\mathbf{X}$ because $f(\mathbf{X})$ is quadratic. Also, the given expansion is exact because higher-order derivatives are zero. Thus, $\nabla f(\mathbf{X}) = \mathbf{0}$ yields $\mathbf{X} = \mathbf{X}^0 - \mathbf{H}^{-1}\nabla f(\mathbf{X}^0)$. Because $\mathbf{X}$ satisfies $\nabla f(\mathbf{X}) = \mathbf{0}$, $\mathbf{X}$ must be optimum regardless of the choice of initial $\mathbf{X}^0$.

Set 19.2a

2. Optimal solution: $x_1 = 0, x_2 = 3, z = 17$.
4. Let $w_j = x_j + 1, j = 1, 2, 3, v_1 = w_1 w_2, v_2 = w_1 w_3$. Then,
   Maximize $z = v_1 + v_2 - 2w_1 - w_2 + 1$
   subject to $v_1 + v_2 - 2w_1 - w_2 \leq 9, \ln v_1 - \ln w_1 - \ln w_2 = 0,$

$$\ln v_2 - \ln w_1 - \ln w_3 = 0, \text{ all variables are nonnegative.}$$

Set 19.2b

1. Solution: $x_1 = 1, x_2 = 0, z = 4$.
2. Solution: $x_1 = 0, x_2 = .4, x_3 = .7, z = -2.35$.

Set 19.2c

1. Maximize $z = x_1 + 2x_2 + 5x_3$
subject to $2x_1 + 3x_2 + 5x_3 + 1.28y \leq 10$
$9x_1^2 + 16x_3^2 - y^2 = 0$
$7x_1 + 5x_2 + x_3 \leq 12.4, x_1, x_2, x_3, y \geq 0$

CHAPTER 20

Set 20.1a

1. See Figure C.16.

Set 20.1b

1. Case 1: Lower bound is not substituted out.

	x_{12}	x_{13}	x_{24}	x_{32}	x_{34}	
Minimize z	1	5	3	4	6	
Node 1	1	1				= 50
Node 2	−1		1	−1		= −40
Node 3		−1		1	1	= 20
Node 4			−1		−1	= −30
Lower bound	0	30	10	10	0	
Upper bound	∞	40	∞	∞	∞	

Case 2: Lower bound is substituted out.

	x'_{12}	x'_{13}	x'_{24}	x'_{32}	x'_{34}	
Minimize z	1	5	3	4	6	
Node 1	1	1				= 20
Node 2	−1		1	−1		= −40
Node 3		−1		1	1	= 40
Node 4			−1		−1	= −20
Upper bound	∞	10	∞	∞	∞	

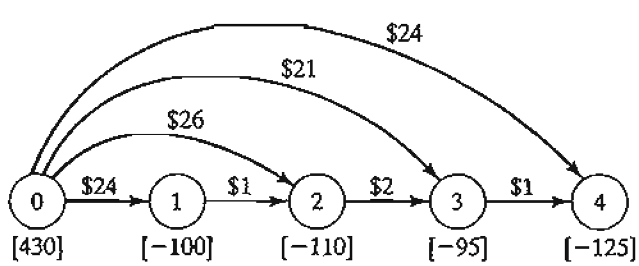


FIGURE C.16

Set 20.1c

1. Optimum cost = \$9895. Produce 210 units in period 1 and 220 units in period 3.
5. Optimal solution: Total student miles = 24,300. Problem has alternative optima.

	Number of students	
	School 1	*School 2*
Minority area 1	0	500
Minority area 2	450	0
Minority area 3	0	300
Nonminority area 2	1000	0
Nonminority area 2	0	1000

Set 20.2a

1. (c) Add the artificial constraint $x_2 \leq M$. Then

$$(x_1, x_2) = \alpha_1(0,0) + \alpha_2(10,0) + \alpha_3(20,10) + \alpha_4(20,M) + \alpha_5(0,M)$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = 1, \alpha_j \geq 0, j = 1, 2, \ldots, 5$$

2. Subproblem 1: $(x_1, x_2) = \alpha_1(0,0) + \alpha_2\left(\frac{12}{5}, 0\right) + \alpha_3(0,12)$
   Subproblem 2: $(x_4, x_5) = \beta_1(5,0) + \beta_2(50,0) + \beta_3(0,10) + \beta_4(0,5)$
   Optimal solution: $\alpha_1 = \alpha_2 = 0, \alpha_3 = 1 \Rightarrow x_1 = 0, x_2 = 12$

$$\beta_1 = .4889, \beta_2 = .5111, \beta_3 = \beta_4 = 0 \Rightarrow x_4 = 28, x_5 = 0.$$

6. Since the original problem is minimization, we must maximize each subproblem. Optimal solution: $(x_1, x_2, x_3, x_4) = \left(\frac{5}{3}, \frac{15}{3}, 0, 20\right), z = 195.$

CHAPTER 22

Set 22.1a

2. Solution: Day 1: Accept if offer is high. Day 2: Accept if offer is medium or high. Day 3: Accept any offer.

Set 22.2a

1. Solution: Year 1: Invest \$10,000. Year 2: Invest all. Year 3: Do not invest. Year 4: Invest all. Expected accumulation = \$35,520.
4. Allocate 2 bikes to center 1, 3 to center 2, and 3 to center 3.

Set 22.3a

3. Solution: First game: Bet \$1. Second game: Bet \$1. Third game: Bet \$1 or none. Maximum probability = .109375.

CHAPTER 23

Set 23.1a

2. Do not fertilize, fertilize when in state 1, fertilize when in state 2, fertilize when in state 3, fertilize when in state 1 or 2, fertilize when in state 1 or 3, fertilize when in state 2 or 3, or fertilize regardless of state.

Set 23.2a

1. Years 1 and 2: Don't advertise if product is successful; otherwise, advertise. Year 3: Don't advertise.
3. If stock level at the start of month is zero, order 2 refrigerators; otherwise, do not order.

Set 23.3a

1. Advertise whenever in state 1.

APPENDIX A

Set A.3a

1. `rest{i in 1..n}:(if i<=n-1 then x[i]+x[i+1] else x[1]+x[n])>=c[i];`

Set A.4a

2. See file A.4a-2.txt

Set A.5a

2. Data for `unitprofit` must be re-read four times with convoluted ordering of data elements.

```
24 5  6 4 4
 6 5  1 4 2
 1 5 -1 4 1
 2 5  0 4 1
```

Set A.5c

1. Error will result because members of sets `paint` and `resource` cannot be read from the double-subscripted table `RMaij`.

APPENDIX C

Partial Answers to Selected Problems[1]

CHAPTER 1

Set 1.1a

4. 17 minutes
5. (a) Jim's alternatives: Throw curve or fast ball.
   Joe's alternatives: Prepare for curve or fast ball.
   (b) Joe wants to increase his batting average.
   Jim wants to reduce Joe's batting average.

CHAPTER 2

Set 2.1a

1. (a) $-x_1 + x_2 \geq 1$
   (c) $x_1 - x_2 \leq 0$
   (e) $.5x_1 - .5x_2 \geq 0$
3. Unused $M1 = 4$ tons/day

Set 2.2a

1. (a and e) See Figure C.1.
2. (a and d) See Figure C.2.

[1]Solved problems in this appendix are designated by * in the text.

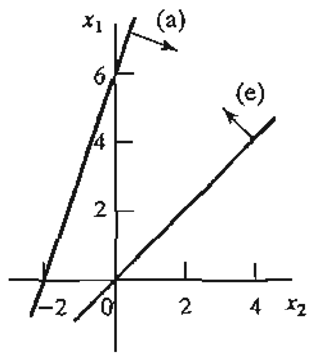


FIGURE C.1

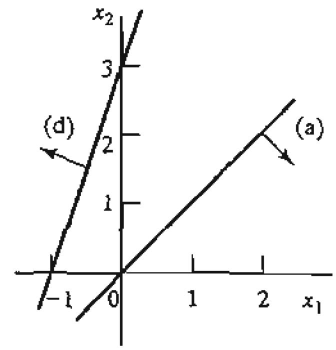


FIGURE C.2

5. Let

x_1 = Number of units of A

x_2 = Number of units of B

Maximize $z = 20x_1 + 50x_2$ subject to

$$-.2x_1 + .8x_2 \le 0, 2x_1 + 4x_2 \le 240$$
$$x_1 \le 100, x_1, x_2 \ge 0$$

Optimum: $(x_1, x_2) = (80, 20), z = \$2,600$

7. Let

x_1 = Dollars invested in A

x_2 = Dollars invested in B

Maximize $z = .05x_1 + .08x_2$ subject to

$$.75x_1 - .25x_2 \ge 0, .5x_1 - .5x_2 \ge 0,$$
$$x_1 - .5x_2 \ge 0, x_1 + x_2 \le 5000, x_1, x_2 \ge 0$$

Optimum: $(x_1, x_2) = (2500, 2500), z = \$325$

11. Let

x_1 = Play hours per day

x_2 = Work hours per day

Maximize $z = 2x_1 + x_2$ subject to

$$x_1 + x_2 \le 10, x_1 - x_2 \le 0$$
$$x_1 \le 4, x_1, x_2 \ge 0$$

Optimum: $(x_1, x_2) = (4, 6), z = 14$

14. Let

x_1 = Tons of $C1$ per hour

x_2 = Tons of $C2$ per hour

Maximize $z = 12000x_1 + 9000x_2$ subject to

$$-200x_1 + 100x_2 \leq 0, 2.1x_1 + .9x_2 \leq 20, x_1, x_2 \geq 0$$

Optimum: $(x_1, x_2) = (5.13, 10.26)$, $z = 153{,}846$ lb

(a) Optimum ratio $C1:C2 = .5$.

(b) Optimum ratio is the same, but steam generation will increase by 7692 lb/hr.

18. Let

x_1 = Number of HiFi1 units

x_2 = Number of HiFi2 units

Minimize $z = 1267.2 - (15x_1 + 15x_2)$ subject to

$$6x_1 + 4x_2 \leq 432, 5x_1 + 5x_2 \leq 412.8$$

$$4x_1 + 6x_2 \leq 422.4, x_1, x_2 \geq 0$$

Optimum: $(x_1, x_2) = (50.88, 31, 68)$, $z = 28.8$ idle min.

Set 2.2b

1. (a) See Figure C.3

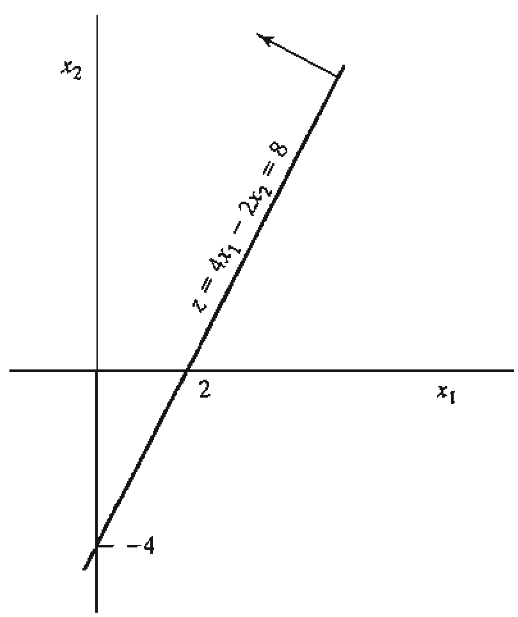


FIGURE C.3

5. Let

x_1 = Thousand bbl/day from Iran

x_2 = Thousand bbl/day from Dubai

Minimize $z = x_1 + x_2$ subject to

$$-.6x_1 + .4x_2 \le 0, .2x_1 + .1x_2 \ge 14$$
$$.25x_1 + .6x_2 \ge 30, .1x_1 + .15x_2 \ge 10$$
$$.15x_1 + .1x_2 \ge 8, x_1, x_2 \ge 0$$

Optimum: $x_1 = 55, x_2 = 30, z = 85$

7. Let

x_1 = Ratio of scrap A alloy

x_2 = Ratio of scrap B alloy

Minimize $z = 100x_1 + 80x_2$ subject to

$$.03 \le .06x_1 + .03x_2 \le .06, .03 \le .03x_1 + .06x_2 \le .05$$
$$.03 \le .04x_1 + .03x_2 \le .07, x_1 + x_2 = 1, x_1, x_2 \ge 0$$

Optimum: $x_1 = .33, x_2 = .67, z = \$86,667$

Set 2.3a

3. Let

x_{ij} = Portion of project i completed in year j

$$\text{Maximize } z = .05(4x_{11} + 3x_{12} + 2x_{13}) + .07(3x_{22} + 2x_{23} + x_{24})$$
$$+ .15(4x_{31} + 3x_{32} + 2x_{33} + x_{34}) + .02(2x_{43} + x_{44})$$

subject to

$$x_{11} + x_{12} + x_{13} = 1, x_{43} + x_{44} = 1$$
$$.25 \le x_{22} + x_{23} + x_{24} + x_{25} \le 1$$
$$.25 \le x_{31} + x_{32} + x_{33} + x_{34} + x_{35} \le 1$$
$$5x_{11} + 15x_{31} \le 3, 5x_{12} + 8x_{22} + 15x_{32} \le 6$$
$$5x_{13} + 8x_{23} + 15x_{33} + 1.2x_{43} \le 7$$
$$8x_{24} + 15x_{34} + 1.2x_{44} \le 7, 8x_{25} + 15x_{35} \le 7$$
$$\text{all } x_{ij} \ge 0$$

Optimum: $x_{11} = .6, x_{12} = .4, x_{24} = .255, x_{25} = .025$

$$x_{32} = .267, x_{33} = .387, x_{34} = .346, x_{43} = 1, z = \$523,750$$

Set 2.3b

2. The model can be generalized to account for any input currency p and any output currency q. Define x_{ij} as in Example 2.3-2 and r_{ij} as the exchange rate from currency i to currency j. The associated model is

Maximize $z = y$ subject to

$$\text{capacity: } x_{ij} \leq c_i, \text{ for all } i \neq j$$

$$\text{Input currency } p\text{: } I + \sum_{j \neq p} r_{jp} x_{jp} = \sum_{j \neq p} x_{pj}$$

$$\text{Output currency } q\text{: } y + \sum_{j \neq q} x_{qj} = \sum_{j \neq q} r_{jq} x_{jq}$$

$$\text{Currency } i \neq p \text{ or } q\text{: } \sum_{j \neq i} r_{ji} x_{ji} = \sum_{j \neq i} x_{ij}$$

$$\text{all } x_{ij} \geq 0$$

Rate of return: 1.8064% for \$ → \$, 1.7966% for \$ → €, 1.8287% for \$ → £, 2.8515% for \$ → ¥, and 1.0471% for \$ → KD. Wide discrepancies in ¥ and KD currencies may be attributed to the fact that the given exchange rates may not be totally consistent with the other rates. Nevertheless, the problem demonstrates the advantage of targeting accumulation in different currencies.
[*Note*: Interactive AMPL (file ampl2.3b-2.txt) or Solver (file solver2.3b-2.xls) is ideal for solving this problem. See Section 2.4.]

Set 2.3c

2. Let

x_i = Dollars invested in project i, $i = 1, 2, 3, 4$

y_j = Dollars invested in bank in year j, $j = 1, 2, 3, 4$

Maximize $z = y_5$ subject to

$$x_1 + x_2 + x_4 + y_1 \leq 10{,}000$$

$$.5x_1 + .6x_2 - x_3 + .4x_4 + 1.065y_1 - y_2 = 0$$

$$.3x_1 + .2x_2 + .8x_3 + .6x_4 + 1.065y_2 - y_3 = 0$$

$$1.8x_1 + 1.5x_2 + 1.9x_3 + 1.8x_4 + 1.065y_3 - y_4 = 0$$

$$1.2x_1 + 1.3x_2 + .8x_3 + .95x_4 + 1.065y_4 - y_5 = 0$$

$$x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4, y_5 \geq 0$$

Optimum solution:

$$x_1 = 0, x_2 = \$10{,}000, x_3 = \$6000, x_4 = 0$$

$$y_1 = 0, y_2 = 0, y_3 = \$6800, y_4 = \$33{,}642$$

$$z = \$53{,}628.73 \text{ at the start of year 5}$$

5. Let x_{iA} = amount invested in year i using plan A, $i = 1, 2, 3$
x_{iB} = amount invested in year i using plan B, $i = 1, 2, 3$
Maximize $z = 3x_{2B} + 1.7x_{3A}$ subject to

$$x_{1A} + x_{1B} \le 100 \text{ (start of year 1)}$$

$$-1.7x_{1A} + x_{2A} + x_{2B} = 0 \quad \text{(start of year 2)}$$

$$-3x_{1B} - 1.7x_{2A} + x_{3A} = 0 \quad \text{(start of year 3)}$$

$$x_{iA}, x_{iB} \ge 0, i = 1, 2, 3$$

Optimum solution: Invest \$100,000 in plan A in year 1 and \$170,000 in plan B in year 2. Problem has alternative optima.

Set 2.3d

3. Let x_j = number of units of product j, $j = 1, 2, 3$
Maximize $z = 30x_1 + 20x_2 + 50x_3$ subject to

$$2x_1 + 3x_2 + 5x_3 \le 4000$$

$$4x_1 + 2x_2 + 7x_3 \le 6000$$

$$x_1 + .5x_2 + .33x_3 \le 1500$$

$$2x_1 - 3x_2 = 0$$

$$5x_2 - 2x_3 = 0$$

$$x_1 \ge 200, x_2 \ge 200, x_3 \ge 150$$

$$x_1, x_2, x_3 \ge 0$$

Optimum solution: $x_1 = 324.32, x_2 = 216.22, x_3 = 540.54, z = \$41{,}081.08$

7. Let x_{ij} = Quantity produced by operation i in month j, $i = 1, 2$, $j = 1, 2, 3$
I_{ij} = Entering inventory of operation i in month j, $i = 1, 2$, $j = 1, 2, 3$

Minimize $z = \sum_{j=1}^{3}(c_{1j}x_{1j} + c_{2j}x_{2j} + .2I_{1j} + .4I_{2j})$ subject to

$$.6x_{11} \le 800, .6x_{12} \le 700, .6x_{13} \le 550$$

$$.8x_{21} \le 1000, .8x_{22} \le 850, .8x_{23} \le 700$$

$$x_{1j} + I_{1,j-1} = x_{2j} + I_{1j}, x_{2j} + I_{2,j-1} = d_j + I_{2j}, j = 1, 2, 3$$
$$I_{1,0} = I_{2,0} = 0, \text{all variables} \geq 0$$
$$d_j = 500, 450, 600 \text{ for } j = 1, 2, 3$$
$$c_{1j} = 10, 12, 11 \text{ for } j = 1, 2, 3$$
$$c_{2j} = 15, 18, 16 \text{ for } j = 1, 2, 3$$

Optimum: $x_{11} = 1333.33$ units, $x_{13} = 216.67$, $x_{21} = 1250$ units, $x_{23} = 300$ units, $z = \$39{,}720$.

Set 2.3e

2. Let x_s = lb of screws/package, x_b = lb of bolts/package, x_n = lb of nuts/package, x_w = lb of washers/package
Minimize $z = 1.1x_s + 1.5x_b + \left(\frac{70}{80}\right)x_n + \left(\frac{20}{30}\right)x_w$ subject to
$$y = x_s + x_b + x_n + x_w$$
$$y \geq 1, x_s \geq .1y, x_b \geq .25y, x_n \leq .15y, x_w \leq .1y$$
$$\left(\tfrac{1}{10}\right)x_b \leq x_n, \left(\tfrac{1}{50}\right)x_b \leq x_w$$
All variables are nonnegative
Solution: $z = \$1.12, y = 1, x_s = .5, x_b = .25, x_n = .15, x_w = .1$

5. Let x_A = bbl of crude A/day, x_B = bbl of crude B/day, x_r = bbl of regular/day x_p = bbl of premium/day, x_j = bbl of jet fuel/day
Maximize $z = 50(x_r - s_r^+) + 70(x_p - s_p^+) + 120(x_j - s_j^+)$
$- (10s_r^- + 15s_p^- + 20s_j^- + 2s_r^+ + 3s_p^+ + 4s_j^+)$
$- (30x_A + 40x_B)$ subject to
$$x_A \leq 2500, x_B \leq 3000, x_r = .2x_A + .25x_B, x_p = .1x_A + .3x_B, x_j = .25x_A + .1x_B$$
$$x_r + s_r^- - s_r^+ = 500, x_p + s_p^- - s_p^+ = 700, x_j + s_j^- - s_j^+ = 400, \text{All variables} \geq 0$$
Solution:
$$z = \$21{,}852.94, x_A = 1176.47 \text{ bbl/day}, x_B = 1058.82, x_r = 500 \text{ bbl/day}$$
$$x_p = 435.29 \text{ bbl/day}, x_j = 400 \text{ bbl/day}, s_p^- = 264.71$$

Set 2.3f

1. Let $x_i(y_i)$ = Number of 8-hr (12-hr) buses starting in period i
Minimize $z = 2\sum_{i=1}^{6} x_i + 3.5\sum_{i=1}^{6} y_i$ subject to
$$x_1 + x_6 + y_1 + y_5 + y_6 \geq 4, x_1 + x_2 + y_1 + y_2 + y_6 \geq 8,$$
$$x_2 + x_3 + y_1 + y_2 + y_3 \geq 10, x_3 + x_4 + y_2 + y_3 + y_4 \geq 7,$$
$$x_4 + x_5 + y_3 + y_4 + y_5 \geq 12, x_5 + x_6 + y_4 + y_5 + y_6 \geq 4$$

All variables are nonnegative

Solution: $x_1 = 4, x_2 = 4, x_4 = 2, x_5 = 4, y_3 = 6$, all others $= 0$, $z = 49$.

Total number of buses = 20. For the case of 8-hr shift, number of buses = 26 and comparable $z = 2 \times 26 = 52$. Thus, (8-hr + 12-hr) shift is better.

5. Let x_i = Number of students starting in period i ($i = 1$ for 8:01 A.M., $i = 9$ for 4:01 P.M.)

Minimize $z = x_1 + x_2 + x_3 + x_4 + x_6 + x_7 + x_8 + x_9$ subject to

$$x_1 \geq 2, x_1 + x_2 \geq 2, x_1 + x_2 + x_3 \geq 3,$$

$$x_2 + x_3 + x_4 \geq 4, x_3 + x_4 \geq 4, x_4 + x_6 \geq 3,$$

$$x_6 + x_7 \geq 3, x_6 + x_7 + x_8 \geq 3, x_7 + x_8 + x_9 \geq 3$$

$$x_5 = 0, \text{ all other variables are nonnegative}$$

Solution: Hire 2 at 8:01, 1 at 10:01, 3 at 11:01, and 3 at 2:01. Total = 9 students

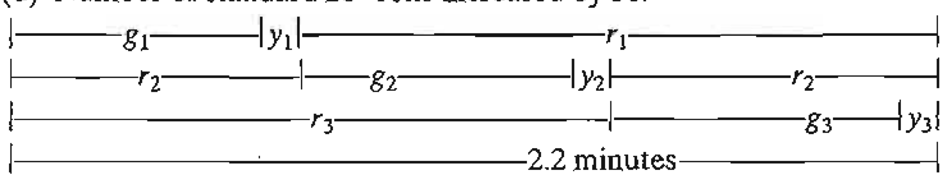
Set 2.3g

1. (a) $1150L$ ft^2

   (b) (3,0,0), (1,1,0), (1,0,1), and (0,2,0) with respective 0, 3, 1, and 1 trim loss per foot.

   (c) Number of standard 20′-rolls decreased by 30.

   (d) Number of standard 20′-rolls increased by 50.

6. 

Let g_i, y_i, and r_i be the durations of green, yellow, and red lights for cars exiting highway i. All time units are in seconds. No cars move on yellow.

maximize $z = 3(500/3600)g_1 + 4(600/3600)g_2 + 5(400/3600)g_3$ subject to

$$(500/3600)g_1 + (600/3600)g_2 + (400/3600)g_3 \leq (510/3600)(2.2 \times 60 - 3 \times 10)$$

$$g_1 + g_2 + g_3 + 3 \times 10 \leq 2.2 \times 60, g_1 \geq 25, g_2 \geq 25, g_3 \geq 25$$

Solution: $g_1 = 25$ sec., $g_2 = 43.6$ sec., $g_3 = 33.4$ sec. Booth income = \$58.04/hr

Set 2.4a

2. (d) See file solver2.4a-2(d).xls in folder AppenCFiles.

Set 2.4b

2. (c) See file ampl2.4b-2(c).txt in folder AppenCFiles.

   (f) See file ampl2.4b-2(f).txt in folder AppenCFiles.

CHAPTER 3

Set 3.1a

1. 2 tons/day and 1 ton/day for raw materials $M1$ and $M2$, respectively.
4. Let x_{ij} = units of product i produced on machine j.
   Maximize $z = 10(x_{11} + x_{12}) + 15(x_{21} + x_{22})$ subject to

$$x_{11} + x_{21} - x_{12} - x_{22} + s_1 = 5$$
$$-x_{11} - x_{21} + x_{12} + x_{22} + s_2 = 5$$
$$x_{11} + x_{21} + s_3 = 200$$
$$x_{12} + x_{22} + s_4 = 250$$
$$s_i, x_{ij} \geq 0, \text{ for all } i \text{ and } j$$

Set 3.1b

3. Let x_j = units of product $j, j = 1, 2, 3$.
   Maximize $z = 2x_1 + 5x_2 + 3x_3 - 15x_4^+ - 10x_5^+$
   subject to

$$2x_1 + x_2 + 2x_3 + x_4^- - x_4^+ = 80$$
$$x_1 + x_2 + 2x_3 + x_5^- - x_5^+ = 65$$
$$x_1, x_2, x_3, x_4^-, x_4^+, x_5^-, x_5^+ \geq 0$$

   Optimum solution: $x_2 = 65$ units, $x_4^- = 15$ units, all others $= 0$, $z = \$325$.

Set 3.2a

1. (c) $x_1 = \frac{6}{7}, x_2 = \frac{12}{7}, z = \frac{48}{7}$.
   (e) Corner points $(x_1 = 0, x_2 = 3)$ and $(x_1 = 6, x_2 = 0)$ are infeasible.
3. Infeasible basic solutions are:

$$(x_1, x_2) = \left(\tfrac{26}{3}, -\tfrac{4}{3}\right), \quad (x_1, x_3) = (8, -2)$$
$$(x_1, x_4) = (6, -4), \quad (x_2, x_3) = (16, -26)$$
$$(x_2, x_4) = (3, -13), (x_3, x_4) = (6, -16)$$

Set 3.3a

3. (a) Only (A, B) represents successive simplex iterations because corner point A and B are adjacent. In all the remaining pairs the associated corner points are not adjacent.
   (b) (i) Yes. (ii) No, C and I are not adjacent. (iii) No, path returns to a previous corner point, A.
5. (a) x_3 enters at value 1, $z = 3$ at corner point D.

Set 3.3b

3.

New basic variable	x_1	x_2	x_3	x_4
Value	1.5	1	0	.8
Leaving variable	x_7	x_7	x_8	x_5

6. (b) x_2, x_5, and x_6 can increase value of z. If x_2 enters, x_8 leaves and $\Delta z = 5 \times 4 = 20$. If x_5 enters, x_1 leaves and $\Delta z = 0$ because x_5 equals 0 in the new solution. If x_6 enters, no variable leaves because all the constraint coefficients of x_6 are less than or equal to zero. $\Delta z = \infty$ because x_6 can be increased to infinity without causing infeasibility.
9. Second best value of $z = 20$ occurs when s_2 is made basic.

Set 3.4a

3. (a) Minimize $z = (8M - 4)x_1 + (6M - 1)x_2 - Ms_2 - Ms_3 = 10M$
   (b) Minimize $z = (3M - 4)x_1 + (M - 1)x_2 = 3M$
6. The starting tableau is

Basic	x_1	x_2	x_3	x_4	Solution
z	-1	-12	0	0	-8
x_3	1	1	1	0	4
x_4	1	4	0	1	8

Set 3.4b

1. Always minimize the sum of artificial variables because the sum represents the amount of infeasibility in the problem.
7. Any nonbasic variable having nonzero objective coefficients at end of Phase I cannot become positive in Phase II because it will mean that the optimal objective value in Phase I will be positive; that is, infeasible Phase I solution.

Set 3.5a

1. (a) $A \rightarrow B \rightarrow C \rightarrow D$.
   (b) 1 at A, 1 at B, $C_2^4 = 6$ at C, and 1 at D.

Set 3.5b

1. Alternative basic optima: $\left(0, 0, \frac{10}{3}\right)$, $(0, 5, 0)$, $\left(1, 4, \frac{1}{3}\right)$. Nonbasic alternative optima: $\left(\alpha_3, 5\alpha_2 + 4\alpha_3, \frac{10}{3}\alpha_1 + \frac{1}{3}\alpha_3\right)$, $\alpha_1 + \alpha_2 + \alpha_3 = 1, 0 \le \alpha_i \le 1, i = 1, 2, 3$.

Set 3.5c

2. (a) Solution space is unbounded in the direction of x_2.
   (b) Objective value is unbounded because each unit increase in x_2 increases z by 10.

Set 3.5d

1. The most that can be produced is 275 units.

Set 3.6a

2. Let

   x_1 = number of Type 1 hats per day

   x_2 = number of Type 2 hats per day

   Maximize $z = 8x_1 + 5x_2$ subject to

$$2x_1 + x_2 \leq 400$$
$$x_1 \leq 150, x_2 \leq 200$$
$$x_1, x_2 \geq 0$$

   (a) See Figure C.4: $x_1 = 100$, $x_2 = 200$, $z = \$1800$ at point B.
   (b) \$4 per Type 2 hat in the range (200, 500).
   (c) No change because the dual price is \$0 per unit in the range $(100, \infty)$.
   (d) \$1 worth per unit in the range (100, 400). Maximum increase = 200 Type 2.

Set 3.6b

3. (a) $0 \leq \frac{c_1}{c_2} \leq 2$.
   (b) New $\frac{c_1}{c_2} = 1$. Solution remains unchanged.

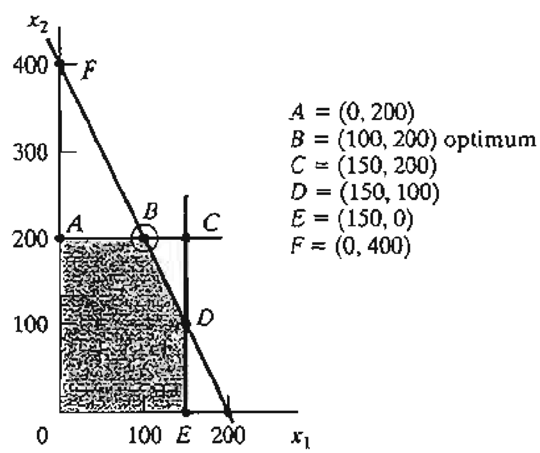


FIGURE C.4

Set 3.6c

2. (a) Yes, because additional revenue per min = \$1 (for up to 10 min of overtime) exceeds additional cost of \$.83/min.
   (b) Additional revenue is \$2/min (for up to 400 min of overtime) = \$240 for 2 hr. Additional cost for 2 hr = \$110. Net revenue = \$130.
   (c) No, its dual price is zero because the resource is already abundant.
   (d) $D_1 = 10$ min. Dual price = \$1/min for $D_1 \leq 10$. $x_1 = 0, x_2 = 105, x_3 = 230$, net revenue = $(\$1350 + \$1 \times 10 \text{ min}) - \left(\frac{\$40}{60} \times 10 \text{ min}\right) = \$1353.33$.
   (e) $D_2 = -15$. Dual price = \$2/min for $D_2 \geq -20$. Decrease in revenue = \$30. Decrease in cost = \$7.50. Not recommended.
6. Let

$$x_1 = \text{radio minutes}, x_2 = \text{TV minutes}, x_3 = \text{newspaper ads}$$

Maximize $z = x_1 + 50x_2 + 10x_3$ subject to

$$15x_1 + 300x_2 + 50x_3 + s_1 = 10{,}000, x_3 - S_2 = 5,$$

$$x_1 + s_3 = 400, -x_1 + 2x_2 + s_4 = 0, x_1, x_2, x_3 \geq 0,$$

$$s_1, s_2, s_3, s_4 \geq 0$$

   (a) $x_1 = 59.09$ min, $x_2 = 29.55$ min, $x_3 = 5$ ads, $z = 1561.36$
   (b) From TORA, $z + .158s_1 + 2.879S_2 + 0s_3 + 1.364s_4 = 156.364$. Dual prices for the respective constraints are .158, −2.879, 0, and 1.36. Lower limit set on newspaper ads can be decreased because its dual price is negative (= −2.879). There is no advantage in increasing the upper limit on radio minutes because its dual price is zero (the present limit is already abundant).
   (c) From TORA, $x_1 = 59.9091 + .00606D_1 \geq 0, x_3 = 5, s_3 = 340.90909 + .00606D_1 \geq 0, x_2 = 29.54545 + .00303D_1 \geq 0$. Thus, dual price = .158 for the range $-9750 \leq D_1 \leq 56{,}250$. A 50% increase in budget ($D_1 = \$5000$) is recommended because the dual price is positive.
11. (a) Scarce: resistor and capacitor resource; abundant: chip resource.
   (b) Worths per unit of resistor, capacitor, and chips are \$1.25, \$.25, and \$0.
   (e) Change $D_3 = 350 - 800 = -450$ falls outside the feasibility range $D_3 \geq -400$. Hence problem must be solved anew.
13. (b) Solution $x_1 = x_2 = 2 + \frac{\Delta}{3}$ is feasible for all $\Delta > 0$. For $0 < \Delta \leq 3$, $r_1 + r_2 = \frac{\Delta}{3} \leq 1 \Rightarrow$ feasibility confirmed. For $3 \leq \Delta < 6$, $r_1 + r_2 = \frac{\Delta}{3} > 1 \Rightarrow$ feasibility not confirmed. For $\Delta > 6$, the change falls outside the ranges for D_1 and D_2.

Set 3.6d

2. (a) x_1 = Cans of A1, x_2 = Cans of A_2, x_3 = Cans of BK.
   Maximize $z = 80x_1 + 70x_2 + 60x_3$ subject to

$$x_1 + x_2 + x_3 \leq 500, x_1 \geq 100, 4x_1 - 2x_2 - 2x_3 \leq 0$$

Optimum: $x_1 = 166.67, x_2 = 333.33, x_3 = 0, z = 36666.67$.

(b) From TORA, reduced cost per can of BK = 10. Price should be increased by more than 10 cents.

(c) $d_1 = d_2 = d_3 = -5$ cents. From TORA, the reduced costs for the nonbasic variables are

$$x_3: 10 + d_2 - d_3 \geq 0, \text{ satisfied}$$

$$s_1: 73.33 + .67d_2 + .33d_1 \geq 0, \text{ satisfied}$$

$$s_3: 1.67 - .17d_2 + .17d_1 \geq 0, \text{ satisfied}$$

Solution remains the same.

5. (a) x_i = Number of units of motor $i, i = 1, 2, 3, 4$.
Maximize $z = 60x_1 + 40x_2 + 25x_3 + 30x_4$ subject to

$$8x_1 + 5x_2 + 4x_3 + 6x_4 \leq 8000, x_1 \leq 500, x_2 \leq 500,$$

$$x_3 \leq 800, x_4 \leq 750, x_1, x_2, x_3, x_4 \geq 0$$

Optimum: $x_1 = 500, x_2 = 500, x_3 = 375, x_4 = 0, z = \$59,375$

(b) From TORA, $8.75 + d_2 \geq 0$. Type 2 motor price can be reduced by up to \$8.75.

(c) $d_1 = -\$15, d_2 = -\$10, d_3 = -\$6.25, d_4 = -\$7.50$. From TORA,

$$x_4: 7.5 + 1.5d_3 - d_4 \geq 0, \text{ satisfied}$$

$$s_1: 6.25 + .25d_3 \geq 0, \text{ satisfied}$$

$$s_2: 10 - 2d_3 + d_1 \geq 0, \text{ satisfied}$$

$$s_3: 8.75 - 1.25d_3 + d_2 \geq 0, \text{ satisfied}$$

Solution remains the same, but z will be reduced by 25%.

(d) Reduced cost of $x_4 = 7.5$. Increase price by more than \$7.50.

Set 3.6e

5. The dual price for the investment constraint $x_{1A} + x_{1B} \leq 100$ is \$5.10 per dollar invested for *any* amount of investment.

9. (a) Dual price for raw material A is \$10.27. The cost of \$12.00 per lb exceeds the expected revenue. Hence, purchase of additional raw material A is not recommended.

(b) Dual price for raw material B is \$0. Resource is already abundant and no additional purchase is warranted.

CHAPTER 4

Set 4.1a

2. Let y_1, y_2, and y_3 be the dual variables.
   Maximize $w = 3y_1 + 5y_2 + 4y_3$ subject to

$$y_1 + 2y_2 + 3y_3 \leq 15, 2y_1 - 4y_2 + y_3 \leq 12$$

$$y_1 \geq 0, y_2 \leq 0, y_3 \text{ unrestricted}$$

4. (c) Let y_1 and y_2 be the dual variables.
   Minimize $z = 5y_1 + 6y_2$ subject to

$$2y_1 + 3y_2 = 1, y_1 - y_2 = 1$$

$$y_1, y_2 \text{ unrestricted}$$

5. Dual constraint associated with the artificial variables is $y_2 \geq -M$.
   Mathematically, $M \to \infty \Rightarrow y \geq -\infty$, which is the same as y_2 being unrestricted.

Set 4.2a

1. (a) $\mathbf{AV}_1$ is undefined.
   (e) $\mathbf{V}_2\mathbf{A} = (-14 \quad -32)$

Set 4.2b

1. (a) Inverse $= \begin{pmatrix} \frac{1}{4} & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{8} & \frac{3}{4} & 0 & 0 \\ \frac{3}{8} & -\frac{5}{4} & 1 & 0 \\ \frac{1}{8} & -\frac{3}{4} & 0 & 1 \end{pmatrix}$

Set 4.2c

3. Let y_1 and y_2 be the dual variables.
   Minimize $w = 30y_1 + 40y_2$ subject to

$$y_1 + y_2 \geq 5, 5y_1 - 5y_2 \geq 2, 2y_1 - 6y_2 \geq 3$$

$$y_1 \geq -M(\Rightarrow y_1 \text{ unrestricted}), y_2 \geq 0$$

   Solution: $y_1 = 5, y_2 = 0, w = 150$.

6. Let y_1 and y_2 be the dual variables.
   Minimize $w = 3y_1 + 4y_2$ subject to

$$y_1 + 2y_2 \geq 1, 2y_1 - y_2 \geq 5, y_1 \geq 3$$

$$y_2 \text{ unrestricted}$$

   Solution: $y_1 = 3, y_2 = -1, w = 5$

8. (a) $(x_1, x_2) = (3, 0), z = 15, (y_1, y_2) = (3, 1), w = 14$. Range $= (14, 15)$
9. (a) Dual solution is infeasible, hence cannot be optimal even though $z = w = 17$.

Set 4.2d

2. (a) Feasibility: $(x_2, x_4) = (3, 15) \Rightarrow$ feasible.
   Optimality: Reduced costs of $(x_1, x_3) = (0, 2) \Rightarrow$ optimal.

4.

Basic	x_1	x_2	x_3	x_4	x_5	Solution
z	0	0	$-\frac{2}{5}$	$-\frac{1}{5}$	0	$\frac{12}{5}$
x_1	1	0	$-\frac{3}{5}$	$\frac{1}{5}$	0	$\frac{3}{5}$
x_2	0	1	$\frac{4}{5}$	$-\frac{3}{5}$	0	$\frac{6}{5}$
x_5	0	0	-1	1	1	0

Solution is optimal and feasible.

7. Objective value: From primal, $z = c_1x_1 + c_2x_2$, and from dual, $w = b_1y_1 + b_2y_2 + b_3y_3$. $b_1 = 4, b_2 = 6, b_3 = 8, c_1 = 2, c_2 = 5 \Rightarrow z = w = 34$.

Set 4.3a

2. (a) Let (x_1, x_2, x_3, x_4) = daily units of SC320, SC325, SC340, and SC370
   Maximize $z = 9.4x_1 + 10.8x_2 + 8.75x_3 + 7.8x_4$ subject to

$$10.5x_1 + 9.3x_2 + 11.6x_3 + 8.2x_4 \le 4800$$
$$20.4x_1 + 24.6x_2 + 17.7x_3 + 26.5x_4 \le 9600$$
$$3.2x_1 + 2.5x_2 + 3.6x_3 + 5.5x_4 \le 4700$$
$$5x_1 + 5x_2 + 5x_3 + 5x_4 \le 4500$$
$$x_1 \ge 100,\ x_2 \ge 100,\ x_3 \ge 100,\ x_4 \ge 100$$

   (b) Only soldering capacity can be increased because it has a positive dual price (= .4944).
   (c) Dual prices for lower bounds are ≤ 0 ($-.6847$, -1.361, 0, and -5.3003), which means that the bounds have an adverse effect on profitability.
   (d) Dual price for soldering is $.4944/min valid in the range (8920, 10201.72), which corresponds to a maximum capacity increase of 6.26% only.

Set 4.3b

2. New fire truck toy is profitable because its reduced cost $= -2$.
3. Parts PP3 and PP4 are not part of the optimum solution. Current reduced costs are .1429 and 1.1429. Thus, rate of deterioration in revenue per unit is $.1429 for PP3 and $1.1429 for PP4.

Set 4.4a

1. (b) No, because point E is feasible and the dual simplex must stay infeasible until optimum is reached.
4. (c) Add the artificial constraint $x_1 \leq M$. Problem has no feasible solution.

Set 4.5a

4. Let Q be the weekly feed in lb (= 5200, 9600, 15000, 20000, 26000, 32000, 38000, 42000, for weeks 1, 2, ..., and 8). Optimum solution: Limestone = .028Q, corn = .649Q, and soybean meal = .323Q. Cost = .81221Q.

Set 4.5b

1. (a) Additional constraint is redundant.

Set 4.5c

2. (a) New dual values $= \left(\frac{1}{2}, 0, 0, 0\right)$. Current solution remains optimal.
   (c) New dual values $= \left(-\frac{1}{8}, \frac{11}{4}, 0, 0\right)$. $z - .125s_1 + 2.75s_2 = 13.5$. New solution: $x_1 = 2, x_2 = 2, x_3 = 4, z = 14$.

Set 4.5d

1. $\frac{p}{100}(y_1 + 3y_2 + y_3) - 3 \geq 0$. For $y_1 = 1, y_2 = 2$, and $y_3 = 0$, $p \geq 42.86\%$.
3. (a) Reduced cost for fire engines $= 3y_1 + 2y_2 + 4y_3 - 5 = 2 > 0$. Fire engines are not profitable.

CHAPTER 5

Set 5.1a

4. Assign a very high cost, M, to the route from Detroit to dummy destination.
6. (a and b) Use M = 10,000. Solution is shown in bold. Total cost = \$49,710.

	1	2	3	Supply
Plant 1	600	700	400; **25**	25
Plant 2	320; **23**	300; **17**	350	40
Plant 3	500	480; **25**	450; **5**	30
Excess Plant 4	1000; **13**	1000	M	13
Demand	36	42	30	

(c) City 1 excess cost = \$13,000.

9. Solution (in million gallons) is shown in bold. Area 2 will be 2 million gallons short. Total cost = $304,000.

	A1	A2	A3	Supply
Refinery 1	12 / **4**	18 / **2**	M	6
Refinery 2	30	10 / **4**	8 / **1**	5
Refinery 3	20	25	12 / **6**	6
Dummy	M	50 / **2**	50	2
Demand	4	8	7	

Set 5.2a

2. Total cost = $804. Problem has alternative optima.

		Sharpening service			
Day	*New*	*Overnight*	*2-day*	*3-day*	Disposal
Monday	24	0	6	18	0
Tuesday	12	12	0	0	0
Wednesday	2	14	0	0	0
Thursday	0	0	20	0	0
Friday	0	14	0	0	4
Saturday	0	2	0	0	12
Sunday	0	0	0	0	22

5. Total cost = $190,040. Problem has alternative optima.

Period	Capacity	Produced amount	Delivery
1	500	500	400 for (period) 1 and 100 for 2
2	600	600	200 for 2, 220 for 3, and 180 for 4
3	200	200	200 for 3
4	300	200	200 for 4

Set 5.3a

1. (a) Northwest: cost = $42. Least-cost: cost = $37. Vogel: cost = $37.

Set 5.3b

5. (a) Cost = $1475.
   (b) $c_{12} \geq 3, c_{13} \geq 8, c_{23} \geq 13, c_{31} \geq 7$.

Set 5.4a

5. Use the code (city, date) to define the rows and columns of the assignment problem. Example: The assignment (D, 3)-(A,7) means leaving Dallas on Jun 3 and returning from Atlanta June 7 at a cost of $400. Solution is shown in bold. Cost = $1180. Problem has alternative optima.

	(A, 7)	(A, 12)	(A, 21)	(A, 28)
(D, 3)	400	300	300	**280**
(D, 10)	**300**	400	300	300
(D, 17)	300	**300**	400	300
(D, 25)	300	300	**300**	400

6. Optimum assignment: I-*d*, II-*c*, III-*a*, IV-*b*.

Set 5.5a

4. Total cost = $1550. Optimum solution summarized below. Problem has alternative optima.

	Store 1	Store 2	Store 3
Factory 1	50	0	0
Factory 2	50	200	50

CHAPTER 6

Set 6.1a

1. For network (i): (a) 1-3-4-2. (b) 1-5-4-3-1. (c and d) See Figure C.5.
4. Each square is a node. Adjacent squares are connected by arcs. Each of nodes 1 and 8 has the largest number of emanating arcs, and hence must appear in the center. Problem has more than one solution. See Figure C.6.

FIGURE C.5

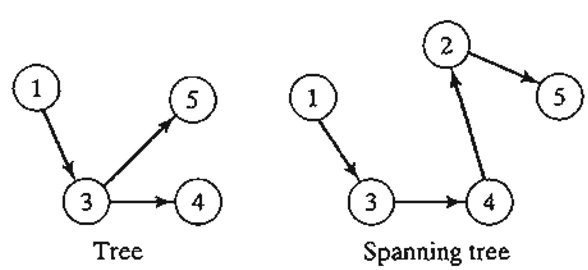
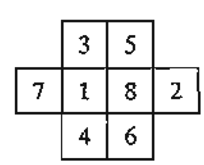


FIGURE C.6



Set 6.2a

2. (a) 1-2, 2-5, 5-6, 6-4, 4-3. Total length = 14 miles.
5. High pressure: 1-2-3-4-6. Low pressure: 1-5-7 and 5-9-8.

Set 6.3a

1. Buy new car in years 1 and 4. Total cost = \$8900. See Figure C.7.
4. For arc (i, v_i)-$(i + 1, v_{i+1})$, define $p(q)$ = value(number of item i). Solution: Select one unit of each of items 1 and 2. Total value = \$80. See Figure C.8.

Set 6.3b

1. (c) Delete all nodes but 4, 5, 6, 7, and 8. Shortest distance = 8 associated with routes 4-5-6-8 and 4-6-8.

Set 6.3c

1. (a) 5-4-2-1, distance = 12.
4. Figure C.9 summarizes the solution. Each arc has unit length. Arrows show one-way routes. Example solution: Bob to Joe: Bob-Kay-Rae-Kim-Joe. Largest number of contacts = 4.

Set 6.3d

1. (a) Right-hand side of equations for nodes 1 and 5 are 1 and −1, respectively, all others = 0. Optimum solution: 1-3-5 or 1-3-4-5, distance = 90.

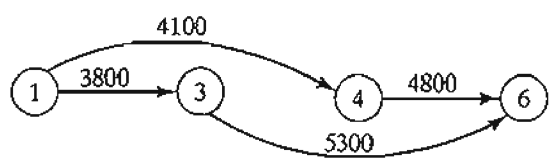


FIGURE C.7

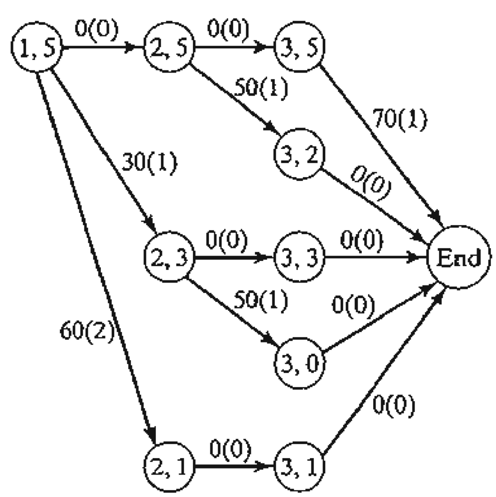
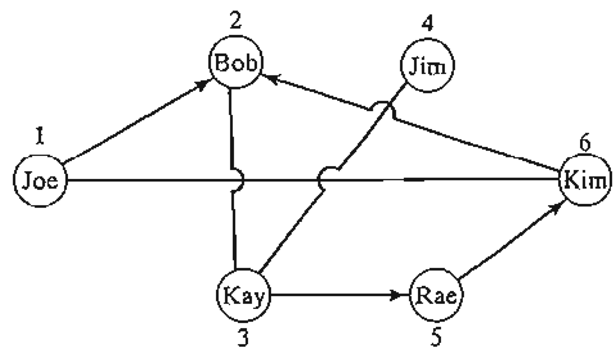


FIGURE C.8

FIGURE C.9



Set 6.4a

1. Cut 1: 1-2, 1-4, 3-4, 3-5, capacity = 60.

Set 6.4b

1. (a) Surplus capacities: arc (2-3) = 40, arc (2-5) = 10, arc (4-3) = 5.
   (b) Node 2: 20 units, node 3: 30 units, node 4: 20 units.
   (c) No, because there is no surplus capacity out of node 1.
7. Maximum number of chores is 4. Rif-3, Mai-1, Ben-2, Kim-5. Ken has no chore.

Set 6.5a

3. See Figure C.10.

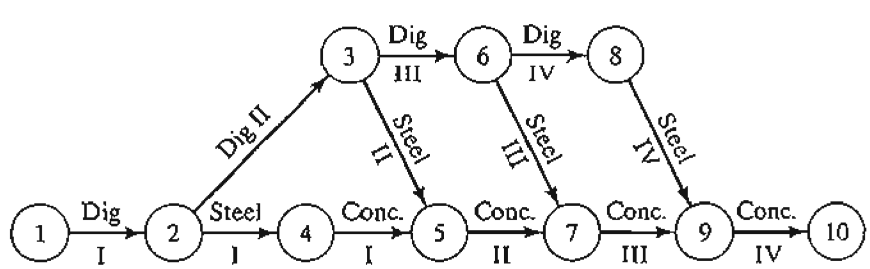
Set 6.5b

1. Critical path: 1-3-4-5-6-7. Duration = 19.

Set 6.5c

3. (a) 10. (b) 5. (c) 0.
5. (a) Critical path: 1-3-6, duration = 45 days.
   (b) A, D, and E.

FIGURE C.10



(c) Each of C, D, and G will be delayed by 5 days. E will not be affected.
(d) Minimum equipment = 2 units.

CHAPTER 7

Set 7.1a

2. (1, 0) and (0, 2) are in Q, but $\lambda(1,0) + (1-\lambda)(0,2) = (\lambda, 2-2\lambda)$ does not lie in Q for $0 < \lambda < 1$.

Set 7.1b

2. (b) Unique solution with $x_1 > 1$ and $0 < x_2 < 1$. See Figure C.11.
   (d) An infinite number of solutions.
   (f) No solution.
3. (a) Basis because det $\mathbf{B} = -4$.
   (d) Not a basis because a basis must include exactly 3 independent vectors.

Set 7.1c

1.

$$\mathbf{B}^{-1} = \begin{pmatrix} .3 & -.2 \\ .1 & .1 \end{pmatrix}$$

Basic	x_1	x_2	x_3	x_4	Solution
z	1.5	−.5	0	0	21.5
x_3	0	.5	1	0	2
x_4	.5	0	0	1	1.5

Solution is feasible but nonoptimal.

4. Optimal $z = 34$.
   Maximize $z = 2x_1 + 5x_2$ subject to $x_1 \le 4$, $x_2 \le 6$, $x_1 + x_2 \le 8$, $x_1, x_2 \ge 0$

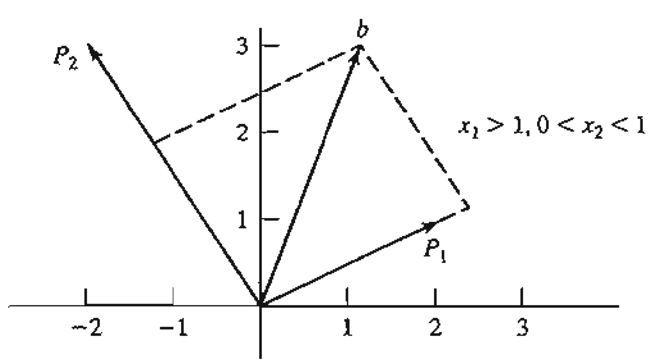


FIGURE C.11

Set 7.2a

1. (a) $\mathbf{P}_1$ must leave.
   (b) $\mathbf{B} = (\mathbf{P}_2, \mathbf{P}_4)$ is a feasible basis.
2. For the basic vector $\mathbf{X}_B$, we have

$$\{z_j - c_j\} = \mathbf{c}_B\mathbf{B}^{-1}\mathbf{B} - \mathbf{c}_B = \mathbf{c}_B\mathbf{I} - \mathbf{c}_B = \mathbf{c}_B - \mathbf{c}_B = \mathbf{0}$$

7. Number of adjacent extreme points is $n - m$, assuming nondegeneracy.
10. In case of degeneracy, number of extreme points is less than the number of basic solutions, else they are equal.
11. (a) new $x_j = \frac{1}{\alpha}$ old x_j.
    (b) new $x_j = \frac{\beta}{\alpha}$ old x_j.

Set 7.2b

2. (b) $(x_1, x_2, x_3) = (1.5, 2, 0), z = 5.$

Set 7.3a

2. (b) $(x_1, x_2, x_3, x_4, x_5, x_6) = (0, 1, .75, 1, 0, 1), z = 22.$

Set 7.4a

2. Maximize $w = \mathbf{Yb}$ subject to $\mathbf{YA} \leq \mathbf{c}, \mathbf{Y} \geq \mathbf{0}$.

Set 7.4b

5. Method 1: $(b_1, b_2, b_3) = (4, 6, 8) \Rightarrow$ dual objective value $= 34$.
   Method 2: $(c_1, c_2) = (2, 5) \Rightarrow$ primal objective value $= 34$.
7. Minimize $w = \mathbf{Yb}$ subject to $\mathbf{YA} = \mathbf{C}$, $\mathbf{Y}$ unrestricted.

Set 7.5a

1. $-\frac{2}{7} \leq t \leq 1$
2. (a)

Basic solution	Applicable range of t
$(x_2, x_3, x_6) = (5, 30, 10)$	$0 \leq t \leq \frac{1}{3}$
$(x_2, x_3, x_1) = (\frac{25}{4}, \frac{90}{4}, 5)$	$\frac{1}{3} \leq t \leq \frac{5}{2}$
$(x_2, x_4, x_1) = (\frac{5}{2}, 15, 20)$	$\frac{5}{2} \leq t \leq \infty$

5. $\{z_j - c_j\}_{j=1,4,5} = \left(4 - \frac{3t}{2} - \frac{3t^2}{2}, 1 - t^2, 2 - \frac{t}{2} + \frac{t^2}{2}\right)$. Basis remains optimal for $0 \leq t \leq 1$.

Set 7.5b

1. (a) $t_1 = 10, \mathbf{B}_1 = (\mathbf{P}_2, \mathbf{P}_3, \mathbf{P}_4)$
2. At $t = 0, (x_1, x_2, x_4) = (.4, 1.8, 1)$. It remains basic for $0 \le t \le 1.5$. No feasible solution for $t > 1.5$.

CHAPTER 8

Set 8.1a

1. G_5: Minimize $s_5^+, 55x_p + 3.5x_f + 5.5x_s - .0675x_g + s_5^- - s_5^+ = 0$.
3. Let x_1 = No. of in-state freshmen, x_2 = No. of out-of-state freshmen, x_3 = No. of international freshmen.

   G_i: Minimize $s_i^-, i = 1, 2, \ldots, 5$, subject to $x_1 + x_2 + x_3 + s_1^- - s_1^+ = 1200$,

   $$2x_1 + x_2 - 2x_3 + s_2^- - s_2^+ = 0, -.1x_1 - .1x_2 + .9x_3 + s_3^- - s_3^+ = 0,$$

   $$.125x_1 - .05x_2 - .556x_3 + s_4^- - s_4^+ = 0, -.2x_1 + .8x_2 - .2x_3 + s_5^- - s_5^+ = 0$$

   All variables are nonnegative
5. Let x_j = No. of production runs in shift $j, j = 1, 2, 3$.
   Minimize $z = s_1^- + s_1^+$, subject to $-100x_1 + 40x_2 - 80x_3 + s_1^- - s_1^+ = 0$,

   $$4 \le x_1 \le 5, 10 \le x_2 \le 20, 3 \le x_3 \le 20$$

Set 8.2a

1. Objective function: Minimize $z = s_1^- + s_2^- + s_3^- + s_4^+ + s_5^+$
   Solution: $x_p = .0201, x_f = .0457, x_s = .0582, x_g = 2$ cents, $s_5^+ = 1.45$
   Gasoline tax is \$1.45 million short of goal.
4. x_1 = lb of limestone/day, x_2 = lb of corn/day, x_3 = lb of soybean meal/day.
   Objective function: Minimize $z = s_1^- + s_2^+ + s_3^- + s_4^- + s_5^+$
   Solution: $x_1 = 166.08$ lb, $x_2 = 2778.56$ lb, $x_3 = 3055.36$ lb, $z = 0$. Problem has alternative optima. All goals are satisfied but goals 3 and 4 are overachieved.
7. x_j = No. of units of product $j, j = 1, 2$.
   Assign a relatively high weight to the quota constraints.
   Objective function: Minimize $z = 100s_1^- + 100s_2^- + s_3^+ + s_4^+$
   Solution: $x_1 = 80, x_2 = 60, s_3^+ = 100$ minutes, $s_4^+ = 120$ minutes.
   Production quota can be met with 100 minutes of overtime for machine 1 and 120 minutes of overtime for machine 2.

Set 8.2b

2. G_1 solution: $x_p = .01745, x_f = .0457, x_s = .0582, x_g = 21.33$, $s_4^+ = 19.33$, all others = 0. Goals G_1, G_2, and G_3 are satisfied. G_4 is not.

G_4 problem: Same constraints as G_1 plus $s_1^- = 0, s_2^- = 0, s_3^- = 0$.

G_4 solution: $x_p = .0201, x_f = .0457, x_s = .0582, x_g = 2, s_5^+ = 1.45$. All other variables $= 0$. Goal G_5 is not satisfied.

G_5 problem: Same as G_4 plus $s_4^+ = 0$.

$G5$ solution: Same as G_4, which means that goal 5 cannot be satisfied ($s_5^+ = 1.45$).

CHAPTER 9

Set 9.1a

3. x_{ij} = No. of bottles of type i assigned to individual j, where $i = 1$ (full), 2 (half full), 3 (empty).

   Constraints:

$$x_{11} + x_{12} + x_{13} = 7, x_{21} + x_{22} + x_{23} = 7, x_{31} + x_{32} + x_{33} = 7$$

$$x_{11} + .5x_{21} = 3.5, x_{12} + .5x_{22} = 3.5, x_{13} + .5x_{23} = 3.5$$

$$x_{11} + x_{21} + x_{31} = 7, x_{12} + x_{22} + x_{32} = 7, x_{13} + x_{23} + x_{33} = 7$$

   All x_{ij} are nonnegative integers

   Solution: Use a dummy objective function.

	No. bottles assigned to individual		
Status	*1*	*2*	*3*
Full	1	3	3
Half full	5	1	1
Empty	1	3	3

6. y = Original sum of money. x_j = Amount taken on night j, $j = 1, 2, 3$.

   x_4 = Amount given to each mariner by first officer.

   Minimize $z = y$ subject to $3x_1 - y = 2$, $x_1 + 3x_2 - y = 2$, $x_1 + x_2 + 3x_3 - y = 2$, $y - x_1 - x_2 - x_3 - 3x_4 = 1$. All variables are nonnegative integers.

   Solution: $y = 79 + 81n, n = 0, 1, 2, \ldots$

10. Side 1: 5, 6, and 8 (27 minutes). Side 2: 1, 2, 3, 4, and 7 (28 minutes). Problem has alternative optima.

12. $x_{ij} = 1$ if student i selects course j, and zero otherwise, c_{ij} = associated preference score, C_j = course j capacity. Maximize $z = \sum_{i=1}^{10}\sum_{j=1}^{6} c_{ij}x_{ij}$ subject to

$$\sum_{j=1}^{6} x_{ij} = 2, i = 1, 2, \ldots, 10, \sum_{i=1}^{10} x_{ij} \leq C_j, j = 1, 2, \ldots, 6$$

   Solution: Course 1: students (2, 4, 9), 2: (2, 8), 3:(5, 6, 7, 9), 4:(4, 5, 7, 10), 5: (1,3, 8, 10), 6: (1,3). Total score = 1775.

Set 9.1b

1. Let $x_j = 1$ if route j is selected and 0 otherwise. Total distance of route (ABC, 1, 2, 3, 4, ABC) = 10 + 32 + 4 + 15 + 9 = 80 miles.
   Minimize $z = 80x_1 + 50x_2 + 70x_3 + 52x_4 + 60x_5 + 44x_6$ subject to

   $$x_1 + x_3 + x_5 + x_6 \geq 1, x_1 + x_3 + x_4 + x_5 \geq 1, x_1 + x_2 + x_4 + x_6 \geq 1,$$
   $$x_1 + x_2 + x_5 \geq 1, x_2 + x_3 + x_4 + x_6 \geq 1, x_j = (0, 1), \text{for all } j.$$

   Solution: Select routes (1, 4, 2) and (1, 3, 5), $z = 104$. Customer 1 should be skipped in one of the two routes.
2. Solution: Committee is formed of individuals a, d, and f. Problem has alternative optima.
7. $x_t = 1$ if transmitter t is selected, 0 otherwise. $x_c = 1$ if community c is covered, 0 otherwise. c_t = cost of transmitter t. S_c = set of transmitters covering community c. P_j = population of community j.

   Maximize $z = \sum_{c=1}^{15} P_c x_c$ subject to

   $$\sum_{t \in S_c} x_t \geq x_c, c = 1, 2, \ldots, 15, \sum_{t=1}^{7} c_t x_t \leq 15$$

   Solution: Build transmitters 2,4,5,6, and 7. All but community 1 are covered.

Set 9.1c

2. Let x_j = Number of widgets produced on machine j, $j = 1, 2, 3$. $y_j = 1$ if machine j is used and 0 otherwise. Minimize $z = 2x_1 + 10x_2 + 5x_3 + 300y_1 + 100y_2 + 200y_3$ subject to $x_1 + x_2 + x_3 \geq 2000$, $x_1 - 600y_1 \leq 0$, $x_2 - 800y_2 \leq 0$, $x_3 - 1200y_3 \leq 0$, $x_1, x_2, x_3 \geq 500$ and integer, $y_1, y_2, y_3 = (0, 1)$.
   Solution: $x_1 = 600$, $x_2 = 500$, $x_3 = 900$, $z = \$11{,}300$.
3. Solution: Site 1 is assigned to targets 1 and 2, and site 2 is assigned to targets 3 and 4. $z = 18$.
10. x_e = Number of Eastern (one-way) tickets, x_u = Number of US Air tickets, x_c = Number of Continental tickets. e_1, and e_2 binary variables. u and c nonnegative integers. Maximize $z = 1000(x_e + 1.5x_u + 1.8x_c + 5e_1 + 5e_2 + 10u + 7c)$ subject to $e_1 \leq x_e/2$, $e_2 \leq x_e/6$, $u \leq x_u/6$, and $c \leq x_c/5$, $x_e + x_u + x_c = 12$.
    Solution: Buy 2 tickets on Eastern and 10 tickets on Continental. Bonus = 39000 miles.

Set 9.1d

1. Let x_{ij} = Integer amount assigned to square (i, j). Use a dummy objective function with all zero coefficients.

Constraints:

$$\sum_{j=1}^{3} x_{ij} = 15, i = 1, 2, 3, \sum_{i=1}^{3} x_{ij} = 15, j = 1, 2, 3,$$

$$x_{11} + x_{22} + x_{33} = 15, x_{31} + x_{22} + x_{13} = 15,$$

$$(x_{11} \geq x_{12} + 1 \text{ or } x_{11} \leq x_{12} - 1), (x_{11} \geq x_{13} + 1 \text{ or } x_{11} \leq x_{13} - 1),$$

$$(x_{12} \geq x_{13} + 1 \text{ or } x_{12} \leq x_{13} - 1), (x_{11} \geq x_{21} + 1 \text{ or } x_{11} \leq x_{21} - 1),$$

$$(x_{11} \geq x_{31} + 1 \text{ or } x_{11} \leq x_{31} - 1), (x_{21} \geq x_{31} + 1 \text{ or } x_{21} \leq x_{31} - 1),$$

$$x_{ij} = 1, 2, \ldots, 9, \text{ for all i and } j$$

Solution:

2	9	4
7	5	3
6	1	8

Alternative solutions are direct permutations of rows and/or columns.

3. x_j = Daily number of units of product j.
Maximize $z = 25x_1 + 30x_2 + 22x_3$ subject to

$$\begin{pmatrix} 3x_1 + 4x_2 + 5x_3 \leq 100 \\ 4x_1 + 3x_2 + 6x_3 \leq 100 \end{pmatrix} \text{ or } \begin{pmatrix} 3x_1 + 4x_2 + 5x_3 \leq 90 \\ 4x_1 + 3x_2 + 6x_3 \leq 120 \end{pmatrix}$$

$$x_1, x_2, x_3 \geq 0 \text{ and integer}$$

Solution: Produce 26 units of product 1, 3 of product 2, and none of product 3, and use location 2.

Set 9.2a[2]

2. (a) $z = 6, x_1 = 2, x_2 = 0$.
(d) $z = 12, x_1 = 0, x_2 = 3$.

3. (a) $z = 7.25, x_1 = 1.75, x_2 = 1$.
(d) $z = 10.5, x_1 = .5, x_2 = 2$.

9. Equivalent 0-1 ILP:
Maximize $z = 18y_{11} + 36y_{12} + 14y_{21} + 28y_{22} + 8y_{31} + 16y_{32} + 32y_{33}$
subject to $15y_{11} + 30y_{12} + 12y_{21} + 24y_{22} + 7y_{31} + 14y_{32} + 28y_{33} \leq 43$
All variables are binary.
Solution: $z = 50, y_{12} = 1, y_{21} = 1$, all others $= 0$. Equivalently, $x_1 = 2, x_2 = 1$.
The 0-1 version required 41 nodes. The original requires 29.

[2]Use TORA integer programming module to generate the B&B tree.

Set 9.2b

1. (a) Legitimate cut because it passes through an integer point and does not eliminate any feasible integer point. You can verify this result by plotting the cut on the LP solution space.
6. (a) Optimum integer solution: $(x_1, x_2, x_3) = (2, 1, 6)$, $z = 26$.
Rounded solution: $(x_1, x_2, x_3) = (3, 1, 6)$, which is infeasible.

Set 9.3a

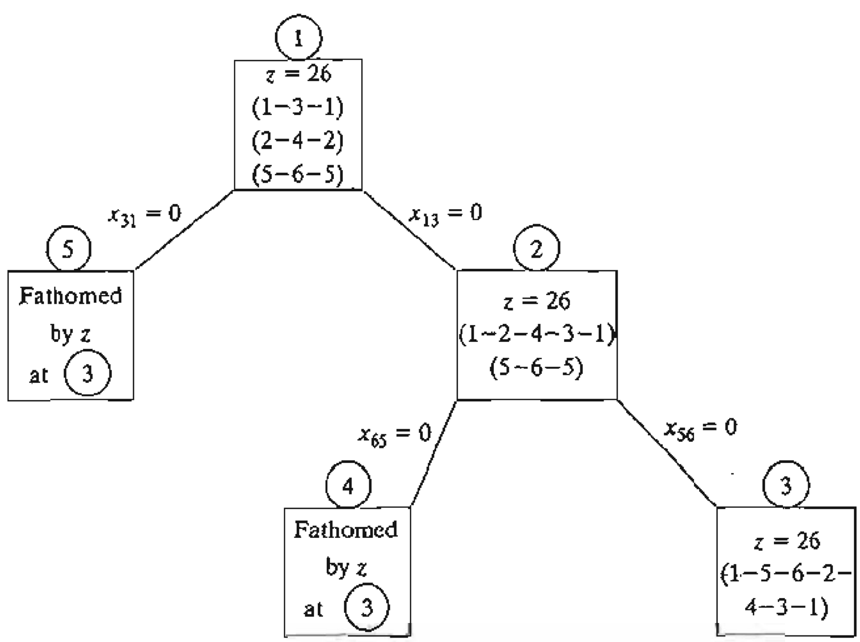
1. The table below gives the number of distinct employees who enter/leave the manager's office when we switch from project i to project j. The objective is to find a "tour" through all projects that will minimize the total traffic.

	1	2	3	4	5	6
1	—	4	4	6	6	5
2	4	—	6	4	6	3
3	4	6	—	4	8	7
4	6	4	4	—	6	5
5	6	6	8	6	—	5
6	5	3	7	5	5	—

Set 9.3c

2. See Figure C.12.

FIGURE C.12



CHAPTER 10

Set 10.1a

1. Solution: Shortest distance = 21 miles. Route: 1-3-5-7.

Set 10.2a

3. Solution: Shortest distance = 17. Route: 1-2-3-5-7.

Set 10.3a

2. (a) Solution: Value = 120. $(m_1, m_2, m_3) = (0, 0, 3), (0, 4, 1), (0, 2, 2)$, or $(0, 6, 0)$.
5. Solution: Total points = 250. Select 2 courses from I, 3 from II, 4 from III, and 1 from IV,
7. Let $x_j = 1$ if application j is accepted, and 0 otherwise. Equivalent knapsack model is
   Maximize $z = 78x_1 + 64x_2 + 68x_3 + 62x_4 + 85x_5$ subject to
   $$7x_1 + 4x_2 + 6x_3 + 5x_4 + 8x_5 \le 23, x_j = (0, 1), j = 1, 2, \ldots, 5$$
   Solution: Accept all but the first application. Value = 279.

Set 10.3b

1. (a) Solution: Hire 6 for week 1, fire 1 for week 2, fire 2 for week 3, hire 3 for week 4, and hire 2 for week 5.
3. Solution: Rent 7 cars for week 1, return 3 for week 2, rent 4 for week 3, and no action for week 4.

Set 10.3c

2. Decisions for next 4 years: Keep, Keep, Replace, Keep. Total cost = \$458.

Set 10.3d

3. (a) Let x_i and y_i be the number of sheep kept and sold at the end of period i and define $z_i = x_i + y_i$.
   $$f_n(z_n) = \max_{y_n = z_n}\{p_n y_n\}$$
   $$f_i(z_i) = \max_{y_i \le z_i}\{p_i y_i + f_{i+1}(2z_i - 2y_i)\}, i = 1, 2, \ldots, n - 1$$

CHAPTER 11

Set 11.3a

2. (a) Total cost per week = \$51.50.
   (b) Total cost per week = \$50.20, $y^* = 239.05$ lb.

4. (a) Choose policy 1 because its cost per day is \$2.17 as opposed to \$2.50 for policy 2.
   (b) Optimal policy: Order 100 units whenever the inventory level drops to 10 units.

Set 11.3b

2. Optimal policy: Order 500 units whenever level drops to 130 units. Cost per day = \$258.50.
4. No advantage if $\text{TCU}_1(y_m) \le \text{TCU}_2(q)$, which translates to no advantage if the discount factor does not exceed .9344%.

Set 11.3c

1. AMPL/Solver solution: $(y_1, y_2, y_3, y_4, y_5) = (4.42, 6.87, 4.12, 7.2, 5.8)$, cost = \$568.12,
4. Constraint: $\sum_{i=1}^{4} \frac{365 D_i}{y_i} \le 150.$

   Solver/AMPL solution: $(y_1, y_2, y_3, y_4) = (155.3, 118.82, 74.36, 90.09)$, cost = \$54.71.

Set 11.4a

1. (a) 500 units required at the start of periods 1, 4, 7, and 10.

Set 11.4b

3. Produce 173 units in period 1, 180 in period 2, 240 in period 3, 110 in period 4, and 203 in period 5.

Set 11.4c

1. (a) No, because inventory should not be held needlessly at end of horizon.
   (b) (i) $0 \le z_1 \le 5, 1 \le z_2 \le 5, 0 \le z_3 \le 4; x_1 = 4, 1 \le x_2 \le 6, 0 \le x_3 \le 4.$
   (ii) $5 \le z_1 \le 14, 0 \le z_2 \le 9, 0 \le z_3 \le 5; x_1 = 0, 0 \le x_2 \le 9, 0 \le x_3 \le 5.$
2. (a) $z_1 = 7, z_2 = 0, z_3 = 6, z_4 = 0$. Total cost = \$33.

Set 11.4d

1. Use initial inventory to satisfy the entire demand of period 1 and 4 units of period 2, thus reducing demand for the four periods to 0, 22, 90, and 67, respectively. Optimal solution: Order 112 units in period 2 and 67 units in period 4. Total cost = \$632.

Set 11.4e

1. Solution: Produce 210 units in January, 255 in April, 210 in July, and 165 in October.

CHAPTER 12

Set 12.1a

1. (a) .15 and .25, respectively. (b) .571. (c) .821.
2. $n \gtrsim 23$.
3. $n > 253$.

Set 12.1b

3. $\frac{5}{32}$.
4. Let p = probability Liz wins. Probability John wins is $3p$, which equals the probability Jim will win. Probability Ann wins is $6p$. Because one of the four wins, $p + 3p + 3p + 3p + 6p = 1$.

   (a) $\frac{3}{13}$.

   (b) $\frac{7}{13}$.

   (c) $\frac{6}{13}$.

Set 12.1c

3. (a) .375. (b) .6.
7. .9545.

Set 12.2a

2. (a) $K = 20$.
3. $P\{\text{Demand} \geq 1100\} = .3$.

Set 12.3a

3. (a) $P\{50 \leq \text{copies sold} \leq 70\} = .6667$.

   (b) Expected number of unsold copies = 2.67

   (c) Expected net profit = \$22.33

Set 12.3b

1. Mean = 3.667, variance = 1.556.

Set 12.3c

1. (a) $P(x_1 = 1) = P(x_2 = 1) = .4, P(x_1 = 2) = P(x_2 = 2) = .2, P(x_1 = 3) = P(x_2 = 3) = .4$.

   (b) No, because $P(x_1, x_2) \neq P(x_1)P(x_2)$.

Set 12.4a

1. $\left(\frac{1}{2}\right)^{10}$.
3. .0547.

Set 12.4b

1. .8646.
3. (a) $P\{n = 0\} = 0$.
   (b) $P\{n \geq 3\}$; 1.

Set 12.4c

1. $\lambda = 12$ arrivals/min. $P\{t \leq 5 \text{ sec}\} = .63$.

Set 12.4d

2. .001435.

CHAPTER 13

Set 13.1a

1. Weights for A, B, and C = (.44214, .25184, .30602).

Set 13.1b

2. CR > .1 for all matrices except **A**. $(w_S, w_J, w_M) = (.331, .292, .377)$. Select Maisa.
4. All matrices are consistent. $(w_H, w_P) = (.502, .498)$. Select H.

Set 13.2a

2. (a) See Figure C.13.
   (b) EV(corn) = −\$8250, EV(soybeans) = \$250. Select soybeans.
6. (a) See Figure C.14.
   (b) EV(game) = −\$.025. Do not play the game.

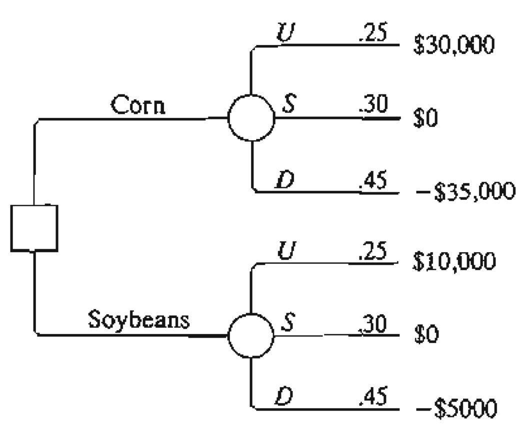


FIGURE C.13

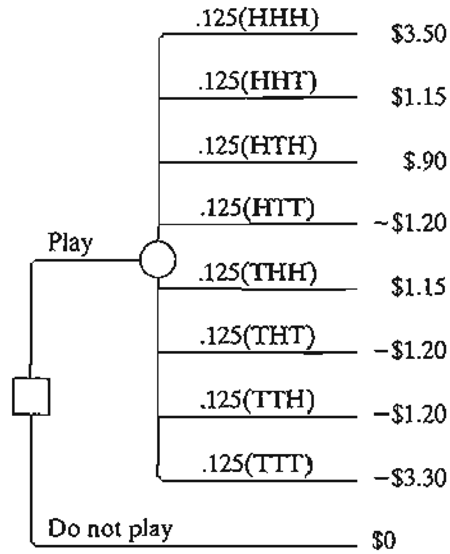


FIGURE C.14

12. Optimum maintenance cycle = 8 years. Cost per year = \$397.50.
15. Optimum production rate = 49 pieces per day.
19. Level must be between 99 and 151 gallons.

Set 13.2b

2. Let z be the event of having one defective item in a sample of size 5.
   Answer: $P\{A|z\} = .6097$, $P\{B|z\} = .3903$.
4. (a) Expected revenue if you self-publish = \$196,000.
   Expected revenue if you use a publisher = \$163,000.
   (b) If survey predicts success, self-publish, else use a publisher.
7. (b) Ship lot to B if both items are bad, else ship lot to A.

Set 13.2c

1. (a) Expected value = \$5, hence there is no advantage.
   (b) For $0 \leq x < 10$, $U(x) = 0$, and for $x = 10$, $U(x) = 100$.
   (c) Play the game.
2. Lottery: $U(x) = 100 - 100p$, with $U(-\$1{,}250{,}000) = 0$ and $U(\$900{,}000) = 100$.

Set 13.3a

1. (a) All methods: Study all night (action a_1).
   (b) All methods: Select actions a_2 or a_3.

Set 13.4a

2. (a) Saddle-point solution at (2, 3). Value of game = 4.
3. (a) $2 < v < 4$.

Set 13.4b

1. Each player should mix strategies 50-50. Value of game = 0.
2. Police payoff matrix:

	100%A	50%A-50%B	100%B
A	100	50	0
B	0	30	100

Strategy for Police: Mix 50-50 strategies 100%A and 100%B.
Strategy for Robin: Mix 50-50 strategies A and B. Value of game = \$50 (= expected fine paid by Robin).

Set 13.4c

1. (a) Payoff matrix for team 1:

	AB	AC	AD	BC	BD	CD
AB	1	0	0	0	0	−1
AC	0	1	0	0	−1	0
AD	0	0	1	−1	0	0
BC	0	0	−1	1	0	0
BD	0	−1	0	0	1	0
CD	−1	0	0	0	0	1

Optimal strategy for both teams: Mix AB and CD 50-50. Value of the game = 0.

3. (a) (m, n) = (Number of regiments at location 1, No. of regiments at locations 2). Each location has a payoff of 1 if won and −1 if lost. For example, Botto's strategy (1, 1) against the enemy's (0, 3) will win location 1 and lose location 2, with anet payoff of $1 + (-1) = 0$. Payoff matrix for Colonel Blotto:

	3,0	2,1	1,2	0,3
2,0	−1	−1	0	0
1,1	0	−1	−1	0
0,2	0	0	−1	−1

Optimal strategy for Blotto: Blotto mixes 50-50 strategies (2-0) and (0-2), and the enemy mixes 50-50 strategies (3-0) and (1-2). Value of the game = $-.5$, and Blotto loses. Problem has alternative optima.

CHAPTER 14

Set 14.1a

1. (a) Order 1000 units whenever inventory level drops to 537 units.

Set 14.1b

2. Solution: $y^* = 317.82$ gallons, $R^* = 46.82$ gallons.
3. Solution: $y^* = 316.85$ gallons, $R^* = 58.73$ gallons. In Example 14.1-2, $y^* = 319.44$ gallons, $R^* = 93.61$ gallons. Order quantity remains about the same as in Example 14.1-2, but R^* is smaller because the demand pdf has a smaller variance.

Set 14.2a

3. $.43 \leq p \leq .82$
6. 32 coats.

Set 14.2b

1. Order $9-x$ if $x < 4.53$, else do not order.

Set 14.3a

2. Order $4.61-x$ if $x < 4.61$, else do not order.

CHAPTER 15

Set 15.1a

1. (a) Productivity = 71%.
   (b) The two requirements cannot be met simultaneously.

Set 15.2a

1.

Situation	Customer	Server
(a)	Plane	Runway
(b)	Passenger	Taxi
(h)	Car	Parking space

Set 15.3a

1. (b) (i) $\lambda = 6$ arrivals per hour, average interarrival time $= \frac{1}{6}$ hour.
(c) (i) $\mu = 5$ services per hour, average service time $= .2$ hour.

3. (a) $f(t) = 20e^{-20t}, t > 0$.
(b) $P\left\{t > \frac{15}{60}\right\} = .00674$.

7. Jim's payoff is 2 cents with probability $P\{t \leq 1\} = .4866$ and -2 cents with probability $P\{t \geq 1\} = .5134$. In 8 hours, Jim pays Ann $= 17.15$ cents.

10. (a) $P\{t \leq 4 \text{ minutes}\} = .4866$.
(b) Average discount percentage $= 6.208$.

Set 15.4a

1. $p_{n\geq5}(1 \text{ hour}) = .55951$.

4. (a) $p_2(t = 7) = .24167$.

6. (a) Combined $\lambda = \frac{1}{10} + \frac{1}{7}$, $p_2(t = 5) = .219$.

Set 15.4b

2. (a) $p_0(t = 3) = .00532$.
(c) $p_{n\leq17}(t = 1) = .9502$.

5. $p_0(4) = .37116$.

8. (a) Average order size $= 25 - 7.11 = 17.89$ items.
(b) $p_0(t = 4) = .00069$.

Set 15.5a

3. (a) $p_{n\geq3} = .4445$.
(b) $p_{n\leq2} = .5555$.

6. (a) $p_j = .2, j = 0, 1, 2, 3, 4$.
(b) Expected number in shop $= 2$ customers.
(c) $p_4 = .2$.

Set 15.6a

1. (a) $L_q = 1p_6 + 2p_7 + 3p_8 = .1917$ car.
(c) $\lambda_{lost} = .1263$ car per hour. Average number lost in 8 hr $= 1.01$ cars.
(d) No. of empty spaces $= c - (L_s - L_q) = c - \sum_{n=0}^{8} np_n + \sum_{n=c+1}^{8} (n - c)p_n$.

Set 15.6b

2. (a) $p_0 = .2$.
(b) Average monthly income $= \$50 \times \mu t = \$375$.
(c) Expected payment $= \$40 \times L_q = \$128$.

5. (a) $p_0 = .4$.
   (b) $L_q = .9$ car.
   (c) $W_q = 2.25$ min.
   (d) $p_{n\geq 11} = .0036$.
6. (d) No. of spaces is at least 13.

Set 15.6c

1. $P\{\tau > 1\} = .659$.
5. \$37.95 per 12-hour day.

Set 15.6d

1. (a) $p_0 = .3654$.
   (b) $W_q = .207$ hour.
   (c) Expected number of empty spaces $= 4 - L_q = 3.212$.
   (d) $p_5 = .04812$.
   (e) 40% reduction lowers W_s to about 9.6 min ($\mu = 10$ cars/hr).
4. (a) $p_8 = .6$.
   (b) $L_q = 6.34$ generators.
   (c) Probability of finding an empty space cannot exceed .4 regardless of belt capacity. This means that the best utilization of the assembly department is 60%.
7. (a) $1 - p_5 = .962$.
   (b) $\lambda_{lost} = \lambda p_5 = .19$ customer per hour.

Set 15.6e

2. For $c = 2$, $W_q = 3.446$ hour and for $c = 4$, $W_q = 1.681$ hour, an improvement of over 51%.
5. Let K be the number of waiting-room spaces. Using TORA, $p_0 + p_1 + \cdots + p_{K+2} \geq .999$ yields $K \geq 10$.
7. (a) $p_{n\geq 4} = .65772$.
   (e) Average number of idle computers $= .667$ computer.

Set 15.6f

2. (c) Utilization $= 81.8\%$.
   (d) $p_2 + p_3 + p_4 = .545$.
4. (a) $p_{40} = .00014$.
   (b) $p_{30} + p_{31} + \text{L} + p_{39} = .02453$.
   (d) Expected number of occupied spaces $= L_s - L_q = 20.043 - .046 \approx 20$.
   (f) Probability of not finding a parking space $= 1 - p_{n\leq 29} = .02467$. Number of students who cannot park in an 8-hour period is approximately 4.

Set 15.6g

2. (a) Approximately 7 seats.
   (b) $p_{n\geq 8} = .2911$.

Set 15.6h

1. (b) Average number of idle repairpersons = 2.01.
   (d) $P\{2 \text{ or } 3 \text{ idle servers}\} = p_0 + p_1 = .34492$.
4. (a) $L_s = 1.25$ machines.
   (b) $p_0 = .33342$.
   (c) $W_s = .25$ hour.
6. $\lambda = 2$ calls per hour per baby, $\mu = .5$ baby per hour, $R = 5$, $K = 5$.
   (a) Number of awake babies $= 5 - L_s = 1$ baby.
   (b) $p_5 = .32768$.
   (c) $p_{n\leq 2} = .05792$.

Set 15.7a

2. (a) $E\{t\} = 14$ minutes and $\text{var}\{t\} = 12$ minutes^2. $L_s = 7.8672$ cars.
4. $\lambda = .0625$ prescriptions per minute, $E\{t\} = 15$ minutes, $\text{var}\{t\} = 9.33$ minutes^2.
   (a) $p_0 = .0625$.
   (b) $L_q = 7.3$ prescriptions
   (c) $W_s = 132.17$ minutes.

Set 15.9a

2. Use $(M/M/1)$:$(GD/10/10)$. Cost per hour is \$431.50 for repairperson 1 and \$386.50 for repairperson 2.
4. (b) $\mu = \lambda + \sqrt{\frac{c_2\lambda}{c_1}}$
   (c) Optimum production rate = 2725 pieces per hour.

Set 15.9b

2. (a) Hourly cost per hour is \$86.4 for two repairpersons and \$94.80 for three.
   (b) Schedule loss per breakdown $= \$30 \times W_s = \$121.11$ for two repairpersons and \$94.62 for three.
4. Rate of breakdowns per machine, $\lambda = .36125$ per hour, $\mu = 10$ per hour. Model $(M/M/3)$:$(GD/20/20)$ yields $L_s = .70529$ machine. Lost revenue = \$36.60 and cost of three repairpersons = \$60.

Set 15.9c

1. (a) Number of repairpersons ≥ 5.
   (b) Number of repairpersons ≥ 4.

CHAPTER 16

Set 16.1a

4. (a) $P\{H\} = P\{T\} = .5$. If $0 \leq R \leq .5$, Jim gets \$10.00. If $.5 < R \leq 1$, Jan gets \$10.00.
7. Lead time sampling: If $0 \leq R \leq .5, L = 1$ day. If $.5 < R \leq 1, L = 2$ days. Demand per day sampling: If $0 \leq R \leq .2$, demand $= 0$ unit. If $.2 < R \leq .9$, demand $= 1$ unit. If $.9 < R \leq 1$, demand $= 2$ units. Use one R to sample L. If $L = 1$, use another R to sample demand for one day, else if $L = 2$, use one R to generate demand for day 1 and then another R to generate demand for day 2.

Set 16.2a

1. (a) Discrete.

Set 16.3a

4. See Figure C.15.

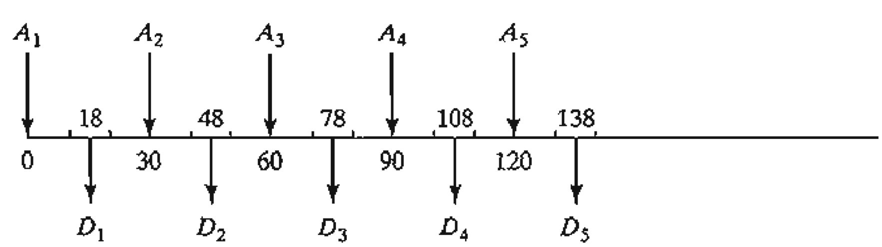
Set 16.3b

1. $t = -\frac{1}{\lambda}\ln(1 - R), \lambda = 4$ customers per hour.

Customer	R	t (hr)	Arrival time
1	—	—	0
2	0.0589	0.015176	0.015176
3	0.6733	0.279678	0.294855
4	0.4799	0.163434	0.458288

2. $t = a + (b - a)R$.
4. (a) $0 \leq R < .2: d = 0, .2 \leq R < .5: d = 1, .5 \leq R < .9: d = 2, .9 \leq R \leq 1: d = 3$.
9. If $0 \leq R \leq p$, then $x = 0$, else $x = \left(\text{largest integer} \leq \frac{\ln(1 - R)}{\ln q}\right)$.

FIGURE C.15



Set 16.3c

1. $y = -\frac{1}{5}\ln(.0589 \times .6733 \times .4799 \times .9486) = .803$ hour.
6. $t = x_1 + x_2 + x_3 + x_4$, where $x_i = 10 + 10R_i, i = 1, 2, 3, 4$.

Set 16.4a

1. In Example 16.4-1, cycle length = 4. With the new parameters, cycling was not evident after 50 random numbers were generated. The conclusion is that judicious selection of the parameters is important.

Set 16.5a

2. (a) Observation-based.
   (b) Time-based.
3. (a) 1.48 customers.
   (b) 7.4 hours.

Set 16.6a

2. Confidence interval: $15.07 \leq \mu \leq 23.27$.

CHAPTER 17

Set 17.1a

2. S1: Car on patrol
   S2: Car responding to a call
   S3: Car at call scene
   S4: Apprehension made.
   S5: Transport to police station

	S1	S2	S3	S4	S5
S1	0.4	0.6	0	0	0
S2	0.1	0.3	0.6	0	0
S3	0.1	0	0.5	0.4	0
S4	0.4	0	0	0	0.6
S5	1	0	0	0	0

Set 17.2a

2. Initial probabilities:

S1	S2	S3	S4	S5
0	0	1	0	0

Input Markov chain:

	S1	S2	S3	S4	S5
S1	0.4	0.6	0	0	0
S2	0.1	0.3	0.6	0	0
S3	0.1	0	0.5	0.4	0
S4	0.4	0	0	0	0.6
S5	1	0	0	0	0

Output (2-step or 2 patrols) transition matrix ($\mathbf{P}^2$)

	S1	S2	S3	S4	S5
S1	0.22	0.42	0.36	0	0
S2	0.13	0.15	0.48	0.24	0
S3	0.25	0.06	0.25	0.2	0.24
S4	0.76	0.24	0	0	0
S5	0.4	0.6	0	0	0

Absolute 2-step probabilities = $(0\ 0\ 1\ 0\ 0)\mathbf{P}^2$

State	Absolute (2-step)
S1	0.25
S2	0.06
S3	0.25
S4	0.2
S5	0.24

$P\{\text{apprehension, S4, in 2 patrols}\} = .2$

Set 17.3a

1. (a) Using excelMarkovChains.xls, the chain is periodic with period 3.

(b) States 1, 2, and 3 are transient, State 4 is absorbing.

Set 17.4a

1. (a)

Input Markov chain:

	S	C	R
S	0.8	0.2	0
C	0.3	0.5	0.2
R	0.1	0.1	0.8

Steady state probabilities:
$(\pi_1, \pi_2, \pi_3) = (\pi_1, \pi_2, \pi_3)\mathbf{P}$
$\pi_1 + \pi_2 + \pi_3 = 1$

Output Results:

State	*Steady state*	*Mean return time*
S	0.50	2.0
C	0.25	4.0
R	0.25	4.0

Expected revenues $= 2 \times .5 + 1.6 \times .25 + .4 \times .25 = \$1{,}500$

(b) Sunny days will return every $\mu_{SS} = 2$ days—meaning two days on no sunshine.

5. (a)

Input Markov chain:

	never	some	always
never	0.95	0.04	0.01
some	0.06	0.9	0.04
always	0	0.1	0.9

(b)

Output Results

State	*Steady state*	*Mean return time*
never	0.441175	2.2666728
some	0.367646	2.7200089
always	0.191176	5.2307892

44.12% never, 36.76% sometimes, 19.11% always

(c) Expected uncollected taxes/year $= .12(\$5000 \times .3676 + 12{,}000 \times .1911)$
$\times\ 70{,}000{,}000 = \$34{,}711{,}641{,}097.07$

14. (a) State = (i, j, k) = (No. in year −2, No. in year −1, No. in current year), i, j, k = (0 or 1)

Example: (1-0-0) this year links to (0-0-1) if a contract is secured next yr.

	0-0-0	1-0-0	0-1-0	0-0-1	1-1-0	1-0-1	0-1-1	1-1-1
0-0-0	0.1	0	0	0.9	0	0	0	0
1-0-0	0.2	0	0	0.8	0	0	0	0
0-1-0	0	0.2	0	0	0	0.8	0	0
0-0-1	0	0	0.2	0	0	0	0.8	0
1-1-0	0	0.3	0	0	0	0.7	0	0
1-0-1	0	0	0.3	0	0	0	0.7	0
0-1-1	0	0	0	0	0.3	0	0	0.7
1-1-1	0	0	0	0	0.5	0	0	0.5

(b)

State	Steady state
0-0-0	0.014859
1-0-0	0.066865
0-1-0	0.066865
0-0-1	0.066865
1-1-0	0.178306
1-0-1	0.178306
0-1-1	0.178306
1-1-1	0.249629

$$\text{Expected nbr. of contracts in 3 yrs} = 1(0.066865 + 0.066865 + 0.066865) + 2(0.178306 + 0.178306 + 0.178306) + 3(0.249629) = 2.01932$$

$$\text{Expected nbr. of contracts/yr} = 2.01932/3 = 0.67311$$

Set 17.5a

1. (a) Initial probabilities:

1	2	3	4	5
1	0	0	0	0

Input Markov chain:

1	2	3	4	5
0	.3333	.3333	.3333	0
.3333	0	.3333	0	.3333
.3333	.3333	0	0	.3333
.5	0	0	0	.5
0	.3333	.3333	.3333	0

State	Absolute (3-step)	Steady state
1	.07407	.214286
2	.2963	.214286
3	.2963	.214286
4	.25926	.142857
5	**.07407**	**.214286**

(b) $a_5 = .07407$
(c) $\pi_5 = .214286$
(d) $\mu_{15} = 4.6666.$

	$(\mathbf{I} - \mathbf{N})^{-1}$ 1	2	3	5	Mu
1	2	1	1	.6667	4.6666
2	1	1.625	.875	.3333	3.8333
3	1	.875	1.625	.3333	3.8333
4	1	.5	.5	1.3333	3.3333

5. (a) Input Markov chain:

	A	B	C
A	.75	.1	.15
B	.2	.75	.05
C	.125	.125	.75

(b)

State	Steady state
A	.394737
B	.307018
C	.298246

A: 39.5%, B: 30.7%, C: 29.8%

(c)

$(\mathbf{I}-\mathbf{N})^{-1}$

	A	C
A	5.71429	3.42857
C	2.85714	5.71429

Mu

	B
A	**9.14286**
C	8.57143

	1	2
A	5.88235	2.35294
B	4.70588	5.88235

	C
A	**8.23529**
B	1.5882

A → B: 9.14 years
A → C: 8.23 years

Set 17.6a

2. (a) States: 1wk, 2wk, 3wk, Library

Matrix **P**:

	1	2	3	lib
1	0	0.3	0	0.7
2	0	0	0.1	0.9
3	0	0	0	1
lib	0	0	0	1

(b)

$(\mathbf{I}-\mathbf{N})^{-1}$

	1	2	3
1	1	0.3	.03
2	0	1	.01
3	0	0	1

Mu

	lib
1	1.33
2	1.1
3	1

I keep the book 1.33 wks on the average.

8. (a)

Matrix **P**:

	1	2	3	4	F
1	0.2	0.8	0	0	0
2	0	0.22	0.78	0	0
3	0	0	0.25	0.75	0
4	0	0	0	0.3	0.7
F	0	0	0	0	1

(b)

	$(I-N)^{-1}$ 1	2	3	4		Mu F
1	1.25	1.282	1.333	1.429	1	5.29
2	0	1.282	1.333	1.429	2	4.04
3	0	0	1.333	1.429	3	2.76
4	0	0	0	1.429	4	1.43

(c) To be able to take Cal II, the student must finish in 16 weeks (4 transitions) or less. Average number of transitions needed = 5.29. Hence, an average student will not be able to finish Cal I on time.

(d) No, per answer in (c).

10. (a) states:0, 1, 2, 3, D (delete)

Matrix **P**:

	0	1	2	3	D
0	0.5	0.5	0	0	0
1	0.4	0	0.6	0	0
2	0.3	0	0	0.7	0
3	0.2	0	0	0	0.8
D	0	0	0	0	1

(b) A new customer stays 12 years on the list.

	$(I-N)^{-1}$ 0	1	2	3		Mu D
0	5.952	2.976	1.786	1.25	0	12
1	3.952	2.976	1.786	1.25	1	9.96
2	2.619	1.31	1.786	1.25	2	6.96
3	1.19	0.595	0.357	1.25	3	3.39

(c) 6.96 years.

CHAPTER 18

Set 18.1a

1. (a) No stationary points.

(b) Minimum at $x = 0$.

(e) Inflection point at $x = 0$, minimum at $x = .63$, and maximum at $x = -.63$.

4. $(x_1, x_2) = (-1, 1)$ or (2,4).

Set 18.2a

1. (b) $(\partial x_1, \partial x_2) = (2.83, -2.5)\, \partial x_2$

Set 18.2b

3. Necessary conditions: $2\left(x_i - \frac{x_n^2}{x_i}\right) = 0, i = 1, 2, \ldots, n - 1$. Solution is $x_i = \sqrt[n]{C}$, $i = 1, 2, \ldots, n$. $\partial f = 2\delta \sqrt[n]{C^{2-n}}$.
6. (b) Solution $(x_1, x_2, x_3, x_4) = \left(-\frac{5}{74}, -\frac{10}{74}, \frac{155}{74}, \frac{60}{74}\right)$, which is a minimum point.

Set 18.2c

2. Minima points: $(x_1, x_2, x_3) = (-14.4, 4.56, -1.44)$ and (4.4, .44,.44).

CHAPTER 19

Set 19.1a

2. (c) $x = 2.5$, achieved with $\Delta = .000001$.
   (e) $x = 2$, achieved with $\Delta = .000001$.

Set 19.1b

1. By Taylor's expansion, $\nabla f(\mathbf{X}) = \nabla f(\mathbf{X}^0) + \mathbf{H}(\mathbf{X} - \mathbf{X}^0)$. The Hessian $\mathbf{H}$ is independent of $\mathbf{X}$ because $f(\mathbf{X})$ is quadratic. Also, the given expansion is exact because higher-order derivatives are zero. Thus, $\nabla f(\mathbf{X}) = \mathbf{0}$ yields $\mathbf{X} = \mathbf{X}^0 - \mathbf{H}^{-1}\nabla f(\mathbf{X}^0)$. Because $\mathbf{X}$ satisfies $\nabla f(\mathbf{X}) = \mathbf{0}$, $\mathbf{X}$ must be optimum regardless of the choice of initial $\mathbf{X}^0$.

Set 19.2a

2. Optimal solution: $x_1 = 0, x_2 = 3, z = 17$.
4. Let $w_j = x_j + 1, j = 1, 2, 3, v_1 = w_1 w_2, v_2 = w_1 w_3$. Then,
   Maximize $z = v_1 + v_2 - 2w_1 - w_2 + 1$
   subject to $v_1 + v_2 - 2w_1 - w_2 \leq 9, \ln v_1 - \ln w_1 - \ln w_2 = 0,$

$$\ln v_2 - \ln w_1 - \ln w_3 = 0, \text{ all variables are nonnegative.}$$

Set 19.2b

1. Solution: $x_1 = 1, x_2 = 0, z = 4$.
2. Solution: $x_1 = 0, x_2 = .4, x_3 = .7, z = -2.35$.

Set 19.2c

1. Maximize $z = x_1 + 2x_2 + 5x_3$
   subject to $2x_1 + 3x_2 + 5x_3 + 1.28y \leq 10$
   $9x_1^2 + 16x_3^2 - y^2 = 0$
   $7x_1 + 5x_2 + x_3 \leq 12.4,\ x_1, x_2, x_3, y \geq 0$

CHAPTER 20

Set 20.1a

1. See Figure C.16.

Set 20.1b

1. Case 1: Lower bound is not substituted out.

	x_{12}	x_{13}	x_{24}	x_{32}	x_{34}	
Minimize z	1	5	3	4	6	
Node 1	1	1				= 50
Node 2	−1		1	−1		= −40
Node 3		−1		1	1	= 20
Node 4			−1		−1	= −30
Lower bound	0	30	10	10	0	
Upper bound	∞	40	∞	∞	∞	

Case 2: Lower bound is substituted out.

	x'_{12}	x'_{13}	x'_{24}	x'_{32}	x'_{34}	
Minimize z	1	5	3	4	6	
Node 1	1	1				= 20
Node 2	−1		1	−1		= −40
Node 3		−1		1	1	= 40
Node 4			−1		−1	= −20
Upper bound	∞	10	∞	∞	∞	

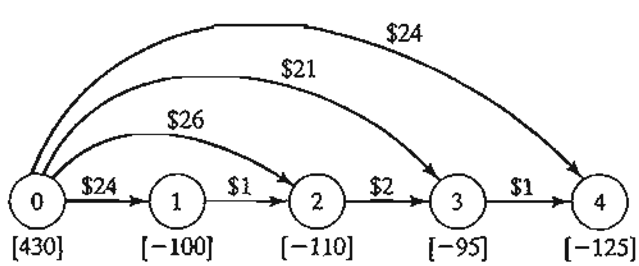


FIGURE C.16

Set 20.1c

1. Optimum cost = \$9895. Produce 210 units in period 1 and 220 units in period 3.
5. Optimal solution: Total student miles = 24,300. Problem has alternative optima.

	Number of students	
	School 1	*School 2*
Minority area 1	0	500
Minority area 2	450	0
Minority area 3	0	300
Nonminority area 2	1000	0
Nonminority area 2	0	1000

Set 20.2a

1. (c) Add the artificial constraint $x_2 \le M$. Then

$$(x_1, x_2) = \alpha_1(0,0) + \alpha_2(10,0) + \alpha_3(20,10) + \alpha_4(20,M) + \alpha_5(0,M)$$

$$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 = 1, \alpha_j \ge 0, j = 1, 2, \ldots, 5$$

2. Subproblem 1: $(x_1, x_2) = \alpha_1(0,0) + \alpha_2\left(\frac{12}{5}, 0\right) + \alpha_3(0,12)$
   Subproblem 2: $(x_4, x_5) = \beta_1(5,0) + \beta_2(50,0) + \beta_3(0,10) + \beta_4(0,5)$
   Optimal solution: $\alpha_1 = \alpha_2 = 0, \alpha_3 = 1 \Rightarrow x_1 = 0, x_2 = 12$

$$\beta_1 = .4889, \beta_2 = .5111, \beta_3 = \beta_4 = 0 \Rightarrow x_4 = 28, x_5 = 0.$$

6. Since the original problem is minimization, we must maximize each subproblem. Optimal solution: $(x_1, x_2, x_3, x_4) = \left(\frac{5}{3}, \frac{15}{3}, 0, 20\right), z = 195.$

CHAPTER 22

Set 22.1a

2. Solution: Day 1: Accept if offer is high. Day 2: Accept if offer is medium or high. Day 3: Accept any offer.

Set 22.2a

1. Solution: Year 1: Invest \$10,000. Year 2: Invest all. Year 3: Do not invest. Year 4: Invest all. Expected accumulation = \$35,520.
4. Allocate 2 bikes to center 1, 3 to center 2, and 3 to center 3.

Set 22.3a

3. Solution: First game: Bet \$1. Second game: Bet \$1. Third game: Bet \$1 or none. Maximum probability = .109375.

CHAPTER 23

Set 23.1a

2. Do not fertilize, fertilize when in state 1, fertilize when in state 2, fertilize when in state 3, fertilize when in state 1 or 2, fertilize when in state 1 or 3, fertilize when in state 2 or 3, or fertilize regardless of state.

Set 23.2a

1. Years 1 and 2: Don't advertise if product is successful; otherwise, advertise. Year 3: Don't advertise.
3. If stock level at the start of month is zero, order 2 refrigerators; otherwise, do not order.

Set 23.3a

1. Advertise whenever in state 1.

APPENDIX A

Set A.3a

1. `rest{i in 1..n}:(if i<=n-1 then x[i]+x[i+1] else x[1]+x[n])>=c[i];`

Set A.4a

2. See file A.4a-2.txt

Set A.5a

2. Data for `unitprofit` must be re-read four times with convoluted ordering of data elements.

```
24 5  6 4 4
 6 5  1 4 2
 1 5 -1 4 1
 2 5  0 4 1
```

Set A.5c

1. Error will result because members of sets `paint` and `resource` cannot be read from the double-subscripted table `RMaij`.