

* Block design:- Let there are v treatments which is arranged in b blocks such that,

(1) each block contains k_i plots that is block size is k_i .

(2) each treatment is replicated s_i times that is replication size s_i , such an arrangement of layout of plots is called block design.

where $i = 1, 2, \dots, v$

$j = 1, 2, \dots, b$

→ There are two types of block design

(1) Complete block design.

(2) Incomplete block design.

* Complete block design:-

If each of the blocks of a design contains each of the v treatments then such design is called complete block design.

ex:-

		v_1	v_2	v_3	
b_1		1	2	3	here $v=3, k=3, b=2$
b_2		2	3	1	

The necessary condition is $k \geq v$ and sufficient condition is $k < v$.

* Incomplete block design:-

If any one block of a design does not contain all the treatments then design becomes incomplete block design. That is $k < v$

Ex^o

1	2
2	3

1	2
2	3
3	3

block $b = 2$

treatment $v = 3$

type $k = 2$

block $b = 3$

treatment $v = 3$

$k = 2$

* Binary design and non-binary design:-

A connected design is said to be binary if the incidence matrix N is defined as,

$$N = (n_{ij})_{v \times b} = \begin{cases} 0 & \text{if } i^{\text{th}} \text{ treatment does not occur in } j^{\text{th}} \text{ block} \\ 1 & \text{if } i^{\text{th}} \text{ treatment occurs in } j^{\text{th}} \text{ block.} \end{cases}$$

ex^o block $b = 6$, treatment $v = 4$, $k = 2$,
 $s_i = s = 3$ that is treatment repeated 3 times.

1 2
1 3
1 4
2 3
2 4
3 4

	V/b	1	2	3	4	5	6
	1	1	1	1	0	0	0
$N =$	2	1	0	0	1	1	0
	3	0	1	0	1	0	1
	4	0	0	1	0	1	1

This is binary block design.

* Randomized block design:- (RBD)

A block design is said to be randomized block design if V treatments are assigned in b block such that each block contain each treatments once and each treatment is replicated in $r (r=b)$ blocks.

Ex: Randomized block design with
 $V=4, b=3, K=4$

1	2	3	4	V/b	1	2	3
2	4	1	3	1	1	1	1
3	4	1	2	$N = 2$	1	1	1
				3	1	1	1
				4	1	1	1

* Properties of block design:-

- (1) connectedness
- (2) Balancedness
- (3) orthogonality

$\Rightarrow$ connectedness:- A block design is said to be connected if all the elementary treatment contrasts are estimable.

* Matrix C of block design:-

- C matrix of a block design is defined as,

- $C = \text{diagonal}(g_1, g_2, \dots, g_v) - \frac{1}{N'} \text{diag}(K_1, K_2, \dots, K_b)$

$$C_{v \times v} = \begin{pmatrix} g_1 & 0 & 0 \\ 0 & g_2 & 0 \\ 0 & 0 & g_3 \\ 0 & 0 & g_v \end{pmatrix}_{v \times v} \quad - N' \times b$$

- where N is an incidence matrix of a block design.

- N' is the transpose of N matrix.

- $\text{diag}(g_1, \dots, g_v) \Rightarrow$ diagonal matrix of

• $\text{diag}(K_1, \dots, K_b) \rightarrow$ diagonal matrix of block sizes

$$C_{v \times v} = \begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & s_n \end{bmatrix}_{v \times v}$$

$$N_{v \times b} \begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_2 & 0 \\ 0 & 0 & K_b \end{bmatrix}_{b \times b}$$

$N_{b \times v}$

* Properties of C matrix as a matrix

- C matrix is positive semi definite matrix.
- C matrix is doubly centroid matrix
- A diagonal elements of C matrix are always positive and may be zero.
- off diagonal elements of a C matrix are always negative or zero.
- rank of the C matrix is $v-1$.
- C matrix is always singular matrix
- C matrix never null a matrix. C matrix must be greater than zero.
- In a design this C matrix is called information matrix and incidence

* Balanced Design:-

A design is said to be balanced if all the diagonal elements of C -matrix are same and off diagonal elements are also another constant.

OR

A design is said to be balanced design if C -matrix is written as,

$$C = \theta \left[I_v - \frac{1}{v} E_{vv} \right]$$

where, θ is non-zero eigen value of C -matrix with multiplicity $(v-1)$. I_v is an identity matrix of order v and E_{vv} is a matrix of order v and all the elements are unique.

* Orthogonal:-

An BD is said to be orthogonal if $\text{cov}(\underline{Q}, \underline{P}) = 0$ where $\underline{P} = \underline{B} - N' \underline{R}' \underline{T}$ and $\underline{Q} = \underline{I} - N \underline{K}' \underline{B}$

OR

An BD is said to be orthogonal if the incidence matrix of BD is expressed as $N = \frac{gk}{n}$.

* Balanced Incomplete block design:

BIBD is an incomplete block design where v treatments are arranged in b blocks having k plots in each block ($1 < k < v$) such that

- (i) each treatment is replicated in s blocks and
- (ii) A pair of treatments occurs together in λ blocks.

	1	2	3
1	1	2	4
2	2	3	5
3	3	4	6
4	4	5	7
5	5	6	1
6	6	7	2
7	7	1	3

In this design $v=7$, $b=7$, $s=3$, $k=3$, $\lambda=1$

→ Parameters of BIBD:-

BIBD has five parameters v, b, s, k, λ

→ Parametric selection:-

- (i) $V_s = bK$
- (ii) $\lambda(V-1) = S(K-1)$
- (iii) $b \geq 1$ ($\because$ fishes's inequality)

* Prove that: $\lambda(V-1) = S(K-1)$

$$\begin{aligned}
 NN' E_{V1} &= N(E_N N') \\
 &= N(K E_{1b})' \quad \{ \because E_N N' = K E_{1b} \} \\
 &= K' N E_{1b}
 \end{aligned}$$

$$S = K S E_{V1} \quad (\because N E_{1b} = S E_{V1}) \quad \text{--- (i)}$$

$$NN' = \begin{bmatrix} n_{11} & n_{12} & \dots & n_{1b} \\ n_{21} & n_{22} & \dots & n_{2b} \\ \vdots & \vdots & & \vdots \\ n_{v1} & n_{v2} & \dots & n_{vb} \end{bmatrix} \begin{bmatrix} n_{11} & n_{21} & \dots & n_{v1} \\ n_{12} & n_{22} & \dots & n_{v2} \\ \vdots & \vdots & & \vdots \\ n_{1b} & n_{2b} & \dots & n_{vb} \end{bmatrix}$$

$$= \begin{bmatrix} n_{11}^2 + n_{12}^2 + \dots + n_{1b}^2 & n_{11}n_{21} + n_{12}n_{22} + \dots + n_{1b}n_{v1} \\ n_{21}n_{11} + n_{22}n_{12} + \dots + n_{2b}n_{1b} & n_{21}^2 + n_{22}^2 + \dots + n_{2b}^2 \\ \vdots & \vdots \\ n_{v1}n_{11} + n_{v2}n_{12} + \dots + n_{vb}n_{1b} & n_{v1}n_{21} + n_{v2}n_{22} + \dots + n_{vb}n_{2b} \end{bmatrix}$$

$$\begin{bmatrix} n_{11}n_{v1} + n_{12}n_{v2} + \dots + n_{1b}n_{vb} \\ n_{21}n_{v1} + \dots + n_{2b}n_{vb} \\ \vdots \\ n_{v1}n_{v1} + \dots + n_{vb}n_{vb} \end{bmatrix}$$

$$= \left[\begin{array}{ccc} \sum_{j=1}^b n_{ij}^2 & \sum_{j=1}^b n_{im} n_{mj} & \sum_{j=1}^b n_{ij} n_{xi} \\ \sum_{j=1}^b n_{mj} n_{ij} & \sum_{j=1}^b n_{mj}^2 & \sum_{j=1}^b n_{mj} n_{xi} \\ \sum_{j=1}^b n_{xj} n_{ij} & \sum_{j=1}^b n_{xj} n_{mj} & \sum_{j=1}^b n_{xj}^2 \end{array} \right]$$

In a binary design $\sum_{j=1}^b n_{ij}^2 = r$ and

$$\sum_{j=1}^b n_{ij} n_{mj} = \lambda \text{ for all } i \neq j \text{ and } m \neq$$

$$\therefore NN' = \begin{bmatrix} r & \lambda & \dots & \lambda \\ \lambda & r & \dots & \lambda \\ \vdots & \vdots & \ddots & \vdots \\ \lambda & \lambda & \dots & r \end{bmatrix}_{b \times b}$$

$$= \begin{bmatrix} r & 0 & \dots & 0 \\ 0 & r & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & r \end{bmatrix} + \begin{bmatrix} 0 & \lambda & \dots & \lambda \\ \lambda & 0 & \dots & \lambda \\ \vdots & \vdots & \ddots & \vdots \\ \lambda & \lambda & \dots & 0 \end{bmatrix}$$

$$= \begin{bmatrix} s & 0 & \dots & 0 \\ 0 & s & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & s \end{bmatrix} + \begin{bmatrix} \lambda & \lambda & \dots & \lambda \\ \lambda & \lambda & \dots & \lambda \\ \vdots & \vdots & \ddots & \vdots \\ \lambda & \lambda & \dots & \lambda \end{bmatrix}$$

$$\begin{bmatrix} \lambda & 0 & \dots & 0 \\ 0 & \lambda & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda \end{bmatrix}$$

$$NN' = sI_v + \lambda E_{vv} - \lambda I_v$$

where I_v is a matrix of order v and E_{vv} is a matrix with all elements unit.

$$\begin{aligned} NN' &= (s - \lambda)I_v + \lambda E_{vv} \\ NN' E_{v1} &= [(s - \lambda)I_v + \lambda E_{vv}] E_{v1} \\ &= (s - \lambda)E_{v1} + \lambda v E_{v1} \\ &= (s - \lambda)E_{v1} + \lambda v E_{v1} \\ &= [(s - \lambda) + \lambda v] E_{v1} \quad \dots (ii) \end{aligned}$$

comparing (i), (ii)

$$\begin{aligned} [(s - \lambda) + \lambda v] E_{v1} &= K s E_{v1} \\ \therefore s + \lambda v - \lambda &= K s \\ \therefore s + (v - 1)\lambda &= K s \\ \therefore -s + K s &= \lambda(v - 1) \\ \therefore \lambda(v - 1) &= s(K - 1) \end{aligned}$$

* Base Inequality:

Theorem:

For any BIBD $b \geq v + s - k$

Proof: let us consider a BIBD with parameters v, b, s, k and λ .

Then we know that $vs = bk$

we also know that in a BIBD $k \leq v$

i.e. $(v - k) \geq 0$ similarly $s \geq k$,

i.e. $(s - k) \geq 0$

$$\Rightarrow (v - k)(s - k) \geq 0$$

$$\Rightarrow vs - vk - ks + k^2 \geq 0 \quad (\because vs = bk)$$

$$\Rightarrow bk - ks - vk + k^2 \geq 0$$

$$\Rightarrow k(b - s - v + k) \geq 0$$

$$k \neq 0 \text{ or } (b - s - v + k) \geq 0$$

$$\therefore b \geq v + s - k$$

* Theorem:- show that a BIBD is connected
if $R(C) = v - 1$.

Proof:

consider a BIBD with parameters v, b, s, k, λ .

Now the information matrix C for a

$$C = SI_v - NK^{-1}N'$$

[$\because$ every incomplete block design
 $C = \text{diag}(s_1, \dots, s_v) - N \text{diag}(k_1^{-1}, \dots, k_v^{-1}) N'$]

In a BIBD $s_1 = s_2 = \dots = s$
 $k_1 = k_2 = \dots = k$

$$= \text{diag}(s, \dots, s) - N \text{diag}(k^{-1}, \dots, k^{-1}) N'$$

$$= \begin{pmatrix} s & 0 & \dots & 0 \\ 0 & s & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & s \end{pmatrix} - N \begin{pmatrix} 1/k & 0 & \dots & 0 \\ 0 & 1/k & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 1/k \end{pmatrix} N'$$

$$= SI_v - \frac{N I_v N'}{k}$$

$$= SI_v - NK^{-1}N'$$

N is the incidence matrix of BIBD
 N' is a transpose of matrix N
 we know,

$$C = SI_v - \frac{NN'}{k} \text{ because } s_1 = s_2 = \dots = s_v$$

$$= SI_v - \frac{[(s-\lambda)I_v + \lambda E_{vv}]}{k}$$

$$= \left[8 - \frac{(8-\lambda)}{K} \right] I_v - \frac{\lambda}{K} E_{vv}$$

$$= \left[\frac{8K - 8 + \lambda}{K} \right] I_v - \frac{\lambda}{K} E_{vv}$$

$$= \left[\frac{8(K-1) + \lambda}{K} \right] I_v - \frac{\lambda}{K} E_{vv}$$

$$= \left[\frac{\lambda(v-1) + \lambda}{K} \right] I_v - \frac{\lambda}{K} E_{vv} \quad (\because \lambda(v-1) = 8(K-1))$$

$$= \frac{\lambda v}{K} I_v - \frac{\lambda}{K} E_{vv}$$

$$= \frac{\lambda}{K} [v I_v - E_{vv}]$$

$$= \frac{\lambda v}{K} \left[I_v - \frac{E_{vv}}{v} \right] \quad \text{--- (1)}$$

now,

$$\left[I_v - \frac{E_{vv}}{v} \right]^2 = \left[I_v - \frac{E_{vv}}{v} \right] \left[I_v - \frac{E_{vv}}{v} \right]$$

$$= I_v - \frac{E_{vv}}{v} - \frac{E_{vv}}{v} + \frac{E_{vv}}{v} - \frac{E_{vv}}{v}$$

$$= I_v - \frac{2}{v} E_{vv} + \frac{1}{v^2} v E_{vv}$$

$$= I_v - \frac{E_{vv}}{v}$$

This shows that $(I_v - \frac{E_{vv}}{v})$ is a idempotent matrix.

Rank of any idempotent matrix = trace of a matrix = sum of the diagonal elements.

$$C = \frac{\lambda v}{k} \left[I_v - \frac{E_{vv}}{v} \right]$$

$$\text{Rank}(C) = \text{Rank} \left[I_v - \frac{E_{vv}}{v} \right]$$

$$= \text{Rank}(I_v) - \frac{1}{v} \text{Rank}(E_{vv})$$

$$= v - \frac{1}{v} v$$

$$\text{Rank}(C) = v - 1.$$

* Theorem:- For any Symmetrical BIBD (v, k, λ) must be a Perfect Square for even v .

Proof:- let us consider a BIBD with parameters v, b, r, k, λ . let N it's Incidence

matrix and N' is its transpose, we know that,

$$\begin{aligned} NN' &= (s + \lambda(v-1)) (s-\lambda)^{v-1} \\ &= [s + s(k-1)] (s-\lambda)^{v-1} \quad (\because \lambda(v-1) = s(k-1)) \\ &= [s + sk - s] (s-\lambda)^{v-1} \\ &= [sk] (s-\lambda)^{v-1} \\ &= sk (s-\lambda)^{v-1} \end{aligned}$$

$$|N| |N'| = s^2 (s-\lambda)^{v-1}$$

$$|N| |N| = s^2 (s-\lambda)^{v-1} \quad (\because N = N')$$

$$|N|^2 = s^2 (s-\lambda)^{v-1} \therefore |N| = s (s-\lambda)^{\frac{v-1}{2}}$$

This shows that for any even value of v , $(s-\lambda)$.

* Resolvable BIBD :-

A BIBD is said to be s -resolvable if b blocks are arranged in s groups such that each group contains one and only one treatment, each group will contain b/s blocks. Any two treatments common between two blocks of the same group are constant while any two treatments common between two blocks of the different groups are another constant.

Ex:-1

$$v=4, b=6, s=3, k=2, \lambda=1$$

$$D = \begin{matrix} 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 2 & 3 \\ 2 & 4 \\ 3 & 4 \end{matrix}$$

here, $b=6, s=3 \therefore b/s = 2$ blocks

$$\begin{matrix} 1 & 2 & 1 & 3 & 1 & 4 & \rightarrow \text{block-1} \\ 3 & 4 & 2 & 4 & 2 & 3 & \rightarrow \text{block-2} \end{matrix}$$

$$v=4, s=3, k=2$$

* Attine Resolvable BIBD:-

A resolvable BIBD is said to be attine resolvable BIBD if number of treatment common between any two blocks of same group is constant. Similarly any two block of different group are another constant.

* d -attine resolvable BIBD:-

An attine resolvable BIBD is said to be d -attine resolvable BIBD if in each group

* Theorem :- For any BIBD show that efficiency factor $E < 1$.

Proof:- Consider a BIBD with parameters v, b, r, k, λ .

We know that for any BIBD $vr = bk$ and $\lambda(v-1) = r(k-1)$ holds true. Efficiency factor of a BIBD or a BIBD is designed as,

$$E = \frac{v(\hat{t}_i - \hat{t}_j)_{RBD}}{v(\hat{t}_i - \hat{t}_j)_{BIBD}}$$

where $(\hat{t}_i - \hat{t}_j)$ is the estimate of treatment effect.

$$E = \frac{\left(\frac{1}{r} + \frac{1}{r}\right) \sigma^2}{\frac{2k}{\lambda v} \sigma^2}$$

$$= \left(\frac{2}{r} \times \frac{\lambda v}{2k}\right)$$

$$\therefore E = \frac{\lambda v}{rk}$$

We have property $\lambda(v-1) = r(k-1)$

$$\therefore E = \frac{\lambda v}{BK}$$

$$= \left(\frac{\lambda}{B} \right) \left(\frac{v}{K} \right)$$

$$= \left(\frac{K-1}{v-1} \right) \left(\frac{v}{K} \right)$$

$$= \frac{\frac{K-1}{K}}{\frac{v-1}{v}}$$

$$E = \frac{1 - \frac{1}{K}}{1 - \frac{1}{v}}$$

In a BIBD $K < v$ always.

Again $1 - \frac{1}{K}$ is always less than 1 and K is a positive and greater than 2.

Similarly, $1 - \frac{1}{v}$ is also less than 1 again $1 - \frac{1}{v}$

for v is greater than 2.

therefore $\frac{1 - \frac{1}{K}}{1 - \frac{1}{v}} < 1$

$\therefore$ So, $E < 1$ always.

* Youden Square Design:-

A design is said to be LSD if s lattice are arranged in s rows and s column such that,

- (i) each row contain one and only one treatment
- (ii) each column contain one and only one treatment.

Ex:-

1	2	3	4	5	6	7	
2	3	4	5	6	7	1	→ row wise
4	5	6	7	1	2	3	

1	2	4	
2	3	5	
3	4	6	
4	5	7	→ column wise
5	6	1	
6	7	2	
7	1	3	

$v = 7$
 $s = 3$
 $k = 3$
 $b = 7$
 $\lambda = 1$

let us, letter wise,

A	B	C	D
B	C	D	A
C	D	A	B
D	A	B	C

* Construction of BIBD :-

Method :- (1)

BIBD with v series v , $b = vC_2$, $s = v-1$,
 $k = 2$, $\lambda = 1$

→ Step :- 1 Take v treatments write down all possible combination of v treatment taking two treatments together.

→ Step :- 2 Here there will be vC_2 combinations two treatments are taken together and is kept in one block so, these vC_2 treatment combinations are kept in vC_2 blocks. Each block contain 2 treatments and each treatment is replicated s times and finally a pair of treatment occurs together in λ block.

Ex:- $v = 7$, $b = 7C_2 = 21$, $s = 7-1 = 6$, $k = 2$, $\lambda = 1$
 treatments are $t_1, t_2, t_3, t_4, t_5, t_6, t_7$

$t_1 t_2$	$t_2 t_3$	$t_3 t_4$	$t_4 t_5$	$t_5 t_6$	$t_6 t_7$
$t_1 t_3$	$t_2 t_4$	$t_3 t_5$	$t_4 t_6$	$t_5 t_7$	
$t_1 t_4$	$t_2 t_5$	$t_3 t_6$	$t_4 t_7$		
$t_1 t_5$	$t_2 t_6$	$t_3 t_7$			
$t_1 t_6$	$t_2 t_7$				
$t_1 t_7$					

Here each treatments are replicated 6 times
 $\therefore s=6$ and $k=2$

Each pair of treatment occurs only one time.

$$\therefore v=7, b=21, s=6, k=2, \lambda=1$$

* Method:- (II)

A matrix H_n of order n is said to be Hadamard matrix if $H_n H_n' = n I_n = H_n' H_n$

First Hadamard matrix is $H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$$\text{Now, } H_2 H_2' = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= 2 I_2$$

Now,

$$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$H_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

{-1 as +1 and +1 as 0}

Step:1:- considers a Hadamard matrix of order (size) n .

Step:2:- Delete first row and first column of Hadamard matrix H_n

Step:3: change -1 as +1 and 1 as 0

Step:4: considers the remaining row and column of hadamard matrix as the Incidence matrix N .

Step:5:- This Incidence matrix is the Incidence matrix of a BIBD with parameters

$$v = n-1, b = n-1, r = n/2, k = n/2, \lambda = n/4$$

Exo 0

$$H_4 = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$$

-1 as +1 and 1 as 0.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = N \text{ which is incidence matrix of a BTBD } \begin{bmatrix} 1 & 3 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}$$

Parameters of this BTBD are,

$$v = 3(n-1), \quad b = 3, \quad s = n/2 = 4/2 = 2, \\ k = n/2 = 2, \quad \lambda = n/4 = 4/4 = 1$$

$$H_8 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 & 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \end{bmatrix}$$

-1 as 1 and 1 as 0

$$\begin{bmatrix} -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 & -1 & -1 & 1 \\ 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ -1 & 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & -1 & 1 & 1 & -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

= N (Incidence matrix)

$$\begin{matrix} = & 1 & 3 & 5 & 7 \\ & 2 & 3 & 6 & 7 \\ & 1 & 2 & 5 & 6 \\ & 4 & 5 & 6 & 7 \\ & 1 & 3 & 4 & 6 \\ & 2 & 3 & 4 & 5 \\ & 1 & 2 & 4 & 7 \end{matrix}$$

Here, $\# \text{element}, v = \# \{1, 2, 3, 4, 5, 6, 7\}$

$$b = n - 1 = 8 - 1 = 7$$

$$g = n/2 = 8/2 = 4$$

$$k = n/2 = 8/2 = 4$$

$$n = m_1 + m_2 + \dots + m_k = 0$$

* Method:- (III)

BIBD with series $v = 4\lambda + 3$, $b = 4\lambda + 3$
 $s = 2\lambda + 1 = k$ and λ where $4\lambda + 3$ is a
 prime number.

Step:-1 let $4\lambda + 3$ is a prime number
 for any $\lambda > 0$ first of all find
 out the primitive elements of
 $GF(4\lambda + 3)$
 $\{GF = \text{Galois Field}\}$. The elements
 of $GF(4\lambda + 3)$ are $0, 1, 2, \dots, 4\lambda + 3 - 1$
 $= 4\lambda + 2$.

Step:-2:- Find the primitive element
 for $GF(4\lambda + 3)$ that is, if $\alpha^{4\lambda + 3 - 1}$
 with reduced mod $(4\lambda + 3)$ the
 α is primitive element. Next
 write all the elements as the
 powers of primitive elements
 consider either even or
 primitive elements consider
 either even power with reduce
 mod $4\lambda + 3$ and keep them
 in block.

Denote this block as a key block
 Develop this key block with
 reduced mod $v = 4\lambda + 3$. This
 way we get BIBD with
 Parameters $v = 4\lambda + 3 = b$

Ex^o- $\lambda=1, V=4(1)+3=7, b=7$

$s=2(1)+1=3, k=3$

$\therefore V(=7)$ is a prime number

Element of $GF(7)$ are 0, 1, 2, 3, 4, 5, 6

2^0	2^1	2^2	2^3	2^4	2^5	2^6
2	2	2	2	2	2	2
1	2	4	1			

$\therefore 2^6 \neq 1$ so 2 is not be the primitive element of $GF(7)$

3^0	3^1	3^2	3^3	3^4	3^5	3^6
3	3	3	3	3	3	3
1	3	2	27	6×3	4×3	5×3
			6	18	12	15
				4	5	1

Here, $3^6=1$ with reduced mod 7,
so, 3 is primitive element of $GF(7)$

key by block	3^1	3^3	3^5
b_1	3	6	5
b_2	4	7	6
b_3	5	1	7
b_4	6	2	1
b_5	7	3	2
b_6	1	4	3
b_7	2	5	4

even no

²	⁴	⁶
3	3	3
2	4	1
3	5	2
4	6	3
5	7	4
6	1	5
7	2	6
1	3	7

here, $v=7$, $b=7$, $s=3$, $k=3$, $\lambda=1$

* Method :- Using Block Section

Step: 1 :- consider a BIBD with Parameters v_1, b_1, s_1, k_1 and λ_1

Step: 2 :- Delete any one block from this BIBD.

Step: 3 :- so the remaining blocks are now $b-1$.

Step: 4 :- Take one block and see which treatment are present in this block. now select those treatment from that block which are absent in deleted block and then keep these treatments in another block.

Step: 5:- Continue step 4 for remaining blocks.

Step: 6:- since K_1 treatments are deleted from $\mathcal{V}_1$, so the treatments for new design will be $(\mathcal{V}_1 - K_1)$.

Step: 7:- Each block will contain $(K - \lambda)$ treatment so, the new BIBD exist with Parameters $V = V_1 - K_1$, $b = b_1 - 1$, $K = K_1 - \lambda$, $S = S_1$, $\lambda = \lambda_1$.

ex: $V = b = 1, S = K = 5, \lambda = 2$

2	8	10	7	6		2	8	10
3	9	11	8	7		3	11	8
4	10	1	9	8		4	10	8
5	11	2	10	9		11	2	10
6	1	3	11	10	$\Rightarrow$	3	11	10
7	2	4	1	11		2	4	11
8	3	5	2	1		8	3	2
9	4	6	3	2		4	3	2
10	5	7	4	3		10	4	3
11	6	8	5	4		11	8	4
1 7 9 6 5								

2 $\rightarrow$ 1 8 $\rightarrow$ 4

3 $\rightarrow$ 2 10 $\rightarrow$ 5

4 $\rightarrow$ 3 11 $\rightarrow$ 6

$$t_{\text{seatsment}} = 6$$

$$\text{here, } t_{\text{seatsments}} = V + K = 11 - 5 = 6$$

$$\text{blocks } b' = \{b-1\} = 10$$

$$g' = g = 5$$

$$K' = K - \lambda = 5 - 2 = 3$$

$\lambda' = \lambda = 2 \Rightarrow$ The resulting BIBD is

1	4	5	$\rightarrow$	b_1
2	6	4		b_2
3	5	4		b_3
6	1	5		b_4
2	6	5		b_5
1	2	6		b_6
4	2	1		b_7
3	2	1		b_8
5	3	2		b_9
6	4	3		b_{10}

* Factorial Experiment:-

→ Why we need factorial?

we have basic design CRD, RBD, LSD in this design v treatment are arranged in b blocks.

→ v treatment are arranged in b blocks and treatment are at different level.

ex:-

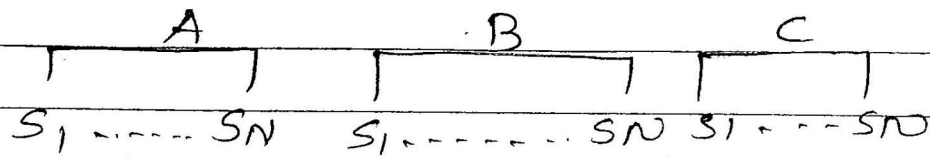
Fertilizers N, P, K we want the response of these fertilizers on certain crop.

			N		P		K	
N	P	K						
			40gm/H	60gm/H	100gm/H	120gm/H	80gm/H	100gm/H

when any treatment which called factor has different level and different dosages then basic design can not be used for such cases another design is used is called factorial because treatment considers as a factor.

* What is factorial design?

Let A, B, C... are N factors let further this factors are at $S_1, S_2, \dots, S_n$ levels respectively.



The combinations of all the levels of each factor is called factorial design.

The combinations are $S_1' \times S_2' \times \dots \times S_N'$

→ If $S_1 = S_2 = \dots = S_N$ then this factorial design is called symmetrical factorial which is given by

$$S_1' \times S_1' \times \dots \times S_1' = S_1^{1+1+1+\dots+N} = S_1^N = \text{Symmetrical factorial.}$$

* Difference :- Basic design and factorial design.

→ Basic design we write only treatment and factorial design we write treatment combinations.

→ In factorial design treatment are at different level so, level due to factor → S^N .

→ In factorial design main effect and interaction effect.

→ If there are n factors each are at s levels then symmetrical

* 2^3 factorial experiment:

$$M = \left\{ \begin{matrix} n_0 P_0 K_0 \\ n_0 P_0 K_1 \\ n_0 P_1 K_0 \end{matrix} + \begin{matrix} n_1 P_1 K_1 \\ n_1 P_0 K_0 \\ n_1 P_0 K_1 \end{matrix} + \begin{matrix} n_1 P_0 K_0 \\ n_1 P_0 K_1 \\ n_1 P_1 K_0 \end{matrix} + \begin{matrix} n_1 P_0 K_1 \\ n_1 P_1 K_0 \\ n_1 P_1 K_1 \end{matrix} \right\} + \left\{ -C \begin{matrix} n_0 P_0 K_1 \\ n_0 P_0 K_0 \\ n_0 P_1 K_0 \end{matrix} + \begin{matrix} n_0 P_1 K_0 \\ n_0 P_1 K_1 \end{matrix} \right\}$$

no P, K 8

n, p_0, k_0

$$n_i \quad p_o \quad k_i \quad P = \{n_i p_i k_i + n_o p_i k_o + n_o p_i k_i + n_i p_i k_o\} t$$

$$n, P, K_0 \quad \} - (n_0 P_0 K_0 + n_0 P_0 K_1 + n_1 P_0 K_0 + n_1 P_0 K_1)$$

m, p, k 8

$\Rightarrow$ Interaction effect :-

$n=2, 2^2=4$ treatment combination
 $n_0 P_0, n_0 P_1, n_1 P_0, n_1 P_1$

$$NP = \underbrace{(n_1 P_1) + (-n_0 P_1)}_? + \underbrace{\{-n_1 P_0 + (-n_0 P_0)\}}_?$$

$$NP = \frac{(m_1 p_1) + (-m_1 p_0)}{2} + \frac{\{1 - (m_0 p_1)\} + \{m_0 p_0\}}{2}$$

Interaction effect is defined the average of all treatment combination of factorial experiment in two groups such that level of 1 factor is same while level of another factor are different.

$$NP = \frac{(n_{1P_1} - n_{1P_0}) - (n_{0P_1} + n_{0P_0})}{4}$$

$$= \frac{(n_{1P_1} + n_{0P_0}) - (n_{1P_0} + n_{0P_1})}{4}$$

Bose defined as contrast of the contrast of all treatment combination 2^3 factorial experiment in two groups such that level of one factor is same while level of another factor are different.

Suppose 2^3 factorial experiment treatment combination are $n_{0P_0K_0}$, $n_{0P_0K_1}$, $n_{0P_1K_0}$, $n_{0P_1K_1}$, $n_{1P_0K_0}$, $n_{1P_0K_1}$, $n_{1P_1K_0}$, $n_{1P_1K_1}$.

$$NP = [(n_{1P_1K_1} + n_{1P_1K_0}) - (n_{0P_1K_1} + n_{0P_1K_0})] - [(n_{1P_0K_1} + n_{1P_0K_0}) - (n_{0P_0K_1} + n_{0P_0K_0})]$$

$$NK = [(n_{1P_1K_1} + n_{1P_0K_1}) - (n_{0P_1K_1} + n_{0P_0K_1})] - [(n_{1P_1K_0} + n_{1P_0K_0}) - (n_{0P_1K_0} + n_{0P_0K_0})]$$

$$PK = [(n_{1P_1K_1} + n_{0P_1K_1}) - (n_{1P_0K_1} + n_{0P_0K_1})] - [(n_{1P_1K_0} + n_{0P_1K_0}) - (n_{1P_0K_0} + n_{0P_0K_0})]$$

ANOVA TABLE :-

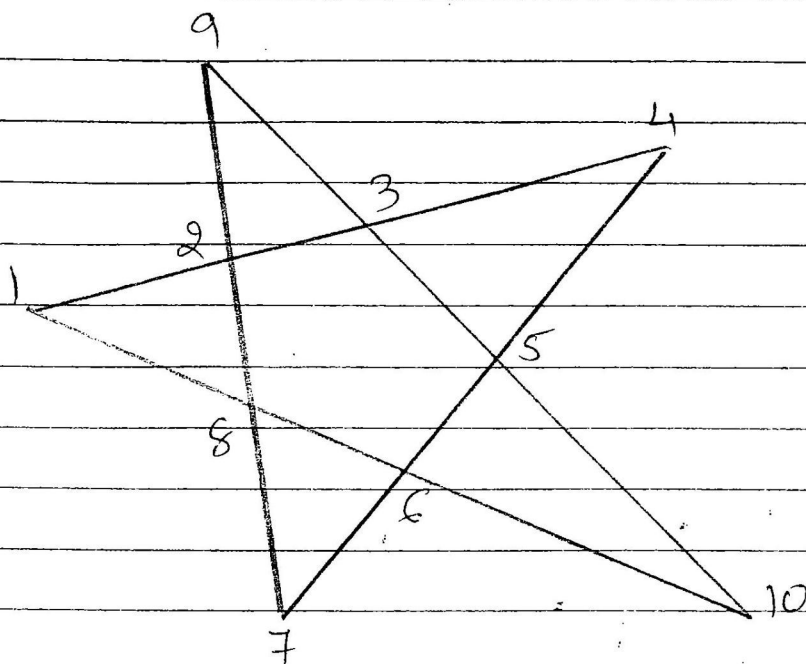
SV	df	S.S.	MSS	F-ratio	Conclusion
replication	$b-1$	$\sum R_i^2 - CF$ $\frac{1}{4} = S_x^2$	$\frac{S_x^2}{b-1} = \sigma_b^2$	$\frac{\sigma_b^2}{\sigma_c^2}$	$F_c < F_t$
treatment	$b-1 = 7$				at α level
main N	1	S_N^2	$\frac{S_N^2}{1} = \sigma_N^2$	$\frac{\sigma_N^2}{\sigma_c^2}$	of significance
P	1	S_P^2	$\frac{S_P^2}{1} = \sigma_P^2$	$\frac{\sigma_P^2}{\sigma_c^2}$	then test is
contrast NP	1	S_{NP}^2	$\frac{S_{NP}^2}{1} = \sigma_{NP}^2$	$\frac{\sigma_{NP}^2}{\sigma_c^2}$	insignificant otherwise
K	1	S_K^2	$\frac{S_K^2}{1} = \sigma_K^2$	$\frac{\sigma_K^2}{\sigma_c^2}$	significant
NK	1	S_{NK}^2	$\frac{S_{NK}^2}{1} = \sigma_{NK}^2$	$\frac{\sigma_{NK}^2}{\sigma_c^2}$	In first case accept
PK	1	S_{PK}^2	$\frac{S_{PK}^2}{1} = \sigma_{PK}^2$	$\frac{\sigma_{PK}^2}{\sigma_c^2}$	H_0 and next case
NPK	1	S_{NPK}^2	$\frac{S_{NPK}^2}{1} = \sigma_{NPK}^2$	$\frac{\sigma_{NPK}^2}{\sigma_c^2}$	reject H_0 .
ERROR	$3(b-1)$	By Subtract S_c^2	$\frac{S_c^2}{3(b-1)} = \sigma_c^2$		
Total	$2^3 \cdot b - 1$				

* Partially balanced incomplete block design:-

BIBD is not available for all treatments as it has to satisfy the following condition $VS = bk, \lambda(V-1) = S(K-1), b > 1$. Again lattice design is available only for the square number of treatments and cubic number of treatments while youden square design required large number of replication size so to have an IB with all possible treatments and a smaller replication size, we require another class of incomplete block design. For such design Bose and Nair (1936) introduce the concept of PBIBD, which is available for all possible number of treatments and a smaller number of replication size.

* Association schemes:-

- Find out first and second associated.
- First associated $\Rightarrow$ point which are on the same line.
- Second associated $\Rightarrow$ point which are on the different line.



1st associate

- 1 $\rightarrow$ 2, 3, 4, 8, 6, 10
- 2 $\rightarrow$ 8, 7, 9, 1, 3, 4
- 3 $\rightarrow$ 1, 2, 4, 5, 9, 10
- 4 $\rightarrow$ 1, 2, 3, 5, 6, 7
- 5 $\rightarrow$ 3, 9, 10, 4, 6, 7
- 6 $\rightarrow$ 4, 5, 7, 11, 8, 10
- 7 $\rightarrow$ 2, 8, 9, 4, 5, 6
- 8 $\rightarrow$ 1, 6, 10, 2, 7, 9
- 9 $\rightarrow$ 2, 8, 7, 10, 3, 9
- 10 $\rightarrow$ 1, 8, 6, 3, 5, 9

2nd associate

- 5, 7, 9
- 5, 6, 10
- 6, 7, 8
- 8, 9, 10
- 1, 2, 8
- 2, 3, 9
- 1, 3, 10
- 3, 4, 5
- 1, 4, 6
- 2, 4, 7

$$n_1 = 6$$

$$n_2 = 3$$

$$\therefore n_1 + n_2 = 6 + 3 = 9$$

$$\therefore V - 1 = 10 - 1 = 9$$

$$\therefore n_1 + n_2 = V - 1$$

* Parameters of PBIBD :-

On the basis of association schemes, PBIBD has two types of Parameters

(1) Primary Parameters: v, b, s, k, λ_i ($i=1, \dots, m$)

(2) Secondary Parameters: $n_i, p_{ij}^{(k)}$ ($i \neq j = 1, 2, \dots, v, k = 1, 2, \dots, m$)

→ Parametric Relation :-

$$(1) \quad v s = b k$$

$$(2) \quad \sum_{i=1}^m n_i = v - 1$$

$$(3) \quad \sum_{i=1}^m n_i \lambda_i = s(k-1)$$

* Prove that :- $\sum_{i=1}^m n_i = v - 1$

For any PBIBD with m^{th} association schemes we know that $\sum_{i=0}^m B_i = E_{vv}$

now post multiply both side by E_v ,

$$\sum_{i=0}^m B_i E_v = E_{vv} E_v$$

where B_i is a matrix of order $v \times v$ and is called association

associate has n_i treatments

$$\therefore \sum_{i=0}^m n_i E_{v_i} = V E_{v_i} \Rightarrow \sum_{i=0}^m n_i = V$$

$$\Rightarrow n_0 + \sum_{i=1}^m n_i = V \text{ where } n_0 = 1$$

$$\Rightarrow \sum_{i=1}^m n_i = V - 1$$

* Prove that : $\sum_{i=1}^m n_i \lambda_i = S(K-1)$

For any PBIB Design with m^{th} association schemes we know that,

$$\sum_{i=0}^m B_i = E_{vv} \text{ ——— (1)}$$

$$NN' = \sum_{i=0}^m \lambda_i B_i \text{ ——— (2)}$$

where λ_i is number of times a pair of treatments occurs together in i^{th} associate class. Now post multiply by E_{v_i} of both side of (2),

$$\therefore NN'E v_1 = \sum_{i=0}^m \lambda_i B_i E v_1$$

$$\therefore N (N'E v_1) = \left(\sum_{i=0}^m \lambda_i B_i \right) E v_1$$

$$\therefore NK E b_1 = \sum_{i=0}^m \lambda_i n_i E v_1$$

$$\therefore KN E b_1 = \sum_{i=0}^m n_i \lambda_i E v_1$$

$$\therefore K \mathcal{B} E v_1 = \sum_{i=0}^m n_i \lambda_i E v_1$$

$$\therefore K \mathcal{B} = \sum_{i=0}^m n_i \lambda_i$$

$$\therefore K \mathcal{B} = n_0 \lambda_0 + \sum_{i=1}^m n_i \lambda_i$$

$$\therefore \sum_{i=1}^m n_i \lambda_i = K \mathcal{B} - n_0 \lambda_0$$

$$= K \mathcal{B} - \mathcal{B} = \mathcal{B}(K-1)$$

$$\therefore \sum_{i=1}^m n_i \lambda_i = \mathcal{B}(K-1)$$

* Cyclic BIBD

A non-group divisible PBIBD is called cyclic PBIBD if the set of first associates of the treatment numbered i is obtained by adding $i=1$ to the numbers in the set of first associates of the treatment numbered 1 and subtracting v whenever the sum exceeds v .

<u>ex%</u>	1 st	2 nd
1 $\rightarrow$	2, 4	3
2 $\rightarrow$	1, 3	4
3 $\rightarrow$	2, 4	1
4 $\rightarrow$	1, 3	2

In cyclic PBIBD treatments are arranged in such a way that it will be in the increasing sequence in each block starting with $i, i+1, i+2, \dots, i+(n-1); i$ where $i=1$

* What is meant by confounding?
Explain 3 types of confounding

Defⁿ:- confounding of factorial experiment is defined as reduction of block size in such a way that one block is divided into two or more blocks such that treatment comparison of that main or interaction effect is mixed up with block effect.

Types of confounding

1. Total
2. Partial
3. Balanced.

1. Total confounding

In a confounding factorial experiment all the main effects or interaction effects are same in all the replication. The experiment is called total confounding or complete confounding.

ex: 2^3 factorial

Replication-1		Replication-2		Replication-3	
B-1	B-2	B-3	B-4	B-5	B-6
AbobCo	Aob1Co	AobobCo	Aob1G	A, b1G	Aob1G
AobobG	Aob1G	A, b1G	Aob1Co	AobobCo	A, b1G
A, b1Co	A, b1Co	AobobG	A, b1Co	A, b1Co	A, b1Co
A, b1G	A, b1G	A, b1Co	A, b1Co	AobobG	Aob1G

AB is confounded AB is confounded AB is confounded

2. Partial confounding

A confounded factorial experiment is called partial confounding if different main effects and interaction effects are confounded in different replications.

ex: 2^3 Factorial

Replication-1		Replication-2		Replication-3	
B-1	B-2	B-3	B-4	B-5	B-6
AobobCo	Aob1Co	AobobCo	AobobG	AobobCo	AobobG
AobobG	Aob1G	Aob1Co	Aob1G	Aob1G	Aob1Co
A, b1Co	A, b1Co	A, b1G	A, b1Co	A, b1Co	A, b1Co
A, b1G	AobobG	A, b1G	A, b1Co	A, b1Co	A, b1G

In Partial confounding the advantage is that we do not lose the entire information on that confounded main effect and interaction effect.

Main effect or interaction effect which is confounded in one replication is not confounded in another replication so we can estimate fraction of information of that main or interaction effect from those replication in.

Partial confounding of 2^3 into 2^2 block size with AB, AC, BC, ABC confounded into 8-replication.

3. Balanced confounding.

When all the effect of same order are confounded with equal no. of times the confounding is said to be balanced confounding or balanced partial confounding.

In the effect of certain order are confounded are equal no. of times in the replications this

System of confounding is known as balanced partial confounding.

2^3 factorial experiment

Replication-1		Replication-2		Replication-3	
B-1	B-2	B-3	B-4	B-5	B-6
aoboc ₀	aob ₁ c ₀	aoboc ₀	aoboc ₁	aob ₁ c ₀	aoboc ₀
aoboc ₁	aob ₁ c ₁	aob ₁ c ₀	aob ₁ c ₁	aob ₁ c ₁	aob ₁ c ₁
a ₁ b ₁ c ₀	a ₁ boc ₀	a ₁ boc ₁	a ₁ boc ₀	a ₁ boc ₀	a ₁ boc ₁
a ₁ b ₁ c ₁	a ₁ boc ₁	a ₁ b ₁ c ₁	a ₁ b ₁ c ₀	a ₁ b ₁ c ₀	a ₁ b ₁ c ₀

AB is confounded | AC is confounded | BC is confounded

It is balanced confounding because AB, AC, BC have same order 1 and also they are equal no. of times replicated.

* Ghash and Biscous method:-

Consider a BIBD with parameters $v=b=4\lambda+3$, $r=k=2\lambda+1$, $\lambda=1$. This BIBD can be constructed using primitive element using GF. From this method we are constructing develop BIBD by key block either from taking even powers or odd powers of the primitive element.

of GF $4\lambda + 3$. these will be two BIBD Design. Since both have same block size, treatment, each treatment is replicated same no. of time further each pair of treatment occurs together λ times

considers one block of both design and cross it with respectively. this way cross the times for all the b blocks of the two design this gives a complete balanced cross design plan these will $b \times k$ crosses altogether each occurs λ times and every plot occurs λ times.

Ex: $v = b = 7, s = k = 3, \lambda = 1$

3 is primitive element of GF(7)

3^0	3^1	3^2	3^3	3^4	3^5	3^6
1	3	2	6	4	5	7
3	2	6	4	5	7	1
2	6	4	5	7	1	3
6	4	5	7	1	3	2
5	7	1	3	2	6	4
4	1	3	2	6	4	5

$\Rightarrow$ 1×3 2×6 4×5
 2×4 3×7 5×6
 3×5 4×1 6×7
 4×6 5×2 7×1
 5×7 6×3 1×2
 6×1 7×4 2×3
 7×2 1×5 3×4

		1	2	3	4	5	6	7	
	1	1	1	1	1	1	1	0	6
	b 2	0	1	1	1	1	1	1	6
N =	1 3	1	0	1	1	1	1	1	6
	o 4	1	1	0	1	1	1	1	6
	c 5	1	1	1	0	1	1	1	6
	k 6	1	1	1	1	0	1	1	6
	7	1	1	1	1	1	0	1	6
		6	6	6	6	6	6	6	

$$\sum m_{ij}^2 = 6$$

$$\sum m_{ij} m_{mj} = 5$$

This is binary design.

- each line will occur 2^2 times = 6
- each cross will occur λ times $\lambda = 1$
- Total no. of crosses will be $b \times k$ times = 21

* Ex:- show that a design with parameters $V=4, b=6, r=3, k=2, \lambda=1$ is balanced, connected or orthogonal?

→

1	2	$b=6$
1	3	$r=4$
1	4	$s=3$
2	3	$k=2$
2	4	$\lambda=1$
3	4	$n = r \times s = 12$ or
		$b \times k = 12$

		1	2	3	4	5	6
	1	1	1	1	0	0	0
$N =$	2	1	0	0	1	1	0
	3	0	1	0	1	0	1
	4	0	0	1	0	1	1

$r \times b$
 4×6

	1	1	0	0
$N' =$	1	0	1	0
	1	0	0	1
	0	1	1	0
	0	1	0	1
	0	0	1	1

6×4

now,

$NN' =$	3	1	1	1
	1	3	1	1
	1	1	3	1
	1	1	1	3

$$C = \text{diag}(3) - \frac{N K^{-1} N'}{K}$$

$$= \text{diag}(3) - \frac{N N'}{K} \quad (K = K_1 = K_2 = \dots = K)$$

$$= \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} - \frac{N N'}{K}$$

$$C = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3/2 & -1/2 & 1/2 & 1/2 \\ 1/2 & 3/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 3/2 & 1/2 \\ 1/2 & 1/2 & 1/2 & 3/2 \end{bmatrix}$$

$$= \begin{bmatrix} 3/2 & -1/2 & -1/2 & -1/2 \\ -1/2 & 3/2 & -1/2 & -1/2 \\ -1/2 & -1/2 & 3/2 & -1/2 \\ -1/2 & -1/2 & -1/2 & 3/2 \end{bmatrix}$$

$$= 2 \begin{bmatrix} 3/4 & -1/4 & -1/4 & -1/4 \\ -1/4 & 3/4 & -1/4 & -1/4 \\ -1/4 & -1/4 & 3/4 & -1/4 \\ -1/4 & -1/4 & -1/4 & 3/4 \end{bmatrix} \quad \text{--- (1)}$$

Since all the diagonal elements are constant and again all the off diagonal elements are another constant so design is balanced.

$$C = \Theta \left[I_v - \frac{1}{v} E_{vv} \right]$$

$$= \Theta \left\{ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \right\}$$

$$= \Theta \begin{bmatrix} 3/4 & -1/4 & -1/4 & -1/4 \\ -1/4 & 3/4 & -1/4 & -1/4 \\ -1/4 & -1/4 & 3/4 & -1/4 \\ -1/4 & -1/4 & -1/4 & 3/4 \end{bmatrix} \quad \text{--- (2)}$$

from (1), (2) we get $\Theta = 2$
 $\therefore$ Eigen Value = 2 with multiply
 $(v-1) = 3$.
 so, design is balanced.

$$\Rightarrow E_{vv}C = 0, \therefore |C| = 0 \text{ so, } CE_{vv} = 0$$

C is a singular matrix
 so, $\text{Rank}(C) = v-1 = 3$
 $\therefore$ design is connected.

$$\Rightarrow B = (3 \ 3 \ 3 \ 3)'$$

$$K = (2 \ 2 \ 2 \ 2 \ 2 \ 2)$$

$$g_k' = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 2 \\ 2 \\ 2 \end{bmatrix}_{6 \times 1} \quad \begin{bmatrix} 3 & 3 & 3 & 3 \end{bmatrix}_{1 \times 4}$$

$$= \begin{bmatrix} 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 6 & 6 \end{bmatrix}_{4 \times 6}$$

$$\frac{g_k'}{n} = \begin{bmatrix} 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 \end{bmatrix}$$

$\therefore N \neq \frac{g_k'}{n} \therefore$ design is not an orthogonal.