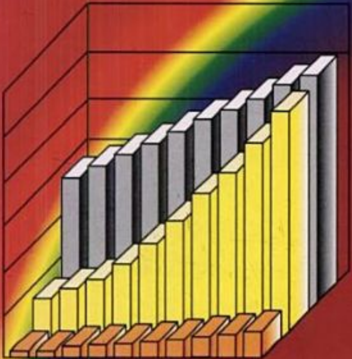
Programmed Statistics

(Questions-Answers)

Second Edition

Premier12

B.L. AGARWAL



- Learning Statistics
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Contents

Chapter **1**

Statistics in General

SECTION-A

Short Essay Type Questions

Q. 1 Define statistics as given be Sir R.A. Fisher.

Ans. Sir R.A. Fisher defined statistics as, "The science of statistics is essentially a branch of applied mathematics and may be regarded as mathematics applied to observational data."

Q. 2 Give the definitions of statistics given by A.L. Bowley, Lovitt, W.A. Wallis and H.V. Roberts and C.H. Meyers.

Ans. The definitions given by statisticians named in the question are quoted below:

A.L. Bowley:

(i) Statistics is the device for abbreviating and classifying the statements and making clear the relations.

(ii) Statistics is the science of measurement of social phenomenon regarded as a whole in all its manifestations.

(iii) Statistics is the numerical statement of facts in any department of enquiry, placed in relation to each other.

Lovitt: Statistics is the science which deals with the collecting, classifying, presenting, comparing and interpreting numerical data collected to throw light on any sphere of enquiry.

W.A. Wallis and H.V. Roberts: Statistics is not a body of substantive knowledge, but a body of methods obtaining knowledge.

Cecil H. Meyers: Statistics may be defined as a science of numerical information which employs the processes of measurement and collection, classification, analysis, decision-making and communication of results in a manner understandable and verifiable by other.

Q. 3 Give the statements given by A.L. Bowley and William A. Spurr and Charles P. Bonini.

Ans. A.L. Bowley: Great numbers are not counted correctly to a unit, they are estimated.

William A. Spurr and Charles P. Bonini: Not all numbers are statistical, Logarithms, for instance, are mere abstract numbers. Statistical data are concrete numbers, which represent objects.

Q. 4. Quote statements about statistics made by A.E. Waugh, A.L. Boddington, Whipple, Tippet, W.I. King, Marshall, Yule and Kendall, R.A. Fisher, A.M. Mood, Disraeli and Darrel Huf.

A.E. Waugh: The purpose of statistical methods is to simplify great bodies of numerical data.

A.L. Boddington: The essence of statistics is not mere counting but comparison.

Whipple: Statistics enables one to enlarge his Horizon.

L.H.C. Tippet:

(i) Planning is the order of the day and without statistics planning is inconceivable.

(ii) Statistics is both a science and an art.

W.I. King:

(i) The science of statistics is a most useful servant, but only of great value to those who understand its proper use.

(ii) The science of statistics is the method of judging collective, natural or social phenomena from the results obtained by the analysis of enumeration or collection of estimates.

Marshall: Statistics are the straw out of which I like every other economist have to make bricks.

Yule and Kendall: Statistics is not a science, it is a scientific method.

R.A. Fisher: Statistics is a branch of applied mathematics which specialises in data.

A.M. Mood: Statistics provides tools and techniques for research workers.

Disraeli: There are three kinds of lies: lies, damned lies and statistics.

Darrel Huf: A well wrapped statistics is better than Hitlers biglie it misleads, yet it cannot be pinned on you.

Q. 5. Which definition of statistics is considered to be the best.

Ans. The definition of statistics given by R.A. Fisher is considered to be the best and most exact.

Q. 6. Give in a few words the statistical perspective.

Ans. Statistical perspective is the invaluable compendium which gather all the facts, figures, objective survey and fascinating remembrances to a assimilable record.

Q. 7. Mention main divisions of statistics.

Ans. Following are the main divisions of statistics:

(i) *Mathematical or theoretical statistics*: It covers development of statistical distributions, experimental designs, sampling designs, etc.

(ii) *Statistical methods or functions*: It covers collection, tabulation, analysis and interpretation of data, etc.

(iii) *Descriptive statistics*: Classification and diagramatic representation of data.

(iv) *Inferential statistics*: To draw conclusion about population on the basis of sample drawn from it.

(v) *Applied statistics*: It mainly covers population, census, national income, production, business statistics, industrial statistics, quality control, biostatistics, etc.

Q. 8. What are the limitations of statistics?

Ans. Broadly the limitations of statistics are:

(i) Statistics deals with quantitative data only. Even qualitative information is converted into numerical data by the method of ranking, scoring or scaling.

(ii) Statistics is true on an average only.

(iii) Statistics deals with the masses, not an individual. No statistics is applicable for a single observation.

(iv) Statistical results are correct in a general sense. They are always subject to certain amount of error.

(v) Statistics is only a means to draw conclusions about masses or population but not a panacea to all sort of problems.

(vi) Statistics can be misused in many ways.

Q. 9 What are different types of investigations?

Ans. There are two types of investigations, namely:

(i) investigation through census method

(ii) investigation through sample methods.

Q. 10 What does census method imply?

Ans. Census method means to include each and every unit or object of the population under reference for enquiry or observation. For example, to know the national income, we have to include every individual or unit which contributes towards the national income.

Q. 11 What is meant by investigation through sample method?

Ans. In sample method, an investigator has to select some units from the population about which conclusions have to be drawn and take observations on the selected units. The results obtained from sample values are applicable to the population as a whole. For instance, to know the average age at marriage, an investigator selects an adequate number of married couples and arrive at an average age of marriage which is considered to be the average age of marriage for the whole population.

Q. 12. What are four main functions of statistics?

Ans. Four functions of statistics are:

(i) Collection of data;

(ii) Presentation of data;

(iii) Analysis of data; and

(iv) Interpretation of results.

Q. 13 Give different methods of collection of data.

Ans. Following are the methods of collection of data:

(i) Direct personal enquiry method;

(ii) Indirect oral investigation;

(iii) By filling of schedules;

(iv) By mailed questionnaires;

(v) Information from local agents and correspondents;

(vi) By old records; and

(vii) By direct observational method.

Q. 14 Name two kinds of statistical data and describe them in brief.

Ans. Two kinds of statistical data are:

(i) *Primary data*: Primary data are those which are collected from the units or individuals directly and these data have never been used for any purpose earlier.

(ii) *Secondary data*: The data, which had been collected by some individual or agency and statistically treated to draw certain conclusions. Again the same data are used and analysed to extract some other information, are termed as secondary data.

Q. 15 What are the requisites of a reliable data?

Ans. The requisites of a reliable data are:

(i) It should be complete;

(ii) It should be consistent;

(iii) It should be accurate; and

(iv) It should be homogeneous in respect of unit of information.

Q. 16 What precautions should be taken in the planning of a statistical survey?

Ans. Following precautions are to be taken in the planning of a survey:

(i) *Purpose*: First a clear-cut objective of the survey should be spelled out.

(ii) *Scope of survey*: Different aspects to be covered to achieve the fixed objectives should clearly be explained.

(iii) *Definition of terms*: All the terms involved in a survey should be defined without ambiguity so that no unit is likely to fall in more than one category.

(iv) *Stating the hypothesis*: Hypothesis to be tested from the data collected by way of survey should be laid down in accordance with the objectives of the survey.

Q. 17 Give briefly the characteristics of a good questionnaire or a schedule.

Ans. Characteristics of a good questionnaire or a schedule are:

(i) Number of questions should be such that it extracts all information required for the report.

(ii) Each question should have almost all alternative answers.

(iii) The question should be clear and without any ambiguity.

(iv) All questions should be mutually exclusive in nature.

(v) Some very personal questions be avoided.

(vi) Questionnaire or schedule should not be very lengthy and time-consuming.

Q. 18 Name five fields where statistics is inevitable.

Ans. Broadly five fields where statistics is inevitable can be named as follows:

(i) Scientific research;
(ii) Economic analysis;
(iii) Planning;
(iv) Business and commerce; and
(v) Forecasting and projection.

Q. 19 Mention different kinds of statistical investigations.

Ans. Different kinds of statistical investigations are:

(i) Surveys or experiments;
(ii) Surveys through census or sample enquiry;
(iii) Confidential or open enquiry;
(iv) Direct or indirect enquiry;
(v) Original or repetitive enquiry;
(vi) Regular or ad hoc enquiry; and
(vii) Limited or extensive enquiry.

Q. 20 What is an absolute biased error?

Ans. When the figures are rounded straight way to the nearest lowest unit of rounding or to the nearest highest unit of rounding, the difference between actual and estimated (rounded) values in the two cases are called biased errors. For example, if we round the value 357 to the nearest 100, the nearest lower value is 300 and nearest higher value is 400. In case I, Absolute biased error = 357 – 300 = 57.

In case II,

Absolute biased error = 357 – 400 = – 43.

If there are two or more values in a set, the sum of absolute errors is taken,

Q. 21 What is an absolute unbiased error?

Ans. If the values are rounded as per the rules of rounding, *i.e.*, a given value is rounded to nearest lower value of the unit of rounding in case it is less than half of the unit of rounding and to next higher value of the unit of rounding if it is more than half of the unit of rounding, it is called unbiased error. The difference between the actual value and the estimated (rounded) value is called absolute unbiased error. If, there are two or more values in a set, then the sum of the absolute unbiased error is taken.

Q. 22 How do you estimate an average unbiased absolute error (A.E.).

Ans. The Formula is,

Average A.E. (unbiased) = Average error × $\sqrt{n}$

where, n = number of items in the set and Average error = The mean of the minimum and maximum values which are likely to be left over or increased in the process of rounding. For example, if we are rounding a value to the nearest 100, the chances are that the lowest value which may be left out is 0 and maximum value which may be added is 50. Hence, the average error is (0 + 50)/2 = 25.

Q. 23 Enunciate the law of statistical regularity.

Ans. The law states that a reasonably large number of items selected at random from a large group of items will, on the average, be representative of the large group or population. This law is governed by the theory of probability.

Q. 24 State the law of inertia of large numbers.

Ans. The law of inertia states that the large aggregates are more stable than small ones. According to Professor A.L. Bowley, great numbers and averages resulting from them, such as we always obtain measuring social phenomena have a great inertia.

Q. 25 What is the law of persistence of small number?

Ans. The law of persistence of small numbers states that the ratio of the small number of items having some distinguished characteristics to the total number of units in the population remains constant even through the population size is immensely increased.

Q. 26 Give an example of the law of persistence of small numbers.

Ans. Suppose a school admits all good students. Even then some students will be of poor intelligence. The ratio of such students to the total number of students will remain the same even if the number of students in the school is doubled, trebled, etc.

Q. 27 State the law of decreasing variation?

Ans. Law of decreasing variation indicates that the variation in a sample tends to reduce as the sample size increases.

Q. 28 How is the law of decreasing variation helpful in sample surveys?

Ans. It is the law of decreasing variation which puts an investigator on sound footing to decide about the adequate sample size which is a true representative of the population.

Q. 29 What do you understand by approximation of values or figures?

Ans. To express a value or figure to a round figure which is easy to write and understand is called approximation. This is mostly done from convenience point of view. It helps in comparison of values tremendously.

Q. 30 Give different methods of approximation with a brief description.

Ans. Different methods of approximation are:

(i) *By adding figure*: In this methods the given value of figure is always increased to next higher value of unit of rounding. For instance, a value 21, 357.4 will be approximated to 21,358 up to unit place, 21, 360 up to tenth place, 21,400 up to hundredth place and 22,000 up to thousandth place.

(ii) *By discarding figures.* It is a process just reverse to the adding figures. In this method, the given value is decreased to next lower value of unit of rounding. For example, the value 21,357.4 is approximated to 21,357 up to unit place, 21,350 up to tenth place, 21,300 up to hundredth place and 21,000 up to thousandth place.

(iii) *Approximation to the whole number*: This method is also known as rounding of figures and is an usually accepted method. This method is the best one as it minimises the error of approximation. In this method, a value is raised to the next higher value of the unit of rounding if it is more than half of the unit of rounding and is left over if it is less than half of the unit of rounding.

Q. 31 What are different sources of statistical errors?

Ans. Following are the four sources of statistical errors:

(i) Errors of origin;

(ii) Errors of inadequacy;

(iii) Errors of manipulation; and

(iv) Errors of interpretation.

Q. 32 Explain briefly the possible error.

Ans. In rounding of values to the nearest of units, tens or hundreds or thousands, etc., a value less than half of the unit of rounding is left over and greater than half of the unit of rounding is increased to the next higher unit. Thus, the possible error is ± 1/2 × unit of rounding. For example, if the number 23 is rounded to the nearest ten, its value is 20 and possible error is ± 5. Hence the value will lie between 20 ± 5, *i.e.*, between 15 and 25.

Q. 33 What are different sources of primary data?

Ans. Data obtained from original experiments or surveys, *i.e.*, the data collected by investigators or enumerators is known as primary data. Also the census data, data released in the Reserve Bank of India bulletins, data published by other authorities in original form are considered as primary data.

Q. 34 What are different sources of secondary data?

Ans. Published thesis, research papers, project reports, summarised census report, monthly abstracts of CSO and different publications of trade and commerce associations, etc., are the various sources of secondary data.

Q. 35 What kind of deficiencies of data are checked through editing?

Ans. Data are edited to remove mainly four deficiencies which are:

(i) Completeness of data;

(ii) Consistency of data;

(iii) Accuracy of data; and

(iv) Homogeneity of data.

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s) in the blanks:

1. The statement, "Not all numbers are statistical; logarithms for instance, are merely abstract numbers. Statistical data are concrete numbers, which represents objects", was given by ________ and ________.
2. "Great numbers are not counted correctly to a unit, they are estimated", is the statement of ________.
3. The definition, "Statistics is the science which deals with the collection, classification and tabulation of numerical facts as the basis for explanation, description and comparison of phenomena", was given by ________.
4. "Statistics is a body of methods for making wise decisions in the face of uncertainty" is the definition of statistics given by ________ and ________.
5. Statistics is both, a ________ and an ________ .
6. The credit of the statement, "A statistician is a practitioner of the art and science of statistics" goes to ________.
7. "The purpose of statistical methods is to simplify great bodies of numerical data" is the statement given by ________.
8. The definition, "The essence of statistics is not mere counting but comparison", was given by ________.
9. The statement, "Statistics enables one to enlarge his horizon", goes in the name of ________.
10. The statement, "Planning is the order of the day and without statistics planning is inconceivable" was given by ________.
11. The author of the statement, "The science of statistics is a most useful servant, but only of great value to those who understand its proper use", was due to ________.
12. Statistics can prove ________.
13. Use of statistical methods is most dangerous in the hands of ________.
14. The statement, "On average a factory labour has become younger in 1991 as compared to 1981", is ________.
15. Statistics deals with only ________.
16. Statistical analysis helps in the ________ of results.
17. Statistics is not applicable to ________ observation.
18. Not a ________ but data are the subject-matter of statistics.
19. Statistics are numerical ________ of facts, but all numerical statements are not ________.
20. By statistics we mean quantitative data affected to a marked extent by ________ of causes. (*Yule & Kendall*)
21. Statistics does not study ________.
22. Statistics is not a science, its a ________.
23. Statistics are the straw out of which I like every other economist, have to make ________. (*Marshall*)
24. Statistics is the arithmetic of human ________.
25. Planning on the basis of inadequate and inaccurate statistics is ________ than no planning at all. (*Third-Five-Year Plan*, Planning Commission).
26. Statistics is liable to be ________.
27. The data collected from published reports is known ________ data.
28. Data obtained by conducting a survey is called ________ data.
29. Before analysis, the data should be ________.

30. _________ units are better than arbitrary units.

31. _________ are used in a mailed enquiry method.

32. Mailed enquiry method cannot by adopted if the respondents are _________.

33. In personal enquiry method, the response is better than _________ method.

34. Pretesting is essential for preparing a good _________ or _________.

35. Population figures published by the Census Commissioner are _________ data.

36. Mistakes and statistical errors are _________.

37. If a quantity is such that all errors tend to be in the same direction, they are called _________ errors.

38. The errors caused by the carelessness of the investigators are called _________ errors.

39. Formula for the estimation of biased absolute error is _________.

40. Formula for the estimation of unbiased absolute error is _________.

41. Biased relative error can be estimated by the formula _________.

42. Unbiased relative error can be estimated by the formula _________.

43. A survey in which information is collected from each and every individual of the population is known as _________.

44. Assigning number digits to various responses whether quantitative or qualitative is called _________.

45. A figure 16,318.7 rounded to the nearest tenth place is _________.

46. The figure 32,627 rounded to the nearest hundredth place is _________.

47. The figure 32,627 approximated to the thousandth place by the method of discarding figure is _________.

48. The figure 45,067 approximated to thousandth place by the method of adding figure is _________.

49. The figure 13.85 rounded to one decimal place is _________.

50. The figure 13.75 rounded to one decimal place is _________.

51. Government cannot do proper planning without the help of _________.

52. Observations collected through surveys or experiments are classified as _________.

53. To know the area under cultivation of wheat, the appropriate type of investigation is _________.

54. To know the average yield of a crop, an appropriate investigation type will be _________.

55. The compendium which gathers all the facts and fascinating memories in a assimilable record is known as _________.

SECTION-C

Multiple Choice Questions

Select the correct alternative out of given ones:

Q. 1 The statement, "Statistics is both a science and an art", was given by:

(a) R.A. Fisher
(b) Tippet
(c) L.R. Connor
(d) A.L. Bowley

Q. 2 Who stated that statistics is a branch of applied mathematics which specialises in data?

(a) Horace Secrist
(b) R.A. Fisher
(c) Ya-lun-chou
(d) L.R. Connor

Q. 3 The word 'statistics' is used as:

(a) Singular
(b) Plural
(c) Singular and plural both
(d) none of the above

Q. 4 "Statistics provides tools and techniques for research workers", was stated by:

(a) John I. Griffin
(b) W.I. King
(c) A.M. Mood
(d) A.L. Boddington

Q. 5 Out of various definitions given by the following workers, which definition is considered to be most exact?

(a) R.A. Fisher
(b) A.L. Bowley
(c) M.G. Kendall
(d) Cecil H. Meyers.

Q. 6. Who stated that there are three kinds of lies: lies, damned lies and statistics.

(a) Mark Twin
(b) Disraeli
(c) Darrell Huff
(d) none of the above

Q. 7 Who gave the statement, "A well wrapped statistics is better than Hitler's 'biglie', it misleads, yet it cannot be pinned on you."

(a) Mark Twin
(b) W.A. Neiswanger
(c) Darrell Huff
(d) G.W. Snedecor

Q. 8 Which of the following represents data?

(a) a single value
(b) only two values in a set
(c) a group of values in a set
(d) none of the above

Q. 9 Statistics deals with:

(a) qualitative information
(b) quantitative information
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 10 Statistical results are,

(a) cent per cent correct
(b) not absolutely correct
(c) always incorrect
(d) misleading

Q. 11 If '*a*' is the actual value and '*e*' is its estimated value, the absolute error is:

(a) $a - e$
(b) $| a - e |$
(c) a/e
(d) $(a - e)/e$

Q. 12 If '*a*' is the actual value and '*e*' is its estimated value, the formula for relative error is:

(a) a/e
(b) $(a - e)/e$
(c) $| a - e |/e$
(d) $(a - e)/a$

Q. 13 Data taken from the publication, '*Agricultural Situation in India*' will be considered as:

(a) primary data
(b) secondary data
(c) primary and secondary data
(d) neither primary nor secondary data

Q. 14 Mailed questionnaire method of enquiry can be adopted if respondents:

(a) live in cities
(b) have high income
(c) are educated
(d) are known

Q. 15 The statement, "Designing of an appropriate questionnaire itself wins half the battle", was given by:

(a) A.R. Ilersic
(b) W.I. King
(c) H. Huge
(d) H. Secrist

Q. 16 Statistical data are collected for,

(a) collecting data without any purpose
(b) a given purpose
(c) any purpose
(d) none of the above

Q. 17 Relative error is always:

(a) positive

(b) negative
(c) positive and negative both
(d) zero

Q. 18 Statistical error refers to:

(a) Original value – Approx. value
(b) Actual value – Estimated value
(c) $\dfrac{\text{Actual value} - \text{Estimated value}}{\text{Estimated value}}$
(d) $\dfrac{\text{Actual value} - \text{Estimated value}}{\text{Actual value}}$

Q. 19 Method of complete enumeration is applicable for:

(a) Knowing the production
(b) Knowing the quantum of export and import
(c) Knowing the population
(d) all the above

Q. 20 A statistical population may consist of:

(a) an infinite number of items
(b) a finite number of items
(c) either of (a) and (b)
(d) none of (a) and (b)

Q. 21 Which of the following example does not constitute an infinite population?

(a) Population consisting of odd numbers
(b) Population of weights of newly born babies
(c) Population of heights of 15-year-old children
(d) Population of head and tails in tossing a coin successively.

Q. 22 Which of the following can be classified as hypothetical population?

(a) All labourers of a factory
(b) Female population of a country
(c) Population of real numbers between 0 and 100
(d) students of the world

Q. 23 A study based on complete enumeration is known as:

(a) sample survey
(b) pilot survey
(c) census survey
(d) none of the above

Q. 24 If the actual value of a unit is 415 and its estimated value is 400, the absolute error is:

(a) −15
(b) 15
(c) 0.0375
(d) − 0.0361

Q. 25 If the estimated value of an item is 50 and its actual value is 60, the relative error is:

(a) −20
(b) 0.16
(c) 1.2
(d) 0.20

Q. 26 Who originally gave the formula for the estimation of errors?

(a) L.R. Connor
(b) W.I. King
(c) A.L. Bowley
(d) A.L. Boddington

Q. 27 Boddington gave the formula for the estimation of errors of the type:

(a) Absolute error biased
(b) Absolute error unbiased
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 28 Boddington's formula for estimation of absolute error (A.E) is:

(a) Total A.E/ $\sqrt{n}$
(b) Average A.E × $\sqrt{n}$
(c) Total A.E/n
(d) Average A.E × n

(n = No. of items)

Q. 29 Boddington's formula for estimation of relative error is:

(a) Total A.E $\sqrt{n}$
(b) Average A.E/n
(c) $\dfrac{\text{Average A.E} \times n}{e}$
(d) $\dfrac{\text{Average A.E} \times \sqrt{n}}{e}$

[e = estimated value]

Q. 30 Bowley's formula for absolute unbiased error is:

(a) $\frac{1}{\sqrt{n}}$ Average (A.E)

(b) $\frac{2}{3\sqrt{n}}$ (Average A.E)

(c) $\frac{2\sqrt{n}}{3}$ (Average A.E)

(d) $\frac{3}{2\sqrt{n}}$ (Average A.E)

Q. 31 Statistical results are:

(a) absolutely correct
(b) not true
(c) true on average
(d) universally true

Q. 32 The statistical law(s) based on trial and error methods is/are:

(a) law of statistical regularity
(b) law of inertia of large numbers
(c) both laws (a) and (b)
(d) none of the laws (a) and (b)

Q. 33 The figure 32,64,616.8 approximated to the tenth place by the method of discarding figure is:

(a) 32,64,615.8
(b) 32,64,616
(c) 32,64,620
(d) 32,64,610

Q. 34 The figure 32,64,616.8 approximated to the tenth place by adding figure is:

(a) 32, 64, 615
(b) 32, 64, 616
(c) 32, 64, 620
(d) 32, 64, 610

Q. 35 The figure 26,476 approximated to the hundredth place by discarding figure is:

(a) 26,400
(b) 26,500
(c) 27,000
(d) 25,000

Q. 36 The figure 47, 616 approximated to hundredth place by adding figure is:

(a) 47,630
(b) 47,620
(c) 47,700
(d) 47,600

Q. 37 The figure 43,572.6 approximated to the thousandth place by discarding figure is:

(a) 43,500
(b) 43,000
(c) 44,000
(d) 44,500

Q. 38 The value 43,572.6 approximated to the thousandth place by adding figure is:

(a) 43,500
(b) 43,000
(c) 44,000
(d) 44,600

Q. 39. The figure 45,986 approximated to the ten thousandth place by the method of discarding figure is:

(a) 40,000
(b) 46,000
(c) 45,500
(d) 45,000

Q. 40 The figure 45,986 approximated to ten thousandth place by the method of adding figure is:

(a) 50,000
(b) 46,000
(c) 40,000
(d) none of the above

ANSWERS

SECTION-B

(1) William A. Spurr and Charles P. Bonini (2) A. L. Bowley (3) Lovitt (4) W.A. Wallis and H.V. Roberts (5) science, art (6) C.H. Meyers (7) A.E. Waugh (8) A.L. Boddington (9) Whipple (10) Tippet (11) W.I. King (12) anything (13) inexperts (14) acceptable (15) quantitative data (16) interpretation (17) single (18) datum (19) statements; statistics (20) multiplicity (21) individuals (22) scientific method (23) bricks (24) welfare (25) worse (26)

misused (27) secondary (28) primary (29) edited (30) Physical (31) Questionnaires (32) illiterate (33) mailed enquiry (34) schedule; questionnaire (35) primary (36) not same (37) biased (38) unbiased (39) Average A.E × No. of items (40) Average A.E × $\sqrt{\text{No. of items}}$ (41) Biased A.E/Estimated value (42) $\frac{\text{Unbiased A.E}}{\text{Estimated value}}$ (43) census survey (44) coding (45) 16,320 (46) 32,600 (47) 32,000 (48) 46,000 (49) 13.8 (50) 13.8 (51) statistics (52) primary data (53) census method (54) sample method (55) statistical perspective.

SECTION-C

(1) b (2) b (3) c (4) c (5) a (6) b
(7) c (8) c (9) b (10) b (11) b (12) c
(13) b (14) c (15) c (16) b (17) c (18) b
(19) d (20) c (21) c (22) c (23) c (24) b
(25) d (26) d (27) b (28) b (29) d (30) b
(31) c (32) c (33) d (34) c (35) a (36) c
(37) b) (38) c (39) a (40) a

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Chapter **2**

Classification, Tabulation and Frequency Distribution

SECTION-A

Short Essay Type Questions

Q. 1 What is meant by classification?

Ans. Classification is the process of arranging things or items in groups or classes according to their resemblance and affinities and give expression to the units of attributes that may subsist amongst the diversity of individuals.

Q. 2 What are the modes of classification?

Ans. Different modes of classification are:

(i) *Geographical classification*: classification is according to place, area or region.

(ii) *Chronological classification*: It is according to the lapse of time, e.g., monthly, yearly, etc.

(iii) *Qualitative classification*: Data are classified according to the attributes of the subjects or items, e.g., sex, qualification, colour, etc.

(iv) *Quantitative classification*: Data are classified according to the magnitude of the numerical values, e.g., age, income, height, weight, etc.

Q. 3 What are the objectives of classification?

Ans. Broadly, there are six objectives of classification:

(i) To present the facts in a simple manner.

(ii) To highlight items which possess or do not possess certain attributes or qualities.

(iii) To provide help in making comparison between items.

(iv) To find out mutual relationship between certain measures and their effects.

(v) To present the data in a manner which is suitable for further treatment.

(vi) To provide basis for tabulation.

Q. 4 What do you understand by qualitative classification?

Ans. It is the classification on the basis of certain attributes or some qualities of items which cannot be measured quantitatively.

Q. 5 Describe in brief different kinds of classification.

Ans. Different types of classification are:

(i) Classification according to attributes.

(ii) Simple or two-fold or dichotomous classification

(iii) Multiple classification.

(iv) Quantitative classification, *i.e.*, the classification according to variate values.

Q. 6 What do you understand by multifactor classification?

Ans. Classification criteria based on two or more factors (attributes) is known as multifactor classification. In this type of classification, first the data are classified into two or more classes on the basis of one factor. For each component classification, further classification is done on the basis of second factor and so on.

Q. 7 What do you understand by open end(s) in group data?

Ans. If in grouped classes, the lower limit of the beginning class is not specified and/or the upper limit of the highest (last) class is not specified, it is known as grouped data with open end class(es).

Q. 8 What are different characteristics of classification? Describe each characteristic in five lines.

Ans. Different characteristics of classification are:

(i) *Exhaustive*: The classes should be such that they cover every item of the set, *i.e.*, they should also be complete and non-overlapping. For instance, for marital status the classes should be married, unmarried, widow, widower, divorcee, deserted.

(ii) *Stability*: Classification should be uniform or standardised so that the results are comparable at different occasions or in different studies.

(iii) *Flexibility*: Classification should be amenable according to different situations or requirements of study.

(iv) *Homogeneity*: The units of measurement of all classes should be same. Also like units only be accommodated in one class.

(v) *Suitability*: Classification be done according to the objective of the study only. For instance, to study the financial status of people, it will be useless to classify them according to their skin colour or their hair colour, etc.

(vi) *Arithmetic accuracy*: The sum of number of units in all classes should be equal to the total number of units. Also in case of observations, the sum of observations in all classes should be equal to the sum of all observations.

Q. 9 How can one determine the number of classes for a frequency distribution?

Ans. In quantitative classification, the number of classes depends upon the class interval. So a formula was suggested by H.A. Sturges to determine the class interval and also the number of classes. The formula is,

$$i = \frac{L - S}{1 + 3.322 \log_{10} n}$$

where,

i = class interval

L = Largest observation

S = Smallest observation

and n = total number of observations in the set.

Also, the denominator, $1 + 3.322 \log_{10} n$ is equal to the number of classes.

Q. 10 Describe in brief the grouping error.

Ans. If the classes are formed in such a way that the frequencies are evenly distributed throughout the class interval, it is justified to assume that the frequencies are centered at the mid-value of the class. But in cases where such an assumption is not valid, it leads to error which is known as grouping error. Grouping error affects the accuracy of the results.

Q. 11 Clarify the difference between exclusive and inclusive class intervals.

Ans. Following are the differences between exclusive and inclusive class intervals:

(i) In exclusive class intervals, the upper limit of a class is the lower limit of the next class. Also the upper limit of a class is not included in that class.

(ii) In inclusive class intervals the upper limit of a class instead is not the lower limit of the next class. The lower limit is generally greater by unit measurement.

(iii) In inclusive method, both the limits of a class are included.

(iv) To simplify the calculation procedure, inclusive classes are converted into exclusive classes.

(v) Inclusive classes approach is suitable in case of data given in whole numbers. In rest of the cases exclusive class approach is suitable.

Q. 12 If mid-values of the classes are known, how can the classes be formed?

Ans. Find the difference between two consecutive mid-values. Subtract half of the difference from the mid-value and again add it to the mid-value. The values obtained on subtracting and adding half of the difference are the lower and upper limits of the class of which the mid-value has been used. If m is the mid-value of a class and i is difference between two consecutive mid-values, the lower and upper class limits are $\left(m-\frac{i}{2}\right)$ and $\left(m+\frac{i}{2}\right)$ respectively.

Q. 13 Illustrate exclusive and inclusive class intervals.

Ans. In exclusive class intervals uppers limit of the class is not included, e.g., in the class 10-20 those values are included which are 10 or more and less than 20. Similarly in the inclusive class intervals, both the limits of a class are included. The classes may be of the type, 5 – 5.99, 10 – 14.99, etc.

Q. 14 Distinguish between real limits and apparent class limits of a distribution in grouped data.

Ans. If the distribution is for a discrete variable, the real and apparent class limits are same, e.g., 5-10, 11-16, 17-22, ... since there is no recorded value between 10 & 11, 16 & 17, etc.

But if the class intervals are exclusive, in that case either the upper limit or the lower limit is to be excluded since the value can be included in one class only. For instance, let the classes be 5-10, 10-15, 15-20, etc. Suppose the upper limits are excluded. In that case, the real limits are 5-9, 10-14, 15-19, etc. Of course there is no point in between 9 & 10, 14 & 15, etc. If the lower limits are excluded, the real limits are 6-10, 11-15, 16-20, etc.

Q. 15 What do you understand by tabulation?

Ans. It is the process of presenting data collected through survey, experiment or record in rows and columns so that it can more easily be understood and can be used for further statistical analysis.

Q. 16 What are different parts of a standard table?

Ans. There are five parts of a table.

(i) Title, (ii) captions and stubs, (iii) Body; (iv) Prefatorial, and (v) Source note.

Q. 17 What are the objectives of tabulation of data?

Ans. The objectives of tabulation are:

(i) To clarify the object of investigation.

(ii) To reduce complexity of data

(iii) To economise space

(iv) To depict the relation among data if it exists.

(v) To facilitate analysis of data.

Q. 18 What are the requisites of a standard table?

Ans. Requisites of a standard table are:

(i) It should be suitable for the purpose.

(ii) Clarity and completeness of table is necessary.

(iii) Table should be of adequate size.

(iv) Units of measurements should be specified.

(v) Logical arrangement of items.

(vi) Totals and sub-totals be given.

Q. 19 What are the main purposes of tabulation?

Ans. The main purposes of tabulation are:

(i) To present the haphazard data in simple and concised manner.

(ii) To save space.

(iii) To show the trend of data, if any.

(iv) To facilitate comparison of data.

(v) To detect errors and omissions of data, if any.

(vi) To facilitate the process of statistical analysis.

(vii) To know the source of data.

Q. 20 What is the difference between classification and tabulation?

Ans. Classification is meant for arranging the data into characteristics or groups where each group has the number of item attached to it. In case of variables, it is given in the form of frequency distribution.

Tabulation is the logical and systematic arrangement of data in rows and columns. In a table, data may be presented in modified form as well, e.g., in per cent, proportion, total or average values, etc.

Q. 21 What is an original table (classification table)?

Ans. In an original table, the data are presented in the same form in which they are collected.

Q. 22 What is a derivative table?

Ans. In a derivative table the data are not presented in its original form but the values based on original observations are presented. For instance, totals for different classifications, the means, percentage, ratios or proportions, etc., are presented in a derivative table.

Q. 23 Distinguish between frequency and cumulative frequency.

Ans. *Frequency*: Number of times a variate value is repeated is called its frequency.

Cumulative frequency: This is the number of observations corresponding to less than (more than) or equal to a specified value.

Q. 24 Differentiate between a time series and a spatial series.

Ans. (i) Time series is an ordered data arranged in sequence of time period. Time periods may be weekly, monthly, yearly, quinquennially, decadal, etc. Time series is also known as *historic series.* (ii) Spatial series is one in which the data are arranged according to the place or space. The place or space may be localities, cities, states, countries, etc.

Q. 25 Explain briefly the stem and leaf display of data.

Ans. It is a method of presentation of data in which each value is divided into two parts. One part consists of one or more leading digits as stem and remaining of the digits as leaf.

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s) in the blanks:

1. Classification is the ________ of facts that are distinguished by some significant ________.
2. For a good classification, the class should be ________ and ________.
3. Classification can be done according to ________.
4. Quantitative classification leads to ________.
5. Yearwise recording of data of food production will be called ________ classification.
6. The census data published for citywise population in India will be known as ________ classification.
7. The data recorded according to standard of education like illiterate, primary, secondary, graduate, technical, etc., will be known as ________ classification.
8. Distribution of families according to their size will be classified as ________classification.
9. The difference between the upper and lower limit of a class is called ________.
10. The average of the upper and lower limits of a class is known as ________.
11. There is a general assumption that the class frequency is centered at the ________ of the class.
12. Departure from the assumption that the frequencies are evenly distributed over the class interval leads to ________ error.
13. Formula for determining the number of classes was given by ________.
14. H.A. Sturges formula for determining the number of classes is ________.
15. Number of classes depend on ________.
16. H.A. Sturges formula for finding out the class interval is ________.
17. The number of classes and class interval for the distribution of marks from 0 to 100 of 50 students of a class should be ________ and ________ respectively.

18. Class boundaries are also sometimes called ________ limits.

19. Mid-values of the classes are also called ________.

20. Frequency density of a class is the frequency ________ of class.

21. In the mid-value of a class interval is 20 and the difference between two consecutive mid-values in 5, the class limits are ________ and ________.

22. An arrangement of data in rows and columns is known as ________.

23. Tables help in ________ of data.

24. Tabulation makes the data easily ________.

25. Tabulation follows ________.

26. ________ facts cannot be presented in the form of a table.

27. A general purpose table is also known as ________ or ________ table.

28. A general table is a ________ of a large amount of data.

29. The table which do not present the data but the results of analysis are called ________ tables.

30. Headings of the columns of a table are known as ________.

31. Headings of the rows of a table are called ________.

32. The portion of the table in which data are presented is designated as ________ of a table.

33. The manner in which the frequencies are distributed according to variate values is known as ________.

34. Frequency distributions are often constructed with the help of ________.

35. Relative frequency is the ratio of a frequency to the ________ of the distribution.

36. Percentage frequency is the ________ multiplied by 100.

37. Frequencies added successively in an ordered series giving the number of items up to that value are called ________.

38. A frequency distribution with upper limits of classes and corresponding cumulative frequencies is known as ________ type distribution.

39. A frequency distribution with lower limits of classes and corresponding cumulative frequencies is known as ________ type distribution.

40. The graphs of less than type and more than type distributions intersect at ________.

41. A grouped series, in which either the lower limit of the first group or the upper limit of the last group is missing or both, is called an ________.

42. A series arranged in accordance with each and every observation is known as ________.

43. The distribution of frequencies according to individual variate values is called ________ distribution.

44. A series of data with exclusive classes along with the corresponding frequencies is called ________ distribution.

45. Given the following frequency distribution, fill in the missing frequencies:

Class intervals	*Frequency*	*Cumulative Frequency*
10-20	5	5
20-30	10	15
30-40	12	–
40-50	–	–
50-60	7	–
60-70	4	52

SECTION-C

Multiple Choice Questions

Select the correct alternative out of given ones:

Q. 1 Numerical data presented in descriptive form are called
(a) classified presentation
(b) tabular presentation
(c) graphical presentation
(d) textual presentation

Q. 2 Whether classification is done first or tabulation?
(a) Classification follows tabulation.
(b) Classification precedes tabulation.
(c) Both are done simultaneously.
(d) No criterion.

Q. 3 For the mid-values given below,
25, 34, 43, 53, 61, 70
The first class of the distribution is:
(a) 24.5-34.5
(b) 25-34
(c) 20-30
(d) 20.5-29.5

Q. 4 In an exclusive type distribution, the limits excluded are:
(a) lower limits
(b) upper limits
(c) either of the lower or upper limit
(d) lower limit and upper limits both

Q. 5 A series showing the sets of all distinct values individually with their frequencies is known as:
(a) grouped frequency distribution
(b) simple frequency distribution
(c) cumulative frequency distribution
(d) none of the above

Q. 6 A series showing the sets of all values in classes with their corresponding frequencies is known as:
(a) grouped frequency distribution
(b) simple frequency distribution
(c) cumulative frequency distribution
(d) none of the above

Q. 7 If the number of students in a school is 200 and maximum and minimum marks earned are 90 and 10 respectively, for the distribution of marks, the class interval (rounded) is:
(a) 10
(b) 9
(c) 12
(d) none of above
[Given $\log_{10}2 = 0.3010$, $\log_{10}3 = 0.4771$]

Q. 8 If the lower and upper limits of a class are 10 and 40 respectively, the mid-points of the class is:
(a) 25.0
(b) 12.5
(c) 15.0
(d) 30.0

Q. 9 In a grouped data, the number of classes preferred are:
(a) minimum possible
(b) adequate
(c) maximum possible
(d) any arbitrarily chosen number.

Q. 10 Class interval is measured as:
(a) The sum of the upper and lower limit
(b) half of the sum of lower and upper limit
(c) half of the difference between upper and lower limit
(d) the difference between upper and lower limit.

Q. 11 The class interval of the continuous grouped data:

10-19
20-29
30-39
40-49
50-59

is:
(a) 9 (b) 10 (c) 14.5 (d) 4.5

Q. 12 A grouped frequency distribution with uncertain first or last classes is known as:

(a) exclusive class distribution
(b) inclusive class distribution
(c) open end distribution
(d) discrete frequency distribution

Q. 13 The distribution,

Values	*Frequency*
Less than 5	5
Less than 10	12
Less than 15	21
Less than 20	27
Less than 25	31
Less than 30	33

is of the type:

(a) inclusive class type
(b) exclusive class type
(c) discrete type
(d) none of the above

Q. 14 Data can be well displayed or presented by way of:

(a) stem and leaf display
(b) cross classification
(c) two or more dimensional table
(d) all the above

Q. 15 A simple table represents:

(a) only one factor or variable
(b) always two factors or variables
(c) two or more number of factors or variables
(d) all the above

Q. 16 A complex table represents:

(a) only one factor or variable
(b) always two factors or variables
(c) two or more factors or variables
(d) all the above

Q. 17 The headings of the rows given in the first column of a table are called:

(a) stubs
(b) captions
(c) titles
(d) prefatory notes.

Q. 18 The column heading of a table are known as:

(a) sub-titles
(b) stubs
(c) reference notes
(d) captions

Q. 19 The series,

Place	*No. of accidents per day*
Delhi	10
Kolkata	15
Mumbai	18
Chennai	17
Indore	7

is of the type:

(a) Spatial
(b) Geographical
(c) Industrial
(d) Time series

Q. 20 The series,

Year	*Production of food (M. Tonnes)*
1987	160
1988	168
1989	170
1990	172
1991	174
1992	176

is categorised as:

(a) individual series
(b) continuous series
(c) discrete series
(d) time series

Q. 21 The income of five persons is as follows:

Person	*Income (Rs/Month)*
Mr A	1,700
Mr B	2,300
Mr C	7,000
Mr D	8,500
Mr E	5,400

The above series is of the type:

(a) Individual series
(b) Discrete series
(c) Continuous series
(d) Time series

Q. 22 The series,

Marks	*No. of Students*
20-30	5
30-40	14
40-50	24
50-60	12
60-70	9
70-80	2

is of the type:

(a) Discrete series
(b) Continuous series
(c) Individual series
(d) none of the above

Q. 23 A frequency distribution can be:

(a) discrete
(b) continuous
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 24. In an individual series, each variate value:

(a) has same frequency
(b) has frequency one
(c) has varied frequency
(d) has frequency two

Q. 25 Which of the following statements is true?

(a) An individual series is a particular case of discrete series
(b) An individual series is a particular case of continuous series
(c) An individual series is a special case of discrete and continuous series
(d) There is nothing like individual series

Q. 26 Frequency of a variable is always:

(a) in percentage
(b) a fraction
(c) an integer
(d) none of the above

Q. 27 The data given as, 5, 7, 12, 17, 79, 84, 91 will be called as:

(a) a continuous series
(b) a discrete series
(c) an individual series
(d) time series

Q. 28 The following frequency distribution,

x :	12,	17,	24,	36,	45,	48,	52
f :	2,	5,	3,	8,	9,	6,	1

is classified as:

(a) continuous distribution.
(b) discrete distribution.
(c) cumulative frequency distribution
(d) none of the above

Q. 29 In an ordered series, the data are:

(a) in ascending order
(b) in descending order
(c) either (a) or (b)
(d) neither (a) or (b)

Q. 30 The following frequency distribution,

Classes	*Frequency*
0-10	3
0-20	8
0-30	14
0-40	20
0-50	25

is known as:

(a) continuous frequency distribution
(b) discrete frequency distribution
(c) cumulative distribution in more than type
(d) cumulative distribution in less than type

Q. 31 The following frequency distribution,

Classes	*Frequency*
0-15	17
0-10	8
0-5	3

is classified as:

(a) cumulative distribution in less than type
(b) cumulative distribution in more than type
(c) discrete frequency distribution
(d) cumulative frequency distribution

Q. 32 Classification is applicable in case of:

(a) quantitative characters
(b) qualitative characters
(c) both (a) and (b)
(d) none of the above

ANSWERS

Section-B

(1) grouping; characteristics (attributes) (2) exhaustive; mutually exclusive (3) attributes (4) frequency distribution (5) Chronological (6) geographical (7) qualitative (8) quantitative (9) class interval (10) mid-value (11) mid-value (12) grouping (13) H.A. Sturges (14) $1 + 3.322 \log_{10} n$ (15) class interval (16) $(L - S) /(1 + 3.322 \log_{10} n)$ where L = largest value, S = smallest value and n = No. of values (17) 7 and 15 (18) mathematical (19) class marks (20) per unit (21) 17.5 and 22.5 (22) tabulation (23) analysis (24) understandable (25) classification (26) qualitative (27) primary or reference (28) repository (29) specific purpose (30) captions (31) stubs (32) body (33) frequency distribution (34) tally marks (35) total frequency (36) relative frequency (37) cumulative frequency (38) less than (39) more than (40) median (41) open end series (42) individual series (43) discrete frequency (44) continuous frequency (45) freq = 14; as. cu-freq-27, 41, 48.

SECTION-C

(1) d (2) b (3) d (4) c (5) b (6) a
(7) b (8) a (9) b (10) d (11) b (12) c
(13) b (14) d (15) a (16) c (17) a (18) d
(19) b (20) d (21) a (22) b (23) c (24) b
(25) a (26) c (27) c (28) b (29) c (30) d
(31) b (32) c

Suggested Reading

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Chapter **3**

Diagramatic and Graphical Representation

SECTION-A

Short Essay Type Questions

Q. 1 What are the advantages of diagramatic representation of data?

Ans. Following are the advantages of diagramatic representation of data:

(i) Diagrams give a bird's-eye view of complex data.
(ii) They have long lasting impression.
(iii) Easy to understand even by a common man.
(iv) They save time and labour.
(v) They facilitate comparison.

Q. 2 Give the names of diagrams which are one-dimensional.

Ans. Bar diagrams and line diagrams are one-dimensional. Different types of bar diagrams are:

(i) simple bar diagram
(ii) multiple bar diagram
(iii) sub-divided or component bar diagram
(iv) percentage bar diagram.

Q. 3 Name diagrams which are two-dimensional.

Ans. Rectangles, circles and pie diagrams are two-dimensional diagrams.

Q. 4 What are the diagrams categorised under three-dimensional diagrams?

Ans. Cubes, cylinders and spheres are categorised as three-dimensional diagrams.

Q. 5 What types of diagram are known as non-dimensional diagrams?

Ans. Pictograms are known as non-dimensional diagrams.

Q. 6 What do you understand by bar diagram and a sub-divided bar diagram?

Ans. A bar diagram represents the magnitude of a single factor according to time periods, places, items, etc. But when the magnitude of the factor is given with its sub-factors, each bar is further sub-divided into components in proportion to the magnitude of the sub-factors.

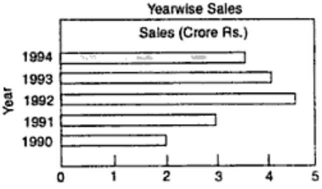

Fig. 3.1. Bar diagram

Such a diagram is known as sub-divided bar diagram.

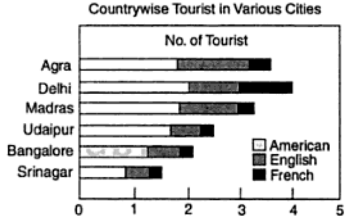

Fig. 3.2. Sub-divided bar diagram

Q. 7 When do you prefer a multiple bar diagram (compound bar diagram).

Ans. To depict a number of related factors for comparison in various years or at a number of places, multiple bar diagrams are preferable.

Q. 8 What is a multiple bar diagram?

Ans. In a multiple bar diagram, adjoining bars are drawn according to the number of factors and their heights in proportion to the values of the factors in the same order for each period or place. Each bar of a group is shown by different patterns or colours to make them easily distinguishable and this pattern is retained in all the groups. A constant distance is maintained between groups of bars drawn for periods or places. Such a diagram is known as multiple or compound bar diagram (Fig. 3.3).

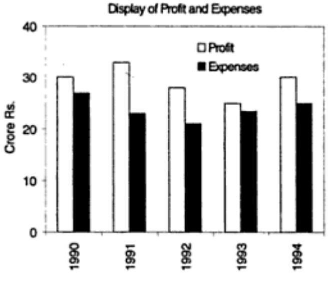

Fig. 3.3. Multiple bar diagram

Q. 9 What do you understand by deviation bar diagram?

Ans. Deviation bar diagrams are suitable to show the net deviations during various years or according to different countries or places, etc. In the deviation bar diagrams, positive deviations are shown to the right side of the base line and negative deviations are shown to the left side of the same base line. For instance, the gaps between imports and exports, profit and loss in different years or from different countries can be very well displayed through deviation bar diagrams (Fig. 3.4).

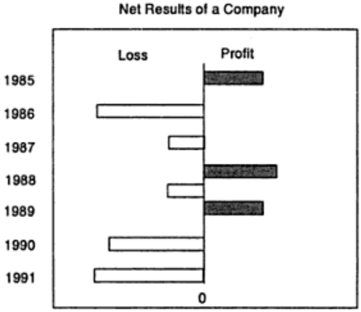

Fig. 3.4. Deviation bar diagram

Q. 10 Write a short note on duo-directional bar diagrams.

Ans. Duo-directional bar diagrams are used to exhibit the two aspects of a single factor at a glance given for different periods or places. In this type of diagram, one part of the bar remains above the base line and the other below the base line. The heights of the bars below and above the line are in proportion to the values of the two aspect separately whereas the bar as a whole represents the factor. For example, we want to show the price of certain item in different years. The price consists of two parts, the cost and the profit. So profit may be taken above the line and cost below the line. It is a sort of sub-divided bar diagram (Fig. 3.5).

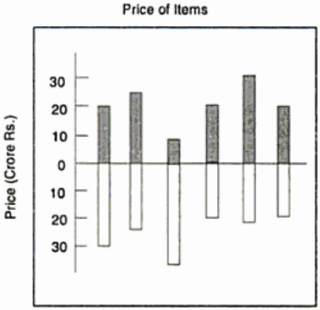

Fig. 3.5. Duo-directional bar diagram

Q. 11 Write a brief note on paired bar diagram.

Ans. When two related factors having different units of measurements are to be displayed for comparison in various periods or places, paired bar diagrams are suitable. In this diagram usually, the periods or places are shown in a strip and horizontal bars for each factor are drawn to the right and left of the vertical strip or vertical bars are drawn below and above the horizontal strip. For instance, area and production of paddy in different years in India can be very well displayed through a paired bar diagram (Fig. 3.6).

Q. 12 What are sliding bar charts? Explain in brief.

Ans. Sliding chart is a bilateral chart in which two components of a factor are represented by two parts of the bar. One part is on the left and the other is on the right of the base line. The scale may be the absolute numbers or in percentages. Such a chart is suitable in situations such as a numbers arrested in a criminal case or patients operated for different diseases. What percentage or number of suspects have been cleared and what are still under trial. How many patients are cured and how many of them still suffer. The two components can be better displayed by a sliding bar diagram. The base line may be representing the type of operations, kind of court cases, etc. The percentage or numbers in the two components, cured and not cured, cases cleared and not cleared may be changing from time to time. For each type of operation crime (factor), a separate sliding bar will be drawn.

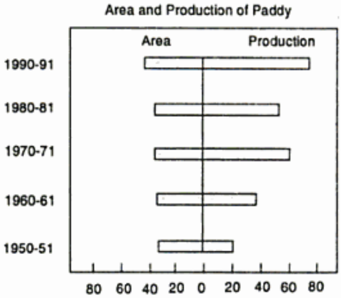

Fig. 3.6. Paired bar diagram

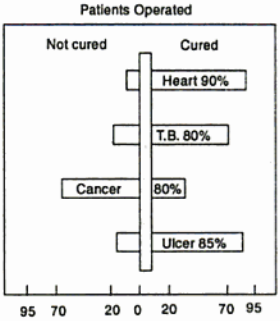

Fig. 3.7. Sliding bar chart

Q. 13 What is a broken bar diagram?

Ans. Often an investigator comes across cases where some figures are very large as compared to others. In this situation, if the scale is chosen for proper portray of small values by bars, the bars for large values will expand to a unpalatable size. Again, if the scale is chosen for proper display of large values by bars, the bars for small value will become non-existent. Hence, to remove this discrepancy, broken bars are constructed.

First, a small scale is taken and bars are erected at all periods or places up to the highest small values and/or a round off value. Then with a gap another base line and a new scale for large values is chosen. Bars are constructed for remaining value on the new base line. Such a bar diagram is known as *broken bar diagram*. These diagrams be interpreted very carefully to avoid any wrong conclusions.

Admissions in Various Subjects

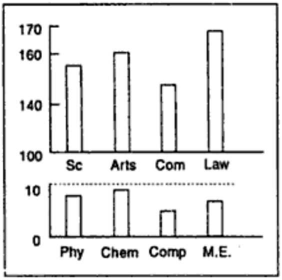

Fig. 3.8. Broken bar diagram

Q. 14 What is the basis of comparison in bar charts?

Ans. The basis of comparison in bar charts is linear or one directional.

Q. 15 What is the difference between bar diagrams (charts) and column charts (diagrams)?

Ans. In the bar diagrams, the bars are arranged horizontally on a vertical base line, whereas in the column charts, bar are arranged vertically on a horizontal base line.

Q. 16 What are the differences between a bar diagram and a rectangular diagram?

Ans. The differences are:

(i) In bar diagram all bars are of equal width.

(ii) The width of bars in bar diagram is chosen arbitrarily for qualitative characters like places, states, countries, etc., or according to the equal class intervals.

(iii) In a rectangular diagram, there is always a variable and its magnitude. The width of the rectangles are according to variate values and heights of the rectangles are according to their respective magnitudes.

(iv) In bar diagram, comparisons are based on the heights of bars only whereas in rectangular diagrams, comparisons are based on the area of the rectangles.

Q. 17 Explain a line diagram in two lines.

Ans. A line diagram is a one-dimensional diagram in which the height of the line represents the frequency corresponding to the value of the item or a factor.

No. of Accidents on Week Days

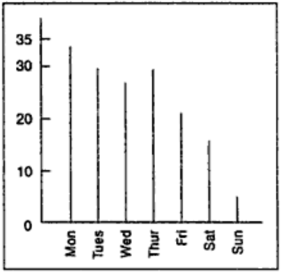

Fig. 3.9. Line diagram

Q. 18 Explain in brief a pie-chart.

Ans. A pie-chart is a circular diagram which is usually used for depicting the components of a single factor. The circle is divided into segments which are in proportion to the size of the components. They are shown by different patterns or colours to make them attractive (Fig. 3.10).

India's imports from various sources (per cent) 1985-86

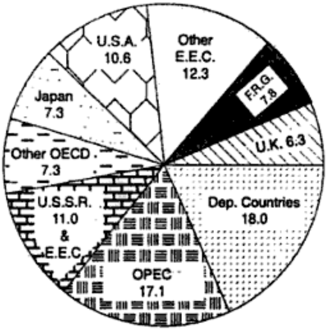

Fig. 3.10. Pie chart

Q. 19 Explain a histogram.

Ans. A histogram is a bar diagram which is suitable for frequency distributions with continuous classes. The width of all bars is equal to class interval and heights of the bars are in proportion to the frequencies of the respective classes. In this diagram bars touch each other but one bar never overlaps the other (Fig. 3.11).

Distribution of Wages

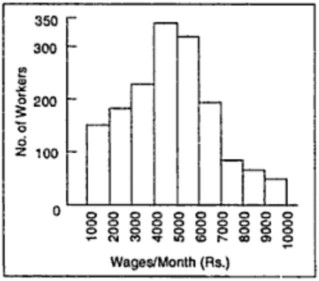

Fig. 3.11. Histogram

Q. 20 Describe a frequency polygon.

Ans. When the mid-points of the tops of the adjacent bars of a histogram are joined in order, then the graph of lines so obtained is called a frequency polygon.

Frequency Distribution of Wages

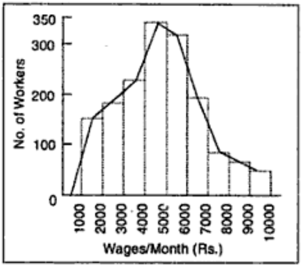

Fig. 3.12. Frequency polygon

Q. 21 Discuss a frequency curve in brief.

Ans. A frequency curve is a graphical representation of frequencies corresponding to their variate values by a smooth curve. A smoothened frequency polygon represents a frequency curve.

Distribution of Marks

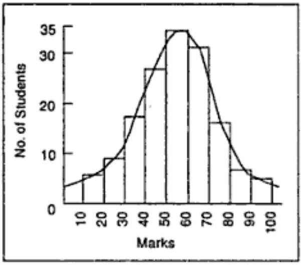

Fig. 3.13. Frequency curve

Q. 22 What is a false line in reference to a graph?

Ans. When the variation in the magnitudes of a variable is small as compared to the variate values, the vertical scale chosen as zero at the origin fails to depict the fluctuation prominently as desired by the investigator. Hence a line parallel to abscissa (X-axis) is drawn a little above it and the point joining the ordinate (Y-axis) is taken as origin which usually represents the minimum value (an approximate value by discarding figure) from amongst the magnitudes to be taken on the vertical scale. Then proper scale is chosen measuring distances from this false base line. Such a process makes the fluctuations conspicuous.The graph so obtained is called *gee whiz* graph. False line on the graph is usually shown by a saw-tooth line.

Q. 23 What is the range curve (chart)?

Ans. To depict the spread of highest and lowest values of a time series, the band formed by the line graphs of highest and lowest values is known as *range curve.* Range curve can also be represented through bar segments drawn at each period which are of magnitudes of the differences between highest and lowest values. These bars start from the lowest price and go up to the highest value.

Q. 24 How do you draw a histogram when the widths of all classes are not equal?

Ans. When widths of all classes of a frequency distribution are not equal, heights of the bars are taken in proportion to the frequency density (frequency per unit interval).

Q. 25 What is a ratio chart or a semi-logarithmic graph?

Ans. It is a line graph obtained by plotting the points (x, log y) in such a way that the x-values are taken along X-axis on a natural scale and values log y are taken along Y-axis. Hence, ratio chart is a graph of the points (x, log y) plotted on an ordinary graph paper. It is also called semi-logarithmic graph in the sense that log-values are used only for y (Fig. 3.14).

Yield Per Hectare of Kharif Pulses

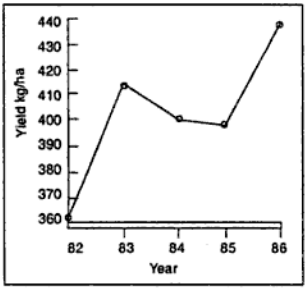

Fig. 3.14. Ratio chart

Q. 26. What do you understand by a graph?

Ans. A graph is a display of points and lines. In a graph of paired values (x, y), so called the co-ordinates of a point, are plotted on a graph paper by suitably choosing the scales along X-axis (abscissa) and Y-axis (ordinate). The plotted points are joined by straight lines in their sequence of occurrence. The figure so obtained is called a graph. The graph depicts the trend, fluctuations, variability, etc., very prominently (Fig. 3.15).

Two or more graphs made on the same graph paper having a common scale along X-axis and Y-axis facilitate the comparison of data tremendously.

Population and Availability of Pulses

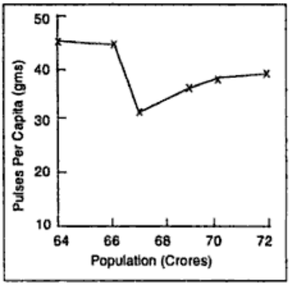

Fig. 3.15. Graph

Q. 27 Describe an ogive curve in brief.

Ans. It is graph plotted for the variate values and their corresponding cumulative frequencies of a frequency distribution. Its shape is just like elongated

S. An ogive curve is prepared either for more than type or less than type distribution.

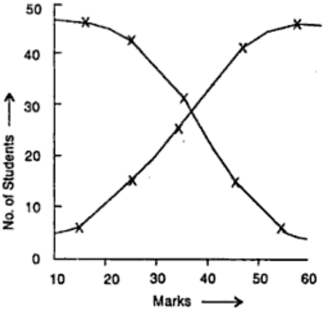

Fig. 3.16. Ogive curve

Q. 28 What are the uses of ogive curve?

Ans. Ogive curve is useful in finding out quartiles, deciles, percentiles, etc.

Q. 29 At what point the ogives for more than type and less than type distribution intersect?

Ans. The ogives for more than type and less than type distributions intersect at the median.

Q. 30 Explain briefly a Lorenz curve.

Ans. Lorenz Curve is a special type of cumulative frequency curve which is used to portray the data to indicate whether a factor is equally distributed in relation to the other factor for certain segment of the population. It was originally developed by M.O. Lorenz and is named after him.

Q. 31 How can one draw a Lorenz curve?

Ans. Following steps are involved while drawing a Lorenz curve:

1. The variate values giving information about the segment of population are ignored.
2. Cumulative totals for the magnitudes or frequencies of the two other factors are found out separately.
3. Cumulative totals are expressed as percentage of their respective grand totals.
4. Paired cumulative percentages are plotted on a graph paper choosing same scale along axes from 0 to 100.
5. Plotted points are joined by a smooth free hand curve. This curve always starts from the origin and terminates at the end point (100, 100) (Fig. 3.17).

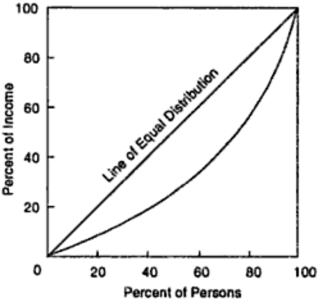

Fig. 3.17. Lorenz curve

Q. 32 What is the importance of the Lorenz curve?

Ans. In case of equal distribution of both the factors in the segment of population, the graph will be a straight line. But the Lorenz curve farther from straight line shows the inequality of distribution of two factors.

Q. 33 How are the data portrayed by pictograms?

Ans. In pictograms, the data are displayed by the pictures of the items to which the data pertain. A single picture represents a fixed number. For example, the population is shown by man, milk production by milk cans, fleets of aeroplane by the pictures of aeroplanes, etc.

Population of India in 1993 is 89.6 crore. This can be shown by a pictogram having the pictures of nine men, each man representing 10 crore. Pictograms are the least satisfactory type of diagrams. They are inaccurate too. Even then they are preferred by novice and dilettante people. Display of data through pictograms was initiated by Dr. Otto Neurath in 1923 (Fig. 3.18).

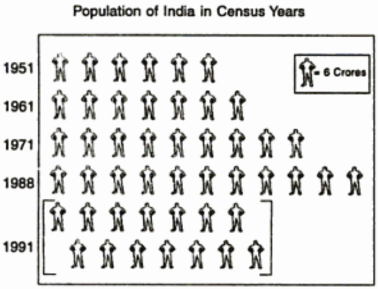

Fig. 3.18. Pictograms

Q. 34 What are column charts with circular base line?

Ans. It is a sort of bar diagram on circular base. Such column charts are very attractive and appealing to display monthly data of a year or hourly data of a day, etc. In this diagram, the periods are taken on a circle just like the dial of clock and equidistant concentric circles represent certain frequency. For instance, let the inner circle represent a frequency equal to 10, the second circle would represent 20 and so on.

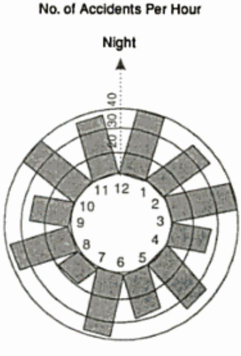

Fig. 3.19. Column chart on circular base

Columns (bars) are drawn on each period of the circular dial whereas the length of the column, measured through concentric circles, is equal to the frequency of that period. Such a chart is known as column chart on circular base (Fig. 3.19).

Q. 35 Describe briefly a step bar or column chart.

Ans. It is a technique for portraying components divisions of an aggregate or components percentages. In step bar chart, item names are taken along vertical base line and bars are taken horizontally on an arithmetic scale. In a step column chart, item names are taken along a horizontal base line and bars are taken vertically on an arithmetic scale. In this type of charts, the next bar or column starts where previous bar or column ends. Step bar or column charts are used as a substitute of pie-charts, although, these charts are not as appealing as the pie-charts.

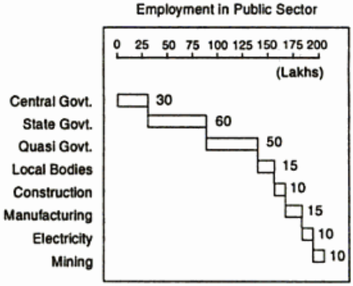

Fig. 3.20. Step bar chart

Q. 36 What are overlapping bar or column charts?

Ans. These are digression designed multiple bar or column charts. In this type of charts, one column (bar) penetrates into the next column (bar) by half of its width. Each column or bar is shown by different patterns or colours. Of course, such charts save space as the spread of group of columns or bars is sizeably reduced (Fig. 3.21).

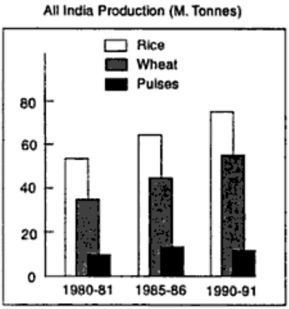

Fig. 3.21. Overlapping bar chart

Q. 37 Write six lines about trilinear chart.

Ans. The trilinear charts enables to display three variables simultaneously in the form of components or elements of a quantity as a whole. It is a 100 per cent chart, *i.e.*, the sum of the three components is always equal to 100 per cent. The trilinear chart is drawn in the form of a equilateral triangle by caliberating each side in percentage divisions ranging from 0 to 100. The lines are drawn parallel to the sides of the triangle in the manner of co-ordinates.

Q. 38 How do you define a chart, diagram or graph?

Ans. It is technique for portraying numerical facts through dots, lines, areas, volumes, other geometrical forms and symbols.

Q. 39 What are the advantages of charts, diagrams and graphs?

Ans. Charts, diagrams and graphs facilitate the presentation of quantitative values in a simple and easily understandable form. They facilitate comparison and analysis of data. Graphs in particular show fluctuations, trends and relationships between the variables. In comparison to tables and textual forms, charts and graphs are better and easily understood by a commoner.

Q. 40 What factors are responsible for the choice of a chart, diagram and graph?

Ans. The main factors responsible for the choice of a chart or graph are:

(1) Objective of the display of data.

(2) Type of data.

(3) Size of chart or graph.

(4) Audience for whom a chart or graph is prepared.

(5) Funds available.

(6) Predilection of the scientist.

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s) in the blanks:

1. Diagrams are another form of __________.
2. It is comparatively easier to understand __________ than numerical figures.
3. The ogives of less than type and more than type distributions intersect at __________.
4. Histogram and historigram are __________.
5. In a histogram bars __________ each other.
6. A histogram is __________ for geographical classification.
7. Frequency polygon can be drawn with the help of __________ .
8. __________ diagram can be sketched for geographical series.
9. Sub-divided bar diagram depicts the distribution of __________ of a factor.
10. Paired bar diagrams are suitable for the data of two related factors having __________ units of measurements.
11. __________ bar diagram is suitable for showing the differences between budget

provisions and actual expenditure of PWD in the last ten years.

12. Histograms can be drawn only for __________ distributions.

13. Pie-chart is always __________.

14. Squares are ______________ dimensional diagrams.

15. Cylinders are _____________ dimensional diagrams.

16. Pictograms are ____________ dimensional diagrams.

17. A straight line in a graph indicates the __________.

18. A zig-zag graph shows the __________ of a series.

19. The heights of bars with unequal class intervals are proportional to __________.

20. In rectangular diagrams, comparison is based on __________ of the rectangles.

21. Line diagram is suitable when there are __________ variate values in the frequency distributions.

22. Multiple bar diagram has a __________ of bars for each year or place.

23. When more than one factor is to be displayed for comparison during various years, _______ bar diagram is suitable.

24. Two related factors having different units of measurements can be displayed suitably by __________ diagram.

25. A graph of variate values and corresponding frequencies is known as __________.

26. A smoothened frequency polygon is known as __________.

27. The graph of the points (x, log y) is known as __________.

28. Ratio charts are also known as __________ charts.

29. Lorenz curve was initially given by ________.

30. Lorenz curve indicates the ____________ of distributions of two factors of a population.

31. When Lorenz curve turns out to be a straight line, it its concluded that the two factors have __________ distribution.

32. More the distance of Lorenz curve from the line of equal distribution, more is the __________ in the distributions of two factors.

33. Pictograms are __________ satisfactory.

34. Pictograms were originated by __________.

35. Column charts on circular base are just like __________.

36. There is no zero base line in a semi-logarithmic graph since __________.

SECTION-C

Multiple Choice Questions

Select the correct alternative out of given ones:

Q. 1 Charts and graphs are the presentation of numerical facts by means of:
(a) points and lines
(b) area and other geometrical forms
(c) symbols
(d) all the above

Q. 2 Graphs and charts facilitate:
(a) comparison of values
(b) to know the trend
(c) to know relationship
(d) all the above

Q. 3 The purpose served by diagrams and chart is:
(a) simple presentation of data
(b) to avoid tabulation
(c) to avoid textual form
(d) all the above

Q. 4 Choice of a particular chart depends on:

(a) the purpose of study
(b) the nature of data
(c) the type of audience
(d) all the above

Q. 5 Rectilinear co-ordinate chart is also referred as:
(a) Cartesian co-ordinate graph
(b) rectangular graph
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 6 Trilinear chart is used to portray simultaneously:
(a) two variables
(b) three variables
(c) four variables
(d) any number of variables

Q. 7 The shape of a trilinear charts is that of a:
(a) cone
(b) cube
(c) equilateral triangle
(d) pyramid

Q. 8 Which of the followings is a one-dimensional diagram?
(a) Bar diagram
(b) Pie-chart
(c) Cylinder
(d) A graph

Q. 9 Which of the followings is not a two-dimensional diagram?
(a) Square diagram
(b) Multiple bar diagram
(c) Rectangular diagram
(d) Pie-chart

Q. 10 Which of the following statement is not correct?
(a) The bars in a histogram touch each other
(b) The bar in a column chart touch each other
(c) There are bar diagrams which are known as broken bar diagrams
(d) Multiple bar diagrams also exist

Q. 11 Non-dimensional diagrams are also known as:
(a) cubes
(b) spheres
(c) pictograms
(d) all the above

Q. 12 Which of the statement is correct?
(a) Histograms and historigrams are similar in look
(b) Cube and square diagrams are similar in look
(c) Pie-chart and ogives
(d) none of the above

Q. 13 Ogive curve occur for,
(a) more than type distribution
(b) less than type distribution
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 14 In an ogive curve, the points are plotted for:
(a) The values and frequencies
(b) The values and cumulative frequencies
(c) Frequencies and cumulative frequencies
(d) None of the above

Q. 15 A semi-logarithmic graph or ratio chart is obtained by plotting the points:
(a) $\{x, y\}$
(b) $\{\log x, \log y\}$
(c) $\{x, \log y\}$
(d) $\{x, \log y/x\}$

Q. 16 Ogives for more than type and less than type distributions intersect at:
(a) mean
(b) median
(c) mode
(d) origin

Q. 17 When the values are large in magnitude in a chronological series and variation amongst values is small, a graph is better drawn by choosing:
(a) a false base line
(b) wide scale
(c) narrow scale
(d) none of the above

Q. 18 In a bar diagram, the base line is:
(a) horizontal
(b) vertical
(c) false base line

(d) any of the above

Q. 19 In a column chart, the base line is:
(a) horizontal
(b) vertical
(c) at an angle of 45°
(d) false base line

Q. 20 In a column chart, bars are:
(a) horizontal
(b) vertical
(c) slanting
(d) none of the above

Q. 21. In a bar diagram, the bars are:
(a) horizontal
(b) vertical
(c) slanting
(d) none of the above

Q. 22 In case of frequency distribution with classes of unequal widths, the heights of bars of a histogram are proportional to:
(a) class frequency
(b) class intervals
(c) frequencies in percentage
(d) frequency densities

Q. 23 Yearwise production of rice, wheat and maize for the last ten years can be displayed by:
(a) simple column chart
(b) subdivided column chart
(c) broken bar diagram
(d) multiple column chart

Q. 24 Profit and loss of a firm during various years can be displayed through:
(a) simple bar diagram
(b) duo-directional bar diagram
(c) deviation bar chart
(d) multiple bar diagram

Q. 25 When for some countries, the magnitudes are small and for other, the magnitudes are very large, to portray the data, it is preferred to construct:
(a) deviation bar diagram
(b) duo-directional bar diagram
(c) broken bar diagram
(d) any of the above

Q. 26 With the help of histogram we can prepare:
(a) frequency polygon
(b) frequency curve
(c) frequency distribution
(d) all the above

Q. 27 Historigram is suitable for:
(a) time series data
(b) chronological distribution
(c) none of (a) or (b)
(d) both (a) and (b)

Q. 28 When we have the number of court cases of different categories and information about number of cases settled, the information can be better portrayed through:
(a) sliding bar diagram
(b) histogram
(c) paired bar diagram
(d) column chart

Q. 29 To show the maximal and minimal values in a time series, the suitable chart is:
(a) deviation bar diagram
(b) range curve
(c) historigram
(d) all the above

Q. 30 With the help of ogive curve, one can determine:
(a) median
(b) deciles
(c) percentiles
(d) all the above

Q. 31 Lorenz curve is an indicator for the distribution of two factors:
(a) being equal
(b) being unequal
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 32 Greater the distance of Lorenz curve from the line of equal distribution:
(a) more is the inequality
(b) less is the inequality
(c) has to tell nothing about inequality
(d) none of the above

Q. 33 Pictograms are:
(a) very accurate
(b) least accurate

(c) mostly used
(d) scientifically correct

Q. 34 Pictograms are generally used by:
(a) cartographers
(b) dilettante
(c) scientist
(d) all the above

Q. 35 Pictograms are suitable for the data in:
(a) counts
(b) intervals
(c) fraction
(d) none of the above

Q. 36 Pictograms are shown by:
(a) dots
(b) lines
(c) circles
(d) pictures

Q. 37 In a column chart on circular base, bars are:
(a) vertical
(b) horizontal
(c) slanting
(d) curved

Q. 38 If there is an increase in a series at constant rate, the graph will be a:
(a) convex curve
(b) parabola
(c) concave curve
(d) a straight line from left bottom to right top

Q. 39 If there is a decrease in a series at constant rate, the graph will be a:
(a) hyperbola
(b) a straight line from left top to right bottom
(c) a convex curve
(d) none of the above

Q. 40 A semi-logarithmic graph of a series increasing by a constant amount will be a:
(a) straight line at angle of 45°
(b) a convex upward curve
(c) a concave upward curve
(d) a convex downward curve

Q. 41 The suitable chart to emphasise the difference between two time series, of which one is at a higher level, is:
(a) range chart
(b) deviation bar chart
(c) paired bar diagrams
(d) band chart

Q. 42 When there is a pronounced skewness, the desirable scale to plot the frequency distribution is:
(a) arithmetic scale
(b) multiple scale
(c) logarithmic scale
(d) any of the above

Q. 43 When there are a large number of values in an individual series, preference for portraying the data goes to:
(a) bar diagram
(b) column chart
(c) line chart
(d) scatter diagram

Q. 44 An alternative chart to pie-chart is:
(a) step bar diagram
(b) rectangular chart
(c) sphere
(d) none of the above

Q. 45 The graph of the successive points of a distribution joined by straight lines in statistical terminology is known as:
(a) frequency distribution
(b) frequency curve
(c) trend
(d) cumulative distribution curve

Q. 46 A deviational or bilateral chart with 100 per cent component columns is also known as:
(a) duo-directional chart
(b) floating column chart
(c) sub-divided column chart
(d) range chart

Q. 47 Pie-chart represents the components of a factor by:
(a) percentages
(b) angles
(c) sectors
(d) circles

Q. 48 The immigration and outmigration of people in a number of countries and also the net migration can be better displayed by:

(a) duo-directional column chart
(b) gross-deviation column chart
(c) net deviation column chart
(d) range chart

Q. 49 Common form of rectilinear co-ordinate graph is:

(a) band chart
(b) surface chart
(c) stratum chart
(d) all the above

Q. 50 An arithmetic chart can:

(a) have only one amount scale
(b) have multiple amount scale
(c) be without any amount scale
(d) none of the above

Q. 51 It is necessary to find commutative frequencies in order to draw a/an:

(a) histogram
(b) frequency polygon
(c) Ogive curve
(d) column chart

Q. 52 If we plot the points of a less than type or more than type frequency distribution, the shape of graph is:

(a) Ogive curve
(b) scatter diagram
(c) zig-zag curve
(d) parabola

Q. 53 Histogram is suitable for the data presented as:

(a) continuous grouped frequency distribution
(b) discrete grouped frequency distribution
(c) individual series
(d) all the above

Q. 54 A histogram can be drawn for the distribution with unequal class intervals by considering:

(a) class frequency
(b) height of bars proportional to class intervals
(c) height of bars proportional to frequency density
(d) all the above

Q. 55 In a histogram with equal class intervals; heights of bar are proportional to:

(a) mid-values of the classes
(b) frequencies of respective classes
(c) either (a) or (b)
(d) neither (a) nor (b)

Q. 56 The data relating to the number of registered allopathic and homeopathic doctors in six different states can be most appropriately represented by diagram:

(a) line graph
(b) histogram
(c) pie-diagram
(d) double bar diagram

Q. 57 Histogram can be used only when:

(a) class intervals are equal or unequal
(b) class intervals are all equal
(c) class intervals are unequal
(d) frequencies in class interval are equal.

Q. 58 The most appropriate diagram to represent the data relating to the monthly expenditure on different items by a family is:

(a) histogram
(b) pie-diagram
(c) frequency polygon
(d) line graph

Q. 59 The gross income, taxes and net income of a manufacturer during different years can better be represented by:

(a) deviation bar diagram
(b) broken bar diagram
(c) paired bar diagram
(d) duo-directional bar diagram

Q. 60 Proportion of males and females in India in different occupations in the year 2000 can most properly be represented by:

(a) sliding bar diagram
(b) deviation bar diagram
(c) sub-divided bar diagram
(d) multiple bar diagram

ANSWERS

Section-B

(1) tabulation (2) diagrams (3) median (4) not same (5) touch (6) not suitable (7) histogram (8) Bar (9) Components (10) different (11) Deviation (12) continuous frequency (13) circular (14) two (15) three (16) non (17) trend (18) fluctuations (19) frequency densities (20) area (21) too many (22) group (23) multiple (24) paired bar (25) frequency polygon (26) frequency curve (27) ratio chart (28) semi-logarithmic (29) M.O. Lorenz (30) inequality (31) equal (32) variability (33) least (34) Dr Otto Neurath (35) bar diagrams (36) log 0 = – ∞.

SECTION-C

(1) d	(2) d	(3) d	(4) d	(5) c	(6) b
(7) c	(8) a	(9) b	(10) b	(11) c	(12) a
(13) c	(14) b	(15) c	(16) b	(17) a	(18) b
(19) a	(20) b	(21) a	(22) d	(23) d	(24) c
(25) c	(26) d	(27) d	(28) a	(29) b	(30) d
(31) b	(32) a	(33) b	(34) b	(35) a	(36) d
(37) c	(38) d	(39) b	(40) b	(41) d	(42) c
(43) c	(44) a	(45) b	(46) b	(47) c	(48) b
(49) d	(50) b	(51) c	(52) a	(53) a	(54) c
(55) b	(56) d	(57) b	(58) b	(59) d	(60) a

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Chapter **4**

Measures of Central Tendency

SECTION-A

Short Essay Type Questions

Q. 1 What do you understand by measure of central tendency?

Ans. It is a single value within the range of data which represents a group of individual values in a simple and concise manner so that the mind can get a quick understanding of the general size of the individuals in the group. Since the value lies within the range of data, it is known as a measure of central tendency (value).

Q. 2 Quote the statements about central values given by John I. Griffin, R.A. Fisher and A.L. Bowley.

Ans. The Statements of the prominent statisticians about the central values are:

John I. Griffin: An average may be thought of as a measure of central value.

R.A. Fisher: The inherent inability of human to grasp in its entirely a large body of numerical data compels us to seek relatively few constants that will adequately describe the data.

A.L. Bowley: Averages are statistical constants which enable us to comprehend in a single effort the significance of the whole.

Q. 3 Name different measures of central values.

Ans. Different measures of central values are:

(i) Arithmetic mean; (ii) Geometric mean; (iii) Harmonic mean, (iv) Median; (v) Mode; (vi) Quartiles, (vii) Octiles; (vii) Deciles; (ix) Percentiles, etc.

Q. 4 What are the properties of a good measure of central tendency?

Ans. The desired properties of a good measure of central tendency are:

(i) It should be based on all observations of a set of values.

(ii) It should be rigorously defined.

(iii) It should be easily computable.

(iv) It should be least affected by extreme values.

(v) It should fluctuate least from sample to sample drawn from the same population.

Q. 5 What are the major classifications of averages?

Ans. Major classifications of averages are:

(i) Mathematical averages—arithmetic mean, geometric mean, harmonic mean.

(ii) Positional averages—median, mode, quartiles, quintiles, octiles, deciles, percentiles.

(iii) Commercial averages—moving average, progressive average, composite average.

Q. 6 What are the functions of averages?

Ans. Various functions of averages are:

(i) It presents a simple and concised picture of large and complicated set of data.

(ii) It makes possible and easier to compare two or more groups of data.

(iii) It provides a study of different groups of data.

(iv) It facilitates the interpretation of data.

(v) It helps in taking decisions. For example, one can tell whether the income of a family is above or below the average income.

Q. 7 What are the limitations of an average in general?

Ans. The general limitations of an average are:

(i) A mean represents a group as a whole, not an individual.

(ii) Often an average is a value which does not exist in the set of data.

(iii) Sometimes an average gives a value which is not feasible, e.g., average size of a family is 3.62.

(iv) An average is incapable to throw any light on the constitution of the series.

Q. 8 Define arithmetic mean (average).

Ans. Arithmetic mean (A.M.) of a set of data may be defined as the sum of the values divided by the number of values in the set. Its formula is, A.M. = $\Sigma x_i/N$ for $i = 1, 2, 3, \ldots, N$.

Q. 9 What are the merits of arithmetic mean?

Ans. Merits of arithmetic mean are:

(i) It is easy to calculate.

(ii) It is rigidly defined.

(iii) It is based on all observations.

(iv) Observations are not to be arranged in order.

(v) It provides sound basis for comparison of two series.

(vi) It can be calculated if partial calculations are available.

(vii) It is less susceptible to sampling fluctuations.

(viii) It is most suitable for further algebraic treatment.

Q. 10 What are the demerits of arithmetic mean?

Ans. Demerits of arithmetic mean are:

(i) It is too much affected by extreme values.

(ii) Mostly it does not correspond to any value of the set of observations.

(iii) It cannot be calculated for frequency distribution with open end classes.

(iv) It does not convey any information about the spread or trend of data.

(v) It is not a suitable measure of central value in case of highly skewed distribution.

Q. 11 Prove that the sum of deviations from mean is zero.

Ans. Suppose $x_1, x_2, \ldots, x_n$ are n observations in a set. The sum of deviations from mean $\bar{x}$ is $\sum_{i=1}^{n}(x_i - \bar{x})$.

$$\sum_{i=1}^{n}(x_i - \bar{x}) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x} = \sum_{i=1}^{n} x_i - n\bar{x}$$

$$= \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i = 0$$

Q. 12 Give the formula for calculating mean of a set of values from coded data.

Ans. The formula for calculating mean from coded data is,

$$\bar{x} = a + \frac{\sum_{i=1}^{k} f_i x_i'}{\sum_{i=1}^{n} f_i} \times c$$

where, a is the constant value which is subtracted from each observation and c is a constant which divides each value obtained after subtracting a.

Also f_i is the frequency of the value x_i and $x_i' = (x_i - a)/c$.

Q. 13 What is the effect of change of origin and scale on arithmetic mean?

Ans. Arithmetic mean is increased or decreased by the constant value added or subtracted from each observation respectively. Also it is $\frac{1}{c}$ time of the original mean if each observation is divided by c and c times the original mean if each observation is multiplied by c.

Q. 14 What is the difference between sample mean and population mean?

Ans. Population mean is based on the values of each and every item of the population, whereas sample mean is based on the values of items selected in the sample from the population. If we have a random sample, then the sample mean is an estimate of population mean.

Q. 15 There are $(n + 1)$ observations in a sample. If $\bar{x}_1$ is the mean of the first n numbers and $\bar{x}_2$ is the mean of the last n numbers, prove that

$$\bar{x}_2 = \bar{x}_1 + \frac{1}{n}(x_{n+1} - x_1)$$

Ans. Proof: By Formula,

$$\bar{x}_1 = \frac{1}{n}(x_1 + x_2 + \ldots + x_n)$$

$$\bar{x}_2 = \frac{1}{n}(x_2 + x_3 + \ldots + x_{n+1})$$

$$= \frac{1}{n}(x_1 + x_2 + x_3 + \ldots + x_n + x_{n+1} - x_1)$$

$$= \frac{1}{n}(x_1 + x_2 + \ldots + x_n) + \frac{1}{n}(x_{n+1} - \bar{x}_1)$$

$$= \bar{x}_1 + \frac{1}{n}(x_{n+1} - x_1)$$

Q. 16 Express weighted mean in brief.

Ans. If $x_1, x_2, \ldots, x_k$ are the values having weights $w_1, w_2, \ldots, w_k$ respectively, the weighted mean,

$$\bar{x} = \frac{w_1x_1 + w_2x_2 + \ldots + w_kx_k}{w_1 + w_2 + \ldots + w_k}$$

Q. 17 What would be the weighted mean of n natural numbers if weights are the corresponding numbers?

Ans. Using the formula, the weighted mean

$$\bar{x} = \frac{1.1 + 2.2 + \ldots + n.n}{1 + 2 + \ldots + n}$$

$$= \frac{1^2 + 2^2 + \ldots + n^2}{1 + 2 + \ldots + n}$$

$$= \frac{n(2n+1)(n+1)/6}{n(n+1)/2}$$

$$= \frac{2n+1}{3}$$

Q. 18 What is a moving average?

Ans. In this type of average, we get a series of averages out of a series of data. In this we take a fixed number of beginning items and find its average. To get the next average, delete the first item of the previous group and add next one item of the series to it and find its average. Continue this process till all the values are exhausted. This gives a series of averages. For example, suppose there are five items, *a, b, c, d, e* in the series. Taking three items in a group, the moving averages will be,

$$\frac{a+b+c}{3}, \frac{b+c+d}{3}, \frac{c+d+e}{3}.$$

Q. 19 Where do you utilise moving averages?

Ans. Moving averages are utilised mostly in the analysis of time series data to know the trend.

Q. 20 What do you understand by progressive average?

Ans. In progressive average, we find out the average of the first two periods value and then go on adding the value for the third, fourth, fifth period, etc., and calculate the averages of three, four, five, etc., periods successively. Such averages are called *progressive averages*. For example, if *a, b, c, d* and *e* are the values of five successive periods, the progressive averages are:

$$\frac{a+b}{2}, \frac{a+b+c}{3}, \frac{a+b+c+d}{4}, \frac{a+b+c+d+e}{5}.$$

Q. 21 For what purposes the progressive averages are utilised?

Ans. Progressive averages are used to know the average profits in the starting years of commercial institutions.

Q. 22 What is meant by composite average?

Ans. The average of the averages of different series is known as composite average. The formula is,

$$\text{composite average} = \frac{\text{sum of the averages of individual series}}{\text{number of series}}$$

Q. 23 What is combined or pooled mean?

Ans. The mean of the data that would be obtained by combining the values of all individual samples.

Q. 24 Give the formula for finding out the combined (pooled) mean of three samples when their means and sample sizes are given.

Ans. Suppose $\bar{x}_1, \bar{x}_2, \bar{x}_3$ are three sample means base on n_1, n_2 and n_3 observations respectively. Formula for the pooled or combined mean is,

$$\bar{x}_{123} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3}{n_1 + n_2 + n_3}$$

Pooled mean is the mean obtained on combining all samples, considered as a single sample.

Q. 25 Elucidate trimmed mean.

Ans. Suppose we want a 10 per cent trimmed mean, then the mean of the observations by excluding 10 per cent smallest and 10 per cent largest observations is known as 10 per cent trimmed mean. Not necessarily 10 per cent trimmed mean has to be found out, it can be any other chosen percentage which is considered suitable.

Q. 26 In a frequency distribution, the mid value of the first class is 10 and each class has equal class intervals. The cumulative frequencies of the classes are 3, 18, 37, 40 and 45. Give the original frequency distribution and find its mean.

Ans. The first class will be (10 – 2.5) to (10 + 2.5), *i.e.*, 7.5 – 12.5. The second class will be 12.5 – 17.5 and so on. Their corresponding frequencies will be 3, (18 – 3), (37 – 18), (40 – 37) and (45 – 40), *i.e.*, 3.15, 19, 3 and 5. Hence the frequency distribution is:

Classes	*Frequency*
7.5–12.5	3
12.5–17.5	15
17.5–22.5	19
22.5–27.5	3
27.5–32.5	5

In the above distribution, the mid-values of the classes are 10, 15, 20, 25, 30 respectively. Hence, the mean μ of the distribution is,

$$\mu = \frac{10\times3 + 15\times15 + 20\times19 + 25\times3 + 30\times5}{3+15+19+3+5}$$

$$= \frac{860}{45}$$

$$= 19.11$$

Q. 27 Define geometric mean.

Ans. Geometric mean is the n^{th} root of the product of n values of a set of observations. By formula, $G = (x_1, x_2, \ldots, x_n)^{1/n}$.

Q. 28 What are the merits of geometric mean?

Ans. Merits of geometric mean are:

(i) It is least affected by extreme values.
(ii) It is based on all observations of the set.
(iii) It is suitable for further algebraic treatment.

Q. 29 What are the demerits of the geometric mean?

Ans. Demerits of the geometric mean are:

(i) Its calculation is somewhat complicated.
(ii) It cannot be calculated if any of the value in the set is zero.
(iii) If any one or more values are negative, either geometric mean will not be calculable or an absurd value will be obtained.

Q. 30 For what type of data geometric mean is the most appropriate measure of location?

Ans. It is appropriate for averaging the ratios of change, for average of proportions, etc.

Q. 31 Where geometric mean is considered to be the best average theoretically?

Ans. Geometric mean is considered most suitable average for index numbers.

Q. 32 If G_1 is the geometric mean of n_1 observations $x_1, x_2, \ldots, x_{n_1}$ and G_2 is the geometric mean of n_2 observations $y_1, y_2, \ldots, y_{n_2}$ and let G be the geometric mean of all $(n_1 + n_2)$ observations, prove that,

$$\log G = \frac{1}{n_1 + n_2}\{n_1 \log G_1 + n_2 \log G_2\}$$

Ans. By formula, geometric mean of $(n_1 + n_2)$ observations is,

$$G = \left(x_1 \cdot x_2 \ldots x_{n_1} \cdot y_1 \cdot y_2 \ldots\ldots y_{n_2}\right)^{1/n_1+n_2}$$

$$\therefore \quad \log G = \frac{1}{n_1+n_2} \log(x_1 \cdot x_2 \ldots\ldots x_{n_1} \cdot y_1 \cdot y_2 \ldots\ldots y_{n_2})$$

$$= \frac{1}{n_1+n_2}\{\log(x_1 \cdot x_2 \ldots\ldots x_{n_1}) + \log (y_1 \cdot y_2 \ldots\ldots y_{n_2})\}$$

$$= \frac{1}{n_1+n_2}\left\{\sum_{i=1}^{n_1} \log x_i + \sum_{j=1}^{n_2} \log y_j\right\}$$

We know, log G.M. $= \frac{1}{n}\sum_i \log x_i$

or $\sum_i \log x_i = n \log \text{G.M.}$

$$\therefore \quad \log G = \frac{1}{n_1+n_2}\{n_1 \log G_1 + n_2 \log G_2\}$$

Q. 33 Explain briefly the inverse property of geometric mean.

Ans. If $\bar{x}_G$ is the geometric mean of $x_1, x_2, \ldots, x_n$, the geometric mean of the inverse of the observations $1/x_1, 1/x_2, \ldots, 1/x_n$ is $1/\bar{x}_G$. This is known as inverse property of geometric mean.

Q. 34 The arithmetic mean of two numbers is 10 and their geometric mean is 8. Find the numbers.

Ans. Suppose the two numbers are x_1 and x_2. Then by formula,

$$\text{A.M.} = \frac{x_1 + x_2}{2} = 10$$

and $\quad \text{G.M.} = \sqrt{x_1 \cdot x_2} = 8$

$\therefore \quad (x_1 + x_2) = 20$ and $x_1 x_2 = 64$

$$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1 x_2$$

$$= 400 - 4 \times 64 = 144$$

$\therefore \quad (x_1 - x_2) = +12$

Solving $\quad x_1 + x_2 = 20$

and $\quad x_1 - x_2 = 12,$

we get $\quad x_1 = 16, x_2 = 4$

Hence, the numbers are 16 and 4.

Q. 35 Define harmonic mean and give the formula for harmonic mean.

Ans. Harmonic mean is the inverse of the arithmetic mean of the reciprocals of the observations of a set.

The formula for harmonic mean is,

$$\text{H.M.} = \frac{f_1 + f_2 + \ldots + f_k}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \ldots + \frac{f_k}{x_k}}$$

$$= \frac{\Sigma_i f_i}{\Sigma_i f_i / x_i} \text{ for } i = 1, 2, \ldots, k$$

where, f_i is the frequency of x_i for $i = 1, 2, \ldots, k$.

Q. 36 What are the merits of harmonic mean?

Ans. Merits of harmonic mean are:

(i) It is based on all observations of a set.

(ii) It is a good mean for a highly variable series.

(iii) It gives more weightage to the small values and less weightage to the large values.

(iv) It is better than weighted mean since in this, values are automatically weighted.

Q. 37 What are the demerits of harmonic mean?

Ans. Demerits of harmonic mean are:

(i) Its calculation is complicated.

(ii) If any value is zero, it cannot be calculated.

(iii) Its value is generally not a member of the series.

Q. 38 Can we use coding while calculating geometric mean or harmonic mean?

Ans. We cannot use coding in geometric mean or harmonic mean as no easy expression is available which relates the mean of coded data to the mean of the original data.

Q. 39 How will you calculate mean in case of a continuous frequency distribution?

Ans. To calculate the mean of a continuous frequency distribution, first find the mid-values of the classes and use them as variate values. Then use the formula for the weighted mean with frequencies as weights.

Q. 40 Given two values x_1 and x_2, prove that

$$\text{A.M.} \geq \text{G.M.} \geq \text{H.M.}$$

Ans. We know,

$$\text{A.M.} = \frac{x_1 + x_2}{2}, \text{G.M.} = \sqrt{x_1 x_2}, \text{H.M.} = \frac{2x_1 x_2}{x_1 + x_2}$$

Again, $$\left(\sqrt{x_1} - \sqrt{x_2}\right)^2 \geq 0$$

or $$x_1 + x_2 - 2\sqrt{x_1 x_2} \geq 0$$

$$\frac{x_1 + x_2}{2} \geq \sqrt{x_1 x_2}$$

$\therefore$ $$\text{A.M.} \geq \text{G.M.}$$

Again, $$x_1 + x_2 \geq 2\sqrt{x_1 x_2}$$

or, $$(x_1 + x_2)\sqrt{x_1 x_2} \geq 2\left(\sqrt{x_1 x_2}\right)^2$$

$$(x_1 + x_2)\sqrt{x_1 x_2} \geq 2x_1 x_2$$

$$\sqrt{x_1 x_2} \geq \frac{2x_1 x_2}{x_1 + x_2}$$

$$\text{G.M.} \geq \text{H.M.}$$

$\therefore$ $$\text{A.M.} \geq \text{G.M.} \geq \text{H.M.}$$

Q. 41 Express geometric mean in terms of arithmetic mean and harmonic mean.

Ans. The relation between geometric mean, arithmetic mean and harmonic mean is,

$$\text{G.M.} = \sqrt{\text{A.M.} \times \text{H.M.}}$$

Q. 42 Express harmonic mean in terms of geometric mean and arithmetic mean.

Ans. The expression for harmonic mean relating to geometric mean and arithmetic mean is,

$$\text{H.M.} = (\text{G.M.})^2/\text{A.M.}$$

Q. 43 For what type of values harmonic mean is suitable?

Ans. Harmonic mean is suitable when the values are pertaining to the rate of change per unit time such as speed, number of items produced per day, contracts completed per year, etc. In general, harmonic mean is suitable for time, speed, rates, prices, etc.

Q. 44 Define quadratic mean.

Ans. The square root of the arithmetic mean of the square of numbers of a set is known as quadratic mean.

Q. 45 When do you prefer a quadratic mean than a mean.

Ans. When one has some negative numbers in a set, quadratic mean is preferable than any other mean. The reason being that considering the sign of negative numbers, the mean value is considerably reduced. If the signs of negative numbers are ignored, the mean thus obtained depends on an algebraic lie. But the quadratic mean is devoid of the mathematical abuse and has the same scale and units as the original values.

Q. 46 Comment on the value of quadratic mean as compared to arithmetic mean.

Ans. Generally the quadratic mean is greater than the arithmetic mean. The reason being that the difference between a number and its square increases exponentially as the number gets larger. Hence, the square root of the average of squared numbers results into a higher value than arithmetic mean of original numbers.

Q. 47 Find the arithmetic mean (A.M.) and quadratic mean (Q.M.) for the following set of values and comment on the mean values.

X :	2	3	5	8	12

Ans. $$\text{A.M} = \frac{2+3+5+8+12}{5}$$

$$= 6.0$$

Sum of squares of numbers,

$$\Sigma X^2 = 4 + 9 + 25 + 64 + 144 = 246$$

$$\text{A.M. of squared numbers} = \frac{246}{5} = 49.2$$

$$\text{Q.M.} = \sqrt{49.2} = 7.0143$$

Q.M. is greater than A.M. as is expected on the basis of our theoretical knowledge.

Q. 48 Where do you make use of quadratic mean in general.

Ans. Quadratic mean is absolutely utilised in finding the standard deviation of a set of data. As a matter of fact, the standard deviation is nothing but the quadratic mean of the deviations from data mean.

Q. 49 What do you understand by median?

Ans. It is a most preferable measure of location for asymmetric distributions. Median is the value of the variable that divides the ordered set of values into two equal halves. 50 per cent values are to the left of the median and 50 per cent are to the right of the median.

Q. 50 What are the points in favour of median?

Ans. Points in favour of median are:

(i) Median is not influenced by extreme values because it is a positional average.

(ii) Median can be calculated in case of distribution with open end intervals

(iii) Median can be located even if the data are incomplete.

(iv) Median can be located even for qualitative factors such as ability, honesty, etc.

Q. 51 What are the demerits of median?

Ans. Following are the demerits of median:

(i) A slight change in the series may bring drastic change in median value.

(ii) In case of even number of observations or continuous series, median is an estimated values other than any value in the series.

(iii) It is not suitable for further mathematical treatment except its use in mean deviation.

Q. 52 How will you calculate median in case of ungrouped data?

Ans. To calculate median of ungrouped data, following steps be followed:

(i) Arrange the data in order.

(ii) For an individual or discrete series of N items, the value corresponding to $(N + 1)/2^{\text{th}}$ item is the median value when N is odd.

(iii) When N is even, median is the average of

$$\frac{N^{\text{th}}}{2} \text{ and } \left(\frac{N+2}{2}\right)^{\text{th}} \text{ items.}$$

Q. 53 Give the interpolation formula for calculating median of grouped series.

Ans. For grouped data, the formula for median with usual rotation is,

$$M_d = L_0 + \frac{N/2 - c}{f} \times I$$

where, L_0 = lower limit of the median class

c = cumulative frequency just above the median class

f = Frequency of the median class

I = class interval

Q. 54 What is the impact of extreme values of a set on median?

Ans. Median is not affected by the extreme values of a set.

Q. 55 Discuss mode in brief.

Ans. The variate value x having maximum frequency in a distribution is known as its mode.

Q. 56 What was said about the mode by Ya Lun Chou?

Ans. Ya Lun Chou Statement about mode is, "The mode is that value of a series which appears most frequently than any other."

Q. 57 Can there be more than one mode of a distribution ?

Ans. Some distributions have equal peaks and hence mode is not necessarily unique. There will be as many modes of distribution as the number of peaks in it.

Q. 58 How would you calculate mode of a grouped frequency distribution?

Ans. For frequency distribution with classes in ascending order, the formula for calculating the mode is,

$$M_0 = L_0 + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times I$$

$$= L_0 + \frac{f_0 - f_{-1}}{2f_0 - f_{-1} - f_{+1}} \times I$$

where, the class having the maximum frequency is known as model class.

L_0 = Lower limit of the modal class

I = class interval

Δ_1 = Difference of modal frequency from its preceding class frequency

Δ_2 = difference of modal frequency from its following class frequency.

In the second formula,

L_0 and I are same as above.

f_0 = modal class frequency

f_{-1} = Frequency of the preceding class

f_{+1} = Frequency of the following class.

Q. 59 How would you find out the mode of a discrete frequency distribution?

Ans. It can be determined merely by inspection. The value having maximum frequency is the mode of the distribution.

Q. 60 How can one locate mode graphically in case of grouped data?

Ans. Prepare a histogram. Consider a bar having maximum height and the bars to the left and right adjoining to it. Join the top left corner of the highest bar to the top left corner of the right side bar and top right corner of the highest bar to the top right corner of the bar to its left. The abscissa value of the point of intersection of the joining lines is the mode.

Q. 61 Discuss quartiles and their importance.

Ans. Three variate values of the variable, say x, which divide the series (frequency) into four equal parts are called quartiles for the corresponding distribution of x. There are three quartiles namely Q_1, Q_2 and Q_3. Q_1 is called the lower quartile and Q_3, the upper quartile. 25 per cent values are less than Q_1 and 25 per cent values are larger than Q_3 and the rest 50 per cent values lie between Q_1 and Q_3. Q_2 divide the series into two equal parts and hence it is the same as median.

As regards their importance, quartiles are widely used in economics and business. They are also helpful in determining the shape of a distribution.

Q. 62 Give the formula for calculating the quartiles, of a continuous distribution.

Ans. Formula for calculating i^{th} quartile (i = 1, 2, 3) for a continuous distribution having K-classes is,

$$Q_i = l_0 + \frac{\frac{iN}{4} - c}{f} \times I$$

where, Q_i = i^{th} quartile which is to be calculated

l_0 = lower limit of the i^{th} quartile class

N = Total of all frequencies

c = cumulative frequency of the class just above the quartile class

f = frequency of the quartile class

I = class interval.

Q. 63 Discuss deciles in brief.

Ans. The variate values which divide the series (frequency) into ten equal parts are called deciles. Hence, there are in all nine deciles denoted by $D_1, D_2, ..., D_9$. The items before D_1 and after D_9 are 10 per cent. Also, the number of items that lie between any two deciles is also 10 per cent. D_5, the fifth decil divides the series into halves and hence, it is the same as median.

Q. 64 Give the formula for finding the deciles of a continuous distribution.

Ans. The formula for calculating deciles of a continuous distribution is,

$$D_i = l_0 + \frac{\frac{iN}{10} - c}{f} \times I$$

for i = 1, 2, ..., 9.

All notation can be decoded by replacing the word quartile by decile as in the formula for quartiles.

Q. 65 Describe percentiles in brief.

Ans. The values of the variable x which divide its distribution into 100 equal parts are called percentiles. There are in all 99 percentiles and are denoted by $P_1, P_2, ..., P_{99}$ respectively. Area between any two percentiles, before P_1 and after P_{99} is 0.01, *i.e.*, it represents $\frac{1}{100}$ part of the population. 50th percentile is same as median.

Q. 66 Give the formula for calculating percentiles of a continuous distribution.

Ans. Formula for calculating i^{th} percentile is,

$$P_i = l_0 + \frac{\frac{iN}{100} - c}{f} \times I.$$

for $i = 1, 2, ..., 99$.

All notation can be decoded as for quartiles by replacing the word quartile by percentile.

Q. 67 How to determine quartiles, deciles and percentiles in case of a discrete series?

Ans. *Quantiles*: The values of the variable x corresponding to $\left(\frac{N+1}{4}\right)^{th}, \left(\frac{N+1}{2}\right)^{th}$ and $\frac{3(N+1)}{4}$th items of an ordered discrete series are the values of Q_1, Q_2 and Q_3 respectively. The position of the required item can easily be adjudged with the help of cumulative frequencies.

Deciles: Similar to quartiles, the values of the variable x corresponding to $\frac{i(N+1)}{10}$ th item for $i = 1, 2, ..., 9$ of an ordered discrete series is D_i. The position of $\frac{i(N+1)}{10}$ th item can be located with the help of cumulative frequencies.

Percentiles: Just like deciles, the value of the variable x corresponding to $\frac{i(N+1)}{100}$ th item for $i = 1, 2, ..., 99$ of an ordered series is P_i. $\frac{i(N+1)}{100}$ th item in the series can easily be placed with the help of cumulative frequencies.

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s)* in the blanks

1. "The statement, "An average may be thought of as a measure of central value," was given by __________.
2. The statement, "The inherent inability of human to grasp in its entirely a large body of numerical data compels us to seek relatively few constants that will adequately describe the data," was given by __________.
3. "Averages are statistical constants which enable us to comprehend in a single effort the significance of the whole", was stated by __________.
4. Measures of central tendency are also known as __________ or __________.
5. Arithmetic mean is very much affected by __________.
6. Any mean is based on__________ values of a set.
7. An average __________ the data.
8. In an open end distribution __________ cannot be determined.
9. If 5 is subtracted from each observation of a set, then the mean of the observation is reduced by __________.
10. If each value of a set is divided by 10, the mean of observations is __________ of the mean of the original observations.

11. The mean of the values 11, 12, 13, 14 and 15 is __________.

12. The mean of 8 numbers is 15. Afterwards a new number 24 is added. The mean of the nine numbers is __________.

13. The sum of the deviations from mean is __________.

14. The average income of a person on working for the first five days of the week is Rs. 35 per day and if he works for the first six days of the week, his average income per day is Rs. 40. Then, his income for the sixth day is __________.

15. A factory employs 5 persons in the first week of the month and pays Rs. 25 per day per person. In the second week he employs 8 persons and pays Rs. 30 per day per person. In the third week he employs 7 persons and pays Rs. 25 per day per person. The average payment per day per person by the employer is __________.

16. Weighted mean is more __________ than unweighted mean.

17. In case of 15 per cent trimmed mean, only ________ per cent observations are utilised.

18. The arithmetic mean of n natural numbers for 1 to n is __________.

19. Sum of squares of the deviations from mean is always __________.

20. The geometric mean of four numbers 2, 4, 8 and 64 is __________.

21. If $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ are the variate values of two variables X and Y and their geometric means are G_1 and G_2 respectively, then the geometric mean of (x_i/y_i) is __________.

22. Geometric mean cannot be calculated if any value of the set is __________.

23. Geometric mean is suitable when the values are given as __________ or __________.

24. If arithmetic mean of certain values is 9 and their geometric mean is 6, then the harmonic mean is __________.

25. If the harmonic mean of the two numbers a and b is 5 if $a = 5$, then b is __________.

26. Harmonic men is suitable as an average for finding out the __________.

27. If the two observations x_1 and x_2 are such that $x_1 = -x_2$, their harmonic mean is _______.

28. The relation between A.M., G.M. and H.M. is __________.

29. The variate value in a series which divides the series into halves is called __________.

30. The sum of the absolute deviations taken from median is __________.

31. The median of the series 3, 18, 7, 20, 11, 12, 9, 17, 22 is __________.

32. Median is __________ for each and every distribution.

33. Two ogives, the one less than type and the other more than type, intersect at __________.

34. Median is a more suited average for grouped data with __________ classes.

35. The variate value having maximum frequency in a frequency distribution is called ________.

36. The mode of the distribution of values 5, 7, 9, 9, 8, 5, 6, 8, 7, 7, 5, 7, 9, 2, 7 is _________.

37. __________ and __________ are least affected by the extreme values as a measure of central tendency.

38. Out of all the measures of central tendency __________ is the only measure which is not unique.

39. __________ is not appropriate for further algebraic treatment as a measure of central value.

40. The distribution which has only one mode is called __________.

41. The distribution having two modes is called __________.

42. If the frequencies in distribution have ups and downs, the mode is determined by the method of __________.

43. In an individual or discrete series $\left(\frac{N+1}{4}\right)^{\text{th}}$ item is known as first __________.

44. Second quartile is same as __________.

45. Third quartile and __________ percentile are same.

46. Second quartile and __________ decile are equal.

47. The decile which precedes 72th percentile is __________.

48. The percentage of values which lie between P_{30} and P_{56} is __________.

49. 25th percentile is same as __________ quartile.

50. Sixth decile is same as __________ percentile.

51. Percentage of values lying between first and sixth decile is __________.

52. Percentage of values which are greater than 66th percentile is __________.

53. In a frequency distribution fourth decile and __________ percentile are same.

54. The relation between second quartile Q_2, sixth decile D_6 and 80^{th} percentile is __________.

55. Median is same as __________ decile.

56. Median is same as __________ percentile.

57. Percentiles (quartiles, deciles) are always in __________ order.

58. The variate values which divide a series into five equal parts are called __________.

59. If in a series, 20 per cent values are greater than 75, then __________ = 75.

60. If in a discrete series 30 per cent values are less than 35, then __________ decile = 35.

61. If in a discrete series 75 per cent values are greater than 20, then __________ quartile = 20 or __________ percentile = 20.

62. The general name for quartile, quintiles, octiles, deciles and percentiles is __________.

63. The statement, "The mode is that value of a series which appears most frequently than any other", was given by __________.

64. The relation between first quartile and 25th percentile is __________.

65. The relation between 4th decile and 45th percentile is __________.

66. Percentage of values that lie between tenth and eighty fifth percentile is __________.

67. The number of types of statistical facts are __________ or __________.

68. For a symmetrical distribution, median in terms of quartiles is __________.

69. Geometric mean is __________ of the arithmetic mean of logarithmic values of a variable.

70. Harmonic mean is __________ of the arithmetic mean of the reciprocal of the observations of a data set.

71. Quadratic mean is preferred most, if the data set contains some __________ number.

72. If a variable takes some non-negative values $x_1, x_2, \ldots, x_n$, then the inequality that holds between H.M., G.M. and A.M. is __________.

73. Quadratic mean has __________ scale and units as that of original observation.

74. The application of quadratic mean is evidently found in __________.

75. The mean of 20 observations is 7. If each observation is multiplied by 3 and then 5 is added to it, then the mean of the new data set is __________.

76. The mean of a set of 10 observations is 4. Another set of 20 observations is added to it which makes the mean of the combined set equal to 6. The mean of second set is __________.

77. If $2y - 6x = 6$ and the mode of y is 66, then the mode of x is __________.

78. An average is that value of a distribution which represents __________.

79. No average can be regarded as __________ for all times and all data.

80. Mode is __________ defined.

SECTION-C

Multiple Choice Questions

Select the correct alternative out of given ones.

Q. 1 Mean is a measure of:
(a) location (central value)
(b) dispersion
(c) correlation
(d) none of the above

Q. 2 Which of the following is a measure of central value?
(a) Median
(b) Standard deviation
(c) Mean deviation
(d) Quartile deviation

Q. 3 Which of the following represents median?
(a) First quartile
(b) Fiftieth percentile
(c) Sixth decile
(d) None of the above

Q. 4 If a constant value 50 is subtracted from each observation of a set, the mean of the set is:
(a) increased by 50
(b) decreased by 50
(c) is not affected
(d) zero

Q. 5 If a constant 5 is added to each observation of a set, the mean is:
(a) increased by 5
(b) decreased by 5
(c) 5 times the original mean
(d) not affected

Q. 6 If each observation of a set is multiplied by 10, the mean of the new set of observations:
(a) remains the same
(b) is ten times the original mean
(c) is one-tenth of the original mean
(d) is increased by 10

Q. 7 If each value of a series is multiplied by 10, the median of the coded values is:
(a) not affected
(b) 10 times the original median value
(c) one-tenth of the original median value
(d) increased by 10

Q. 8 If each value of a series is multiplied by 10, the mode of the coded values is:
(a) not affected
(b) one-tenth of the original modal value
(c) 10-times of the original modal value
(d) 100-times of the original modal value

Q. 9 If each observation of a set is divided by 2, then the mean of new values:
(a) is two times the original mean.
(b) is decreased by 2.
(c) is half of the original mean
(d) remains the same

Q. 10 Which of the following relations among the location parameters does not hold?
(a) Q_2 = median
(b) P_{50} = median
(c) D_5 = median
(d) D_6 = median

Q. 11 If the grouped data has open end classes, one cannot calculate:
(a) median
(b) mode
(c) mean
(d) quartiles

Q. 12 Geometric mean of two observations can be calculated only if:
(a) both the observations are positive
(b) one of the two observations is zero
(c) one of them is negative
(d) both of them are zero

Q. 13 Geometric mean is better than other means when,
(a) the data are positive as well as negative
(b) the data are in ratios or percentages
(c) the data are binary
(d) the data are on interval scale

Q. 14 Geometric mean is a good measure of central value if the data are:

(a) categorical
(b) on ordinal scale
(c) in ratios or proportions
(d) none of the above

Q. 15 Harmonic mean is better than other means if the data are for:
(a) speed or rates
(b) heights or lengths
(c) binary values like 0 and 1
(d) ratios or proportions

Q. 16 The correct relationship between A.M., G.M. and H.M. is:
(a) A.M. = G.M. = H.M.
(b) G.M. $\geq$ A.M. $\geq$ H.M.
(c) H.M. $\geq$ G.M. $\geq$ A.M.
(d) A.M. $\geq$ G.M. $\geq$ H.M.

Q. 17 Extreme value have no effect on:
(a) average
(b) median
(c) geometric mean
(d) harmonic mean

Q. 18 Geometric mean of two numbers $\frac{1}{16}$ and $\frac{4}{25}$ is:
(a) $\frac{1}{10}$
(b) $\frac{1}{100}$
(c) 10
(d) 100

Q. 19 Expenditure during first five months of a year is Rs. 96 per month and during last seven months is Rs. 120 per month. The average expenditure per month during whole year is:
(a) Rs. 108 per month
(b) Rs. 110 per month
(c) Rs. 100 per month
(d) Rs. 216 per month

Q. 20 Average strength of eleven members = 11.0. Average strength of the first six members = 10.5. Average strength of the last six members = 11.5.
The average strength of the sixth member is:
(a) 10.5
(b) 11.5
(c) 11.0
(d) 10.0

Q. 21 The average of the 7 number 7, 9, 12, x, 5, 4, 11 is 9. The missing number x is:
(a) 13
(b) 14
(c) 15
(d) 8

Q. 22 The mean proportion of 0.16 and 0.01 is:
(a) 0.4
(b) 0.17
(c) 0.085
(d) 0.04

Q. 23 A train covered the first 5 km of its journey at a speed of 30 km/h and next 15 km at a speed of 45 km/h. The average speed of the train was:
(a) 35 km/h
(b) 40 km/h
(c) 32 km/h
(d) 42 km/h

Q. 24 The second of the two samples has 50 item with mean 15. If the whole group has 150 items with mean 16, the mean of the first sample is:
(a) 18.0
(b) 15.5
(c) 16.5
(d) none of the above

Q. 25 For a group of 100 candidates, the mean was found to be 40. Later on it was discovered that a value 45 was misread as 54. The correct mean is:
(a) 40.50
(b) 39.85
(c) 39.80
(d) 39.91

Q. 26 A distribution consists of three groups having 40, 50 and 60 items with means 20, 26 and 15 respectively. The mean of the distribution is:
(a) 20
(b) 18

(c) 22
(d) 25

Q. 27 The average age of 50 students in a bus is 20 years. When the age of conductor is included, the average age is increased by one year. The age of the conductor is:
(a) 51
(b) 55
(c) 71
(d) 50

Q. 28 The average of five numbers is 40 and the average of another four numbers is 50. The average of all numbers taken together is:
(a) 44.44
(b) 45.00
(c) 45.55
(d) 90.00

Q. 29 The average temperature of two cities on first six days of a week is same. The temperature dropped in one city all of a sudden on the seventh day of the week. The average weekly temperature of the two cities differed by 0.5°C. The difference between the six days average daily temperature of two cities and the seventh day temperature of the other city is:
(a) 2.5
(b) 3.0
(c) 3.5
(d) 2.0

Q. 30 What percentage of values is greater than 3rd quartile?
(a) 75 per cent
(b) 50 per cent
(c) 25 per cent
(d) 0 per cent

Q. 31 What percentage of values is less than 3rd decile?
(a) 30 per cent
(b) 70 per cent
(c) 40 per cent
(d) none of the above

Q. 32 What percentage of values lies between 5th and 25th percentiles.
(a) 15 per cent
(b) 30 per cent
(c) 75 per cent
(d) none of the above

Q. 33 There were 25 teachers in a school whose mean age was 30 years. A teacher retired at the age of 60 years and a new teacher was appointed in his place. The mean age of teachers in the school was reduced by one year. The age of the new teacher was:
(a) 25 years
(b) 30 years
(c) 35 years
(d) 40 years

Q. 34 If the A.M. of a set of two observations is 9 and its G.M. is 6. Then the H.M. of the set of observations is:
(a) 4
(b) $3\sqrt{6}$
(c) 3
(d) 1.5

Q. 35 The A.M. of two numbers is 6.5 and their G.M. is 6. The two numbers are:
(a) 9, 6
(b) 9, 5
(c) 7, 6
(d) 4, 9

Q. 36 If M_d, Q, D and P stand for median, quartile, decile and percentile respectively, then which of the following relation between them is true?
(a) $M_d = Q_2 = D_6 = P_{50}$
(b) $M_d = Q_3 = D_5 = P_{75}$
(c) $M_d = Q_2 = D_4 = P_{50}$
(d) $M_d = Q_2 = D_5 = P_{50}$

Q. 37 Which of the following relation is true between 3rd decile and 30th percentile?
(a) $D_3 = P_{70}$
(b) $D_3 = P_{50}$
(c) $D_3 = P_{30}$
(d) $D_7 = P_{30}$

Q. 38 Which of the deciles are less than first quartile?
(a) D_1 and D_3

(b) D_1 and D_2
(c) D_2 and D_3
(d) None of the above

Q. 39 In a factory there are 60 per cent labourers. 30 per cent scribes and 10 per cent executives. On average, the salary of a labourer is Rs. 1600 p.m, of a scribe Rs. 3000 p.m and that of an executive Rs. 8000 p.m. The average salary of a employee in the factory is:
(a) Rs. 4200 p.m
(b) Rs. 1166 p.m
(c) Rs. 2660 p.m
(d) None of the three

Q. 40 The mean of seven observations is 8. A new observation 16 is added. The mean of eight observations is:
(a) 12
(b) 9
(c) 8
(d) 24

Q. 41 If the sum of N observations is 630 and their mean is 42, then the value of N is:
(a) 21
(b) 30
(c) 15
(d) 20

Q. 42 If the two observations are 10 and –10, then their harmonic mean is:
(a) 10
(b) 0
(c) 5
(d) ∞

Q. 43 If the observations are 5 and –5, their geometric mean is:
(a) 5
(b) –5
(c) 0
(d) none of the above

Q. 44 If the two observations are 20 and –20, their arithmetic mean is:
(a) 10
(b) 20
(c) 0
(d) none of the above

Q. 45 Can a quartile, a decile and a percentile be the median?
(a) Only quartile but not decile and percentile
(b) Quartile and decile but not percentile
(c) Decile and percentile but not quartile
(d) Quartile, decile and percentile, all the three

Q. 46 When all the observations are same, then the relation between A.M., G.M. and H.M. is:
(a) A.M. = G.M. = H.M.
(b) A.M. < G.M. < H.M.
(c) A.M. < G.M. < H.M.
(d) A.M. < G.M. < H.M.

Q. 47 The percentage of values used in case of 10 per cent trimmed mean is:
(a) 40 per cent
(b) 60 per cent
(c) 80 per cent
(d) 20 per cent

Q. 48 The average of $2n$ natural numbers from 1 to $2n$ is:
(a) $(n + 1)/2$
(b) $(2n + 1)/2$
(c) $n (n + 1)/2$
(d) $n (2n + 1)/2$

Q. 49 A man goes from his house to his office at the speed of 20 km/h and returns from his office to home at the speed of 30 km/h. His mean speed is:
(a) 24 km/h
(b) $10\sqrt{6}$ km/h
(c) 25 km/h
(d) none of the above

Q. 50 Mode is that value in a frequency distribution which possesses:
(a) minimum frequency
(b) maximum frequency
(c) frequency one
(d) none of the above

Q. 51 The value of the variable corresponding to the highest point of a frequency distribution curve represents:
(a) mean

(b) median
(c) mode
(d) none of the above

Q. 52 A frequency distribution having two modes is said to be:
(a) unimodal
(b) bimodal
(c) trimodal
(d) without mode

Q. 53 If modal value is not clear in a distribution, it can be ascertained by the method of:
(a) grouping
(b) guessing
(c) summarising
(d) trial and error

Q. 54 Shoe size of most of the people in India is No. 8. Which measure of central value does it represent?
(a) mean
(b) second quartile
(c) eighth decile
(d) mode

Q. 55 In a discrete series having $(2K + 1)$ observations, median is:
(a) K^{th} observation
(b) $(K+1)^{\text{th}}$ observation
(c) $(K + 2)/2^{\text{th}}$ observation
(d) $(2K+1)/2^{\text{th}}$ observation

Q. 56 The median of the variate values 11, 7, 6, 9, 12, 15, 19 is:
(a) 9
(b) 12
(c) 15
(d) 11

Q. 57 The median of the variate values 48, 35, 36, 40, 42, 54, 58, 60 is:
(a) 40
(b) 41
(c) 44
(d) 45

Q. 58 To find the median (mode), it is necessary to arrange the data in:
(a) ascending order
(b) descending order
(c) ascending or descending order
(d) any of the above

Q. 59 For a grouped data, the formula for median is based on:
(a) interpolation method
(b) extrapolation method
(c) trial and error method
(d) iterative method

Q. 60 Which of the measure of central tendency is not affected by extreme values?
(a) mode
(b) median
(c) sixth decile
(d) all the above

Q. 61 The middle value of an ordered series is called:
(a) 2nd quartile
(b) 5th decile
(c) 50 percentile
(d) all the above

Q. 62 Formula for directly calculating the mean $\overline{X}$ of an individual series is:

(a) $\overline{X} = \dfrac{\Sigma X}{N}$

(b) $\overline{X} = \dfrac{\Sigma fX}{N}$

(c) $\overline{X} = A + \dfrac{\Sigma\, dx}{n}$ where $dx = X - A$

(d) $\overline{X} = A + \dfrac{\Sigma\, dx}{N}$ where $dx = X - A$

Q. 63 Formula for calculating the mean of an individual series by short-cut method is:

(a) $\overline{X} = \dfrac{\Sigma X}{N}$

(b) $\overline{X} = \Sigma\, fX/N$

(c) $\overline{X} = A + \dfrac{\Sigma\, dx}{N}$

(d) $\overline{X} = A + \dfrac{\Sigma\, fdx}{N}$

Q. 64 Formula for calculating the mean of a discrete series by direct method is:

(a) $\overline{X} = \Sigma X / \Sigma f$

(b) $\overline{X} = \Sigma_i f_i X_i / \Sigma_i f_i$

(c) $\overline{X} = A + \Sigma_i f_i dx_i / \Sigma f_i$

(d) none of the above

Q. 65 Formula for calculating the mean of a discrete series by short-cut method is:

(a) $\overline{X} = \Sigma X / \Sigma f$

(b) $\overline{X} = \Sigma f_x / \Sigma f$

(c) $\overline{X} = A + \Sigma dx / \Sigma f$

(d) $\overline{X} = A + \dfrac{\Sigma f dx}{\Sigma f}$

Q. 66 Rs. 600 per day are paid on a research farm to its 50 daily paid labourers. A worker gets five unpaid holidays in a month. The average income of a daily paid labourer is:

(a) Rs. 250 p.m
(b) Rs. 300 p.m
(c) Rs. 350 p.m
(d) Rs. 400 p.m

Q. 67 The variate values which divide a series (frequency distribution) into five equal parts are called:

(a) quintiles
(b) quartiles
(c) octiles
(d) percentiles

Q. 68 The variate values which divide a series (frequency distribution) into four equal parts are called:

(a) quintiles
(b) quartiles
(c) deciles
(d) percentiles

Q. 69 The variate values which divide a series (frequency distribution) into ten equal parts are called:

(a) quartiles
(b) deciles
(c) octiles
(d) percentiles

Q. 70 The variate values which divide a series (frequency distribution) into 100 equal parts are known as:

(a) octiles
(b) quartiles
(c) percentiles
(d) deciles

Q. 71 The variate values which divide a series (frequency distribution) into eight equal parts are known as:

(a) quartiles
(b) quintiles
(c) deciles
(d) octiles

Q. 72 The number of partition values in case of quartiles is:

(a) 4
(b) 3
(c) 2
(d) 1

Q. 73 The number of partition values in case of quintiles cannot exceed:

(a) 4
(b) 3
(c) 2
(d) 1

Q. 74 For deciles, the total number of partition values are:

(a) 5
(b) 8
(c) 9
(d) 10

Q. 75 For percentiles, the total number of partition values are:

(a) 10
(b) 59
(c) 100
(d) 99

Q. 76 The second decile divides the series in the ratio:

(a) 1:1
(b) 1:2

(c) 1 : 4
(d) 2 : 5

Q. 77 Eightieth percentile divides a frequency distribution in the ratio:
(a) 4 : 1
(b) 4 : 5
(c) 3 : 1
(d) 3 : 2

Q. 78 The first quartile divides a frequency distribution in the ratio:
(a) 4 : 1
(b) 1 : 4
(c) 3 : 1
(d) 1 : 3

Q. 79 Fourth octile divides a frequency distribution in the ratio:
(a) 1 : 1
(b) 1 : 2
(c) 1 : 3
(d) 1 : 4

Q. 80 The first quartile is also known as:
(a) median
(b) lower quartile
(c) mode
(d) third decile

Q. 81 The third quartile is also called:
(a) lower quartile
(b) median
(c) mode
(d) upper quartile

Q. 82 In case of weighted mean, the accuracy or utility of the mean:
(a) decreases
(b) increases
(c) is unaffected
(d) none of the above

Q. 83 In certain situations weighted mean and unweighted mean can:
(a) be equal
(b) never be equal
(c) both (a) and (b)
(d) neither (a) and nor (b)

Q. 84 Weighted mean is more accurate if we use:
(a) estimated weights
(b) arbitrary weights
(c) real weights
(d) any of the above

Q. 85 If .3, .5, .8, .7, and 1.5 are the respective weights of the values 10, 15, 20, 25 and 30, then the weighted mean is:
(a) 20.0
(b) 23.42
(c) 16.58
(d) none of the above

Q. 86 If for a discrete series, the assumed mean $A = 50, \sum f\,dx = 45$ for $dx = x - A, \sum f = 12$, then the mean of the series is:
(a) 46.25
(b) 7.92
(c) 49.17
(d) 53.75

Q. 87 The mean of the following discrete series (frequency distribution),

x: 7, 12, 16, 22, 25

f: 4, 5, 8, 3, 2

is:
(a) 16.40
(b) 15.09
(c) 20.80
(d) none of the three

Q. 88 If for an individual series, assumed mean, $A = 25, \sum dx = -21$ for $dx = X - A$ and $N = 7$ then the mean of the series is:
(a) 20
(b) 21
(c) 22
(d) 6.57

Q. 89 Given the following less than type frequency distribution of income per month,

Income (Rs.) less than	*No. of persons*
1500	100
1250	80
1000	70
750	55
500	32
250	12

the median class of income is:
(a) 750-1000

(b) 1000-1250
(c) 250-500
(d) 500-750

Q. 90 For the distribution given in question 89, the modal class is:
(a) 250-500
(b) 500-750
(c) 750-1000
(d) none of the above

Q. 91 For the distribution given in question 89, the upper quartile class is:
(a) 500-750
(b) 750-1000
(c) 1000-1250
(d) 1250-1500

Q. 92 For the distribution given in question 89, the sixth decile class is:
(a) 500-750
(b) 750-1000
(c) 1000-1250
(d) 1250-1500

Q. 93 For the distribution given in question 89, 30th percentile class is:
(a) 250-500
(b) 500-750
(c) 750-1000
(d) none of the above

Q. 94 Given the following frequency distribution of income of employees,

Income Rs./month	*No. of employees*
0-250	12
250-500	20
500-750	23
750-1,000	15
1,000-1,250	10
1,250-1,500	20

The median income of employees is:
(a) 625.00
(b) 760.00
(c) 695.65
(d) none of the above

Q. 95 For the frequency distribution given is question 94, the mode of the distribution is:
(a) 23
(b) 666.66
(c) 513.33
(d) 568.18

Q. 96 For the grouped data given in question 94, the third quartile is:
(a) 1,200
(b) 1,125
(c) 1,183.33
(d) none of the above

Q. 97 Sixth decile of the continuous frequency distribution given in question 94 is:
(a) 833.33
(b) 1,000.00
(c) 70.00
(d) none of the above

Q. 98 20th percentile of the grouped data given in question 94 is:
(a) 250
(b) 350
(c) 500
(d) 550

Q. 99 The more than type frequency distribution of age of the patients on a particular day in a hospital is as given below.

Age (years)	*No. of patients*
> 10	152
> 20	128
> 30	113
> 40	77
> 50	36
> 60	22
> 70	5
and up to 80	

the 50th percentile class is:
(a) 30-40
(b) 40-50
(c) 50-60
(d) none of the above

Q. 100 For the more than type distribution given in question 99, the modal class is:
(a) 30-40
(b) 40-50
(c) 50-60
(d) none of the above

Q. 101 For the more than type distribution given in question 99, the first quartile class is:
(a) 30-40
(b) 40-50
(c) 50-60
(d) none of the above

Q. 102 In the ascending series of question 99, fourth decile class is:
(a) 30-40
(b) 40-50
(c) 50-60
(d) none of the above

Q. 103 The mode of the more than type distribution given in question 99 is:
(a) 37.78
(b) 41.56
(c) 42.56
(d) none of the above

Q. 104 The median (second quartile) of the more than type distribution given in question 99 is:
(a) 37.33
(b) 40.00
(c) 40.24
(d) 50.00

Q. 105 The seventh decile of the cumulative distribution of question 99 is:
(a) 42.34
(b) 48.78
(c) 57.66
(d) 47.66

Q. 106 Thirtieth percentile of the more than type distribution of question 99 is:
(a) 31.83
(b) 53.50
(c) 30.88
(d) none of the above

Q. 107 The upper quartile of the distribution given in question 99 is:
(a) 50.00
(b) 49.51
(c) 45.00
(d) 41.00

Q. 108 If we plot the more than type and less than type frequency distributions of the same set of data, their graphs intersect at the point which is known as:
(a) median
(b) mode
(c) mean
(d) none of the above

Q. 109 Mean of a set of values is based on:
(a) all values
(b) 50 per cent values
(c) first and last value
(d) maximum and minimum value

Q. 110 Which mean is most affected by extreme values?
(a) Geometric mean
(b) Harmonic mean
(c) Arithmetic mean
(d) Trimmed mean

Q. 111 The measure of central value which cannot be calculated with open end classes in case of grouped data is:
(a) median
(b) arithmetic mean
(c) mode
(d) third quartile

Q. 112 Harmonic mean gives more weightage to:
(a) small values
(b) large values
(c) positive values
(d) negative values

Q. 113 Harmonic mean gives less weightage to:
(a) positive values
(b) negative values
(c) small values
(d) large values

Q. 114 For further algebraic treatment geometric mean is:
(a) suitable
(b) not suitable
(c) sometimes suitable
(d) none of the above

Q. 115 For further algebraic treatment harmonic mean is:
(a) suitable

(b) not suitable
(c) sometimes suitable
(d) none of the above

Q. 116 For a highly variable series, the most suitable mean is:
(a) arithmetic mean
(b) geometric mean
(c) harmonic mean
(d) none of the above

Q. 117 The percentage of values in a set of values which are less than (more than) the median value is:
(a) 100 per cent
(b) 75 per cent
(c) 50 per cent
(d) 25 per cent

Q. 118 The percentage of values of a set which is beyond the third quartile is:
(a) 100 per cent
(b) 75 per cent
c) 50 per cent
(d) 25 per cent

Q. 119 The percentage of data of a set which are to the left of seventh decile is:
(a) 30 per cent
(b) 50 per cent
(c) 70 per cent
(d) 90 per cent

Q. 120 The percentage of data of a set which is to the right of ninetieth percentile is:
(a) 0 per cent
(b) 10 per cent
(c) 90 per cent
(d) 80 per cent

Q. 121 In a class test, 40 students out of 50 passed with mean marks 6.0 and the overall average of class marks was 5.5. The average marks of students who failed were:
(a) 2.5
(b) 3.0
(c) 4.8
(d) 3.5

Q. 122 Seven persons gambled sitting on a table. Four persons lost on an average Rs. 55 whereas the other three gained on an average Rs. 70. Is the information worth believing?
(a) Yes
b) No
(c) Not certain
(d) None of the above

Q. 123 The average marks of section A are 65 and that of section B are 70. The average of both the sections combined is 67. The ratio of number of students of section A to B is:
(a) 1 : 3
(b) 2 : 3
(c) 3 : 1
(d) 3 : 2

Q. 124 The partition value which divide a series into two equal parts is known as:
(a) second quartile
(b) third quintile
(c) fourth octile
(d) sixth deciles

Q. 125 In a distribution, the value around which the items tend to be most heavily concentrated is called:
(a) mean
(b) median
(c) third quartile
(d) mode

Q. 126 Geometric mean can be used to find out:
(a) population growth
(b) growth rate of GNP
(c) both the above
(d) none of (a) and (b)

Q. 127 Formula for the combined geometric mean 'GM_{12}' of two series of sizes n_1 and n_2 and their G.M.'s GM_1 and GM_2 respectively is:

(a) $GM_{12} = \dfrac{n_1 \log GM_1 + n_2 \log GM_2}{n_1 + n_2}$

(b) $GM_{12} = \dfrac{n_1 \times GM_1 + n_2 \times GM_2}{n_1 + n_2}$

(c) $GM_{12} = \dfrac{GM_1 \log n_1 + GM_2 \log n_2}{n_1 + n_2}$

(d) $GM_{12} = \text{Antilog}\left(\dfrac{n_1 \log GM_1 + n_2 \log GM_2}{n_1 + n_2}\right)$

Q. 128 Sum of the deviations about mean is:
(a) zero
(b) minimum
(c) maximum
(d) one

Q. 129 Sum of the absolute deviations about median is:
(a) zero
(b) maximum
(c) minimum
(d) one

Q. 130 Sum of square of the deviations about mean is:
(a) maximum
(b) minimum
(c) zero
(d) none of the above

Q. 131 Histogram is useful to determine graphically the value of:
(a) mean
(b) median
(c) mode
(d) all the above

Q. 132 Graphically partition values can be determined with the help of:
(a) frequency polygon
(b) bar diagram
(c) line diagram
(d) ogive curve

Q. 133 The suitable measure of central tendency for qualitative data is:
(a) mode
(b) arithmetic mean
(c) geometric mean
(d) median

Q. 134 Two series having the same mean, median and mode may:
(a) have same values
(b) not have same values
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 135 Weighted mean gives a higher value than unweighted mean if:
(a) all the items have equal weights
(b) larger items have higher weights and small items have lower weights.
(c) larger items have lower weights and smaller items have higher weights
(d) none of the above

Q. 136 In a frequency distribution with open ends, one cannot find out:
(a) mean
(b) median
(c) mode
(d) all the above

Q. 137 The average of n observations $x_1, x_2, ..., x_n$ is M. If x_1 is replaced by x', then the new average is:
(a) $M - x_1 + x'$
(b) $\dfrac{M - x_1 + x'}{n}$
(c) $\dfrac{(n-1)M - x_1 + x'}{n}$
(d) $\dfrac{nM - x_1 + x'}{n}$

Q. 138 In case of 15 per cent trimmed mean, the percentage of observations utilised is:
(a) 15 per cent
(b) 30 per cent
(c) 85 per cent
(d) none of the above

Q. 139 If for values of X, A.M. = 25, H.M. = 9, then the G.M. is:
(a) 17
(b) 15
(c) 5.83
(d) 16

Q. 140 The second quartile of the following set of data, 0, 1, −1, −2, 6, 4, 5, 8, 12, 10, 11 is:
(a) 4
(b) 5
(c) 6
(d) 8

Q. 141 The measure of central tendency which remains unaltered by extreme observations is:
(a) mean
(b) mode
(c) harmonic mean
(d) geometric mean

Q. 142 The mean of the squares of first eleven natural numbers is:
(a) 46
(b) 23
(c) 48
(d) 42

Q. 143 The point of intersection of two cumulative frequency curves provides:
(a) mean
(b) mode
(c) median
(d) first quartile

Q. 144 The percentage of items in a frequency distribution lying between upper and lower quartiles is:
(a) 80 per cent
(b) 40 per cent
(c) 50 per cent
(d) 25 per cent

Q. 145 A person has been deputed to find the average income of factory employees. To provide a correct picture of average income, he should find out:
(a) geometric mean
(b) weighted mean
(c) progressive mean
(d) arithmetic mean.

Q. 146 To know the trend of a time series data, preferable type of average is:
(a) progressive average
(b) moving average
(c) weighted mean
(d) quadratic mean.

Q. 147 If we know the mean values and number of units of various groups, then one mean value for all the groups can be obtained by finding out:
(a) geometric mean
(b) weighted mean
(c) pooled mean
(d) progressive mean

Q. 148 In a frequency distribution of a large number of values, the mode is:
(a) largest observation
(b) smallest value
(c) observation with maximum frequency
(d) maximum frequency of an observation

Q. 149 The relation between quadratic mean (Q.M.) and arithmetic mean (A.M.) is:
(a) Q.M. = A.M.
(b) Q.M. > A.M.
(c) Q.M. < A.M.
(d) Q.M. ≠ A.M.

Q. 150 A set of values contains a few negative values. The average of set can better be measured by:
(a) arithmetic mean
(b) geometric mean
(c) progressive mean
(d) quadratic mean

ANSWERS

Section-B

(1) John I. Griffin (2) R.A. Fisher (3) A.L. Bowley (4) typical value, summary measure (5) extreme values (6) variate (7) summarises (8) mean (9) 5 (10) $\frac{1}{10}$th (11) 13 (12) 16 (13) zero (14) Rs. 65 (15) Rs. 27 (16) accurate (17) 70 (18) $(n + 1)/2$ (19) minimum (20) 8 (21) Antilog (G_1/G_2) (22) zero (23) ratio; proportions (24) 4 (25) 5 (26) average speed (27) infinity (28) G.M. = $\sqrt{\text{A.M.}\times\text{H.M.}}$ (29) median (30) minimum (31) 12 (32) unique (33) median (34) open end (35) mode (36) 7 (37) median; mode (38) mode (39) mode (40) unimodal (41) bimodal (42) grouping (43) quartile (44) median (45) 75th (46) fifth (47) D_7 (48) 26 per cent (49) lower or first (50) 60th (51) 50 per cent (52) 34 per cent (53) 40th (54) $Q_2 < D_6 < P_{80}$ (55) fifth (56) fiftieth (57) ascending (58) quintiles (59) P_{80} (60) third (61) first; twenty-fifth (62) fractiles

(63) Ya Lun Chou (64) $Q_1 = P_{25}$ (65) $D_4 < P_{45}$ (66) 75 per cent (67) numerical; analytical (68) $(Q_1+Q_3)/2$ (69) antilog (70) reciprocal (71) negative (72) H.M. ≤ G.M. ≤ A.M. (73) same (74) standard deviation (75) 26 (76) 7 (77) 21 (78) entire data (79) best (80) not rigidly.

SECTION-C

(1) a (2) a (3) b (4) b (5) a (6) b
(7) b (8) c (9) c (10) d (11) c (12) a
(13) b (14) c (15) a (16) d (17) b (18) a
(19) b (20) c (21) c (22) d (23) b (24) c
(25) d (26) a (27) c (28) a (29) a (30) c
(31) a (32) d (33) c (34) a (35) d (36) d
(37) c (38) b (39) c (40) b (41) c (42) d
(43) d (44) c (45) d (46) a (47) c (48) b
(49) a (50) b (51) c (52) b (53) a (54) d
(55) b (56) d (57) d (58) d (59) a (60) d
(61) d (62) a (63) c (64) b (65) d (66) b
(67) a (68) b (69) b (70) c (71) d (72) b
(73) a (74) c (75) d (76) c (77) a (78) d
(79) a (80) b (81) d (82) b (83) a (84) c
(85) b (86) d (87) b (88) c (89) d (90) b
(91) c (92) b (93) a (94) c (95) d (96) b
(97) a (98) b (99) b (100) b (101) d (102) a
(103) d (104) c (105) d (106) a (107) b (108) a
(109) a (110) c (111) b (112) a (113) d (114) b
(115) b (116) c (117) c (118) d (119) c (120) b
(121) d (122) b (123) d (124) a (125) d (126) c
(127) d (128) a (129) c (130) b (131) c (132) d
(133) d (134) c (135) b (136) a (137) d (138) d
(139) b (140) b (141) b (142) a (143) c (144) c
(145) b (146) b (147) c (148) c (149) b (150) d

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Chapter 5

Measures of Dispersion, Skewness and Kurtosis

SECTION-A

Short Essay Type Questions

Q. 1 What do you understand by dispersion of a set of values?

Ans. Experience tells that in many situations, the spread of values is different but their central values are same. All the more, a central value provides no information about the scattering of values in a set of data. Hence, certain measures are evolved which reflect on the scattering of values in numerical terms are known as measures of dispersion.

Q. 29 Quote the statements about the term dispersion given by (i) Reigleman, (ii) W.I. King, (iii) Spiegel, (iv) B.C. Brookes and W.F.L. Dick, (v) A.L. Bowley.

Ans. The statements given by various workers are as follows:

(i) **Reigleman:** Dispersion is the extent to which the magnitude or qualities of the items differ, that is the degree of diversity.

(ii) **W.I. King:** The term dispersion is used to indicate the facts that within a given group, the items differ from another in size or in other words there is a lack of uniformity in their size.

(iii) **Spiegel:** The degree to which numerical data tend to spread about an average value is called the variation or dispersion of data.

(iv) **B.C. Brookes and W.F.L. Dick:** Dispersion or spread is the degree of the scatter or variation of the variable about a central value.

(v) **A.L. Bowley:** Dispersion is the measure of the variation of the items.

Q. 3 Name different measures of dispersion.

Ans. Following are the different measures of dispersion:

(i) range, (ii) Interquartile range and quartile deviation, (iii) Mean deviation, (iv) Median absolute deviation, (v) Variance (vi) Standard deviation, and (vii) Coefficient of variation.

Q. 4 In what manner, John I. Griffin and, C.T. Clark and L.L. Schkade defined measures of dispersion.

Ans. The definitions given by the statisticians named in the question are as given below:

John I. Griffin: A measure of variation or dispersion describes the degree of scatter shown by the observations and is usually measured as an average deviation about some central value or by any order statistic.

Clark and Schkade: Measures of dispersion are measures of scatter about an average.

Q. 5 What are requisites of a good measure of dispersion?

Ans. Main requisites of an ideal measure of dispersion can be given as follows:

(i) It should be based on all the observations.

(ii) Its unit should be same as the unit of measurement of items.

(iii) It should be rigidly defined.

(iv) It should follow general rules of mathematics.

(v) It should not be subjected to complicated and tedious calculations.

Q. 6 What are the uses of dispersion?

Ans. Main uses of dispersion are:

(i) It tells about the reliability of a measure of central value.

(ii) It makes possible to compare two series of data in respect of their variability.

(iii) Measure of dispersion provides the basis for the control of variability.

(iv) It has a wide application in almost all fields of statistics.

Q. 7 Define range.

Ans. The difference between the largest and smallest values of a set of data is called its range. Range is shown as lowest value-largest value.

Q. 8 Give the merits of range.

Ans. Merits of range are:

(i) It is the easiest measure of dispersion.

(ii) It can always be found out visually *i.e.*, it involves no calculations.

(iii) It is one of the largely used measure of dispersion.

Q. 9 Give demerits of range.

Ans. Demerits of range are:

(i) It depends on two extreme values of a series. Thus, it gives no information about the observations lying between smallest and largest values.

(ii) It is highly susceptible to sampling fluctuations.

(iii) It is not suitable for further mathematical treatment.

(iv) Addition or deletion of a single value may change the entire complex of range.

Q. 10 Define coefficient of range.

Ans. It is a pure number given as the ratio of difference between the largest and smallest values to the sum of the largest and smallest values of a set of data. Numerically, coefficient of range is $(L - S)/(L + S)$. Lesser the coefficient of range, better it is.

Q. 11 Express interquartile (I.Q.) range.

Ans. It is equal to the difference between the upper and lower quartiles. Symbolically, it is equal to $(Q_3 - Q_1)$. This measure of dispersion tells about the range of the middle 50 per cent values of a set of data. In this measure, lower 25 per cent and upper 25 per cent values are excluded. It is not a good measure of dispersion as it tells nothing about the dispersion of values around average. It hardly fulfils any of the requisites of a good measure of dispersion.

Q. 12 Define percentile range.

Ans. The difference between 90th percentile (P_{90}) and 10th percentile (P_{10}) is called percentile range. It is denoted as $P_{90} - P_{10}$. Its value is same as $D_9 - D_1$ where D_1 and D_9 denote 9th and 1st deciles. This measure is more useful in education.

Q. 13 Define quartile deviation (Q.D.) and give its important features.

Ans. It is half of the interquartile range, *i.e.*, $(Q_3 - Q_1)/2$. It is an absolute measure of dispersion. Hence, to compare two series, a relative measure known as coefficient of quartile deviation is given which is symbolically expressed as

$(Q_3 - Q_1)/(Q_3 + Q_1)$.

Q. 14 For a symmetrical distribution, how can one determine the upper and lower quartiles with the help of quartile deviation?

Ans. For a symmetrical distribution, the upper and lower quartiles can be determined by the formula $(Q_2 + Q.D.)$ and $(Q_2 - Q.D.)$ respectively.

Q. 15 What are the good points of quartile deviation?

Ans. Following are the good points of quartile deviation:

(i) It is easy to calculate and understand.

(ii) It can be calculated in case of open end frequency distributions as well.

(iii) It is not affected by 25 per cent upper and 25 per cent lower extreme values.

Q. 16 Throw light on mean deviation (M.D.).

Ans. Range and quartile deviations are positional measures of dispersion, whereas, mean deviation is a measure of dispersion which is based on all values of a set of data. It is defined as the average of the absolute deviations taken from an average usually, the mean, median or mode. It is usually denoted by δ. To clarify whether the average used in mean deviation is mean, median or mode, a suffix is attached to δ such as δ_M, δ_{Md} or δ_{Mo}.

The formula for calculating mean deviation of n observations $X_1, X_2, \ldots, X_n$ is,

$$\delta = \frac{1}{n}\sum_i |X_i - A|$$

for $\quad i = 1, 2, \ldots, n$

Also A may be any chosen constant out of mean, median and mode.

For a frequency distribution in which the variate value x_i occurs f_i times ($i = 1, 2, \ldots, k$), the formula for mean deviation is,

$$\delta = \frac{1}{n}\sum_i f_i |X_i - A|$$

where $\sum_i f_i = n$ and A as defined above.

Here, it is worth emphasising that mean deviation is minimum about the median. That is why median is commonly used as an average value about which the mean deviation is calculated.

Q. 17 Discuss coefficient of mean deviation.

Ans. Mean deviation has the same unit of measurement as that of the variable x. If two series have different units of measurement, the series cannot be compared. Hence for comparing any two series, an unitless measure is given known as *coefficient of mean deviation*. It is the ratio of mean deviation to the average 'A' used in calculating it. Its formula is,

$$\text{Coeff. of M.D.} = \frac{\text{Mean deviation}}{A} \times 100$$

It is multiplied by 100 to express coefficient of mean deviation in percentage.

Q. 18 Write the merits of mean deviation.

Ans. Following are the merits of mean deviation:

1. It utilises all the observations of the set.
2. It is simple to calculate and understand.
3. It is least affected by extreme values.

Q. 19 Mention the demerits of mean deviation.

Ans. Following are the demerits of mean deviation:

1. The foremost weakness of mean deviation is that in its calculation negative differences are considered positive without any sound reasoning.
2. It is not amenable to further algebraic treatment.
3. It cannot be calculated is case of open end(s) frequency distribution.

Q. 20 Define and discuss variance.

Ans. The average of the square of the deviations taken from mean is called variance. The population variance is generally denoted by σ^2 and its estimate (sample variance) by s^2. For N population values $X_1, X_2, \ldots, X_N$ having the population mean μ, the population variance,

$$\sigma^2 = \frac{1}{N}\sum_i (X_i - \mu)^2$$

for $\quad i = 1, 2, \ldots, N$

where $\quad \mu = \Sigma_i X_i / N$

An estimate of σ^2 based on n sample values $x_1, x_2, \ldots, x_n$, the sample variance,

$$s^2 = \frac{1}{n-1}\sum_i (x_i - \bar{x})^2$$

for $i = 1, 2, \ldots, n$

and $\bar{x} = \Sigma_i\, x_i / n.$

For a frequency distribution of sample values x_1, x_2, ..., x_k having frequencies $f_1, f_2, \ldots, f_k$ respectively. The sample variance,

$$s^2 = \frac{1}{n-1}\sum_i f_i (x_i - \bar{x})^2$$

$$= \frac{1}{n-1}\left\{\sum_i f_i x_i^2 - \frac{(\Sigma_i f_i x_i)^2}{n}\right\}$$

where $n = \Sigma_i f_i$ for $i = 1, 2, \ldots, k$

Q. 21 Discuss merits and demerits of variance.

Ans. It possesses all the requisites of a good measure of dispersion except that its unit is square of the unit of measurement of variate values. Hence, many times it becomes difficult to actually adjudge the magnitude of variation. Also variance is sensitive to extreme values. Variance is the backbone of statistics.

Q. 22 Mean deviation is minimum while taking the deviations from median. Then why do we take sum of square of deviations from mean while calculating the variance?

Ans. Deviations from mean are used for variance because sum of squares of deviations from mean is minimum.

Q. 23 How does variance is affected by the change of origin and scale?

Ans. Variance is independent of change of origin, but is affected by the change of scale. If each observation of a data set is multiplied by c, then the variance of new data set, is c^2 times the variance of original data.

Q. 24 What is the difference between absolute and relative dispersion?

Ans. If the unit of a measure of dispersion is in same terms as that of the observations of a series, it is called *absolute measure of dispersion*, e.g., height in cm, weight in kg., income in rupees, etc. In this case two series, having different units of dispersion, cannot be compared. Hence, for comparison of two series having different units of measurement, one requires an unit less measure of dispersion. Such measures are termed as *relative measures or coefficient of* dispersion. For this, a measure of dispersion is usually divided by mean used in its calculation and multiplied by 100. This provides the measure expressed in percentage which is fit for comparison of any two or more series.

Q. 25 Define and describe in brief standard deviation.

Ans. The positive square root of the variance is called standard deviation. The idea of standard deviation was first given by Karl Pearson in 1893.

Symbolically,

$$\sigma = \sqrt{\sigma^2}$$

and

$$s = \sqrt{s^2}$$

It fulfils all the requisites of a good measure of dispersion except that it is sensitive to extreme values. That is why it is known as standard deviation.

Q. 26 Explain median absolute deviation (MAD).

Ans. Standard deviation is affected by extreme values. Hence, the median absolute deviation is an alternative measure of dispersion. Median absolute deviation is defined as the median of the absolute deviation taken from median. It is seldom used as it is not easily amenable to further algebraic treatment. Moreover, it is not involved in distribution function.

The formula for median absolute deviation is,

$$\text{MAD} = \text{Median}\ \left|X_i - X_{M_d}\right|$$

Q. 27 Comment on the variances of median and sample mean based on a sample of size n from a normal population having variance σ^2.

Ans. Variance of sample median for samples of size n from a normal population is $\pi\,\sigma^2/2n$ which is greater than the variance of sample mean $\bar{x}$ which is equal to σ^2/n.

Q. 28 How variability is indicated by a measure of dispersion?

Ans. Lesser the value of a measure of dispersion, more is the degree of nearness of observations. It is also an indicator of homogeneity of values of a series.

Q. 29 What are the different names given to standard deviation?

Ans. Standard deviation is also known as *mean error, mean square error or root mean square deviation from mean.*

Q. 30 What is the effect of adding or subtracting a constant '*c*' from each observation of a set on variance or standard deviation?

Ans. The variance or standard deviation remains the same as they are independent of change of origin.

Q. 31 Standard deviation is equivalent to what type of mean?

Ans. As a matter of fact, standard deviation is nothing but the quadratic mean of the deviations from data mean.

Q. 32 Why are mean deviation (M.D.) and standard deviation (S.D.) not calculated using the same average?

Ans. Mean deviations and standard deviation are not calculated using the same average because M.D. is minimum when the deviations are taken from median whereas mean square deviation is least when deviations are measured from mean.

Q. 33 Give the formula for standard deviation of a population of first *n* natural numbers.

Ans. The formula for standard deviation (σ) of first *n* natural numbers is

$$\sigma = \sqrt{\frac{1}{12}\left(n^2 - 1\right)}$$

Q. 34 How is variance affected if each observation of a set is divided by a constant '*d*'.

Ans. The variance of the new set of values will be $\frac{1}{d^2}$ times the original value of variance.

Q. 35 How is variance affected if each observation of a set is multiplied by '*d*'.

Ans. The variance of the transformed set of values will be d^2 times the variance of the original set of values.

Q. 36 What is coefficient of variation and its importance?

Ans. Coefficient of variation (C.V.) is the ratio of the standard deviation and the mean. Usually it is expressed in percentage. The formula for coefficient of variation is,

$$\text{C.V.} = \frac{\text{S.D.}}{\text{mean}} \times 100.$$

It is a relative measure and is most suitable to compare any two series. As we know, the size of measure of dispersion also depends on the size of measurement. Hence, it is very appropriate measure of dispersion to compare two series which differ largely in respect of their means. All the more, a series or a set of values having lesser coefficient of variation as compared to the other is more consistent.

Q. 37 How can the variance of two groups or series of data can be combined (pooled)?

Ans. Suppose $\sigma_1^2, \sigma_2^2; \overline{X}_1, \overline{X}_2$ and N_1, N_2 are the variances, means and sizes of two groups of values respectively. Also let $\overline{X}_{12}$ be their combined mean. The combined variance of two groups is given by the formula.

$$\sigma_{12}^2 = \frac{N_1\left\{\sigma_1^2 + \left(\overline{X}_1 - \overline{X}_{12}\right)^2\right\} + N_2\left\{\sigma_2^2 + \left(\overline{X}_2 - \overline{X}_{12}\right)^2\right\}}{N_1 + N_2}$$

$$= \frac{N_1\left(\sigma_1^2 + d_1^2\right) + N_2\left(\sigma_2^2 + d_2^2\right)}{N_1 + N_2}$$

where, $\overline{X}_1 - \overline{X}_{12} = d_1$ and $\overline{X}_2 - \overline{X}_{12} = d_2$. The formula for combined variance can be extended to any number of groups.

Q. 38 What is the advantage of combined variance?

Ans. Many times we know the means and variances of individual series or groups of data of known sizes. Then for some statistical analysis, their pooled variance is required. By the formula for pooled variance, it can easily be obtained without original data. Also a lot of time and labour is saved.

Q. 39 Define and discuss moments in brief.

Ans. r^{th} moment may be defined as the average of the r^{th} exponent of the deviations of the variate values of a series about mean. r^{th} moment about an arbitrary mean '*a*' is given as.

$$\mu_r = E(x-a)^r$$

for $\quad r = 1, 2, \ldots, r$

where, E stands for expectation. Mathematical expectation has been discussed in chapter 7. Read it from there. For a series having N values $X_1, X_2, \ldots, X_N$, the r^{th} moment about an arbitrary origin '*a*' is given as.

$$\mu_r^a = \frac{1}{N}\sum(X_i - a)^r$$

for $\quad i = 1, 2, \ldots, N.$

If $a = 0$, the moment is known as r^{th} raw (simple) moment and is given as

$$\mu_r' = \frac{1}{N}\sum X_i^r$$

Again, if $a = \mu$, the r^{th} moment is known as central (absolute) moment and its formula is,

$$\mu_r = \frac{1}{N}\sum(x_i - \mu)^r$$

It is interesting to reveal that an algebraic relation between μ_r^a, μ_r' and μ_r can easily be established. It helps in determining one when others are known.

Q. 40 What are the uses and importance of moments.

Ans. Each frequency distribution is specified by its moments especially the first and second moments. They also help in determining the shape of a distribution since skewness and kurtosis are usually measured with the help of moments. Mean and variance are nothing but first and second moments respectively. In practice, moments of order higher than fourth are rarely required.

Q. 41 What is known as probable error?

Ans. Two-thirds of the standard deviation is known as probable error. It is also equal to quartile deviation. Symbolically,

$$\begin{aligned} \text{P.E.} &= \frac{2}{3}\sigma \\ &= \text{Q.D.} \end{aligned}$$

Q. 42 What does empirical relations exist between quartile deviation, mean deviation and standard deviation?

Ans. Relation between Q.D., M.D. and S.D. is,

$$\text{Q.D.} = \frac{5}{6}\text{M.D.} = \frac{2}{3}\text{S.D.}$$

or $\quad 6\ \text{Q.D.} = 5\ \text{M.D.} = 4\ \text{S.D.}$

Q. 43 Give an empirical relation between range, standard deviation, quartile deviation and mean deviation.

Ans. The empirical relation between range, standard deviation, quartile deviation and mean deviation is as follows:

$$R = 6\text{S.D.} = 9\ \text{Q.D.} = \frac{15}{2}\text{M.D.}$$

Q. 44 Give relations of the first four central moments with the raw moments.

Ans. The relations of the first four moments μ_1, μ_2, μ_3 and μ_4 with the raw moments μ_1', μ_2', μ_3' and μ_4' are:

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$\mu_3 = \mu_3' - 3\mu_2'\,\mu_1' + 2\mu_1'^3$$

$$\mu_4 = \mu_4' - 4\mu_3'\,\mu_1' + 6\mu_2'\,\mu_1'^2 - 3\mu_1'^4$$

Q. 45 Write the relations of the first four moments about the arbitrary mean '*a*' and raw moments.

Ans. Four moments about the arbitrary mean '*a*' *i.e.*, in terms of raw moments can be expressed as:

$$\mu_1^a = \mu_1' - a = \mu - a$$

$$\mu_2^a = \mu_2' - 2a\,\mu_1' + a^2$$

$$\mu_3^a = \mu_3' - 3a\,\mu_2' + 3a^2\mu_1' - a^3$$

$$\mu_4^a = \mu_4' - 4a\,\mu_3' + 6a^2\,\mu_2' - 4a^3\,\mu_1' + a^4$$

Q. 46 What will be the effect of coding (change of origin and scale) of data on moments?

Ans. Let a constant value c is subtracted from each variate value and then divided by h, *i.e.*, the coded value $d = \frac{X - c}{h}$. Then, the raw moments can be determined by the following relations,

$$\mu_1' = \left(\frac{1}{N}\sum_i f_i d_i\right) \times h$$

$$\mu_2' = \left(\frac{1}{N}\sum_i f_i d_i^2\right) \times h^2$$

$$\mu_3' = \left(\frac{1}{N}\sum_i f_i d_i^3\right) \times h^3$$

$$\mu_4' = \left(\frac{1}{N}\sum_i f_i d_i^4\right) \times h^4$$

where, there exists k variate values $X_1, X_2, ..., X_k$ having frequencies $f_1, f_2, ..., f_k$ respectively and for $i = 1, 2 ..., k$. Also $\Sigma f_i = N$.

The relations show that there is no effect of change of origin on the moments but the scale factor appears in multiplication if each value after subtracting c is divided by h. Once we know the raw moments, we can find the central moments using the relations between two types of moments.

Q. 47 What are the special advantages of standard deviation?

Ans. The special advantages of standard deviation are:

(i) Standard deviation carries great importance in sampling methods.

(ii) It is least sensitive to sampling fluctuations.

(iii) With the help of standard deviation, it is possible to ascertain the area under the normal curve.

(iv) It has great utility in testing of hypotheses which other measures of dispersion hardly do.

Q. 48 What is meant by skewness?

Ans. Lack of symmetry of tails (about mean) of a frequency distribution curve is known as skewness. Symmetry of tails means that the frequency of the points at equal distances on both sides of the centre of the curve on X-axis is same. Also, the area under the curve at equidistant intervals on both sides of the centre is also equal. Departure from symmetry leads to skewness. It is adjudged by the elongation of the right and left tails of the curve.

Q. 49 What is positive and negative skewness?

Ans. If the left tail of the frequency curve is more elongated than right tail, it is known as negative skewness and a reverse situation leads to positive skewness.

Q. 50 What purpose is served by measuring skewness?

Ans. Measure of Skewness indicates to what extent and in what direction the distribution of a variable differs from symmetry of a frequency curve. The curve may have positive or negative skewness. Both positive and negative skewness can never occur simultaneously.

Q. 51 What changes occur in the position of mean, median and mode in case of positive and negative skewness?

Ans. In a negative skew curve, the mean and median are pulled to the left whereas in a positive skew curve, the mean and median are pulled to the right. Also it should be kept in mind that median always lies in between mean and mode.

Q. 52 How can we know about skewness?

Ans. Skewness can be known by two methods given below:

(i) Graphically

(ii) By mathematical measures.

Q. 53 Describe graphical method of detecting skewness.

Ans. Draw a frequency curve by plotting the points for variate values and corresponding frequencies. Judge by naked eyes whether the tails of the curve are symmetrical or not. If not symmetrical, the curve is skew. Positive skewness depends whether the right tail is more elongated than left tail. For negative skewness, a reverse situation exists.

Graphical method is just enough to know about skewness. But it does not give the idea about the extent of skewness. Further slight skewness cannot be detected by naked eyes.

Q. 54 Give different formulae for measuring skewness α_3.

Ans. Different formulae for measuring skewness are:

(i) Bowley's formula for measuring skewness in terms of quartiles is:

$$\alpha_3 = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

In many books measure of skewness is denoted by J or S_k.

(ii) Kelley gave the formula in terms of percentiles and deciles.

Kelley's absolute measures of skewness are,

$$S_k = P_{90} + P_{10} - 2P_{50}$$
$$= D_9 + D_1 - 2D_5$$

These formulae are not practically used. Instead, it is measured as coefficient of skewness which is given as,

$$S_k = \frac{P_{90} + P_{10} - 2P_{50}}{P_{90} - P_{10}}$$
$$= \frac{D_9 + D_1 - 2D_5}{D_9 - D_1}$$

Kelley's formulae are seldom used.

(iii) Karl Pearson's measure of skewness,

$$\alpha_3 = \frac{\text{mean} - \text{mode}}{\text{S.D.}}$$

(iv) Karl Pearson's formula for a wide class of frequency distributions in terms of moments is,

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

β_1 gives only the measure of skewness but not the direction of skewness. So another measure γ_1 is defined as,

$$\gamma_1 = \sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \alpha_3$$

Q. 55 How do you interpret the value of measure of skewness?

Ans. If the measure of skewness is zero, it means that the frequency curve is symmetrical. Hence, for a symmetrical curve $\beta_1 = \alpha_3 = \gamma_1 = 0$ which implies that $Q_3 + Q_1 = 2Q_2$; $P_{90} + P_{10} = 2P_{50}$; $D_9 + D_1 = 2D_5$; mean = median = mode. Also, $\mu_3 = 0$.

Again, if the value of S_k, γ_1 or α_3 is positive, it leads to positive skewness. In this situation, the frequency curve has elongated right tail. A negative value of S_k, γ_1 or α_3 leads to negative skewness and the frequency curve has a long tail on the left as compared to the right tail. Greater the magnitude of S_k, γ_1 or α_3, more is the skewness.

Q. 56 For a moderately skew distribution, what relation between mean, median and mode exists?

Ans. For a moderately skew distribution, the relation between mean, median and mode is,

Mean − Mode = 3 (Mean − Median)

Q. 57 What do you understand by Kurtosis?

Ans. Kurtosis means bulginess. Kurtosis relates to the peakedness of a frequency curve as compared to a normally peaked curve. If a frequency distribution curve is more peaked or flat than a normally peaked curve then it is called a *Kurtic curve*. This property of bulginess is called *Kurtosis*.

If a frequency curve is more peaked than normal then it is called a *leptokurtic* curve and if it is less peaked than normal, it is called *platykurtic curve*. In terms of Kurtosis, a normally peaked curve is known as *mesokurtic curve*.

Q. 58 Kurtosis is adjudged around which measure of central tendency?

Ans. Kurtosis is adjudged around mode of the frequency distribution.

Q. 59 How can one know about Kurtosis?

Ans. Kurtosis can be perceived simply by looking a frequency distribution curve. But such a perception becomes difficult if the curve is slightly kurtic.

To overcome this difficulty of subjective judgement, it is mathematically measured as the ratio of fourth moment to the square of the second moment.

Symbolically,

$$\beta_2 = \frac{\mu_4}{{\mu_2}^2}$$

Q. 60 How to ascertain Kurtosis with the help of β_2?

Ans. If the value of β_2 is more than 3, the curves is leptokurtic and if less than 3, the curve is platykurtic. For a mesokurtic curve, $\beta_2 = 3$.

Q. 61 What are standard measures of skewness and kurtosis given by Karl Pearson?

Ans. Karl Pearson gave two convenient quantities as gamma measures defined as

$$\gamma_1 = \sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}}$$

and $$\gamma_2 = \beta_2 - 3 = \frac{\mu_4 - 3\mu_2^2}{\mu_2^2}$$

If γ_1 is negative, the frequency curve is negatively skew and if positive, the curve possesses positive skewness. Whereas β_1 gives only the magnitude but not direction.

Again, if $\gamma_2 > 0$, the curve is leptokurtic, if $\gamma_2 < 0$ the curve is platykurtic. When $\gamma_2 = 0$, the curve is mesokurtic or we can say, there is no Kurtosis.

Q. 62 What is the purpose of measuring skewness and Kurtosis?

Ans. Two measures namely, skewness and Kurtosis determine the shape of the frequency curve which obviously reflects on the type of distribution.

Q. 63 What is the role of averages, measures of dispersion, skewness and Kurtosis?

Ans. Averages, measures of dispersion, skewness and Kurtosis are complementary to each other in understanding frequency distribution.

Q. 64 Why mean deviation and standard deviation are not calculated using the same average?

Ans. The same average is not used in calculating mean deviation and standard deviation because mean deviation is minimum about median and standard deviation is minimum about mean as the sum of squares of the deviations from mean is minimum.

Q. 65 What is Sheppard's correction?

Ans. W.F. Sheppard pointed out that in case of continuous frequency distributions at the time of calculating moments, it is presumed that frequencies are centred at the mid-points of the class intervals. Such a presumption introduces some error in the calculation of moments. Hence, he suggested some corrections in various moments. These corrections are known as Sheppard's correction. They are as follows:

Corrected $\mu_1 = \mu_1$ (no correction needed)

Corrected $\mu_2 = \mu_2 - \frac{I^2}{12}$ (I is the class interval)

Corrected $\mu_3 = \mu_3$ (no correction needed)

Corrected $\mu_4 = \mu_4 - \frac{I^2}{2}\mu_2 + \frac{7}{240}I^4$

Corrections for higher order moments than fourth can be seen in a textbook if needed.

Q. 66 What are first and second coefficient of skewness.

Ans. The coefficients of skewness $\alpha_3 = \sqrt{\beta_1} = \gamma_1$ are the first coefficient of skewness. Another measure of skewness which is mainly useful for measuring slight skewness is known as moment coefficient of skewness and is given by the formula,

$$\text{Moment coeff. of skewness} = \frac{\sqrt{\beta_1}(\beta_2 + 3)}{2(5\beta_2 - 6\beta_1 - 9)}$$

where β_1 and β_2 are already given.

Moment coefficient of skewness is known as *second coefficient of skewness.*

Q. 67 Can second coefficient of skewness be used to determine the mode?

Ans. Yes, mode can be determined by the second coefficient of skewness using the relation,

$$\text{Mode} = -\frac{\sqrt{\beta_1}(\beta_2 + 3)\sigma}{2(5\beta_2 - 6\beta_1 - 9)}$$

$$= -\text{ second coeff. of skewness} \times \text{S.D.}$$

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s) in the blanks:

1. The statement, "The term dispersion is used to indicate the facts that within a group, the items different from one another in size or in other words, there is lack of uniformity in their size" was given by __________.
2. "Dispersion is the extent to which the magnitudes or qualities of the items differ that is the degree of diversity" is the statement about dispersion given by __________.
3. The definition, "Dispersion is the measure of the variation of the items" was given by __________.
4. "Dispersion or spread is the degree of the scatter or variation of the variable about a central value" is the definition of dispersion given by __________.
5. __________ defined dispersion as, "The degree to which numerical data tend to spread about an average value is called the variation or dispersion of the data."
6. Dispersion was explained as, "measures of dispersion are measures of scatter about an average" by __________.
7. __________ definition of dispersion is "A measure of variation or dispersion describes the degree of scatter shown by the observations and is usually measured as an average deviation about some central value or by an order statistic."
8. Measure of dispersion which utilises only two observations is __________.
9. __________ is the measure of dispersion which utilises only extreme values.
10. Coefficient of range is the __________ measure of dispersion.
11. Inter-quartile range is equal to __________.
12. Percentile range is given as __________.
13. Percentile range in terms of deciles is given as __________.
14. Mean deviation is based on __________.
15. Mean deviation is minimum about __________.
16. mean deviation suffers with the lacuna that it considers all differences __________.
17. Coefficient of mean deviation about a central value 'c' is given as __________.
18. The formula for calculating mean deviation about median for a discrete frequency distribution is __________.
19. Standard deviation is the __________ of variance.
20. Other names of standard deviation are __________.
21. Best measure of dispersion is __________.
22. Measure of dispersion suitable for comparing any two series is __________.
23. The relation between variance and standard deviation is __________.
24. __________ the value of coefficient of range, better it is.
25. __________ deviation can be obtained in case of open end intervals.
26. Quartile deviation is not affected by __________ per cent extreme values.
27. Quartile deviation is a __________ measure of dispersion.
28. The relation, $2Q_2 = Q_3 + Q_1$ holds in case of __________ distribution.
29. Mean deviation is calculated by considering __________ deviations.
30. __________ is highly susceptible to sampling fluctuations.
31. Measures of dispersion tell about __________ of central value.
32. Interquartile range is __________ of quartile deviation.

33. Median absolute deviation is the __________ of the absolute deviations taken from median.

34. __________ moment represents variance.

35. The relation between probable error and standard deviation is __________.

36. The ratio of quartile deviation to mean deviation is __________.

37. The ratio of quartile deviation to standard deviation is __________.

38. The moments about mean are called __________ moments.

39. Moments about origin zero are called __________ moments.

40. Moments of odd order about mean for a symmetric distribution are __________.

41. Sheppard's correction adjusts the error due to consideration of frequencies located at __________ of the class intervals.

42. For a symmetric distribution coefficient of skewness is __________.

43. α_3 $\sqrt{\beta_1}$ and γ_1 are __________ coefficients of skewness.

44. Moment coefficient of skewness is called __________ coefficient of skewness.

45. Measure of central tendency which can be determined with the help of second coefficient of skewness is __________.

46. In case of normal distribution, mean, median and mode are __________.

47. Less is the coefficient of variation, __________ consistent is the series.

48. The relation between β_2 and γ_2 is __________.

49. For a leptokurtic curve, the relation between μ_4 and μ_2 is __________.

50. For a mesokurtic curve, β_2 equal to __________.

51. For a platykurtic curve is γ_2 is __________.

52. For a platykurtic curve, the relation between second and fourth moment is __________.

53. β_1 gives the measure of skewness but __________.

54. Kurtosis means __________ of the frequency curve.

55. Skewness means __________ of the frequency distribution curve.

56. Sum of squares of the deviations from mean is __________.

57. The relation that holds between percentiles in case of symmetric distribution is __________.

58. For a symmetric distribution 1st, 5th and 9th deciles are connected as __________.

59. The relation of third central moment with raw moments is __________.

60. There is __________ effect of change of origin on the values of moments.

61. Change of scale __________ the value of moments.

62. __________ can be calculated about an arbitrary origin.

63. With the help of a measure of dispersion, namely, __________, area under the normal curve can be determined.

64. Karl Pearson's formula for measure of skewness is __________.

65. If the coefficient of Kurtosis is equal to 3, the frequency curve is __________.

66. If the coefficient of Kurtosis is greater than 3, the distribution is __________.

67. If the mean, mode and standard deviation of a distribution are 48, 38 and 10 respectively, the distribution is __________ skew.

68. If for an asymmetric distribution, the mean, median and S.D. are 25, 15 and 10, respectively the distribution is __________ skew.

69. If $\beta_1 = 0$ and $\beta_2 = 3$, it is known as frequency __________ curve.

70. Mean is not equal to __________ in case of skew distribution.

71. For a symmetric distribution, upper and lower quartiles are equidistant from __________.

72. If skewness is negative, the mean is ________ mode.

73. For moderately skew distribution, the relation between mean, median and mode is __________.

74. Mean deviation can never be __________.

75. For calculating variance, deviations are taken from __________.

76. Coefficient of variation is usually expressed in __________.

77. Graphical method of measuring dispersion is through __________.

78. It is necessary to __________ the data before a Lorenz curve is drawn.

79. Coefficient of mean deviation and coefficient of variation are __________.

80. If quartile deviation of a series is 30 and median is 45, the coefficient of mean deviation is __________.

81. If mean deviation is 16 and mean 30, the coefficient of variation is __________ per cent.

82. If quartile deviation of certain items is 20 and mean is 50, the coefficient of variation is __________ per cent.

83. If team *A* has mean score 7 and variance 25, team *B* has mean score 6 and variance 9, __________ is more consistent.

84. Standard deviation is a __________ order measure of dispersion.

85. Range is a __________ order measure of dispersion.

86. In any distribution mean deviation is ______ standard deviation.

87. __________ the distance of Lorenz Curve from the line of equal distribution, the greater is the variability in the series of values.

88. The average of squared deviations from mean is called __________.

89. A measure of dispersion is a measure of reliability of an __________.

90. If a distribution has, mean = 7.5, mode = 10 and skewness $\alpha = -0.5$, the variance is __________.

91. If the maximum value in a series is 60 and coefficient of range 0.5, the minimum value of the series is __________.

92. Formula for coefficient of mean deviation is __________.

93. The standard deviation of the five observations 5, 5, 5, 5, 5 is __________.

94. Mean deviation is always __________ quartile deviation.

95. The sum of squares of deviations taken from mean 40 for 9 sample observations is 288. The coefficient of variation is __________.

96. The sum of the deviations from median 24 for 12 observations is 72, the coefficient of mean deviation is __________.

97. Range of a set of values is 16 and its minimum value is 21, the maximum value is __________.

98. If mean and standard deviation of 8 observations in a sample are 9 and 4 and that of second sample of size 4 are 15 and 3, the combined variance of the two samples is __________.

99. For a highly skewed distribution, the best measure of central value is __________.

100. If the mean of a distribution is 15 and variance = 25. Also given that $\beta_1 = 1$, the third moment about origin is __________.

101. Variance of meuian for a sample of size *n* from a normal population is __________.

102. Standard deviation utilises __________ mean.

103. Variance of sample median is __________ than the variance of sample mean.

104. The inequality that holds between arithmetic mean (A.M.) and standard deviation (S.D.) is __________.

105. Standard deviation of first *n* natural numbers is __________.

106. The measure of central tendency about which Kurtosis is marked is __________.

107. Standard deviation gives more weight to __________ values.

108. Standard deviation is useful in finding the descriptive measures like __________ and __________ of a distribution.

109. Variation in data can graphically be perceived by __________.

110. If the sum of deviations from median is not zero, then a distribution will be __________.

111. If the frequencies on either side of mode are not similarly distributed, the frequency distribution curve will be __________.

112. Measure of skewness provides the __________ and __________ of asymmetry present in a distribution.

113. Measure of Kurtosis shows the degree of __________ of a frequency distribution curve.

114. Standard deviation of two values X_1 and X_2 is equal to __________.

115. Change of origin and scale of values for a set makes the calculation of standard deviation __________.

SECTION-C

Multiple Choice Questions

Select the correct alternative out of given ones:

Q. 1 Which of the following is not a measure of dispersion?
(a) mean deviation
(b) quartile deviation
(c) standard deviation
(d) average deviation from mean

Q. 2. Which of the following is a unitless measure of dispersion?
(a) standard deviation
(b) mean deviation
(c) coefficient of variation
(d) range

Q. 3 Which one of the given measures of dispersion is considered best?
(a) standard deviation
(b) range
(c) variance
(d) coefficient of variation

Q. 4. For comparison of two different series, the best measure of dispersion is:
(a) range
(b) mean deviation
(c) standard deviation
(d) none of the above

Q. 5. Correct formula for mean deviation from a constant A of a series in which the variate values $x_1, x_2, ..., x_k$ have frequencies $f_1, f_2, ..., f_k$ respectively is:

(a) $\frac{1}{N}\sum_i (f_i x_i - A)$

(b) $\frac{1}{N}\sum_i f_i (x_i - A)$

(c) $\frac{1}{N}\sum_i |f_i (x_i - A)|$

(d) $\frac{1}{N}\sum_i |f_i x_i - \overline{A}|$

where $i = 1, 2, ..., k$ and $\sum_i f_i = N$

Q. 6. Correct formula for variance of n sample observations $x_1, x_2, ..., x_n$ is:

(a) $\frac{1}{n-1}\sum_i (x_i - \bar{x})^2$

(b) $\frac{1}{n-1}\left(\sum_i x_i^2 - \bar{x}^2\right)$

(c) $\frac{1}{n}\sum_i (x_i - \bar{x})^2$

(d) $\frac{1}{n}\sum_i x_i^2 - \bar{x}^2$

Q. 7 The correct relation between variance and standard deviation (S.D.) of a variable X is:

(a) S.D. = $[\text{Var}(X)]^2$
(b) S.D. = $[\text{Var}(X)]^{1/2}$
(c) S.D. = Var (X)
(d) none of the above

Q. 8 Formula for coefficient of variation is:

(a) $\text{C.V.} = \frac{\text{S.D.}}{\text{mean}} \times 100$

(b) $\text{C.V.} = \frac{\text{mean}}{\text{S.D.}} \times 100$

(c) $\text{C.V.} = \frac{\text{mean} \times \text{S.D.}}{100}$

(d) $\text{C.V.} = \frac{100}{\text{mean} \times \text{S.D.}}$

Q. 9 Out of all measures of dispersion, the easiest one to calculate is:

(a) standard deviation
(b) range
(c) variance
(d) quartile deviation

Q. 10 Formula for range (R) of a set of values X_1, X_2, ..., X_n is:

(a) $R = X_{\min} - X_{\max}$

(b) $R = |X_{\min} - X_{\max}|$

(c) $R = X_{\max} - X_{\min}$

(d) $R = X_n - X_1$

Q. 11 Formula for coefficient of range of the set of observations $X_1, X_2, ..., X_n$ is:

(a) coeff. of range $= \frac{X_{\max} - X_{\min}}{X_{\max} + X_{\min}}$

(b) coeff. of range $= \frac{X_{\max} + X_{\min}}{X_{\max} - X_{\min}}$

(c) coeff. of range $= \frac{X_{\max}}{X_{\min}}$

(d) coeff. of range $= \frac{X_{\max} - X_{\min}}{X_{\max}}$

Q. 12 Coefficient of quartile deviation is given by the formula:

(a) coeff. of $\text{Q.D.} = \frac{Q_3 + Q_1}{Q_3 - Q_1}$

(b) coeff. of $\text{Q.D.} = \frac{Q_3 + Q_1}{Q_1 - Q_3}$

(c) coeff. of $\text{Q.D.} = \frac{Q_3 - Q_1}{Q_1 - Q_3}$

(d) coeff. of $\text{Q.D.} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$

Q. 13 For a symmetrical distribution, $M_d \pm \text{Q.D.}$ covers:

(a) 25 per cent of the observations
(b) 50 per cent of the observations
(c) 75 per cent of the observations
(d) 100 per cent of the observations

M_d = Median and Q.D. = Quartile deviation

Q. 14 Quartile deviation or semi inter-quartile deviation is given by the formula:

(a) $\text{Q.D.} = \frac{Q_3 + Q_1}{2}$

(b) $\text{Q.D.} = Q_3 - Q_1$

(c) $\text{Q.D.} = (Q_3 - Q_1)/2$

(d) $\text{Q.D.} = (Q_3 - Q_1)/4$

Q. 15 Mean deviation is minimum when deviations are taken from:

(a) mean
(b) median
(c) mode
(d) zero

Q. 16 Sum of squares of the deviations is minimum when deviations are taken from:

(a) mean
(b) median
(c) mode
(d) zero

Q. 17 If a constant value 5 is subtracted from each observation of a set, the variance is:

(a) reduced by 5
(b) reduced by 25
(c) unaltered
(d) increased by 25

Q. 18 If each observation of a set is divided by 10, the S.D. of the new observations is:

(a) $\frac{1}{10}$th of S.D. of original obs.
(b) $\frac{1}{100}$th of S.D. of original obs.
(c) not changed
(d) 10 times of S.D. of original obs

Q. 19 If each value of a set is divided by c, then the variance (s^2) of the original sample values from coded values $dx_1, dx_2, \ldots, dx_n$ is obtained by the formula:

(a) $s^2 = \frac{1}{n-1}\sum_i \left(dx_i - \bar{d}x\right)^2 \times c$

(b) $s^2 = \frac{1}{n-1}\sum_i \left(dx_i - \bar{d}x\right)^2 \times c^2$

(c) $s^2 = \frac{1}{n-1}\sum_i \left(dx_i - \bar{d}x\right)^2 \times \frac{1}{c}$

(d) $s^2 = \frac{1}{n-1}\sum_i \left(dx_i - \bar{d}x\right)^2 \times \frac{1}{c^2}$

Q. 20 Which of the following formula for standard deviation of a frequency distribution is not correct?

(a) $\sigma = \sqrt{\frac{1}{N}\sum_i f_i(x_i - \bar{x})^2}$

(b) $\sigma = \sqrt{\frac{1}{N}\sum_i f_i x_i^2 - \bar{x}^2}$

(c) $\sigma = \sqrt{\frac{1}{N}\sum_i f_i x_i^2 - \left(\frac{\Sigma_i f_i x_i}{N}\right)^2}$

(d) $\sigma = \sqrt{\frac{1}{N}\left(\sum_i f_i x_i\right)^2 - \frac{\Sigma_i f_i x_i}{N}}$

where $\Sigma_i f_i = N$ and other notations are as usual.

Q. 21 If a constant 'c' is subtracted from each observation and then divided by d, then the formula for variance of frequency distribution having k groups is:

(a) $\sigma^2 = \left[\frac{1}{N}\sum_{i=1}^{k} f_i x_i'^2 - \left(\frac{\sum_{i=1}^{k} f_i x_i'}{N}\right)^2\right] \times d^2$

(b) $\sigma^2 = \frac{1}{N-1}\left[\sum_{i=1}^{k} f_i (x_i' - \bar{x}')^2\right] \times d^2$

(c) $\sigma^2 = \left[\frac{1}{N}\sum_{i=1}^{k} f_i x_i'^2 - \frac{\left(\sum_{i=1}^{k} f_i x_i'\right)^2}{N}\right] \times d^2$

(d) None of the above

where $x_i' = \frac{x_i - c}{d}$ and $N = \sum_i f_i$

Q. 22 The empirical relationship between quartile deviation (Q.D.) and standard deviation in normal distribution is:

(a) 3 Q.D. ≐ 2 S.D.
(b) 4 Q.D. ≐ 3 S.D.
(c) 6 Q.D. ≐ 5 S.D.
(d) 5 Q.D. ≐ 4 S.D.

Q. 23 The relationship between mean deviation (M.D.) and standard deviation is:

(a) 3 M.D. = 2 S.D.
(b) 5 M.D. = 4 S.D.
(c) 6 M.D. = 5 S.D.
(d) M.D. = S.D.

Q. 24 The empirical relation between quartile deviation (Q.D.) and mean deviation (M.D.) from mean is:

(a) 3 Q.D. $\doteq$ 5 M.D.
(b) 6 Q.D. $\doteq$ 3 M.D.
(c) 5 Q.D. $\doteq$ 6 M.D.
(d) 6 Q.D. $\doteq$ 5 M.D.

Q. 25 An empirical relation between standard deviation, mean deviation about mean and quartile deviation is:
(a) 4 S.D. $\doteq$ 6 M.D. $\doteq$ 5 Q.D.
(b) 4 S.D. $\doteq$ 5 M.D. $\doteq$ 6 Q.D.
(c) 6 S.D. $\doteq$ 5 M.D. $\doteq$ 4 S.D.
(d) 5 S.D. $\doteq$ 4 M.D. $\doteq$ 6 Q.D.

Q. 26 The empirical relation between range (R) and standard deviation is:
(a) $R = 3$ S.D.
(b) $R = 2$ S.D.
(c) $R = 6$ S.D.
(d) $R = 4$ S.D.

Q. 27 An empirical relationship between range and quartile deviation about mean is:
(a) $R = 4$ Q.D.
(b) $R = 9$ Q.D.
(c) $R = 6$ Q.D.
(d) none of the above

Q. 28 An empirical relation between range and mean deviation is:
(a) $R = 10$ M.D.
(b) $2R = 5$ M.D.
(c) $3R = 5$ M.D.
(d) $2R = 15$ M.D.

Q. 29 Which measure of dispersion ensures highest degree of reliability?
(a) range
(b) mean deviation
(c) quartile deviation
(d) standard deviation

Q. 30 Which measure of dispersion ensures lowest degree of reliability?
(a) range
(b) mean deviation
(c) quartile deviation
(d) standard deviation

Q. 31 There are two populations consisting of N_1 and N_2 units, having means $\overline{X}_1$ and $\overline{X}_2$, variances σ_1^2 and σ_2^2 respectively. Let their pooled mean be $\overline{X}_{12}$. Also, supposing $\overline{X}_1 - \overline{X}_{12} = d_1$ and $\overline{X}_2 - \overline{X}_{12} = d_2$. The formula for pooled variance is:

(a) $\sigma^2 = \dfrac{N_1(\sigma_1^2 + d_1^2) + N_2(\sigma_2^2 + d_2^2)}{N_1 + N_2}$

(b) $\sigma^2 = \dfrac{N_2(\sigma_1^2 + d_1^2) + N_1(\sigma_2^2 + d_2^2)}{N_1 + N_2}$

(c) $\sigma^2 = \dfrac{N_1\sigma_1^2 + N_2 d_1^2 + N_2\sigma_2^2 + N_1 d_2^2}{N_1 + N_2}$

(d) $\sigma_2 = \dfrac{N_2\sigma_1^2 + N_1\sigma_2^2 + N_1 d_1^2 + N_2 d_2^2}{N_1 + N_2}$

Q. 32 Average wages of workers of a factory are Rs. 550.00 per month and the standard deviation of wages is 110. The coefficient of variation is:
(a) C.V. = 30 per cent
(b) C.V. = 15 per cent
(c) C.V. = 500 per cent
(d) C.V. = 20 per cent

Q. 33 If the mean deviation of a distribution is 20.20, the standard deviation of the distribution is:
(a) 15.15
(b) 25.25
(c) 30.30
(d) none of the above

Q. 34 If the mean and standard deviation of A and B are as, $\overline{X}_A = 15.0$, $\overline{X}_B = 20.0$ and σ_A^2 = 25 and $\sigma_B^2 = 16$, which of the two series is more consistent.
(a) series A
(b) series B
(c) series A and B are equally consistent
(d) none of the above

Q. 35 If the standard deviation of a distribution is 15, the quartile deviation of the distribution is:

(a) 15.0
(b) 12.5
(c) 10.0
(d) none of the above

Q. 36 If the quartile deviation of a series is 60, the mean deviation of this series is:
(a) 72
(b) 48
(c) 50
(d) 75

Q. 37 In a discrete set of values, the correct relation between mean deviation and standard deviation is:
(a) M.D. > S.D.
(b) M.D. < S.D.
(c) M.D. ≤ S.D.
(d) M.D. ≥ S.D.

Q. 38 If the first 25 per cent observations of a series are 20 or less and last 25 per cent observations of a series are 50 or more, the quartile deviation (semi inter-quartile deviation) is:
(a) 25
(b) 35
(c) 15
(d) 30

Q. 39 The mean and standard deviation of a set of values are 25 and 5, respectively. If a constant value 5 is added to each value, the coefficient of variation of the new set of values is:
(a) 250 per cent
(b) 600 per cent
(c) 20 per cent
(d) 16.6 per cent

Q. 40 If the mean of a series is 10 and its coefficient of variation is 40 per cent, the variance of the series is:
(a) 4
(b) 8
(c) 12
(d) none of the above

Q. 41 which of the following measures of dispersion can attain a negative value?
(a) range
(b) mean deviation
(c) standard deviation
(d) variance

Q. 42 A set of values is said to be relatively uniform if it has:
(a) high dispersion
(b) zero dispersion
(c) little dispersion
(d) negative dispersion

Q. 43 The mean and standard deviation of a set of values from a normal distribution are 66 and 4, respectively. The range in which almost 95 per cent values lie is:
(a) 62 to 70
(b) 62 to 74
(c) 58 to 74
(d) 66 and 74

Q. 44 The measure of dispersion which ignores signs of the deviations from a central value is:
(a) range
(b) quartile deviation
(c) standard deviation
(d) mean deviation

Q. 45 Which measure of dispersion is least affected by extreme values?
(a) range
(b) mean deviation
(c) standard deviation
(d) quartile deviation

Q. 46 Which measure of dispersion is most affected by extreme values?
(a) range
(b) mean deviation
(c) standard deviation
(d) quartile deviation

Q. 47 Range of a set of values is 65 and maximum value in the series is 83. The minimum value of the series is:
(a) 74
(b) 9
(c) 18
(d) none of the above

Q. 48 If the minimum value in a set is 9 and its range is 57, the maximum value of the set is
(a) 33
(b) 66
(c) 48
(d) none of the above

Q. 49 If the values of a set are measured in cm, the unit of variance will be:
(a) no unit
(b) cm
(c) cm^2
(d) cm^3

Q. 50 Which measure of dispersion has a different unit other than the unit of measurement of values:
(a) range
(b) mean deviation
(c) standard deviation
(d) variance

Q. 51 The average of the sum of squares of the deviations about mean is called:
(a) variance
(b) absolute deviation
(c) standard deviation
(d) mean deviation

Q. 52 Quartile deviation is equal to:
(a) interquartile range
(b) double the interquartile range
(c) half of the interquartile range
(d) none of the above

Q. 53 Which measure of dispersion can be calculated in case of open end intervals?
(a) range
(b) standard deviation
(c) coefficient of variation
(d) quartile deviation

Q. 54 Which one property out of the following does not hold good in case of standard deviation?
(a) It is distorted by extreme values
(b) It is not very sensitive to sampling fluctuations as compared to other measures.
(c) It is a unitless measure of dispersion
(d) It is a most used measure of dispersion

Q. 55 If each value of a series is divided by 5, its coefficient of variation is reduced by:
(a) 0 per cent
(b) 5 per cent
(c) 10 per cent
(d) 20 per cent

Q. 56 If each value of a series is multiplied by 10, the coefficient of variation will be increased by:
(a) 5 per cent
(b) 10 per cent
(c) 15 per cent
(d) 0 per cent

Q. 57 If a constant value 10 is subtracted from each value of a series, the coefficient of variation will be:
(a) decreased in comparison to original value
(b) increased in comparison to original value
(c) same as original value
(d) none of the above

Q. 58 If each value of a series is multiplied by a constant '*c*', the coefficient of variation as compared to original value is:
(a) increased
(b) decreased
(c) unaltered
(d) zero

Q. 59 If each value of a set is divided by a constant '*d*', the coefficient of variation will be:
(a) same as original value
(b) less than original value
(c) more than original value
(d) none of the above

Q. 60 For a positive skewed distribution, which of the following inequality holds?
(a) median > mode
(b) mode > mean
(c) mean > median
(d) mean > mode

Q. 61 For a negatively skewed distribution, the correct inequality is:
(a) mode < median
(b) mean < median

(c) mean < mode
(d) none of the above

Q. 62 For a positive skewed frequency curve, the inequality that holds is:
(a) $Q_1 + Q_3 > 2Q_2$
(b) $Q_1 + Q_2 > 2Q_3$
(c) $Q_1 + Q_3 > Q_2$
(d) $Q_3 - Q_1 > Q_2$

Q. 63 For a negatively skewed frequency distribution curve, the third central moment,
(a) $\mu_3 > 0$
(b) $\mu_3 < 0$
(c) $\mu_3 = 0$
(d) μ_3 does not exist

Q. 64 For a symmetrical distribution, the coefficient of skewness:
(a) $\alpha_3 = 1$
(b) $\alpha_3 = 3$
(c) $\alpha_3 = 0$
(d) $\alpha_3 = -1$

Q. 65 For a leptokurtic frequency curve, the measure of Kurtosis,
(a) $\alpha_4 = 0$
(b) $\alpha_4 = -3$
(c) $\alpha_4 < 1$
(d) $\alpha_4 > 3$

Q. 66 In case of a positive skewed distribution, the relation between mean, median and mode that holds is:
(a) median > mean > mode
(b) mean > median > mode
(c) mean = median = mode
(d) none of the above

Q. 67 If a moderately skewed distribution has mean 30 and mode 36, the median of the distribution is:
(a) 30
(b) 28
(c) 32
(d) none of the above

Q. 68 If a moderately skewed distribution has mean 40 and median equal to 30, the mode of the distribution is:
(a) 10
(b) 35
(c) 20
(d) zero

Q. 69 First and third quartiles of a frequency distribution are 30 and 75. Also its coefficient of skewness is 0.6. The median of the frequency distribution is:
(a) 40
(b) 39
(c) 38
(d) 41

Q. 70 If the mean, standard deviation and coefficient of skewness of a frequency distribution are 60, 45 and –0.4, respectively, the mode of the frequency distribution is:
(a) 80
(b) 82
(c) 78
(d) 68

Q. 71 For a moderately skew distribution, the empirical relation between mean (M), median (M_d) and mode (M_0) is:
(a) $3(M - M_0) = M - M_d$
(b) $3(M_d - M) = M_0 - M$
(c) $3(M - M_d) = M - M_0$
(d) $2(M_0 - M) = 3(M_d - M)$

Q. 72 If the first quartile $Q_1 = 15$ and third quartile $Q_3 = 25$, the coefficient of quartile deviation is:
(a) 4
(b) 1/4
(c) 5/3
(d) 3/5

Q. 73 If the first quartile $Q_1 = 20$ and third quartile $Q_3 = 50$, the quartile deviation is:
(a) 35
(b) 15
(c) 2.5
(d) 0.8

Q. 74 For a negatively skewed distribution, the correct relation between mean, median and mode is:

(a) mean = median = mode
(b) median < mean < mode
(c) mean < median < mode
(d) mode < mean < median

Q. 75 If the mode of a frequency distribution $M_0 =$ 16 and its mean $\overline{X} = 16$, the median of the distribution is:
(a) zero
(b) 16
(c) 32
(d) 8

Q. 76 In case of positive skewed distribution, the extreme values lie in the
(a) left tail
(b) right tail
(c) middle
(d) anywhere

Q. 77 The extreme values in a negatively skewed distribution lie in the:
(a) middle
(b) right tail
(c) left tail
(d) whole curve

Q. 78 The relation between variance and standard deviation is:
(a) variance is the square root of standard deviation
(b) standard deviation is the square of the variance
(c) variance is equal to standard deviation
(d) square of the standard deviation is equal to variance

Q. 79 Which of the following statements is true for a measure of dispersion?
(a) mean deviation does not follow algebraic rule
(b) range is a crudest measure
(c) coefficient of variation is a relative measure
(d) all the above statements

Q. 80 For a set of values:
(a) mean deviation is always less than standard deviation
(b) mean deviation is always greater than standard deviation
(c) mean deviation is always equal to standard deviation
(d) none of the above

Q. 81 Variance of the following frequency distribution,

Classes	*Frequency*
2-4	2
4-6	5
6-8	4
8-10	1

is approximately equal to:
(a) 2.5
(b) 2.9
(c) 5.0
(d) none of the above

Q. 82 For the data given is question 81, mean deviations about median is:
(a) 1.43
(b) 1.00
(c) 2.43
(d) 6

Q. 83 Coefficient of variation for the data given in question 81 is:
(a) 48.33 per cent
(b) 206.90 per cent
(c) 195.17 per cent
(d) 30.03 per cent

Q. 84 The range of values for the frequency distribution given in question 81 is:
(a) 2
(b) 10
(c) 8
(d) 6

Q. 85 For the data given in question 81, the coefficient of quartile deviation is:
(a) 4.385
(b) 0.228
(c) 2.6
(d) 11.4

Q. 86 The range of the set of values, 15, 12, 27, 6, 9, 18, 21 is:
(a) 21
(b) 4.5
(c) 0.64
(d) 3

Q. 87 The coefficient of range for the values given in question 86 is:
(a) 1.571
(b) 4.500
(c) 0.636
(d) 0.222

Q. 88 The coefficient of skewness of a series *A* is 0.15 and that of series *B* 0.062. Which of the two series is less skew?
(a) series *A*
(b) series *B*
(c) no decision
(d) none of the above

Q. 89 If the coefficient of Kurtosis γ_2 of a distribution is zero, the frequency curve is:
(a) leptokurtic
(b) platykurtic
(c) mesokurtic
(d) any of the above

Q. 90 If for a distribution, coefficient of Kurtosis $\gamma_2 < 0$, the frequency curve is:
(a) leptokurtic
(b) platykurtic
(c) mesokurtic
(d) any of the above

Q. 91 The value of coefficient of Kurtosis β_2 can be:
(a) less than 3
(b) greater than 3
(c) equal to 3
(d) all the above

Q. 92 The standard deviation of a set of values will be:
(a) positive when the values are positive
(b) positive when the values are negative
(c) always positive
(d) all the above

Q. 93 Sum of square of the deviations is minimum when the deviations are taken from:
(a) mean
(b) median
(c) mode
(d) an arbitrary value.

Q. 94 The relationship between the variance of median [V (med)] and variance of mean [V (mean)] of a normal population is:
(a) V (med) < V (mean)
(b) V (med) = V (mean)
(c) V (med) > V (mean)
(d) 1.57 V (med) = V (mean)

Q. 95 Kurtosis in frequency distribution is adjudged around:
(a) second quartile
(b) arithmetic mean
(c) quadratic mean
(d) mode.

Q. 96 The variance of first *n* natural numbers is:
(a) $\left(n^2+1\right)/12$
(b) $(n+1)^2/12$
(c) $\left(n^2-1\right)/12$
(d) $\left(2n^2-1\right)/8$

Q. 97 If a random variable *X* has mean 3 and standard deviation 5, then the variance of a variable $Y = 2X - 5$ is:
(a) 45
(b) 100
(c) 15
(d) 40

Q. 98 Kurtosis and skewness of a frequency distribution curve are bound by the relation:
(a) they always coexist
(b) either of the two can exist alone
(c) their measures are always positive
(d) their measures are always negative.

Q. 99 All values in a sample are same. Then their variance is:
(a) zero
(b) one
(c) not calculable
(d) all the above

Q. 100 Calculation of pooled variance of two series of sizes n_1 and n_2 requires:
(a) means of individual series
(b) variances of individual series
(c) pooled mean
(d) all the above

ANSWERS

SECTION-B

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SECTION-C

(1) d	(2) c	(3) a	(4) d	(5) c	(6) a
(7) b	(8) a	(9) b	(10) c	(11) a	(12) d
(13) b	(14) c	(15) b	(16) a	(17) c	(18) a
(19) b	(20) d	(21) a	(22) a	(23) b	(24) d
(25) b	(26) c	(27) b	(28) d	(29) d	(30) c
(31) a	(32) d	(33) b	(34) b	(35) c	(36) a
(37) b	(38) c	(39) d	(40) d	(41) a	(42) c
(43) c	(44) d	(45) d	(46) a	(47) c	(48) b
(49) c	(50) d	(51) a	(52) c	(53) d	(54) c
(55) a	(56) d	(57) b	(58) c	(59) a	(60) d
(61) c	(62) a	(63) b	(64) c	(65) d	(66) b
(67) c	(68) a	(69) b	(70) c	(71) c	(72) b
(73) b	(74) c	(75) b	(76) b	(77) c	(78) d
(79) d	(80) a	(81) b	(82) a	(83) d	(84) c
(85) b	(86) a	(87) c	(88) b	(89) c	(90) b
(91) d	(92) c	(93) a	(94) c	(95) d	(96) c
(97) b	(98) b	(99) a	(100) d		

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Chapter **6**

Elementary Probability

SECTION-A

Short Essay Type Questions

Q. 1 Give in brief the concept of probability.

Ans. There are a number of events in day-to-day life about which one is not sure whether it will occur or not. But one is always curious to know what chance is there for a happening or event to occur. For instance, one may be interested to estimate whether it will rain today or not, one would like to evaluate his chance of winning for head in a definite number of tosses, what is the chance that there are four aces in one hand in a game of cards among four players, etc. The numerical evaluation of chance factor of an event is known as probability.

Q. 2 Define an event and give its two examples.

Ans. An event is the collection of possible outcomes which are favourable to an happening out of total outcomes which may be enumerable or denumerable.

Consider a statistical experiment which may consist of finite or infinite number of trials whereas each trial results into an outcome such as tossing three coins at a time or tossing a coin three times. We shall have in all eight outcomes. If we consider the birth weight of children, it will consist of an infinite number of outcomes. The totality of all possible outcomes of an experiment is called *sample space* and is denoted by Ω. At the same time, the collection of all outcomes favourable to a phenomenon or happening is called an *event* and is denoted by *E, A, B, C*, etc. In the experiment of tossing a coin thrice, the sample space consists of eight points and each point of the sample space is usually denoted by ω_i ($i = 1, 2, ...$). The sample space,

$$\Omega = \text{HHH HHT HTH HTT THH THT TTH TTT}$$
$$= \omega_1 \quad \omega_2 \quad \omega_3 \quad \omega_4 \quad \omega_5 \quad \omega_6 \quad \omega_7 \quad \omega_8$$

Event E: there are at least two heads, consists of four points,

$$\text{HHH HHT HTH THH}$$
$$\omega_1 \quad \omega_2 \quad \omega_3 \quad \omega_5$$

Again, event A: there is no head, consists of only one point, *i.e.*,

$$\text{TTT} \equiv \omega_8$$

Q. 3 Definite complementary event, and give an example.

Ans. An event $\overline{A}$ is said to be complementary to an event A in Ω if $\overline{A}$ consists of all those points which are not in A. In tossing a coin three times, Ω consists of eight points $\omega_1, \omega_2, ..., \omega_8$. The event '$A$' that there is no head consists of the point ω_8. The event $\overline{A}$, complementary to A, that there is at least one head consists of the seven points, $\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6$ and ω_7.

Q. 4 Definite a simple or elementary event.

Ans. An event having only one sample point is called simple or elementary event. In the experiment of tossing three coins at a time, the event '*A*' that all the coins turn up with heads consists of only one point HHH. Hence, *A* is a simple event. As a matter of fact each outcome of an experiment is a simple event.

Q. 5 Define equal events.

Ans. Two events *A* and *B* are said to be equal if $A \subset B$ and $B \subset A$. This statement implies that all the points of *A* are also the points of *B* and vice-versa.

Q. 6 What do you understand by transitivity of events.

Ans. If *A*, *B* and *C* are three events such that $A \subset B$ and $B \subset C$, it implies that $A \subset C$. Such a property of events is known as transitivity of events.

Q. 7 Define compound event.

Ans. An event which is not simple or elementary is called a compound event. Every compound event can be uniquely represented by the union of a set of elementary events.

Q. 8 Discuss mutually-exclusive or disjoint events.

Ans. Events are said to be mutually exclusive if the occurrence of one precludes the occurrence of others. For example, in tossing a coin, the events head and tail are mutually-exclusive events since if the coin falls with head upside, tail cannot turn up and vice-versa. In other words, two events *A* and *B* are said to be mutually-exclusive if there is no point in common in between the points belonging to *A* and *B*. Consider the trial of tossing a coin thrice. Let the event *A* is that there are two heads in three tossings of a coin. Event *A* has points: HHT, HTH, THH. Again let the event *B* be that there are at least two tails. Event *B* has points: HTT, THT, TTH, TTT. There is no common point amongst the events *A* and *B*. Hence, *A* and *B* are mutually-exclusive events. All the more, two distinct elementary events are always disjoint.

Q. 9 What do you understand by primary events?

Ans. Mutually-exclusive and exhaustive events defined over a sample space Ω usually constitute primary events.

As a matter of fact each outcome of a conceptual or statistical (random) experiment is an event. All such events are called primary events. For example, in throwing of a die, six possible outcomes 1, 2, 3, 4, 5 and 6 spots turning upside of a die constitute primary events.

Q. 10 What is a derived event?

Ans. Two or more events joined by the conjunction 'or' are called derived events. For two events *A* and *B*, the event *A* or *B* ($A \cup B$) is a derived event.

Q. 11 Explain unision of events.

Ans. Unision of two events *A* and *B* means the event which relates to the occurrence of *A* or *B* or both. It is denoted by $A \cup B$ (*A* union *B*). For instance, in tossing a coin three times, suppose the event *A*: there is at least one head and one tail. Event *A* will consist of the six points,

HHT, HTH, HTT, THH, THT, TTH

i.e., $\omega_2 \quad \omega_3 \quad \omega_4 \quad \omega_5 \quad \omega_6 \quad \omega_7$

Again suppose the event *B*: there are at least two heads.

Event *B* has four points, HHH, HHT, HTH, THH

$\omega_1 \quad \omega_2 \quad \omega_3 \quad \omega_5$

The event $A \cup B$ (*A* or *B*) will consist of the points

$$\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7$$

It contains all the points of *A* and *B* taken only once. The idea of unision of two events can be extended to any number of events.

Q. 12 Discuss intersection of events.

Ans. Intersection of two events *A* and *B* leads to an event which conforms to the occurrence of *A* as well as *B*. Hence, it consists of those points which are common to *A* and *B*. It is denoted as $A \cap B$ (*A* intersection *B*). In the example given in Question No. 11, the event $A \cap B$ consists of the points,

HHT, HTH, THH

$\omega_2 \quad \omega_3 \quad \omega_5$

Event $A \cap B$ consists of three points only. The idea of intersection of two events can be extended to any number of events.

Q. 13 What is meant by an impossible event?

Ans. An event which is certain not to occur is called an impossible event. It is denoted by the empty set ϕ. For example, the event A that in two throws of a die, the sum of the spots is one. Event A is an impossible event as in two throws of a die, the sum of spots has to be at least two.

Further the intersection of two mutually-exclusive events is always an impossible event.

Q. 14 What do understand by exhaustive (partition) set of events?

Ans. A set of disjoint events $A_1, A_2, ..., A_n$ is said to be exhaustive if the union of A_i's ($i = 1, 2, ..., n$) is the sample space, *i.e.*, $\bigcup\limits_{i=1}^{n} A_i = \Omega$. This implies that every point of the sample space belongs to one and only one of the A_i's. In throwing a die, the spots 1, 2, 3, 4, 5 and 6 are exhaustive events.

Q. 15 Give classical (Laplacian, mathematical) definition of probability.

Ans. If Ω is a sample space having N points which arises out of all possible equally likely and independent outcomes of a random experiment and n points in Ω are favourable to an event A, the probability of the event A is given as,

$$P(A) = \frac{n}{N}$$

where $0 \le n \le N$.

This is also known as **a priori probability.**

Q. 16 What are the properties of classical probability of an event A?

Ans. Properties of $P(A)$ are:

(i) Probability is a pure number, *i.e.*, it has no unit.

(ii) $0 \le P(A) \le 1$, *i.e.*, probability can never be negative and cannot exceed unity (one).

(iii) It is a relative measure

(iv) If $A_1, A_2, ..., A_k$ are k mutually exclusive and exhaustive events in the sample space Ω then

$$\sum_{i=1}^{k} P(A_i) = 1 \rightarrow P(\Omega) = 1.$$

(v) $P(\phi) = 0$, *i.e.*, probability of an impossible events is zero

(vi) If $A \subset B, P(A) \le P(B)$ and if $B \subset A$, $P(A) \ge P(B)$.

(vii) If $A + \overline{A} = \Omega$, $P(A) + P(\overline{A}) = P(\Omega) = 1$ or $P(\overline{A}) = 1 - P(A)$.

(viii) If $A \subset B, P(B\overline{A}) = P(B) - P(A) = P(B - A)$.

Q. 17 Narrate addition theorem of probability.

Ans. If A and B are two arbitrary (possible) events in the sample space Ω, the probability of the union of A and B is governed by the law,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

The formula for probability of union of two events can be extended to any number of events. For three arbitrary events A, B and C in Ω, the probability of their union is given as,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

Q. 18 Enunciate demerits of classical probability.

Ans. Classical probability approach cannot be adopted if any of the following situation exists:

(i) If it is not possible to enumerate all possible outcomes. For example, the sample space of all even numbers, total number of male births, etc.

(ii) If the outcomes are not independent.

(iii) If the total number of outcomes is infinite.

(iv) If each and every outcome is not equally likely.

Q. 19 If two events A and B are mutually exclusive, what shall be the probability of their union?

Ans. If two events A and B are mutually exclusive, $P(A \cap B) = 0$ and hence,

$$P(A \cup B) = P(A) + P(B)$$

Q. 20 Define independent events.

Ans. Two events A and B are said to be independent if the occurrence of one does not affect the occurrence of the other.

Q. 21 Define pairwise independent events.

Ans. If $A_1, A_2, ..., A_k$ are k events, they are said to be pairwise independent iff,

$$P(A_i \cap A_j) = P(A_i)P(A_j) \qquad \forall \quad i \neq j$$

where $i, j = 1, 2, ..., k$.

Q. 22 If A and B are independent events, how about the independence of their complements $\overline{A}$ and $\overline{B}$?

Ans. If A and B are independent, their complements are also independent, *i.e.*,

$$\text{if} \quad P(A \cap B) = P(A)P(B)$$

$$\text{then} \quad P(\overline{A} \cap \overline{B}) = P(\overline{A})P(\overline{B})$$

$$= [1 - P(A)][1 - P(B)]$$

$$\text{Also} \quad P(A \cup B) = 1 - P(\overline{A} \cap \overline{B})$$

$$= 1 - P(\overline{A})P(\overline{B})$$

Q. 23 State multiplicative law of probability.

Ans. If two events A and B are independent, the probability of their product (intersection) is equal to the product of their individual probabilities. Notationally,

$$P(AB) = P(A \cap B) = P(A)P(B)$$

This law can be extended to any number of events. For three independent events A, B and C,

$$P(ABC) = P(A \cap B \cap C) = P(A)\, P(B)\, P(C).$$

Q. 24 Give concept and definition of statistical (empirical) probability.

Ans. The definition of statistical probability goes after the name of Richard Von Mises. Many deficiencies of classical definition could be removed by introducing the relative frequency approach. If N is infinite or equilikelyness cannot be guessed, statistical definition is more appropriate. The definition is, "If out of a large number of trials, only n are conducted under essentially homogeneous and identical conditions resulting into n outcomes. Out of n outcomes k are favourable to an event A, the probability of A is,

$$P(A) = \lim_{n \to \infty} \left(\frac{k}{n}\right)$$

Here n is never infinity but the relative frequency $\left(\frac{k}{n}\right)$ stabilises as n is fairly large.

This definition removes some of the deficiencies of classical probability. But n is always finite in physical experiments. Hence, it is not definite that such a limit always exists.

Q. 25 Explain conditional probability.

Ans. Many times the information is available that an event has occurred and one is required to find out the probability of occurrence of another event B utilising the information about A. Such a probability is known as conditional probability and is denoted by $P(B \mid A)$, *i.e.*, the probability of the event B given A. For example, suppose we know that a newly born baby will be a male (A) then one is interested to know the probability of a strile birth (B), *i.e.*, we want to calculate $P(B \mid A)$. The formula is,

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

If A and B are independent, then

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{P(A)\,P(B)}{P(A)} = P(B)$$

Q. 26 Give the multiplication rule for the simultaneous occurrence of two events A and B.

Ans. Using the rule of conditional probability, the relations for simultaneous occurrence of A and B can be established as follows:

$$P(A \text{ and } B) = P(A)\, P(B|A)$$

$$P(B \text{ and A}) = P(B)\, P(A|B)$$

Q. 27 If A, B and C are three events, how can you express the probability of simultaneous events A, B and C.

Ans. Using the property of conditional probability, the relation for the simultaneous occurrence of events A, B and C is,

$$P(A \cap B \cap C) = P(A)\,P(B|A)\,P(C|A \text{ and } B)$$

Q. 28 Discuss Bayes' probability and give formula for its calculation.

Ans. Bayes' probability is also known as *inverse probability*. The idea of inverse probability was given by Sir Thomas Bayes in 1763 which was reprinted in *Biometrika* in the year 1958. The problem of inverse probability arises when we have an outcome and we want to know the probability of its belonging to a specified trial or population out of many alternative trials or populations. To be more specific, let us consider three urns containing white (W), black (B) and red (R) balls as follows:

URN I : 2W, 3B and 4R balls.
URN II : 3W, 1B and 2R balls.
URN III : 4W, 2B and 5R balls.

Two balls are drawn from a urn and they happen to be one white and one red balls. Now the interest lies to know the probability that the balls are drawn from Urn III. Such a probability is Bayes' probability. Bayes' theorem can be stated as follows:

If $E_1, E_2, \ldots, E_k$ are k mutually exclusive events defined in $\mathfrak{B}$ (a collection of events), each being a subset of the sample space Ω such that $\bigcup_{i=1}^{k} E_i = \Omega$ and P $(E_i) > 0$ for $i = 1, 2, \ldots, k$ and if A is any arbitrary event which is associated with E_i's such that $P(A) > 0$, we can evaluate probabilities $P(A|E_i)$ for $i = 1, 2, \ldots, k$. In Bayes' approach we have to find out the probability of E_i given that A has occurred, *i.e.*, $P(E_i|A)$. This is also known as *posteriori* probability. Bayes' formula for posteriori probability is,

$$P(E_i|A) = \frac{P(A|E_i)\,P(E_i)}{\sum_{i=1}^{k} P(A|E_i)\,P(E_i)}$$

Q. 29 Calculate the probability for the example given in question 28.

Ans. Let E_1, E_2 and E_3 be the events the urn I, II and III are selected, respectively, and A be the event that out of two selected balls, one is white and the other is red.

Probabilities of selecting a urn are,

$$P(E_1) = \frac{1}{3}, P(E_2) = \frac{1}{3} \text{ and } (E_3) = \frac{1}{3}.$$

Probability of selecting 1W and 1R red balls from urn I is,

$$P(A|E_1) = \frac{2C_1 \times 4C_1}{9C_2}$$

$$= \frac{2 \times 4 \times 2}{9 \times 8} = \frac{2}{9}$$

similarly from urn II is,

$$P(A|E_2) = \frac{3C_1 \times 2C_1}{6C_2}$$

$$= \frac{3 \times 2 \times 2}{6 \times 5} = \frac{2}{5}$$

and from urn III is,

$$P(A|E_3) = \frac{4C_1 \times 5C_1}{11C_2}$$

$$= \frac{4 \times 5 \times 2}{11 \times 10} = \frac{4}{11}$$

Now the Bayes' probability that ball drawn belong to urn III is,

$$P(E_3|A) = \frac{P(A|E_3)\,P(E_3)}{\sum_{i=1}^{3} P(A|E_i)\,P(E_i)}$$

$$= \frac{4/11 \times 1/3}{\left(\frac{2}{9} \times \frac{1}{3} \times \frac{2}{5} \times \frac{1}{3} + \frac{4}{11} \times \frac{1}{3}\right)}$$

$$= \frac{45}{122}.$$

Q. 30 What is D'Alembert's paradox?

Ans. D'Alembert considered the tossing of two coins. He argued that there are three possible cases namely, (i) both heads, (ii) both tails, (iii) one head and one tail.

He concluded that the probability of getting one head and one tail is 1/3. As a matter of fact, the actual probability of getting one head and one tail in two tosses of a coin is 1/4. This is known as D'Alembert's paradox which has arisen due to the difficulty of deciding equally likely alternatives.

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s) in the blanks

1. Classical probability concept was given by ________.
2. Probability is attached only to _______.
3. There can be established one to one correspondence between events and ______.
4. Events may be simple as well as _______.
5. Two events are mutually exclusive if there is ________ in between them.
6. Intersection of two mutually exclusive events is a _______ event.
7. An event consisting of only one point is called an ________ event.
8. Laplace's probability cannot be calculated if the total number of outcomes is ________.
9. Mathematical probability cannot be calculated if the outcomes are ________.
10. Mathematical probability is also known as ________ probability.
11. Classical probability is not calculable if the ________ number of outcomes is not countable.
12. An event which cannot occur is known as ________ event.
13. Two events A and B are equal if ________.
14. The set of union of disjoint events $A_1, A_2, \ldots,$ A_n such that $\sum_{i=1}^{n} A_i = \Omega$ is called ________ event.
15. If an event is such that its points belong to either of the two sets A and B, the set is the ________ of A and B.
16. If an event consists of only those points which are in A as well as in B, the event represents the _______ of A and B.
17. The idea of relative frequency for calculating probability was given by ________.
18. The probability based on the concept of relative frequency is called ______ probability.
19. In statistical probability n is never _________.
20. An event of the type A or B is a ________ event.
21. Totality of all possible outcomes of a random experiment is called ________.
22. Probability can never be less than _______.
23. Probability can never be greater than _________.
24. Probability of the sample space Ω is equal to _________.
25. Probability of an event is never ________.
26. Probability can vary from _________.
27. The need for probability was originally felt in _________.
28. If two events are independent, the probability that both will occur together is equal to the _________ of their individual probabilities.
29. Addition theorem will be applicable only when the various events belong to the __________.
30. Probability can be expressed as ________.

31. Multiplication theorem is applicable only if the events are ________.

32. When all possible outcomes are included, they are known as ________ cases.

33. If $B \subset A$, the relation between $P(A)$ and $P(B)$ is ________.

34. If $B \subset A$, the probability $\left(A\overline{B}\right)$ is ________.

35. If A and B are two events, the $P\left(A \cap \overline{B}\right)$ is ________.

36. If A and B are two events, the $P\left(\overline{A} \cap B\right)$ is ________.

37. $P(A \cup B)$ can be expressed by the ______ law of probability.

38. If two events A and B are disjoint, the $P(A \cup B) =$ ________.

39. If out of three events A, B and C, the events A and B are mutually exclusive, the formula for $P(A \cup B \cup C)$ is ________.

40. If the A and B are independent, then $(A \cap B)$ is ______.

41. If the events A and B are independent, then $P(B \mid A) =$ ________.

42. For any three events A, B and C, $P(A + B/C) =$ ________.

43. For any three events A, B and C, $P(AB \mid C) + P\left(A\overline{B} \mid C\right) =$ ________.

44. For any two events A and B, $P(AB) =$ ________.

45. If A and B are independent, then $P(A \mid B) =$ ________.

46. If $P(A \cap B) = P(A)\,P(B)$, the events A and B are ________.

47. If A, B and C are pairwise independent events, the probability, $P(A \cap B \cap C)$ ________ or ________ or ________.

48. If an event A is independent of the events B, $B \cup C$ and $B \cap C$, then $P(A \cap C) =$ ________.

49. If an event A is independent of the events B, $B \cup C$ and $B \cap C$, the events A and C are ________.

50. If $A \subset B$, then $P(A \mid C)$ ________ $P(B \mid C)$.

51. For any two events A and B the probability, $P\left[(A \cap B) \cup \left(\overline{A} \cap B\right)\right] =$ ________.

52. If $P(A) = p_1$, $P(B) = p_2$ and $P(A \cap B) = p_3$, then

    (a) $P\left(\overline{A} \cup \overline{B}\right) =$ ________.

    (b) $P\left(\overline{A} \cap B\right) =$ ________.

    (c) $P\left(\overline{A} \cap \overline{B}\right) =$ ________.

    (d) $P\left(\overline{A} \cup B\right) =$ ________.

    (e) $P(A \mid B) =$ ________.

53. For any two events A and B, the probability $P(A \mid B) + P\left(\overline{A} \mid B\right) =$ ________.

54. Bayes' probability is also known as ________ probability.

55. In Bayes' probability we calculate ________ probability.

56. For any two arbitrary events A_1, A_2 and given that an event B has already occurred, the expanded form of $P(A_1 \cup A_2 \mid B)$ is ______.

57. Suppose A and B are two events. Event B has occurred and it is known that $P(B) < 1$ then $P\left(A \mid \overline{B}\right) =$ ________.

58. If A_1 and A_2 are mutually exclusive events then $P(A_1 \mid A_1 \cup A_2) =$ ________.

59. If $\overline{A}$ is the complement of A, the probability of the complement of $\overline{A}$ using involution law is ________.

60. If A and B are two events, the probability of the complement of their union using Demorgan's (dualization) law is ________.

61. If A and B are two events, the probability of

the complements of their intersection using Demorgans or dualization law is ________.

62. In tossing a fair coin turning up of head and tail are _________ outcomes.

63. The turning up of spots, 1, 2, 3, 4, 5 and 6 in rolling of a die are _________ and ________ events.

64. The probability of obtaining a total of 8 in a single throw of two dice is _________.

65. If A is an arbitrary event, then P (A/A) = ___________.

66. For any two arbitrary events A and B, $P\left(\bar{A} \mid B\right) =$ _________.

67. For any given event C, P $(\phi \mid C) =$ ________.

68. For any two events A and B, $P\left(A \cup \bar{B}\right) =$ __________.

69. For any two events A and B, $P\left(\bar{A} \cap \bar{B}\right) =$ __________.

70. If two events A and B are mutually exclusive and $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$, then

(i) Probability that either A or B will occur is __________.

(ii) Probability that neither A nor B will occur is _________.

(iii) Probability that A and B both will occur is __________.

71. Making use of the multiplication theorem, the expression for P $(A \cap B \cap C \cap D)$ is ___________.

72. If no outcome of an experiment is expected to occur more frequently then others, the outcomes are ___________.

73. If an event is not simple, it is a ________ event.

74. The outcomes of a statistical experiment which result in the happening of an event are called __________.

75. If two events are not independent, they are obviously __________.

76. Probability interpreted as a measure of degree of belief of an individual is called ___________.

77. Assigning of equal probabilities to all the elementary events of a statistical experiment is called __________ of probabilities.

78. If P $(A \cap B) = P$ (A) P (B), P $(A \cap C) = P$ (A) P (C) and P $(B \cap C)$ = P (B) P (C), the events A, B and C are ________.

79. If a class has 60 per cent boys and 40 per cent girls and the probability of getting first class of a girl is 0.30 and that of a boy getting first class is 0.25, the probability of a randomly chosen student getting first class is _______.

80. If A and B are two mutually exclusive and exhaustive events and P (B) = $2P$ (A), then P (A) = ________.

SECTION-C

Multiple Choice Questions

Select the correct alternative out of given ones:

Q. 1 The outcome of tossing a coin is a:

(a) simple event

(b) mutually exclusive event

(c) complementary event

(d) compound event

Q. 2 Classical probability is measured in terms of:

(a) an absolute value

(b) a ratio

(c) absolute value and ratio both

(d) none of the above

Q. 3 Probability can take values

(a) $-\infty$ to ∞
(b) $-\infty$ to 1
(c) -1 to 1
(d) 0 to 1

Q. 4 Probability is expressed as:
(a) ratio
(b) proportion
(c) percentage
(d) all the above

Q. 5 Two events are said to be independent if:
(a) each outcome has equal chance of occurrence
(b) there is no common point in between them
(c) one does not affect the occurrence of the other
(d) both the events have only one point

Q. 6 If A and B are two events which have no point in common, the events A and B are:
(a) complementary to each other
(b) independent
(c) mutually exclusive
(d) dependent

Q. 7 Classical probability is also known as:
(a) Laplace's probability
(b) mathematical probability
(c) a priori probability
(d) all the above

Q. 8 Each outcome of a random experiment is called:
(a) Primary event
(b) compound event
(c) derived event
(d) all the above

Q. 9 If A and B are two events, the probability of occurrence of either A or B is given as:
(a) $P(A) + P(B)$
(b) $P(A \cup B)$
(c) $P(A \cap B)$
(d) $P(A)\,P(B)$

Q. 10 If A and B are two events, the probability of occurrence of A and B simultaneously is given as:
(a) $P(A) + P(B)$
(b) $P(A \cup B)$
(c) $P(A \cap B)$
(d) $P(A)\,P(B)$

Q. 11 The limiting relative frequency approach of probability is known as:
(a) statistical probability
(b) classical probability
(c) mathematical probability
(d) all the above

Q. 12 The definition of statistical probability was originally given by:
(a) De Moivre
(b) Laplace
(c) Von-Mises
(d) Pascal

Q. 13 The definition of a priori probability was originally given by:
(a) De Moivre
(b) Laplace
(c) Von-Mises
(d) Feller

Q. 14 If it is known that an event A has occurred, the probability of an event E given A is called:
(a) empirical probability
(b) a priori probability
(c) posteriori probability
(d) conditional probability

Q. 15 Probability by classical approach has:
(a) no lecunae
(b) only one lecuna
(c) only two lecunae
(d) many lecunae

Q. 16 Classical probability is possible in case of:
(a) unequilikely outcomes
(b) equilikely outcomes
(c) either unequilikely or equilikely outcomes
(d) all the above

Q. 17 An event consisting of those elements which are not in A is called:
(a) primary event
(b) derived event
(c) simple event

(d) complementary event

Q. 18 The probability of all possible outcomes of a random experiment is always equal to:

(a) infinity
(b) zero
(c) one
(d) none of the above

Q. 19 The probability of the intersection of two mutually exclusive events is always:

(a) infinity
(b) zero
(c) one
(d) none of the above

Q. 20 The individual probabilities of occurrence of two events A and B are known, the probability of occurrence of both the events together will be:

(a) increased
(b) decreased
(c) one
(d) zero

Q. 21 If $E_1, E_2, ..., E_n$ is a countable sequence of events such that $E_i \supset E_{i+1}$ for $i = 1, 2, ...$ then:

(a) $\lim_{n\to\infty} P(E_n) = 0$

(b) $\lim_{n\to\infty} P(E_n) = \infty$

(c) $\lim_{n\to\infty} P(E_n) = 1$

(d) $\lim_{n\to\infty} P(E_n) =$ impossible value

Q. 22 If A_1, A_2 and A_3 are three mutually exclusive events, the probability of their union is equal to:

(a) $P(A_1)P(A_2)P(A_3)$

(b) $P(A_1)+P(A_2)+P(A_3)$
$-P(A_1\ A_2\ A_3)$

(c) $P(A_1)+P(A_2)+P(A_3)$

(d) $P(A_1)P(A_2)+P(A_1)\times P(A_3)$
$+P(A_2)P(A_3)$

Q. 23 If A_1, A_2 and A_3 are three independent events, the probability of their joint occurrence is equal to:

(a) $P(A_1)P(A_2)P(A_3)$

(b) $1/P(A_1)P(A_2)P(A_3)$

(c) $P(A_1)+P(A_2)+P(A_3)$

(d) $P(A_1 \cap A_2)+P(A_1 \cap A_3)$
$+P(A_2 \cap A_3)$

Q. 24 If two events A and B are such that $A \subset B$ and $B \subset A$, the relation between $P(A)$ and $P(B)$ is:

(a) $P(A) \leq P(B)$
(b) $P(A) \geq P(B)$
(c) $P(A) = P(B)$
(d) none of the above

Q. 25 If A is an event, the conditional probability of A given A is equal to:

(a) zero
(b) one
(c) infinite
(d) indeterminate quantity

Q. 26 If $A \subset B$, the probability, $P(A/B)$ is equal to:

(a) zero
(b) one
(c) $P(A)/P(B)$
(d) $P(B)/P(A)$

Q. 27 If $B \subset A$, the probability $P(A/B)$ is equal to:

(a) zero
(b) one
(c) $P(A)/P(B)$
(d) $P(B)/P(A)$

Q. 28 If two events A and B are such that $A \subset B$, the relation between the conditional probabilities $P(A/C)$ and $P(B/C)$ is:

(a) $P(A/C) = P(B/C)$
(b) $P(A/C) > P(B/C)$
(c) $P(A/C) < P(B/C)$
(d) all the above

Q. 29 For any two events A and B, $P\ (A - B)$ is equal to:

(a) $P\ (A) - P\ (B)$

(b) $P\ (B) - P\ (A)$

(c) $P\ (B) - P\ (AB)$

(d) $P\ (A) - P\ (AB)$

Q. 30 If an event B has occurred and it is known that $P\ (B) = 1$, the conditional probability $P\ (A/B)$ is equal to:

(a) $P\ (A)$

(b) $P\ (B)$

(c) one

(d) zero

Q. 31 If A and B are any two mutually exclusive events, the $P\ (A/A \cup B)$ is equal to:

(a) $P\ (A)/[P\ (A) + P\ (B)]$

(b) $P\ (A \cup B)/[P\ (A) + P\ (B)]$

(c) $P\ (B)/P\ (A \cup B)$

(d) none of the three

Q. 32 If A and B are two independent events, then $P\left(\overline{A} \cap \overline{B}\right)$ is equal to:

(a) $P\left(\overline{A}\right)P\left(\overline{B}\right)$

(b) $1 - P\ (A \cup B)$

(c) $[1 - \mathrm{P}\ (A)][1 - P\ (B)]$

(d) all the above

Q. 33 If $E_1, E_2, \ldots, E_n$ are n mutually exclusive events such that $P\ (E_j) \neq 0$ for $j = 1, 2, \ldots, n$ and A is an arbitrary event contained in $\cup E_j$ with $P\ (A) > 0$ and has been observed, the probability of a particular event E_j given A is given by the formula

(a) $P\left(E_j|A\right) = \dfrac{P\left(E_j\right)P\left(A|E_j\right)}{\sum_j P\left(E_j\right)}$

(b) $P\left(E_j|A\right) = \dfrac{P\left(E_j\right)P\left(A|E_j\right)}{\sum_j P\left(E_j\right)P\left(A|E_j\right)}$

(c) $P\left(E_j|A\right) = \dfrac{P\left(A|E_j\right)}{\sum_j P\left(A|E_j\right)}$

(d) none of the above

Q. 34 If A and B are two events such that AB and $A\overline{B}$ are two mutually exclusive and exhaustive events in which the event A can occur, then

(a) $P\ (A) = 1$

(b) $P(A) = P(AB) + P\left(A\overline{B}\right)$

(c) $P(A) = P\left(\overline{A}\right) + P\left(A\overline{B}\right)$

(d) $P(A) = P\left(\overline{A}B\right) + P\left(A\overline{B}\right)$

Q. 35 If k toffees are distributed at random among n children, the probability that a child will receive exactly r toffees is:

(a) $\dfrac{kc_r(n-1)^{n-r}}{n^k}$

(b) $\dfrac{kc_r(k-1)^{k-r}}{k^n}$

(c) $\dfrac{kc_r(n-1)^r}{n^k}$

(d) $\dfrac{kc_r(n-1)^{n-r}}{k^n}$

Q. 36 The idea of posteriori probabilities was introduced by

(a) Pascal

(b) Peter and Paul

(c) Thomas Bayes

(d) M. Loe've

Q. 37 In a city 60 per cent read newspaper A, 40 per cent read newspaper B and 30 per cent read newspaper C, 20 per cent read A and B, 30 per cent read A and C, 10 per cent read B and C. Also 15 per cent read papers A, B and C. The percentage of people who do not read any of these newspapers is:

(a) 65 per cent

(b) 15 per cent

(c) 45 per cent

(d) none of the above

Q. 38 If a bag contains 4 white and 3 black balls. Two draws of 2 balls are successively made, the probability of getting 2 white balls at first draw and 2 black balls at second draw when the balls drawn at first draw were replaced is:

(a) 3/7
(b) 1/7
(c) 19/49
d) 2/49

Q. 39 In question 38, if the balls are not replaced after the first draw, the probability of 2 white balls at first draw and 2 black balls at second draw is:

(a) $\frac{3}{35}$
(b) $\frac{13}{35}$
(c) $\frac{1}{2}$
(d) none of the above

Q. 40 In tossing three coins at a time, the probability of getting at most one head is:

(a) $\frac{3}{8}$
(b) 7/8
(c) 1/2
(d) 1/8

Q. 41 There is 80 per cent chance that a problem will be solved by a statistics student and 60 per cent chance is there that the same problem will be solved by the mathematics student. The probability that at least the problem will be solved is:

(a) 0.48
(b) 0.92
(c) 0.10
(d) 0.75

Q. 42 The probability of two persons being borned on the same day (ignoring date) is:

(a) 1/49
(b) 1/365
(c) 1/7
(d) none of the above

Q. 43 An urn contains 5 red, 4 white and 3 black balls. The probability of three balls being of different colours when the ball is replaced after each draw is equal to:

(a) 3/144
(b) 4/144
(c) 5/144
(d) 1

Q. 44 In question 43, the probability of three balls being drawn in the order red, white and black when the balls are not replaced after each draw, is equal to:

(a) 1/22
(b) 5/144
(c) 60/144
(d) none of the above

Q. 45 An urn *A* contains 5 white and 3 black balls and *B* contains 4 white and 4 black balls. An urn is selected and a ball is drawn from it, the probability, that the ball is white, is:

(a) 9/8
(b) 9/16
(c) 5/32
(d) 5/16

Q. 46 From a pack of 52 cards, two cards are drawn at random. The probability that one is an ace and the other is a king is:

(a) 2/13
(b) 1/169
(c) 16/169
(d) 8/663

Q. 47 Two dice are rolled by two players *A* and *B*. *A* throws 10, the probability that *B* throws more than *A* is:

(a) 1/12
(b) 1/6
(c) 1/18
(d) none of the above

Q. 48 The data reveals that 10 per cent patients die in a particular type of operation. A doctor performed 9 operations and all of them survived. Whether the 10th patient on being operated:

(a) will survive
(b) will die

(c) may survive or die
(d) none of the above

Q. 49 There are two groups of students consisting of 4 boys and 2 girls; 3 boys and 1 girl. One student is selected from both the groups. The probability of one boy and one girl being selected is:
(a) 1/9
(b) $\frac{5}{12}$
(c) 1
(d) none of the above

Q. 50 In a shooting competition, Mr *X* can shoot at the bulls eye 4 times out of 5 shots and Mr *Y*, 5 times out of six and Mr *Z*, 3 times out of 4 shots. The probability that the target will be hit at least twice is:
(a) 107/120
(b) 47/120
(c) 1/2
(d) none of the above

Q. 51 There are two bags. One bag contains 4 red and 5 black balls and the other 5 red and 4 black balls. One ball is to be drawn from either of the two bags. the probability of drawing a black ball is:
(a) 1
(b) 16/81
(c) 1/2
(d) 10/81

Q. 52 Three dice are rolled simultaneously. The probability of getting 12 spots is:
(a) 1/8
(b) 25/216
(c) 1/12
(d) none of the above

Q. 53 Given that $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}, P(A|B) = \frac{1}{6}$, the probability $P(B|A)$ is equal to:
(a) 1/4
(b) 3/4
(c) 1/8
(d) none of the above

Q. 54 From the probabilities given in question 53, the probability, $P\left(B|\overline{A}\right)$ is equal to:
(a) 1/16
(b) 15/24
(c) 15/16
(d) 5/16

Q. 55 A number is selected randomly from each of the two sets

1, 2, 3, 4, 5, 6, 7, 8
2, 3, 4, 5, 6, 7, 8, 9

The probability that the sum of the numbers is equal to 9 is:
(a) 8/91
(b) 7/72
(c) 14/81
(d) 7/64

Q. 56 A bag contains 3 white and 5 red balls. Three balls are drawn after shaking the bag. The odds against these balls being red is:
(a) 5/28
(b) 5/8
(c) 15/64
(d) 3/5

Q. 57 A bag contains 3 white, 1 black and 3 red balls. Two balls are drawn from the well shaked bag. The probability of both the balls being black is:
(a) 1
(b) zero
(c) 1/7
(d) none of the above

Q. 58 The chance of winning the race of the horse *A* in Durby is $\frac{1}{5}$ and that of horse *B* is $\frac{1}{6}$. The probability that the race will be won by *A* or *B* is:
(a) 1/30
(b) 1/3
(c) 11/30
(d) none of the above

Q. 59 Four cards are drawn from a pack of 52 cards. The probability that out of 4 cards being 2 red and 2 black is:

(a) 325/833
(b) 46/833
(c) 234/574
(d) none of the above

Q. 60 The probability of Mr *R* living 20 years more is $\frac{1}{5}$ and that of Mr *S* is $\frac{1}{7}$. The probability that at least one of them will survive 20 years hence is:
(a) 12/35
(b) 1/35
(c) 13/35
(d) 11/35

Q. 61 For a 60 year old person living up to the age of 70, it is 7 : 5 against him and for another 70 year old person surviving up to the age of 80, it is 5 : 2 against him. The probability that one of them will survive for 10 years more is:
(a) 5/42
(b) 49/84
(c) 59/84
(d) none of the above

Q. 62 If 7 : 6 is in favour of *A* to survive 5 years more and 5 : 3 in four of *B* to survive 5 years more, the probability that at least one of them will survive for 5 years more is:
(a) 35/104
(b) 12/26
(c) 21/26
(d) 43/52

Q. 63 The chance of Ram to stand first in the class is $\frac{1}{3}$ and that of Abdul is $\frac{1}{5}$. The probability that either of the two will stand first in the class is:
(a) 1/15
(b) 8/15
(c) 7/15
(d) none of the above

Q. 64 The probability of throwing an odd sum with two fair dice is:
(a) 1/4
(b) 1/16
(c) 1
(d) 1/2

Q. 65 The probability that there is at least one spot in two rollings of a die is:
(a) 13/36
(b) 5/18
(c) 11/36
(d) 1/18

Q. 66 The probabilities of Mr *J* and Mr *M* not living for one more year are $\frac{1}{9}$ and $\frac{1}{7}$ respectively. The probability of living one more year of either one or both is:
(a) 20/21
(b) 62/63
(c) 14/63
(d) 5/21

Q. 67 A group consists of 4 men, 3 women and 2 boys. Three persons are selected at random. The probability that 2 men are selected is:
(a) 3/28
(b) 7/28
(c) 5/28
(d) 5/14

Q. 68 The probability that a leap year will have 53 sundays is:
(a) 1/7
(b) 2/7
(c) 2/53
(d) 52/53

Q. 69 With a pair of dice thrown at a time, the probability of getting a sum more than that of 9 is:
(a) 5/18
(b) 7/36
(c) 5/6
(d) none of the above

Q. 70 If the chance of *A* hitting a target is 3 times out of 4 and of *B* 4 times out of 5 and of *C* 5 times out of 6. The probability that the target will be hit in two hits is:
(a) 19/24
(b) 23/30

(c) 47/120

(d) none of the above

Q. 71 A consignment of 15 record players contains 4 defectives. The record players are selected at random one by one and examined. The ones examined are not put back. The probability that the 9th piece examined is the last defective one is:

(a) 24/455

(b) 8/195

(c) 96/195

(d) none of the above

Q. 72 The chance that doctor A will diagnose a disease X correctly is 60 per cent. The chance that a patient will die by his treatment after correct diagnosis is 40 per cent, and the chance of death by wrong diagnosis is 70 per cent. A patient of doctor A, who had disease X, died. The probability that his disease was diagnosed correctly is:

(a) 6/25

(b) 7/25

(c) 6/7

(d) 6/13

Q. 73 An urn contains four tickets marked with numbers 112, 121, 211, 222 and one ticket is drawn at random. Let A_i (i = 1, 2, 3) be the event that i^{th} digit of the number of the ticket drawn is 1. Are the events A_1, A_2 and A_3 :

(a) mutually exclusive

(b) dependent

(c) independent

(d) pairwise independent

Q. 74 In question 73, are the events A_1, A_2 and A_3 :

(a) dependent

(b) independent

(c) mutually exclusive

(d) none of the above

Q. 75 An urn contains 5 yellow, 4 black and 3 white balls. Three balls are drawn at random. The probability that no black ball is selected is:

(a) 1/66

(b) 7/55

(c) 2/9

(d) none of the above

Q. 76 A bag contains 3 white and 5 red balls. A game is played such that a ball is drawn, its colour is noted and replaced with two additional balls of the same colour. The selection is made three times. the probability that a white ball is selected at each trial is:

(a) 7/64

(b) 21/44

(c) 105/512

(d) 9/320

Q. 77 Given that $P(A)=\frac{1}{3}$, $P(B)=\frac{3}{4}$ and

$P(A \cup B)=\frac{11}{12}$, probability, $P(B/A)$ is:

(a) 1/6

(b) 4/9

(c) 1/2

(d) none of the above

Q. 78 If A, B and C are three events such that $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.5$ and $P(AB') = 0.2$, $P(BC) = 0.3$, $P(A'B'C') = 0.3$, $P(AB \mid C') = 0.1$, the probability, $P(B' \mid C')$ is equal to:

(a) 3/5

(b) 4/5

(c) 1/5

(d) none of the above

Q. 79 Given the probability in question 78, the probability $P(A \mid B)$ is equal to:

(a) 1/4

(b) 1/2

(c) 1/3

(d) none of the above

Q. 80 If four whole numbers are taken at random and multiplied, the chance that the first digit in their product is 0, 3, 6 or 9 is:

(a) $(2/5)^3$

(b) $(1/4)^3$

(c) $(2/5)^4$

(d) $(1/4)^4$

Q. 81 *A* can hit a target 2 times in 5 shots, *B* 3 times in 5 shots and *C* 4 times in 5 shots. They fire a volley (each try once to hit the target). The probability that two shots hit is:
(a) 24/125
(b) 67/125
(c) 121/125
(d) 58/125

Q. 82 There are four coins in a bag. One of the coins has head on both sides. A coin is drawn at random and tossed five times and fell always with head upward. The probability that it is the coin with two head is:
(a) 3/128
(b) 1/4
(c) 32/35
(d) none of the above

Q. 83 One of the two events is certain to happen. The chance of one event is one-fifth of the other. The odds in favour of the other is:
(a) 1 : 6
(b) 6 : 1
(c) 5 : 1
(d) 1 : 5

Q. 84 One of the two events must happen; given that the chance of one is one-fourth of the other. The odd in favour of the other is:
(a) 1 : 3
(b) 1 : 4
(c) 1 : 5
(d) none of the above

Q. 85 A coin is tossed six times. The probability of obtaining heads and tails alternately is:
(a) 1/64
(b) 1/2
(c) 1/32
(d) none of the above

Q. 86 The odds in favour of certain event are 5 : 4, and odds against another event are 4 : 3. The chance that at least one of them will happen is:
(a) 15/63
(b) 51/63
(c) 47/63
(d) none of the above

Q. 87 *A* and *B* start in a ring with ten other persons. If the arrangement of 12 persons is at random, the chance that there are exactly three persons between *A* and *B* is:
(a) 1/66
(b) 2/11
(c) 4/11
(d) none of the above

Q. 88 Three houses were available in a locality for allotment. Three persons applied for a house. The probability that all the three persons applied for the same house is:
(a) 1/3
(b) 1/9
(c) 1/27
(d) 1

Q. 89 In the problem of question 88, the probability that each of the three applied for a different house is:
(a) 1/9
(b) 1/27
(c) 1
(d) 2/9

Q. 90 *A* speaks truth 4 times out of five and *B* speaks truth 3 times out of four. They agree in the assertion that a white ball has been drawn from a bag containing 10 balls of different colours. The probability that a white ball was really drawn is:
(a) 3/50
(b) 1/27
(c) 1/1350
(d) 81/82

Q. 91 In the problem of question 90 if the bag contains 1 white and 9 red balls, the probability of one white ball being drawn is:
(a) 4/7
(b) 3/50
(c) 9/200
(d) none of the above

Q. 92 If *A* tells truth 4 times out of 5 and *B* tells truth 3 times out of 4. The probability that both expressing the same fact contradict each other is:
(a) 1/20

(b) 3/20
(c) 1/5
(d) none of the above

Q. 93 The probability of drawing a white ball in the first draw and again a white ball in the second draw with replacement from a bag containing 6 white and 4 blue balls is:

(a) 2/10
(b) 6/10
(c) 36/100
(d) 1/3

Q. 94 A fair coin is tossed repeatedly unless a head is obtained. The probability that the coin has to be tossed at least four times is:

(a) 1/2
(b) 1/4
(c) 1/6
(d) 1/8

Q. 95 Out of 20 employees in a company, five are graduates. Three employees are selected at random. The probability of all the three being graduates is:

(a) 1/64
(b) 1/125
(c) 1/114
(d) none of the above

Q. 96 In the problem of question 95, the probability that at least one of them is a graduate is:

(a) 11/114
(b) 137/1368
(c) 137/228
(d) none of the above

Q. 97 (Parzen) In answering a question on a multiple choice test, an examinee either knows the answer with probability p or he guesses with probability $(1 - p)$. Let the probability of answering the question correctly be 1 for an examinee who knows the answer and $\frac{1}{m}$ who guesses (m being the number of multiple choice alternatives). Suppose an examinee answer a question correctly. The probability that he really knows the answer is:

(a) $\dfrac{mp}{1+mp}$

(b) $\dfrac{mp}{1+(m-1)p}$

(c) $\dfrac{(m-1)p}{1+(m-1)p}$

(d) none of the above

Q. 98 A machine part is produced by three factories A, B and C. Their proportional production is 25, 35 and 40 per cent, respectively. Also, the percentage defectives manufactured by three factories are 5, 4 and 3, respectively. A part is taken at random and is found to be defective. The probability that the selected part belongs to factor B is:

(a) 28/503
(b) 4/11
(c) 14/276
(d) none of the above

Q. 99 A card is drawn from a well shuffled pack of 52 cards. A gambler bets that it is either a heart or an ace. What are odds against his winning this wet?

(a) 9 : 4
(b) 4 : 9
(c) 35 : 52
(d) 1 : 3

Q. 100 If one card is selected at random from 100 cards numbered as 00, 01, ..., 99. Suppose x and y are the sum and product of the digits on the selected card. If i is a whole number, the probability $P(x = i/y = 0)$ is equal to:

(a) $\dfrac{1}{19}$

(b) 1/50
(c) 19/100
(d) none of the above

Q. 101 Two dice, each numbered 1 to 6, are thrown together. Consider two events A and B given by

A - even number of the first die

B - number on the second die is greater than 4,

Then $P(A \cup B)$ is:

(a) 2/3
(b) 1/6
(c) 1/2
(d) 3/4

Q. 102 An unbiased coin is tossed four times. The probability that the number of heads exceeds the number of tails is:

(a) 1/12
(b) 3/4
(c) 3/8
(d) 5/16

Q. 103 If $P(A|B) = \frac{1}{4}$ and $P(B|A) = \frac{1}{3}$, then $P(A)/P(B)$ is equal to:

(a) 3/4
(b) 7/12
(c) 4/3
(d) 1/12

Q. 104 For two events E_1, E_2, if $P(E_1) = 1/2$, $P(E_2) = \frac{1}{3}$, $P(E_1 \cup E_2) = \frac{2}{3}$, then $P(E_1 \cap E_2)$ is equal to:

(a) 1/4
(b) 1/6
(c) 2/5
(d) 1/3

Q. 105 For two events A_1 and A_2, if $P(A_1) = \frac{2}{3}$, $P(A_2) = \frac{3}{8}$ and $P(A_1 \cap A_2) = \frac{1}{4}$, then A_1 and A_2 are:

(a) mutually exclusive but not independent
(b) mutually exclusive and independent
(c) independent but not mutually exclusive
(d) not mutually exclusive and not independent

Q. 106. In a library there are 40 per cent mathematics books and remaining 60 per cent science books. It is known that 2 per cent of the mathematics books are in Hindi and 1 per cent of science books are in Hindi. If one book is taken out at random and is found to be in Hindi, the probability that it is a science book is:

(a) 2/9
(b) 3/7
(c) 6/13
(d) 1/4

Q. 107 Take four identical marbles. On the first write symbols A_1, A_2, A_3. On each of the other three write A_1, A_2, A_3 respectively. Put the four marbles in an urn and draw one at random. Let E_i denotes the event that the symbol A_i appears on the drawn marble. Then which of the four statements is true for this problem?

(a) $P(E_1) = P(E_2) = P(E_3) = \frac{1}{4}$

(b) $P(E_1E_2) = P(E_2E_3) = P(E_1E_3) = \frac{1}{4}$

(c) $P(E_1E_2E_3) = \frac{1}{8}$

(d) all the above.

Q. 108 For a post in a factory, husband and wife both applied. The probability of selection of a male is $\frac{1}{5}$ and that of a female is $\frac{1}{3}$. The probability of selection of only one of them is:

(a) 2/15
(b) 4/15
(c) 8/15
(d) 2/5

Q. 109 Consider a family of three children. Also assume that all possible distributions of children have same probabilities. Let H be the event, 'the family has children of both the sexes' and A be the event, 'there is at most one girl'. Then the events H and A are:

(a) mutually exclusive
(b) complementary events
(c) independent events
(d) exhaustive events.

Q. 110 In a certain residential suburb, 70 per cent of all households subscribe to the metropolitan newspaper, 60 per cent subscribe to the local evening paper and 40 per cent households subscribe to both the papers. The probability that a household selected at random subscribes to one of the two newspapers is:

(a) 0.3

(b) 0.5

(c) 0.9

(d) 0.2

ANSWERS

SECTION-B

(1) Laplace (2) event (3) sets (4) compound (5) no common point (6) null (7) elementary (8) infinite (9) not equally likely (10) a priori (11) total (12) null or impossible (13) $A \subset B$ and $B \subset A$ (14) exhaustive (15) union (16) intersection (17) Von Mises (18) statistical or empirical (19) infinite (20) derived (21) sample space (22) zero (23) one (24) one (25) negative (26) 0 to 1 (27) gambling (28) product (29) same sample space (30) ratio or fraction or percentage (31) independent (32) exhaustive (33) $P(A) \geq P(B)$ (34) $P(A) - P(B)$ or $P(A - B)$ (35) $P(A) - P(A \cap B)$ (36) $P(B) - P(A \cap B)$ (37) additive (38) $P(A) + P(B)$ (39) $P(A) + P(B) + P(C) - P(A \cap C) - P(B \cap C)$. (40) $P(A) \times P(B)$ (41) $P(B)$ (42) $P(A \mid C) + P(B|C) - P(AB|C)$ (43) $P(A|C)$ (44) $P(A)\,P(B|A)$ or $P(B)\,P(A|B)$ (45) $P(A)$ (46) independent (47) $P(A)\,P(B)\,P(C|A \cap B)$ or $P(A)\,P(C)\,P(B|A \cap C)$ or $P(B)\,P(C)\,P(A|B \cap C)$ (48) $P(A)\,P(C)$ (49) independent (50) $\leq$ (51) $P(A) + P(B) - 2P(A \cap B)$ (52) (a) $1 - p_3$ (b) $p_2 - p_3$ (c) $1 - p_1 - p_2 + p_3$ (d) $1 - p_1 + p_3$ (e) $\dfrac{p_3}{p_2}$ (53) 1 (54) inverse (55) posteriori (56) $P(A_1|B) + P(A_2|B) - P(A_1 A_2|B)$ (57) $[P(A) - P(AB)]/[1 - P(B)]$ (58) $P(A_1)/[P(A_1) + P(A_2)]$ (59) $P(A)$ (60) $P\left(\overline{A} \cap \overline{B}\right)$ (61) $P\left(\overline{A} \cup \overline{B}\right)$ (62) exhaustive (63) mutually ex-clusive; exhaustive (64) $\dfrac{5}{36}$ (65) 1 (66) $1 - P(A/B)$ (67) zero (68) $1 - P(A \cap B)$ (69) $1 - P(A \cup B)$ (70) (i) 5/6 (ii) 1/6 (iii) 0 (71) $P(A)\,P(B|A)\,P(C|A \cap B)\,P(D|A \cap B \cap C)$ (72) equally likely. (73) Compound (74) favourable cases (75) dependent (76) subjective probability (77) natural assignment (78) pairwise independent (79) 0.27 (80) 1/3

SECTION-C

Answers

(1) a	(2) b	(3) d	(4) d	(5) c	(6) c
(7) d	(8) a	(9) b	(10) c	(11) a	(12) c
(13) b	(14) d	(15) d	(16) b	(17) d	(18) c
(19) b	(20) b	(21) a	(22) c	(23) a	(24) c
(25) b	(26) c	(27) b	(28) c	(29) d	(30) a
(31) a	(32) d	(33) b	(34) b	(35) a	(36) c

(37) b [Hint: $P(A \cup B \cup C) = 0.85$, and required percentage = 100 - 85 = 15 per cent]

(38) d $\left[\text{Hint: } P(2W) = \dfrac{4C_2}{7C_2} = \dfrac{2}{7}, P(2B) = \dfrac{3C_2}{7C_2} = \dfrac{1}{7}, P(2W \text{ and } 2B) = \dfrac{2}{7} \times \dfrac{1}{7} = \dfrac{2}{49}\right]$

(39) a $\left[\text{Hint: } P(2W) = \dfrac{2}{7}, P(2B) = \dfrac{3C_2}{5C_2} = \dfrac{3}{10}, P(2W \text{ and } 2B) = \dfrac{2}{7} \times \dfrac{3}{10} = \dfrac{3}{35}\right]$

(40) c $\left[\text{Hint: } \Omega = 8 \text{ points}, E = \text{HTT, THT, TTH, TTT}, P(E) = \dfrac{4}{8} = \dfrac{1}{2}\right]$

(41) b [Hint $P(E) = 0.8 + 0.6 - 0.8 \times 0.6 = 0.92$]

(42) c $\left[\text{Hint: } P(E) = 7\left(\dfrac{1}{7} \times \dfrac{1}{7}\right) = \dfrac{1}{7}\right]$

(43) c $\left[\text{Hint: } P(E) = \dfrac{5}{12} \times \dfrac{4}{12} \times \dfrac{3}{12} = \dfrac{5}{144}\right]$

(44) a $\left[\text{Hint: } P(E)=\frac{5}{12}\times\frac{4}{11}\times\frac{3}{10}=\frac{1}{22}\right]$

(45) d $\left[\text{Hint: } P(E)=\frac{1}{2}\left(\frac{5}{8}+\frac{4}{8}\right)=\frac{9}{16}\right]$

(46) d $\left[\text{Hint: } P(E)=\frac{4C_1\times 4C_1}{52C_2}=\frac{8}{663}\right]$

(47) a $\left[\text{Hint: } \Omega = 36 \text{ points, } E\text{: } (6, 6), (6, 5), (5, 6),\right.$

$\left. P(E)=\frac{3}{36}=\frac{1}{12}\right]$

(48) c

(49) b $\left[\text{Hint: } P(E)=\left(\frac{4c_1\times 1c_1}{6c_1\times 4c_1}\right)+\left(\frac{2c_1}{6c_1}\times\frac{3c_1}{4c_1}\right)\right.$

$\left.=\frac{5}{12}\right]$

(50) a $\left[\text{Hint: } P(E)=\frac{4}{5}\times\frac{5}{6}\times\frac{1}{4}+\frac{4}{5}\times\frac{1}{6}\times\frac{3}{4}+\frac{1}{5}\times\frac{5}{6}\right.$

$\left.\times\frac{3}{4}+\frac{4}{5}\times\frac{5}{6}+\frac{3}{4}=\frac{107}{120}\right]$

(51) c $\left[\text{Hint: } P(E)=\frac{1}{2}\left(\frac{5}{9}+\frac{4}{9}\right)=\frac{1}{2}\right]$

(52) b $\left[\text{Hint: } \Omega = 6\times 6\times 6 \text{ points, } E = (1, 5, 6),\right.$ (1, 6, 5) ..., (6, 4, 2), (6, 5, 1) = 25 points, $P\ (E)$

$\left.=\frac{25}{216}\right]$

(53) c $\left[\text{Hint: } P(B/A)=\frac{P(A/B)P(B)}{P(A)}=\frac{1}{8}\right]$

(54) d $\left[\text{Hint: } P(B/\overline{A})=\frac{[1-P(A/B)]P(B)}{1-P(A)}=\frac{5}{16}\right]$

(55) d [Hint: Ω = 64 points, E = (1, 8), (2, 7), (3, 6), (4, 5), (5, 4), (6, 3), (7, 2), $\left.P(E)=\frac{7}{64}\right]$

(56) a $\left[\text{Hint: } P(E)=\frac{5C_3}{8C_3}=\frac{5}{28}\right]$

(57) b [Hint: Impossible event]

(58) c $\left[\text{Hint: } P(E)=\left(\frac{1}{5}+\frac{1}{6}\right)=\frac{11}{30}\right]$

(59) a $\left[\text{Hint: } P(E)=\frac{26c_2\times 26c_2}{52c_4}=\frac{325}{833}\right]$

(60) d $\left[\text{Hint: } P(E)=\frac{1}{5}+\frac{1}{7}-\frac{1}{5}\times\frac{1}{7}=\frac{11}{35}\right]$

(61) b $\left[\text{Hint: } P(E)=\frac{5}{12}+\frac{2}{7}-\frac{5}{12}\times\frac{2}{7}=\frac{49}{84}\right]$

(62) d $\left[\text{Hint: } P(E)=\frac{7}{13}+\frac{5}{8}-\frac{7}{13}\times\frac{5}{8}=\frac{43}{52}\right]$

(63) b $\left[\text{Hint: } P(E)=\frac{1}{3}+\frac{1}{5}=\frac{8}{15}\right]$

(64) d $\left[\text{Hint: } \Omega = 36 \text{ points, } E = 18 \text{ pairs of odd}\right.$

$\left.\text{sum, } P(E)=\frac{18}{36}=\frac{1}{2}\right]$

(65) c [Hint: Ω 36, pairs E = first throw 1 and second throw 2, 3, 4, 5, 6 or 1st throw 2, 3, 4, 5, 6 and 2nd throw 1 or both the throws 1, 1. E = 11 points $P\ (E)$ = 11/36]

(66) b $\left[\text{Hint: } P(E)=\frac{8}{9}+\frac{6}{7}-\frac{8}{9}\times\frac{6}{7}=\frac{62}{63}\right]$

(67) d $\left[\text{Hint: } P(E)=\frac{4c_2\times 3c_1}{9c_3}+\frac{4c_2\times 2c_1}{9c_3}=\frac{5}{14}\right]$

(68) b [Hint: A leap year has 366 days, *i.e.*, 52 week and 2 days. 52 sundays are sure. 53rd sunday may be on the remaining two day. The possible pairs of days are (S, M), (M, T), (T, W), (W, Th), (Th, F) (F, Sat), (Sat, S) . E = (S, M) and (Sat, S) . P (E) = 2/7]

(69) d[Hint: Ω = 36 pairs, E = (4, 6) (6, 4), (5, 5), (5, 6), (6, 5) and (6, 6). P (E) = 6/36 = 1/6]

(70) c $\left[\text{Hint:} P(E)=\frac{3}{4}\times\frac{4}{5}\times\frac{1}{6}+\frac{3}{4}\times\frac{1}{5}\times\frac{5}{6}+\frac{1}{4}\times\frac{4}{5}\times\frac{5}{6}=\frac{47}{120}\right]$

(71) b [Hint: A – 3 defectives out of 1st eight examined record players B – 9th piece is defective.

$P(A)=\binom{4}{3}\binom{11}{5}\Big/\binom{15}{8}$, $P(B/A)=\frac{1}{7}$, $P(A\cap B)=$ P (B/A) P (A) = 8/195]

(72) d [Hint: E_1 : disease X is correctly diagnosed by doctor A, E_2 : Patient having disease X dies.

$$P(E_1)=0.6,\ P(\overline{E}_1)=0.4,\ P(E_2/E_1)=0.4,\ P(E_2/\overline{E}_1)=0.7$$

$$P(E_1/E_2)=\frac{P(E_2/E_1)P(E_1)}{P(E_2/E_1)P(E_1)+P(E_2/\overline{E}_1)P(\overline{E}_1)}=\frac{6}{13}\Bigg]$$

(73) d [Hint: $P(A_1)=P(A_2)=P(A_3)=\frac{1}{2}$, $P(A_1\cap A_2)=\frac{1}{4}=P(A_1)P(A_2)$. Similarly, $P(A_2\cap A_3)=P(A_2)P(A_3)$ and $P(A_1\cap A_3)=P(A_1)P(A_3)$ Also $P(A_1\cap A_2\cap A_3)=0$]

(74) d

(75) b $\left[\text{Hint:} P(E)=\binom{8}{3}\Big/\binom{12}{3}=\frac{14}{55}\right]$

(76) a $\left[\text{Hint:} P(E_1\,E_2\,E_3)=P(E_1)P(E_2/E_1)\ P(E_3|E_1\,E_2)=\frac{3}{8}\times\frac{5}{10}\times\frac{7}{12}=\frac{7}{64}\right]$

(77) c $\left[\text{Hint: } P(A\cap B)=\frac{1}{3}+\frac{3}{4}-\frac{11}{12}=\frac{1}{6},\ P(B/A)=\frac{1/6}{1/3}=\frac{1}{2}\right]$

(78) b $\left[\text{Hint: } P\ (B'/C')=\frac{P(B'C')}{P(C')}=\frac{P(B'\cup C')}{P(C')}=\frac{0.6+0.5-0.7}{1-0.5}=\frac{4}{5}\right]$

(79) a [Hint: P (AB') = P (A) – P (AB) = 0.1, P (A/B) = 0.1/0.4 = 1/4]

(80) c [Hint: There are ten digits from 0 to 9 in which a product can end. Prob. of any given digit to be the first digits is $\frac{4}{10}=\frac{2}{5}$ for the given event,

$$P(E)=\frac{2}{5}\times\frac{2}{5}\times\frac{2}{5}\times\frac{2}{5}=\left(\frac{2}{5}\right)^4\Bigg]$$

(81) d $\left[\text{Hint: } P\ (E)=\frac{2}{5}\times\frac{3}{5}\times\frac{1}{5}+\frac{2}{5}\times\frac{4}{5}\times\frac{2}{5}+\frac{3}{5}\ \frac{3}{5}\times\frac{4}{5}=\frac{58}{125}\right]$

(82) c [Hint: Prob. of selecting a coin having both head = 1/4 and it has all five heads $=\frac{1}{4}\times1\times1\times1\times1\times1=\frac{1}{4}$. Prob of selection of a coin having head and tail = 3/4 and it fall with five heads $=\frac{3}{4}\times\left(\frac{1}{2}\right)^5=\frac{3}{128}$. $P(E)=\frac{1/4}{1/4+\frac{3}{128}}=\frac{32}{35}$]

(83) d $\Big[$Hint: $p_1 = x,\ p_2 = \frac{x}{5},\ x + \frac{x}{5} = 1,\ x = \frac{5}{6},\ p_2/p_1 = \frac{1/6}{5/6}$ or $p_2 : p_1 = 1:5\Big]$ (84) b (85) c [P (E) = P (HTHTHT)+ P(THTHTH) $= \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6 = \frac{1}{32}\Big]$

(86) c $\Big[$Hint: $P(E_1) = \frac{5}{9},\ P(\overline{E}_2) = \frac{3}{7},\ P(E_1 \cup \overline{E}_2) = \frac{5}{9} + \frac{3}{7} - \frac{5}{9} \times \frac{3}{7} = \frac{47}{63}\Big]$

(87) b [Hint: Let A be at 12th place, P (A) = 1/12. B can be at 4th or 8th places out of remaining 11 places. $P(B) = \frac{2}{11}$. Also A can be at any of the 12 places. Therefore, $P(E) = \frac{1}{12} \times \frac{2}{11} \times 12 = \frac{2}{11}\Big]$

(88) b $\Big[$Hint: $P(E) = 3 \times \left(\frac{1}{3}\right)^3 = \frac{1}{9}\Big]$

(89) d $\Big[$Hint: $P(E) = \frac{3P_3}{3^3} = \frac{2}{9}\Big]$

(90) d [Hint: Prob. of their true assertion that a white ball is drawn = 3/50. Chance of A statement being wrong $= \frac{1}{9} \times \frac{1}{5} = \frac{1}{45}$ and that of B being wrong $= \frac{1}{9} \times \frac{1}{3} = \frac{1}{27}$. Prob. of A and B to agree in false statement that a white ball is drawn $= \frac{1}{45} \times \frac{1}{27} \times \frac{9}{10} = \frac{1}{27 \times 50}$. $P(E) = \frac{3/50}{3/50 + \frac{1}{27 \times 50}} = \frac{81}{82}\Big]$

(91) a $\left[\text{Hint: } P(E) = \frac{3/50}{\frac{3}{5} + \frac{9}{10} \times \frac{1}{5} \times \frac{1}{9}} = \frac{4}{7}\right]$

(92) d $\left[\text{Hint: } P(E) = \frac{4}{5} \times \frac{1}{4} + \frac{1}{5} \times \frac{3}{4} = \frac{7}{20}\right]$

(93) c $\left[\text{Hint: } P(E) = \frac{6c_1 \times 6c_1}{10c_1 \times 10c_1} = \frac{36}{100}\right]$

(94) b [Hint: P (E) = P (TTTH) + P (TTTTH) $+P$(TTTTTH)$+\ldots = \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots = \frac{1/8}{1-1/2} = \frac{1}{4}\Big]$

(95) c [Hint: P (E) = $5c_3/20c_3$ = 1/114]

(96) c $\Big[$Hint: $P(E) = \frac{1}{20c_3}[5c_1 \times 15c_2 + 5c_2 \times 15c_1 + 5c_3] = \frac{137}{228}\Big]$

(97) b [Hint: A_1 – Examinee knows the answer,
A_2 – Examinee guesses the answer,
B – Answers correctly.

$$P(A_1/B) = \frac{P(A_1)\,P(B/A_1)}{P(A_1)\,P(B/A_1) + P(A_2)\,P(B/A_2)}$$

$$= \frac{p \times 1}{p \times 1 + \frac{1-p}{m}}\Bigg]$$

(98) b [Hint: P (E) =

$$= \frac{35/100 \times 4/100}{\frac{35}{100} \times \frac{4}{100} + \frac{25}{100} \times \frac{5}{100} + \frac{40}{100} \times \frac{3}{100}} = \frac{4}{11}\Bigg]$$

(99) a [Hint: A – card is heart, B – card is ace

$$P(A) = \frac{1}{4},\ P(B) = \frac{1}{13},\ P(A \cap B) = \frac{1}{52}.$$

$$P(A \cup B) = \frac{4}{13} = p,\ q = \frac{9}{13},\ q : p = 9:4]$$

(100) c [Hint: E = 00, 01, 02, 03, 04, 05, 06, 07, 08, 09, 10, 20, 30, 40, 50, 60, 70, 80, 90 = 19 points Ω = 00, 01, ..., 99 = 100 points, P (E) = 19/100]

(101) b (102) d (103) a (104) b (105) c (106) b (107) b (108) d (109) c (110) c

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Chapter **7**

Random Variable, Mathematical Expectation and Probability Distributions

SECTION-A

Short Essay Type Questions

Q. 1 Define a random variable and give its examples.

Ans. A rule that assigns a real number to each outcome (sample point) of a random experiment is called a random variable (r.v.). It is governed by a function of the variable. Hence, a random variable is a real valued function $X(x)$ of the elements of the sample space Ω where x is an element of Ω. Further, the range of the variable will be a set of real values. For example, in tossing a coin, $x = 1$, if the coin falls with head, and $x = 0$ if the coin falls with tail. The height of persons can be given by $X(x) = X$, the height measured in centimeters or inches. A random variable is usually denoted by any of the capital Latin letters $X, Y, Z, U, V, ...$

Q. 2 Discuss different types of variables with examples.

Ans. There are two types of random variables namely (i) discrete random variable, (ii) continuous random variable.

(i) A random variable, say x, which can take a finite number of values in an interval of the domain, is called *discrete random variable.* For example, if we toss a coin, the variable can take only two values 0 and 1 assigned to tail and head respectively, *i.e.,*

$$X(x) = \begin{bmatrix} 0 \text{ if } x \text{ is } T \\ 1 \text{ if } x \text{ is } H \end{bmatrix}$$

In rolling of a die, only six values of the variable X, *i.e.,* 1, 2, 3, 4, 5 and 6 are possible. Hence, the variable X is discrete. Here the variable,

$$X(x) = \{x : x = 1, 2, 3, 4, 5 \text{ and } 6]$$

(ii) A random variable X, which can take any value in its domain or in an interval or the union of intervals on the real line is called *Continuous random variable.* For a continuous variable, the probability of a point x is zero, *i.e.,* $P(X = x) = 0$. But the probability is ascribable in an interval. For instance, the weight of middle-aged people in India lying between 40 kg and 150 kg is a continuous variable. Notationally,

$$X(x) = [x : 40 \leq x \leq 150]$$

The maximum breaking strength 250 kg of a wire is a continuous variable. Notationally,

$$X(x) = [x : 0 \leq x \leq 250]$$

Q. 3 What are the important properties of a random variable?

Ans. The properties of a random variable are:

(i) If X is a random variable and a, b are any two constants, $aX + b$ is also a random variable.

(ii) If X is a random variable, X^2 is also a random variable.

(ii) If X is a random variable, $1/X$ is also a random variable.

(iv) If X and Y are two random variables defined over the same space, $X + Y$, $X - Y$, aX, bY or $aX + bY$ are also random variables where a and b are any two constants except that a and b both are not zero.

(v) If $X_1, X_2, ..., X_n$ are n random variables, $U_n = \max(X_1, X_2, ..., X_n)$ and $V_n = \min(X_1, X_2, ..., X_n)$ are also random variables.

Q. 4 What do you understanding by a distribution function?

Ans. A function $F_X(x)$ of a random variable X for a real value x giving the probability of the event $(X \leq x)$ is called a *cumulative distribution function* (c.d.f.) or simply *distribution function*. Symbolically,

$$F_X(x) = P(X \leq x)$$

Obviously X lies in the interval $(-\infty, x)$.

Q. 5 What are the properties of a distribution function?

Ans. Some important properties of distribution function are:

(i) If a and b are two constant values such that $a < b$ and F is the distribution function, then
$$P(a \leq X \leq b) = F(b) - F(a).$$

(ii) If $F(x)$ is the distribution function of a monovariate X, then $0 \leq F(x) \leq 1$.

(iii) If $X < Y$, then $F(x) < F(y)$.

(iv) If $F(x)$ is the distribution function of a monovariate X, then
$$F(-\infty) = \lim_{x \to -\infty} F(x) = 0$$
$$F(\infty) = \lim_{x \to \infty} F(x) = 1.$$

Q. 6 What is a probability mass function (p.m.f.) or discrete probability distribution?

Ans. If X_1, X_2, ... are the variate values in the sample space Ω of a single dimensional variable X with probabilities of occurrence p_1, p_2, ... respectively, *i.e.*, $p_i = P(X = x_i) = p(x_i)$ such that $p(x_i) \geq 0 \ \forall \ i$ and $\sum_{\text{all } x_i \in \Omega} p(x_i) = 1$, $p(x)$ is called the *probability mass function* and $F(x)$ the *probability distribution function* of the random variable x where,

$$F(x) = \sum_{\text{all } (i : x \leq x_i)} p(x_i)$$

The function $F(x)$ is also called the *step function* and its graph is just like staircase having a jump of magnitude p_i at each i taken along abscissa.

Q. 7 What is meant by probability density function?

Ans. If X is a continuous variable and $f_X(x)$ is a continuous function of X, $f_X(x)\,dx$ gives the probability of the event that X lies in the interval

$$\left(x - \frac{1}{2}dx\right) \text{ and } \left(x + \frac{1}{2}dx\right)$$

i.e.
$$\left(x - \frac{1}{2}dx \leq x \leq x + \frac{1}{2}dx\right),$$

$f_X(x)$ or simply $f(x)$ is called *probability density function* (p.d.f.) or simply *density function*. It is also known as *frequency function* because it also gives the proportion of units lying in the interval $\left(x - \frac{1}{2}dx\right)$ and $\left(x + \frac{1}{2}dx\right)$. When the probability, which is the area under *probability density curve* within the interval, is multiplied by the total number of units in the population, this gives the actual number of units in the prescribed interval. If x has the range $[\alpha, \beta]$, $f(x) \geq 0 \forall x \,\varepsilon [\alpha, \beta]$, then $\int_{\alpha}^{\beta} f(x)dx = 1$. Also for any two values (a, b).

$$p = \int_a^b f(x)dx = F(b) - F(a)$$

Q. 8 Define and discuss mathematical expectation.

Ans. *Definition:* The expected value of a random variable is simply the long run average of this variable over an indefinite number of samples.

If X is a discrete random variable that takes on the values $x_1, x_2, \dots x_n$ with probability $p_1, p_2, \dots, p_n$, respectively, the expected value of X is given as,

$$E(X) = \sum_{i=1}^{n} p_i x_i \quad \text{where} \sum_{i=1}^{n} p_i = 1$$

In general, the expected value of a function $H(x)$ of X is,

$$E\{H(x)\} = \sum_{\text{all } x_i} H(x)\, p(x)$$

Again, if X is a continuous random variable with p.d.f. $f(x)$, the expected value of X is,

$$E(X) = \int_a^b x f(x) dx; \quad a \le x \le b.$$

$E(X)$ is also known as *theoretical average value.* If $g(x)$, a function of X is also a continuous variable and $E[g(x)]$ exists, then

$$E[g(x)] = \int_a^b g(x) f(x) dx$$

Mathematical expectation is extremely used to find out the moments of a distribution and has great importance. Moments have already been discussed in chapter 5.

Q. 9 Give some important results of mathematical expectation.

Ans. Following are some of the important results on mathematical expectation:

(i) $E(X) = \sum_{\text{all } x_i} p_i x_i = \frac{1}{N} \sum_{\text{all } x_i} x_i = \mu_1 = \mu$

(mean) where each x_i has probability $\frac{1}{N}$.

If X be a continuous r.v.,

$$E(X) = \int x f(x) dx = \mu$$

Further, $E(x - \bar{x}) = 0$ where $\bar{x}$ is sample mean and $E(\bar{x}) = \mu$.

(ii) $E(X - \mu)^2 = \sum_{\text{all } x_i} p_i (x_i - \mu)^2$

$$= \frac{1}{N} \sum_{\text{all } x_i} (x_i - \mu)^2 = \mu_2 \text{ (variance)}$$

or if X be a continuous r.v.,

$$E(X - \mu)^2 = \int (x - \mu)^2 f(x) dx = \mu_2$$

(iii) If $X_1, X_2, \dots, X_n$ are n random variables, then

$$E(X_1 + X_2 + \dots + X_n) = E(X_1) + E(X_2) + \dots + E(X_n)$$

It is known as *addition theorem* of expectation.

(iv) If $X_1, X_2, \dots, X_n$ are n independent random variables, then

$$E(X_1 X_2 \dots X_n) = E(X_1) E(X_2) \dots E(X_n)$$

(v) Expectation of a constant is the constant itself, *i.e.*, $E(c) = c$ where c is a constant.

(vi) If c is a constant, then

$$E(cX) = cE(X)$$

Again, if a and c are two constants, then

Also $E(aX + c) = aE(X) + c$

and $V(aX + c) = a^2 V(X)$

(vii) If X and Y are two random variables, c_1 and c_2 are two constants, then

$$E(c_1 X + c_2 Y) = c_1 E(X) + c_2 E(Y)$$

(viii) If $X \le Y$, $E(X) \le E(Y)$.

(ix) If X and Y are two random variables, the covariance between X and Y is given as,

$\text{cov}(X, Y) = E[\{X - E(X)\}\{Y - E(Y)\}]$

$= E(XY) - E(X)E(Y)$.

If X and Y are independent, $\text{cov}(X, Y) = 0$

(x) If X_1 and X_2 are two random variables, then

$$V(X_1+X_2)=V(X_1)+V(X_2)+2\operatorname{cov}(X_1, X_2)$$

Also $$V(c_1X_1+c_2X_2)=c_1^2V(X_1)+c_2^2V(X_2)+2c_1c_2\operatorname{cov}(X_1, X_2)$$

(xi) If X_1 and X_2 are two random variables, then

$$V(X_1-X_2)=V(X_1)+V(X_2)-2\operatorname{cov}(X_1, X_2)$$

Also $$V(c_1X_1-c_2X_2)=c_1^2V(X_1)+c_2^2V(X_2)-2c_1c_2\operatorname{cov}(X_1, X_2)$$

Q. 10 State Cauchy-Schwartz inequality on expectation.

Ans. Cauchy-Schwartz inequality states that if X and Y are two random variables taking real values then

$$E[(XY)^2]\leq E(X^2)\,E(Y^2)$$

Q. 11 State Jenson's inequality on expectation.

Ans. If ϕ is a continuous and convex function and X is a random variable having finite mean μ, *i.e.*, $E(X)=\mu$, then

$$E[\phi(x)]\geq\phi[E(x)]$$

In case ϕ is a continuous and concave function,

$$E[\phi(x)]\leq\phi[E(x)]$$

Q. 12 State Gurland's inequality on expectation.

Ans. If ϕ and ψ be two continuous monotone functions of a variable X which are both non-decreasing or both non-increasing and also their expectations exist, then

$$E[\phi(x)\psi(x)]\geq E[\phi(x)]E[\psi(x)]$$

It implies $E(X^2)\geq[E(X)]^2$, provided X takes non-negative values. Also if one of them is non-increasing and the other is non-decreasing, then

$$E[\phi(x)\psi(x)]\leq E[\phi(x)]E[\psi(x)]$$

Q. 13 Define moment generating function (m.g.f.) and its usage.

Ans. Moment generating function $M_X(t)$ of a random variable X having probability function $f(x)$ is

$$M_X(t)=E\left(e^{tx}\right).$$

For a discrete distribution,

$$M_X(t)=E\left(e^{tx}\right)=\sum_{\text{all }x}e^{tx}f(x)$$

and for continuous distribution,

$$M_X(t)=E\left(e^{tx}\right)=\int e^{tx}f(x)dx$$

Moment generating function is used to find the moments of a distribution. The r^{th} moment about origin is the coefficient of $t^r/r!$ in the expansion of $M_X(t)$.

Note: A m.g.f. may exist without the existence of moments.

Q. 14 Define characteristic function (c.f.) and give its importance.

Ans. The characteristic function of a random variable X having the probability function $f(x)$ is given as,

$$\phi_X(t)=E\left(e^{itx}\right)$$

For a discrete distribution,

$$\phi_X(t)=\sum_{\text{all }x}e^{itx}f(x)$$

and for a continuous distribution,

$$\phi_X(t)=\int e^{itx}f(x)dx$$

The greatest importance of characteristic function is that it exists for each and every distribution whereas moment generating function does not exist for every distribution.

Secondly, once we know the characteristic function, we can identify the distribution.

Also, characteristic function is as good as moment generating function in finding out the moments of a distribution. r^{th} moment about the origin is the coefficient of $\frac{(it)^r}{r!}$ in the expansion of $E(e^{itx})$.

Properties

If X and Y are two independent variables and a, b any two real constants, then

$$M_{aX+bY}(t)=M_{aX}(t)\cdot M_{bY}(t)$$

when $a = b = 1$, $M_{X+Y}(t) = M_X(t) \cdot M_Y(t)$

Similarly, $\phi_{aX+bY}(t) = \phi_{aX}(t) \cdot \phi_{bY}(t)$

if $\quad a = b = 1, \phi_{X+Y}(t) = \phi_X(t) \cdot \phi_Y(t)$

This is known as the additive or reproductive property.

Q. 15 Describe in brief probability generating function (p.g.f.).

Ans. If X is non-negative integer-valued variate, the probability generating function of X is,

$$G_X(t) = E\left(t^x\right) = \sum_{\text{all } x} P(X = x) t^x_{0 \le t \le 1}$$

$$= \sum_{\text{all } x} p(x) t^x$$

$G_X(t)$ is known as p.g.f. since the coefficient of t^x in $E(t^x)$ is $p(x)$.

Probability generating function is closely related to moment generating function as by replacing t in p.g.f. by e^t, we obtain m.g.f. For example, the m.g.f. of binomial distribution is $(q + pe^t)^n$ and p.g.f. is $(q + pt)^n$. Similarly in case of Poisson distribution,

$M_X(t) = e^{\mu\left(e^t - 1\right)}$ whereas $G_X(t) = e^{\mu(t-1)}$.

Probability generating function also holds the *additive property*. If X and Y are two independent variates, the p.g.f. of the sum of variates is equal to the product of p.g.f's of individual variates, *i.e.,*

$$G_{X+Y}(t) = G_X(t) \times G_Y(t).$$

If $X_1, X_2, ..., X_n$ are i.i.d, then

$$G_{X_1+X_2+...X_n}(t) = \left[G_X(t)\right]^n$$

If X is an integer valued variable, p.g.f. can be used to find the moments of the distribution.

Q. 16 Describe in brief the uniform distribution.

Ans. A discrete variable X is said to follow *discrete uniform distribution* if its probability function is,

$$P(X = x) = p(x)$$

$$= \frac{1}{n} \quad \text{for } x = 1, 2, ..., n$$

$$= 0 \text{ otherwise}$$

For example, the outcomes of rolling a fair die or drawing cards successively from a well shuffled deck of cards follow uniform distribution. In this distribution, probability at every point remains the same.

Q. 17 Give Bernoulli distribution and its properties.

Ans. A random variable X, marked by only two values 1 and 0 or 1 and -1 with probability of occurrence p and q ($q = 1 - p$) respectively where $P(x = 1) = p$ and $P(x = 0) = q$, is called a Bernoulli variate and its distribution is called *Bernoulli distribution*. The probability distribution of x is,

$$f(x) = p^x q^{1-x}$$

where $0 \le p \le 1$ and $x = 0$ or 1.

The only parameter of Bernoulli distribution is p.

Properties

(i) Mean of Bernoulli distribution is p.

(ii) Variance of Bernoulli distribution is pq.

(iii) Moment generating function of Bernoulli distribution is $(q + pe^t)$.

(iv) An experiment with two possible outcomes classified as success A and failure $\overline{A}$ is called a *Bernoulli trial* if the $P(A)$ remains the same at repeated trials. For instance in tossing a fair coin again and again, the probability of falling the coin with head upside remains $\frac{1}{2}$.

Hence it is a Bernoulli trial.

Q. 18 Delineate Binomial distribution and its important features.

Ans. Binomial distribution was invented by James Bernoulli which was posthumously published in 1713. Let n (finite) Bernoulli trials be conducted with probability 'p' of a success and 'q' of a failure. The probability of x successes out of n Bernoulli trials is given by

$$f(x) = \binom{n}{x} p^x q^{n-x}$$

where $x = 0, 1, 2, ..., n$

$$0 \le p \le 1 \text{ and } p + q = 1$$

A dichotomous r.v., X having the probability mass function $f(x)=\binom{n}{x}p^x q^{n-x}$ is said to follow *Binomial distribution* and is usually denoted as $b\ (n, p)$ or $b\ (x;\ n, p)$. The probability function $\binom{n}{x}p^x q^{n-x}$ is the $(x+1)^{\text{th}}$ term in the binomial expansion of $(q+p)^n$.

Important features

(i) If $n = 1$, the binomial distribution reduces to Bernoulli distribution. Hence, seldom Bernoulli distribution is known as *point binomial.*

(ii) Binomial distribution has two parameters n and p.

(iii) Mean of binomial distribution is np.

(iv) Variance of binomial distribution is npq.

(v) The moment generating function of binomial distribution is $(q + pe^t)^n$.

(vi) Characteristic function of $b\ (n, p)$ is

$$\left(q+pe^{it}\right)^n.$$

(vii) The first four moments with their relation to cumulants are:

$$\mu=\kappa_1=np,\ \mu_2=\kappa_2=npq,$$

$$\mu_3=\kappa_3=npq(q-p)$$

$$\mu_4=\kappa_4+3\kappa_2^2=npq\left[1+3pq(n-2)\right].$$

(viii) The measure of skewness, $\beta_1=\dfrac{(1-2p)^2}{npq}$ or $\gamma_1=\dfrac{1-2p}{\sqrt{npq}}$. The value of β_1 indicates that binomial distribution is positively skewed if $p<\dfrac{1}{2}$ and symmetrical if $p=\dfrac{1}{2}$.

(ix) $\beta_2=3+\dfrac{1-6pq}{npq}$ or $\gamma_2=\dfrac{1-6pq}{npq}$. The value of measure of kurtosis 'β_2' reveals that as the number of trials $n \to \infty$, the distribution tends to mesokurtic.

(x) Recurrence formula for binomial distribution is,

$$p(x=x+1)=p(x=x)\frac{n-x}{x+1}\cdot\frac{p}{q}$$

This formula is very helpful in calculating the probability of the consecutive successes.

(xi) If $X \sim b\ (n_1, p)$ and $Y \sim b\ (n_2, p)$, then $X+Y \sim b\,(n_1+n_2, p)$. This property is known as *additive* or *reproductive* property of binomial distribution.

(xii) If $Y \sim b\ (y;\ n, p)$, the variable $X = Y/n$, the ratio of number of successes to the total number of trials n, is called a *relative* or *Pseudo* binomial variate.

Q. 19 In five tossings of a fair coin find the chance of getting 3 heads.

Ans. $P\ (3 \text{ heads}) = \binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2=\frac{5}{16}.$

Since $p=\frac{1}{2},\ q=\frac{1}{2}.$

Q. 20 A machine produces 10 per cent defective items. Ten items are selected at random. Find the probability of not more than two items being defective.

Ans. $$P(X\le 2)=\sum_{x=0}^{2}\binom{10}{x}\left(\frac{1}{10}\right)^x\left(\frac{9}{10}\right)^{10-x}$$

$$=\binom{10}{0}\left(\frac{1}{10}\right)^0\left(\frac{9}{10}\right)^{10-0}+\binom{10}{1}\left(\frac{1}{10}\right)^1\left(\frac{9}{10}\right)^{10-1}$$

$$+\binom{10}{2}\left(\frac{1}{10}\right)^2\left(\frac{9}{10}\right)^{10-2}=\left(\frac{24}{9}\right)\left(\frac{9}{10}\right)^{10}$$

Q. 21 The chances of a bomber hitting the target and missing the target are 3 : 2. Calculate the probability that the target will be hit at least once in five sorties.

Ans. $$P(X\ge 1)=\sum_{x=1}^{5}\binom{5}{x}\left(\frac{3}{5}\right)^x\left(\frac{2}{5}\right)^{5-x}$$

or $$P(X \geq 1) = 1 - P(x = 0)$$

$$= 1 - \left(\frac{2}{5}\right)^5 = \frac{3093}{3125}.$$

Q. 22 Explicate Poisson distribution and its properties.

Ans. Poisson distribution was discovered by a French Mathematician-cum-Physicist, Simeon Denis Poisson in 1937. He derived it as a limiting case of binomial distribution. If a dichotomous variable X is such that the constant probability p of success for each trial is very small and the number of trials n is indefinitely large and $np = \mu$ is finite, the probability of x successes is given by the probability mass function,

$$P(X = x) = \frac{e^{-\mu}\mu^x}{x!} \text{ for } x = 0, 1, 2, \ldots$$
$$= 0 \quad \text{otherwise}$$

It is denoted by $P(x; \mu)$.

Some of the example of Poisson variate are:

(a) No. of deaths in a city due to suicides.
(b) No. of defective items in a box of 100 items.
(c) No. of plane accidents per week.

Main properties of Poisson distribution are:

(i) Poisson distribution has only one parameter 'μ'.

(ii) Mean of Poisson distribution (variate) is μ.

(iii) Variance of Poisson distribution (variate) is also μ. This is the only distribution of which the mean and variance are equal.

(iv) Moment generating function of Poisson distribution is $e^{\mu(e^t - 1)}$.

(v) Characteristic function of Poisson distribution is $e^{\mu(e^{it} - 1)}$.

(vi) First three moments of Poisson distribution (variate) are equal, *i.e.*, $\mu_1 = \mu_2 = \mu_3 = \mu$ and $\mu_4 = \mu + 3\mu^2$.

(vii) All the cumulants are equal, *i.e.*, $\kappa_1 = \kappa_2 = \kappa_3 = \kappa_4 = \mu$.

(viii) The measure of skewness, $\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{\mu^2}{\mu^3} = \frac{1}{\mu}$ or $\gamma_1 = \frac{1}{\sqrt{\mu}}$.

(ix) Measure of kurtosis, $\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{1}{\mu} + 3$ or $\gamma_2 = \frac{1}{\sqrt{\mu}}$. These measures show that Poisson distribution is positively skewed and leptokurtic.

(x) For a Poisson variate X with mean μ, the recurrence relationship between $P(X = x)$ and $P(X = x + 1)$ is,

$$P(X = x + 1) = P(X = x) \cdot \frac{\mu}{x + 1}.$$

(xi) If n is large ($n \to \infty$) and p small in a binomial distribution such that np remains constant say, μ, the binomial distribution tends to Poisson distribution $\frac{e^{-\mu}\mu^x}{x!}$.

(xii) Independent Poisson variates possess additive (reproductive) property. This property ensures that the sum of independent Poisson variates is also a Poisson variate. If $X_1, X_2, \ldots, X_n$ are $P(x_1; \mu_1)$, $P(x_2; \mu_2)$, ..., $P(x_n; \mu_n)$, respectively, then $(X_1 + X_2 + \ldots + X_n)$ is $P(\Sigma x_i; \mu_1 + \mu_2 + \ldots + \mu_n)$.

(xiii) If X and Y are two Poisson variates with means μ and λ respectively, the conditional distribution of X given $(X + Y)$ is binomial.

(xiv) If X_1 and X_2 are two independent Poisson variates distributed as $P(x_1; \mu_1)$ and $P(x_2; \mu_2)$ respectively, $(X_1 - X_2)$ is not a Poisson variate. This property of Poisson variates is known as the *subtractive* property.

Q. 23 A gear manufacturing company expects that the chance of a gear being defective is $\frac{1}{200}$. The gears are supplied in boxes of 10 gears. Find the probability that there are two defective gears in a box of 10 gears. Also calculate the number of boxes containing two defective pieces out of 10,000 boxes.

Ans. Since the probability of a gear being defective (X) is small, the variate follows Poisson distribution.

The mean, $\mu = 10 \times \frac{1}{200} = 0.05$

$$P(x=2) = \frac{e^{-0.05}(0.05)^2}{2!}$$

From the table $e^{-0.05} = 0.9512$

$$P(x=2) = \frac{0.9512 \times 0.0025}{2}$$
$$= 0.0012$$

No. of boxes having two defective pieces out of 10,000 boxes is,

$$n = 0.0012 \times 10{,}000 = 12.$$

Q. 24 It has been found that on an average the number of mistakes per typed page of a typist is 1.5. Find the probability that there are 3 or less mistakes.

Ans. Given that $\mu = 1.5$

The variable (no. of mistakes) follow Poisson distribution.

Thus,

$$P(X \le 3) = e^{-1.5}\left\{\frac{(1.5)^0}{0!} + \frac{(1.5)^1}{1!} + \frac{(1.5)^2}{2!} + \frac{(1.5)^3}{3!}\right\}$$

From the table $e^{-1.5} = 0.2231$

$P(X \le 3) = 0.2231\ (1 + 1.5 + 1.125 + 0.5625)$

$= 0.9342$

Q. 25 If a Poisson variate X is such that $P(x = 1) = P(x = 2)$, Work out $P(x = 4)$.

Ans. Using the recurrence formula for Poisson distribution.

$$P(x=2) = P(x=1)\frac{\mu}{1+1}$$

or $\mu = 2$

$$\therefore \quad P(x=4) = \frac{e^{-2}(2)^4}{4!}$$

From the table $e^{-2} = 0.1353$

$$P(x=4) = \frac{0.1353 \times 16}{24}$$
$$= 0.09$$

Q. 26 Explain Negative binomial distribution and its properties.

Ans. In sampling procedure, seldom the sample size is not fixed but is determined as the sample size required to achieve r successes. (This is known as inverse sampling.) Obviously n^{th} individual will always be possessing $(r - 1)$ successes. A random variable X, the number of failures before the r^{th} success occurs in a random experiment which results either in a success or a failure is said to follow a negative binomial distribution and its probability function with probability p of a success and q that of a failure is given by

$$p_X\{nb(x;\, r,\, p)\} = \binom{x+r-1}{r-1} p^r q^x$$

for $\quad x = 0, 1, 2, \ldots,$

$\quad r \ge 0;\ 0 \le p < 1$

Also, $\quad p_X\{nb(x;\, r,\, p)\} = \binom{-r}{x} p^r q^x$

$$\because \quad \binom{x+r-1}{r-1} = \binom{-r}{x}(-1)^x$$

Negative binomial distribution is also known as *Pascal's distribution.*

Properties of negative binomial distribution

(i) Number of successes is fixed and number of trials is a random variable.

(ii) Its means is $r\,(1 - p)/p$.

(iii) Its variance is $r\,(1 - p)/p^2$.

(iv) The frequency curve is J-shaped with maximum frequency at $n = 1 + \frac{r-1}{p}$.

(v) Moment generating function of $nb\,(x;\, r,\, p)$ is $(Q - Pe^t)^{-r}$ where $p = \frac{1}{Q}, q = \frac{P}{Q}$ or $Q - P = 1$.

(vi) Recurrence relationship between negative binomial probabilities $f(x + 1)$ and $f(x)$ is

$$f(x+1) = q \cdot \frac{x+r}{x+1} f(x)$$

(vii) Central moments and cumulants of *nb* (*x; r, p*) are:

$$\mu_1 = \kappa_1 = rP, \mu_2 = \kappa_2 = rPQ,$$

$$\mu_3 = rPQ(Q+P)$$

and $$\mu_4 = \kappa_4 + 3\kappa_2^2 = rPQ(1+6PQ) + 3r^2P^2Q^2$$

$$= rPQ(1+6PQ+3r\,PQ)$$

(viii) Pearson's coefficients of skewness are:

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(Q+P)^2}{rPQ} \text{ or } \gamma_1 = \frac{Q+P}{\sqrt{rPQ}}$$

β_1 or γ_1 are always positive hence negative binomial distribution is positively skewed.

(ix) Pearson's coefficient of kurtosis are:

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{1+6PQ}{rPQ} + 3$$

or $$\gamma_2 = \frac{1+6PQ}{rPQ}$$

The value of $\beta_2 > 3$ or $\gamma_2 > 0$, reveals that negative binomial distribution is leptokurtic.

Q. 27 A marker is to continue shooting at the target until he hits the target 6 times. The probability that he hits the target on any shooting is 0.4. Calculate the probability that the marker will have to shoot 9 times.

Ans. Since the number of successes is fixed, we will use negative binomial distribution. In the given problem,

$$r + x = 9, r = 6, \therefore x = 3$$

Also $$p = 0.4, q = 1 - 0.4 = 0.6$$

Thus, using negative binomial probability,

$$P(x=3) = \binom{x+r-1}{r-1} p^r q^x$$

$$= \binom{9-1}{6-1} (0.4)^6 (0.6)^3$$

$$= \binom{8}{5} (0.4)^6 (0.6)^3$$

$$= 0.0495$$

Q. 28 Elucidate Geometric distribution and give its properties.

Ans. If there are a number of trials such that the probability of success '*p*' at each trial remains the same, the probability that there are *x* failures before the first success is given by $p.q^x$. Hence, a random variable *X* which can take only positive integer values is said to follow geometric distribution if its probability mass function is

$$f(x) = P(X = x) = p \,.\, q^x$$

for $x = 0, 1, 2, \ldots$ and $0 \le p \le 1$

$= 0$ otherwise.

It is usually denoted as geom (*p*).

Since the probabilities of the events $x = 0, 1, 2, \ldots$, etc., are in geometric progression, it is named as geometric distribution.

Also it is a particular case of negative binomial distribution for $r = 1$.

The waiting time *X* follows geometric distribution.

Properties of geometric distribution:

(i) The mean of geometric distribution is q/p.

(ii) The variance of geometric distribution is q/p^2.

(iii) Moment generating function of geometric distribution is $p/(1 - qe^t)$.

(iv) $x = 0$ is the mode of geometric distribution.

(v) Median of geometric variate is $-$ log 2/ log $(1 - p)$.

(vi) Recurrence formula for geometric distribution is $p\,(x + i) = qp(x)$.

(vii) If an event '*E*' has not occurred before the time *k*, then $Y = X - k$ is the additional time required for *E* to occur. The distribution of $Y = t$ for $X \ge k$ is pq^t which is independent of *k*. Since the distribution of *Y* is independent of *k*, the waiting time *k* is forgotten. This property of geometric distribution is known as the *lack of memory or memoryless* property of geometric distribution. This shows that shifting of origin does not affect the geometric distribution.

Q. 29 A man rolls a fair die again and again until he obtains a 5 or 6. Calculate the probability that he will require 5 throws.

Ans. Given that $p=\frac{1}{3}, q=\frac{2}{3}$.

The number of failures before a success = 4.

The variable x follows geom (1/3) distribution. Hence,

$$P(x=5 \text{ or } 6)=\frac{1}{3}\left(\frac{2}{3}\right)^4=\frac{16}{243}$$

Q. 30 Give Polya's distribution.

Ans. If we put in negative binomial probability function the value of r and p as,

$$r=\frac{1}{\beta}, p=\frac{1}{1+\beta\mu}, q=\frac{\beta\mu}{1+\beta\mu}$$

$$f(x)=P(X=x)=\binom{-r}{x}\left(\frac{1}{1+\beta\mu}\right)^{1/\beta}\left(\frac{\beta\mu}{1+\beta\mu}\right)^x$$

$$\text{or } f(x)=\frac{(1+\beta)(1+2\beta)+\ldots+\{1+\beta(x-1)\}}{x!}$$

$$\times\left(\frac{1}{1+\beta\mu}\right)^{1/\beta}\left(\frac{\beta\mu}{1+\beta\mu}\right)^x$$

for $x = 0, 1, 2, \ldots$

This form of Polya's distribution has only two parameters β and μ.

If in Polya's probability function, we put $\beta = 1$, $f(x)$ reduces to,

$$f(x)=\left(\frac{1}{1+\mu}\right)\left(\frac{\mu}{1+\mu}\right)^x$$

since $\frac{2, 3, \ldots, x}{x!}=1$

which is another form of geometric distribution with $p=\frac{1}{1+\mu}$.

Q. 31 Explain Hypergeometric distribution and give its characteristics.

Ans. Suppose N individuals of a population can be categorised either a success (S) and a failure (F) and it contains k successes and obviously $(N - k)$ failures. Let a sample of n elements be drawn at random without replacement. In this process successive trials (draws) are dependent. Suppose x is the number of successes in the sample. The variable X follows hypergeometric distribution and its probability mass function is given as,

$$f(x)=p(X=x)=\frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$$\text{For} \quad p=\frac{k}{N}, f(x)=\frac{\binom{Np}{x}\binom{Nq}{n-x}}{\binom{N}{n}}$$

for $x = 0, 1, 2, \ldots, n$
and $n \le k$

It has three parameters, N, K, n, and is usually denoted as H.G. (x; N, K, n).

Characteristics of hypergeometric distribution:

(i) The mean of H.G. (x; N, K, n) is np where $p=\frac{K}{N}\cdot N$ is not necessarily large.

(ii) The variance of hypergeometric distribution is $\left(\frac{N-n}{N-1}\right)npq$ where $q = 1 - p$

or $\text{var}(X)=\frac{N-n}{N-1}\cdot\frac{nk}{N}\frac{N-k}{N}$.

(iii) Recurrence formula between $f(x + 1)$ and $f(x)$ for H.G. (x; N, K, n) is,

$$f(x+1)=\frac{n-x}{x+1}\frac{k-x}{N-k-n+x+1}f(x)$$

(iv) Hypergeometric distribution tends to binomial distribution if $N \to \infty$ and $\frac{K}{N}\to p$.

(v) Characteristic function of hypergeometric distribution does not yield a convenient form to be worked out for moments. The hypergeometric distribution provides an example of a distribution for which the characteristic function is virtually useless while obtaining the moments of the distribution. However, the method of moments may be fruitful.

Q. 32 Give the example of a distribution for which characteristic function is useless for obtaining the moments.

Ans. The hypergeometric distribution provides an example of a distribution for which characteristic function is virtually useless while obtaining the moments of the distribution. However, the method of factorial moments may be useful.

Q. 33 In an international film festival, a panel of 11 judges was formed to judge the best film. At last two films F_A and F_B were considered to be the best where the opinion of judges got divided. Six judges were in favour of F_A whereas five in favour of F_B. A random sample of five judges was drawn from the panel. Find the probability that out of five judges, three were in favour of film F_A.

Ans. It can be calculated with help of hypergeometric probability in the following manner:

In the given problem,

$N = 11, K = 6, N - K = 5, n = 5, x = 3$

$$P(x=3) = \frac{\binom{6}{3}\binom{5}{2}}{\binom{11}{5}}$$

$$= \frac{100}{231}$$

Q. 34 Describe briefly the multinomial distribution.

Ans. Suppose there are k distinct classes. Let the probability that X_i observations fall in the i^{th} class is p_i, the probability,

$$P(X_1 = x_1, X_2 = x_2, \ldots, X_K = x_K / \Sigma x_i = n)$$

for $i = 1, 2, \ldots, k$.

is given by

$$f(x_1, x_2, \ldots, x_k) = \frac{n!}{x_1!x_2!\ldots x_k!} p_1^{x_1} p_2^{x_2} \ldots p_k^{x_k}$$

Subject to the condition, $\Sigma p_i = 1$.

Then $f(x_1, x_2, \ldots, x_k)$ is the probability function of multinomial distribution. It has parameters n and $p_1, p_2, \ldots, p_k$.

This distribution reduces to binomial distribution when there are only two classes.

Q. 35 What is meant by continuous distribution?

Ans. We know that a continuous variable (variate) will have continuous distribution. A variate X which can take any value in an interval (a, b), *i.e.*, $a \le X \le b$ of arithmetic continuum is a continuous variable. The probability density function $f(x)$ of the random variable X in the interval (a, b) is defined as,

$$f(x) = \begin{bmatrix} 0 & \text{if} & x < a \\ \phi(x) & \text{if} & a \le x \le b \\ 0 & \text{if} & x > b \end{bmatrix}$$

Also $f(x)$ always possesses the following *properties*:

(i) $f(x) \ge 0$.

(ii) The total density (probability) in its entire range is equal to unity, *i.e.*,

$$\int_a^b f(x)dx = 1$$

It implies that the total area under the frequency curve is always unity.

(iii) The distribution function of X is given as,

$$P(X \le x) = F(x) = \int_{-\infty}^{x} f(x)dx.$$

(iv) $\int_a^b f(x)dx = F(b) - F(a)$ for $a \le x \le b$

(v) $F(-\infty) = 0$ and $F(+\infty) = 1$

(vi) For a continuous distribution with p.d.f. $f(x)$ where $a \le X < b$, the following relations hold.

(a) The mean, $\mu = E(X) = \int_a^b x\, f(x)dx$

(b) The median $Md = \int_a^{Md} f(x) d_x$

$$= \int_{Md}^{b} f(x)dx = 1/2$$

(c) The variance, $\mu_2 = E(X^2) - \{E(X)\}^2$

where $E(X^2) = \int_a^b x^2 f(x)dx$

(d) The geometric mean G,

$$\log G = \int_a^b \log x f(x)dx$$

(e) The Harmonic mean H,

$$\frac{1}{H} = \int_a^b \frac{1}{x} f(x)dx$$

(f) The mode is the solution of $f'(x) = 0$ and $f''(x) < 0$ provided the derivatives, $f'(x)$ and $f''(x)$ exist

(g) The point of inflection of a continuous frequency curve $y = f(x)$ is obtained by putting $f''(x) = 0$ and verifying $f'''(x) \neq 0$, provided second and third derivatives exist.

(h) The mean deviation about a constant A,

$$M.D_A = \int_a^b |x - A| f(x)dx$$

(i) The moment generating function,

$$M_X(t) = \int_a^b e^{tx} f(x)dx$$

(j) The cumulant generating function,

$\kappa_X(t) = \log M_X(t)$

(k) i^{th} quartile Q_i $(i = 1, 2, 3)$ is obtained by the relation,

$$\int_a^{Q_i} f(x)dx = \frac{i}{4}$$

(l) i^{th} decile D_i $(i = 1, 2, ..., 9)$ is obtained by the relation,

$$\int_a^{D_i} f(x)dx = \frac{i}{10}$$

(m) i^{th} percentile P_i $(i = 1, 2, ..., 99)$ is obtained by the relation

$$\int_a^{P_i} f(x)dx = \frac{i}{100}$$

(n) r^{th} raw moment,

$$\mu_r' = \int_a^b x^r f(x)dx$$

(o) r^{th} central moment,

$$\mu_r = \int_a^b (x - \mu)^r f(x)dx$$

Q. 36 Discuss a continuous uniform or Rectangular distribution and its properties.

Ans. A random variable X is said to follow continuous uniform or rectangular distribution in an interval (a, b) if its density function is constant over the entire range of the variable X. Its functional form is,

$$f(x) = \begin{bmatrix} \frac{1}{b-a} & \text{if} \quad a \leq x \leq b \\ 0 & \text{otherwise} \end{bmatrix}$$

It is denoted as $U(a, b)$. If $X \sim U(0, 1)$, $f(x) = 1$. The distribution function of the variable $X \sim U(a, b)$ is,

$$F(x) = \begin{bmatrix} 0 & \text{if} \quad -\infty < x < a \\ \frac{x-a}{b-a} & \text{if} \quad a \leq x \leq b \\ 1 & \text{if} \quad b < x < \infty \end{bmatrix}$$

Properties of rectangular distribution:

(i) Its mean is equal to $\frac{b+a}{2}$.

(ii) All the moments of odd order are zero.

(iii) Its variance, $\mu_2 = \frac{(b-a)^2}{12}$.

(iv) Its fourth central moment, $\mu_4 = \frac{(b-a)^4}{80}$.

(v) Median of rectangular distribution is $\frac{b+a}{2}$.

(vi) Mean deviation about mean, $\text{M.D}_{\bar{x}} = \frac{b-a}{4}$.

(vii) Measure of Skewness, $\beta_1 = 0$ or $\gamma_1 = 0$.

(viii) Measure of Kurtosis, $\beta_2 = \frac{9}{5}$ or $\gamma_2 = \beta_2 - 3 = -\frac{6}{5}$. Pearson's coefficients β_1 and β_2 reveal that rectangular distribution is symmetrical and platykurtic.

(ix) Moment generating function of rectangular distribution, $M_X(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}$.

(x) Characteristic function of rectangular distribution, $\phi_X(t) = \frac{e^{itb} - e^{ita}}{it(b-a)}$.

(xi) Mode does not exists as the probability at each point in the interval (a, b) remains the same.

(xii) If X and Y are independently and identically distributed (i.i.d) rectangular or uniform U (0, 1), the distribution of the variates $(x + y)$, $(x - y)$, xy and x/y are as follows:

$$f(x+y) = \begin{cases} x+y, & 0 \le x+y \le 1 \\ 2-(x+y), & 1 \le x+y \le 2 \end{cases}$$

$$f(x-y) = \begin{cases} x-y+1, & -1 \le x-y \le 0 \\ 1-(x-y), & 0 \le x-y \le 1 \end{cases}$$

$$f(xy) = -\log(xy),\ 0 < xy < 1$$

$$f(x/y) = \begin{cases} 1/2, & 0 \le x/y < 1 \\ y^2/2x^2, & 1 < x/y < \infty \end{cases}$$

(xiii) If a variable $X \sim U$ (0, 1), then the variable $y = -2 \log X$ is distributed as χ^2 with 2 d.f.

(xiv) Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with distribution U (0, 1), then the variable

$$y = -2 \sum_{i=1}^{n} \log x_i \quad \text{or} \quad y = 2\log\left(1/\prod_{i=1}^{n} X_i\right) \text{ is}$$

distributed as χ^2 with $2n$ d.f.

Q. 37 When is a variable said to follow exponential distribution? What are the properties of exponential distribution?

Ans. A continuous random variable X is said to follow exponential distribution if for any positive value λ, it has the probability density function,

$$f(x) = \lambda\, e^{-\lambda x} \text{ for } \lambda > 0, x > 0$$

This is usually denoted as Expo (λ). λ is known as the parameter of exponential distribution.

The distribution function of exponential variate X is

$$F(x) = \begin{cases} 1-e^{-\lambda x}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$$

The length of time interval between successive occurrences of events follow exponential distribution provided the number of occurrences in fixed time interval follow Poisson distribution. The variate, elapsed time between two calls at a telephone switch board follows exponential distribution. This plays an important role in the theory of reliability.

The well known properties of exponential distribution are:

(i) Mean of exponential distribution is $1/\lambda$.

(ii) Variance of exponential distribution is $1/\lambda^2$.

(iii) Moments of all order exist. The first four central moments are:

$$\mu_1 = \frac{1}{\lambda},\ \mu_2 = \frac{1}{\lambda^2},\ \mu_3 = \frac{2}{\lambda^3},\ \mu_4 = \frac{9}{\lambda^4}$$

If $\lambda > 1$, mean > variance. If $\lambda < 1$, mean < variance, if $\lambda = 1$, mean = variance.

(iv) The relationship between central moments and cumulants are:

$$\mu_1 = \kappa_1, \mu_2 = \kappa_2, \mu_3 = \kappa_3 \text{ and } \mu_4 = \kappa_1 + \kappa_2^2$$

(v) Pearson's measure of skewness, $\beta_1 = 4$ or $\gamma_1 = \sqrt{\beta_1} = 2$

(vi) Pearson's measure of Kurtosis, $\beta_2 = 9$ or $\gamma_2 = \beta_2 - 3 = 6$.
The values of β_1 or γ_1 and β_2 or γ_2 clearly reveal that exponential distribution is positively skewed and is leptokurtic.

(vii) The median of exponential distribution is $\frac{1}{\lambda}$.

(viii) Moment generating function,

$$M_X(t) = \left(1 - \frac{t}{\lambda}\right)^{-1}$$

(ix) Characteristic function, $\phi_X(t) = \left(1 - \frac{it}{\lambda}\right)^{-1}$

(x) It also possesses the memoryless property just like geometric distribution.

Q. 38 What is the probability density function of Laplace or double exponential distribution? Also give its properties.

Ans. A random variable X is said to follow Laplace or double exponential distribution with parameters λ and μ if its probability density function,

$$f(x) = \frac{1}{2}\lambda e^{-\lambda|x-\mu|}$$

for $-\infty < x < \infty$, $\lambda > 0$ and $\mu < \infty$.

It is denoted as $L(\mu, \lambda)$.

If $\mu = 0$,

$$f(x) = \frac{1}{2}\lambda e^{-\lambda|x|}$$

for $-\infty < x < \infty$, $\lambda > 0$.

and is denoted as $L(\lambda)$.

The variate standing for the longest lasting object out of a large number of apparently identical objects follows double exponential distribution.

Properties of double exponential distribution

(i) The mean of double exponential distribution is μ.

(ii) The variance of double exponential distribution is $\frac{2}{\lambda^2}$.

(iii) First four central moments of double exponential distribution and their relations with cumulants are:

$$\mu_1 = \kappa_1 = \mu, \mu_2 = \kappa_2 = \frac{2}{\lambda^2}, \mu_3 = \kappa_3 = 0$$

$$\mu_4 = \kappa_4 + 3\kappa_2^2 = \frac{24}{\lambda^4}.$$

(iv) Measure of skewness, $\beta_1 = 0$ or $\gamma_1 = \sqrt{\beta_1} = 0$.

(v) Measure of skewness, $\beta_2 = 6$ or $\gamma_2 = \beta_2 - 3 = 3$. The values of β_1 and β_2 *vis-a-vis* γ_1 and γ_2 ensure that double exponential distribution is symmetrical but is leptokurtic.

(vi) Moment generating function of double exponential distribution is $e^{\mu t} \cdot \frac{\lambda^2}{\lambda^2 - t^2}$.

(vii) Its characteristic function is $e^{i\mu t} \cdot \frac{\lambda^2}{\lambda^2 + t^2}$.

(viii) Interquartile range $(Q_3 - Q_1)$ of double exponential distribution is $\frac{2}{\lambda}$ log 2.

Q. 39 Discuss elaborately Normal distribution and its characteristics.

Ans. Normal distribution was first discovered by De-Moivre in 1733 and was also known to Laplace in 1774. Later it was derived by Kark Friedrich Gauss in 1809 and used it for the study of errors in astronomy. Anyhow, the credit of normal distribution has been given to Gauss and is often called Gaussion distribution. Normal distribution is the maximally used probability distribution in the theory of statistics.

A random variable X is said to follow a normal distribution with mean μ and variance σ^2, if its probability density function is

$$f_X\left(x;\mu,\sigma^2\right) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$$

for $\quad -\infty < x < \infty$

$-\infty < \mu < \infty$ and $\sigma > 0$

The variate X is said to be distributed normally with mean μ and variance σ^2 and is denoted as $X \sim N(\mu, \sigma^2)$.

If $\mu = 0$ and $\sigma = 1$, then

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

Here, X is said *standardised normal variate* and is denoted as $X \sim N(0, 1)$. Also, p.d.f. $f_X(x)$ is called standardised normal distribution. If $X \sim N(\mu, \sigma^2)$ and we make a transformation, $Z = \dfrac{X - \mu}{\sigma}$, the distribution of Z is a standardised normal distribution as $Z \sim N(0, 1)$ irrespective of the values of μ and σ^2 in the distribution of X. Z is also called *standard normal deviate*. We can write, $f(z) = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$.

Also, $\int_{-\infty}^{\infty} f(z)dz = 1$. The area under the standard normal curve between the ordinates at $z = 0$ and $z_1 = \phi(z)$ is given as,

$$\phi(z) = \int_0^{z_1} f(z)\,dz.$$

Also $\qquad \phi(z) = \phi(-z)$

Area under the normal curve have been tabulated and can be obtained from appendix Table IV provided in the book entitled, *Basic Statistics* by B.L. Agarwal or any other textbook.

Characteristics of the normal distribution:

(i) The normal distribution curve is bell-shaped and is symmetrical about the line $x = \mu$.

(ii) The mode of the normal curve lies at the point $x = \mu$.

(iii) The area under the normal curve within its range $-\infty$ to ∞ is always unity, *i.e.*,

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx = 1$$

(iv) On either side of the line $x = \mu$, the frequency decreases more rapidly within the range $(\mu \pm \sigma)$ and get slower and slower as we depart farther from the point μ on either side. The area under the normal curve beyond the distance $\pm 3\,\sigma$ is only 0.27 per cent which is very small. As a matter of fact, the area under the normal curve within the range $\mu \pm \sigma$ is 0.6826, in the range $\mu \pm 2\,\sigma$ is 0.9544 and within the range $\mu \pm 3\,\sigma$ is 0.9973. The fact that 99.73 per cent items are covered within the range $\mu \pm 3\,\sigma$ of a normal distribution. It gives rise to the large number theory. Hence, for a sample of size $n \geq 30$, the distribution is taken as normal. The area under standard normal curve, bounded by the line $Z = \pm 1.96$ is 0.95, *i.e.*, 95 per cent. Since the normal curve is symmetrical, 2.5 per cent area lies on the left tail and 2.5 per cent area lies on the left tail and 2.5 per cent on the right tail beyond the points $z = -1.96$ and $z = 1.96$, respectively.

(v) Mean = Median = Mode.

(vi) The normal curve is unimodal.

(vii) All odd order moments of the normal distribution are zero.

(viii) The first raw moment, *i.e.*, mean = μ. Also

$$\mu_2 = \sigma^2,\ \mu_3 = 0,\ \mu_4 = 3\sigma^4 = 3\mu_2^2.$$

(ix) Measure of skewness, $\beta_1 = \dfrac{\mu_3^2}{\mu_2^2} = 0$ or γ_1

$= \sqrt{\beta_1} = 0$.

(x) Measure of Kurtosis, $\beta_2 = \dfrac{\mu_4}{\mu_2^2} = 3$ or $\gamma_2 =$

$\beta_2 - 3 = 0$.

The value of β_1 and β_2 clearly reveal that the normal curve is symmetrical and mesokurtic.

(xi) Quartile deviation, $\text{Q.D.} = \dfrac{Q_3 - Q_1}{2} = \dfrac{2}{3}\sigma$.

(xii) The mean deviation about mean, $\text{M.D.}_\mu =$

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} |x-\mu| e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx = \sqrt{\frac{2}{\pi}}\sigma = \frac{4}{5}\sigma.$$

(xiii) The maximum probability say, Max $[p(x)]$ of the normal curve occurs at the point $x = \mu$ whereas,

$$\text{Max}\left[p(x)\right] = \frac{1}{\sigma\sqrt{2\pi}}$$

As σ increases, $p(x)$ decreases and the curve becomes more and more flat and vice-versa.

(xiv) The moment generating function of the normal distribution, $N(x;\ \mu,\ \sigma^2)$ is

$$M_X(t) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{tx} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}\,dx = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

(xv) Characteristic function of the normal distribution, $N\left(x;\mu,\sigma^2\right)$ is

$$\phi_X(t) = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{itx} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}\,dx = e^{i\mu t - \frac{1}{2}\sigma^2 t^2}$$

(xvi) The points of inflexion of the normal curve are given by $x = \mu \pm \sigma$ and at this point

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-1/2}.$$

(xvii) If $X_1, X_2, \ldots, X_n$ are n independent random variables distributed normally with mean $\mu_1, \mu_2, \ldots, \mu_n$ and variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$ respectively, the sum $(X_1 + X_2 + \ldots + X_n)$ is distributed with mean, $(\mu_1 + \mu_2 + \ldots + \mu_n)$ and variance, $\left(\sigma_1^2 + \sigma_2^2, \ldots + \sigma_n^2\right)$.

(xviii) If X and Y are two normal variates such that $X \sim N(0, 1)$ and $Y \sim N(0, 1)$, the variate X/Y follow Cauchy distribution.

Q. 40 What is the importance of standard normal curve?

Ans. The importance of standard normal curve lies in the fact that one has not to tabulate probability (areas) under the curve between two lines $x = a$ and $x = b$ for different value of μ and σ. Only one table of areas prepared for the standard normal curve is sufficient as for this curve, the mean $\mu = 0$ and S.D., $\sigma = 1$.

Q. 41 Throw light on the importance of normal distribution.

Ans. Normal distribution is most important amongst all known distributions due to the facts that:

(i) Most of the discrete distributions such as binomial, Poisson, etc., tend to normal distribution as n increases, *i.e.*, $n \to \infty$.

(ii) Almost all sampling distributions like t, χ^2, F etc., for their large degrees of freedom conform to normal distribution.

(iii) Users convenience is another reason for wide adoptability of normal distribution.

(iv) One of the greatest reason behind the extensive use and application of normal distribution lies in most celebrated theorem known as *central limit theorem.* The theorem states that.

If $X_1, X_2, \ldots, X_n$ is a random sample of size n from any population with mean μ and variance σ^2, the distribution of sample mean $\bar{x}\left(\bar{x} = \frac{\Sigma x_i}{n}\right)$ is asymptotically normal, with mean μ and variance σ^2/n, as $n \to \infty$.

(v) Many variables which are not normally distributed can be normalised through suitable transformation(s).

Q. 42 If a r.v. $X \sim N(40, 5^2)$, find the probabilities for the values of X specified as (i) $32 < X \le 50$ (ii) $|X - 40| > 5$ (iii) $X \le 25$ (iv) $X \ge 44$ (v) $45 \le X \le 50$ (vi) $31 \le X \le 35$.

Given the areas $\phi(z)$ from 0 to Z:

z	0.8	1.0	1.6	1.8	2.0	3.0
$\phi(z)$	0.28814	0.34134	0.35543	0.46407	0.47725	0.49865

Ans.

(i) $z_1 = \dfrac{32-40}{5} = -1.6,\ z_2 = \dfrac{50-40}{5} = 2.0$

$\phi(z_1) = 0.35543$, $\phi(z_2) = 0.47725$

$P(32 \le x \le 50) = 0.47725 + 0.35543$

$= 0.83268$

(ii) $P(|x - 40| > 5) = P\left(\left|\dfrac{x-40}{5}\right| > \dfrac{5}{5}\right)$

$$= P\,(\,|z| > 1)$$
$$= 2\,P\,(z > 1)$$
$$= 2\,(0.5 - 0.34134)$$
$$= 0.31732$$

(iii) $P(x \le 25) = P\left(\frac{x-40}{5} \le \frac{25-40}{5}\right)$

$$= P\,(z \le -3)$$
$$= P\,(|z| \le 3)$$
$$= 0.5 - \phi\,(|-3|)$$
$$= 0.5 - 0.49865$$
$$= 0.00135$$

(iv) $P(x \ge 44) = P\left(\frac{x-40}{5} \ge \frac{44-40}{5}\right)$

$$= P\,(z \ge 0.8)$$
$$= 0.5 - \phi\,(0.8)$$
$$= 0.5 - 0.28814$$
$$= 0.21186$$

(v) $P\,(45 \le x \le 50)$

$$= P\left(\frac{45-40}{5} \le \frac{x-40}{5} \le \frac{50-40}{5}\right)$$
$$= P\,(1 \le z \le 2)$$
$$= \phi\,(2) - \phi\,(1)$$
$$= 0.47725 - 0.34134$$
$$= 0.13591$$

(vi) $P\,(31 \le x \le 35)$

$$= P\left(\frac{31-40}{5} \le \frac{x-40}{5} \le \frac{35-40}{5}\right)$$
$$= P\,(-1.8 \le Z \le -1)$$
$$= \phi\,(-1.8) - \phi\,(-1)$$
$$= \phi\,(1.8) - \phi\,(1)$$
$$= 0.46407 - 0.34134$$
$$= 0.12273$$

Q. 43 If a variable $X \sim N\,(5, 4)$ and $P\,\{\,|x-5| > c\}$ $= 0.01$, find c.

$$\left[\text{Given} \int_0^{2.58} \frac{1}{\sqrt{2\pi}} e^{-1/2z^2}\,dz = 0.495\right]$$

Ans. $P\left\{\left|\frac{x-5}{2}\right| > \frac{1}{2}c\right\} = 0.01$

$$P\left(|z| > \frac{1}{2}c\right) = 0.01$$

$$2\,P\,(Z > 2.58) = 0.01$$

$$\therefore \qquad \frac{1}{2}c = 2.58$$

or $$c = 5.16$$

Q. 44 If $f(x) = ce^{-(x^2-6x+9)/32}$, $-\infty < x < \infty$ represents a normal distribution, find the value of c, the mean and the variance of the distribution.

Ans. We can write,

$$f(x) = c\,e^{-\frac{1}{2}\left(\frac{x-3}{4}\right)^2}$$

By comparing $f\,(x)$ with normal p.d.f.

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

we find, $\mu = 3$, $\sigma = 4$, and $c = \frac{1}{4\sqrt{2\pi}}$. Thus, $\sigma^2 = 4$.

Q. 45 Give the outline of lognormal distribution and its properties.

Ans. A positive continuous random variable X is said to follow lognormal distribution if the variable $\log_e x$ has normal distribution with mean μ and variance σ^2. Notationally, $\log_e x \sim N\ (\mu, \sigma^2)$. The probability density function of lognormal distribution is

$$f_X\left(\log_e x; \mu, \sigma^2\right) = \frac{1}{\sigma\,x\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(\log_e x-\mu)^2}$$

It has two parameters μ and σ^2.

The hourly median power of radio signals received and transmitted between two places follow log normal distribution.

If $\mu = 0$, $\sigma = 1$,

$$f_X\left(\log_e x\right) = \frac{1}{x\sqrt{2\pi}} e^{-\frac{1}{2}(\log_e x)^2}$$

In this particular case when $\log_e x \sim N(0, 1)$, the distribution is called logarithmico normal distribution.

Properties of lognormal distribution:

(i) Mean of log normal distribution is $e^{\mu+\frac{1}{2}\sigma^2}$

(ii) Its variance is $e^{2\mu+\sigma^2}\cdot e^{\sigma^2-1}$

(iii) Measure of skewness $\gamma_1 > 0$. It means that lognormal distribution curve is positively skewed.

(iv) Measure of Kurtosis $\gamma_2 > 3$. It manifests that lognormal distribution curve is leptokurtic.

(v) If X and Y are independent lognormal variates distributed as $N\left(\mu_1, \sigma_1^2\right)$ and $N\left(\mu_2, \sigma_2^2\right)$, the variate $(X + Y)$ is distributed as lognormal with mean $(\mu_1 + \mu_2)$ and variance $\left(\sigma_1^2+\sigma_2^2\right)$. The variate $(X - Y)$ is distributed as lognormal with mean $(\mu_1 - \mu_2)$ and variance $\left(\sigma_1^2+\sigma_2^2\right)$.

(vi) The distribution of the variates XY and X/Y is also lognormal.

Q. 46 Enunciate Cauchy distribution.

Ans. A continuous random variable X is said to have Cauchy distribution if its probability density function is

$$f_X(x) = \frac{1}{\pi\beta\left\{1+\left(\frac{x-\alpha}{\beta}\right)^2\right\}}$$

for $\quad -\infty < x < \infty$

$-\infty < \alpha < \infty$ and $\beta > 0$.

It has two parameters α and β. Notationally, this distribution is denoted as $C(\alpha, \beta)$. If $\alpha = 0$, $\beta = 1$, then

$$f_X(x) = \frac{1}{\pi\left(1+x^2\right)}$$

Here the variable $X \sim C(0, 1)$.

Properties of Cauchy distribution:

(i) Moment generating function of Cauchy distribution does not exist.

(ii) Its mean does not exist.

(iii) Its variance does not exist.

(iv) Characteristic function of Cauchy distribution is $e^{i\alpha t-\beta|t|}$.

(v) Cauchy distribution curve is unimodal and has its maximum at the point $x = \alpha$.

(vi) Cauchy distribution curve has two points of inflexion, $\left(\alpha+\frac{1}{\sqrt{3}}\beta\right)$ and $\left(\alpha-\frac{1}{\sqrt{3}}\beta\right)$.

(vii) If X and Y are two independent Cauchy variates distributed as $X \sim C(\alpha_1, \beta_1)$ and $Y \sim C(\alpha_2, \beta_2)$, their sum $(X + Y)$ is also a Cauchy variate distributed as $C(\alpha_1 + \alpha_2, \beta_1 + \beta_2)$.

(viii) If $X_1, X_2, \ldots, X_n$ are n i.i.d. Cauchy variates distributed as $X_i \sim C(\alpha, \beta)$ for $i = 1, 2, \ldots, n$, the mean $\bar{x} = \frac{1}{n}\Sigma x_i$ is also a Cauchy variate distributed as $C(\alpha, \beta)$.

Q. 47 Elucidate beta distribution of first kind.

Ans. A continuous random variable X with parameters m and n is said to possess beta distribution of first kind. If the probability density function of the variable X is

$$f_X(x) = \begin{cases} \frac{1}{B(m,n)}x^{m-1}(1-x)^{n-1} & m>0, n>0 \\ & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

It is denoted as $\beta_1(m, n)$.

The distribution function of beta distribution of first kind is

$$F_X(x) = \frac{1}{B(m,n)}\int_0^x x^{m-1}(1-x)^{n-1}dx$$

for $\quad m, n > 0$

$0 \le x < 1$

The cumulative distribution function $F_X(x)$ is also known as *incomplete beta function.*

Properties of beta distribution of first kind:

(i) The mean of beta distribution of first kind is $\frac{m}{(m+n)}$.

(ii) Its variance is $\frac{mn}{(m+n)^2(m+n+1)}$.

(iii) The third and fourth central moments of beta distribution of first kind are:

$$\mu_3 = \frac{2mn(n-m)}{(m+n)^3(m+n+1)(m+n+2)}$$

and

$$\mu_4 = \frac{3mn\left\{mn(m+n-6)+2(m+n)^2\right\}}{(m+n)^4(m+n+1)(m+n+2)(m+n+3)}$$

(iv) Measure of skewness,

$$\beta_1 = \frac{4(m-n)^2(m+n+1)}{mn(m+n+2)^2}$$

(v) Measure of Kurtosis,

$$\beta_2 = \frac{3(m+n+1)\left\{mn(m+n-6)+2(m+n)^2\right\}}{mn(m+n+2)(m+n+3)}$$

(vi) Mode of beta distribution of first kind depends on the values of m and n.
If $m < 1$, $x = 0$ is the modal value.
If $n < 1$, $x = 1$ is the modal value
If $m < 1$, $n < 1$, both hold good simultaneously, beta distribution of first kind is bimodal. One mode occurs at $x = 0$ and the other at $x = 1$. If $m = 1$, $n = 1$, then $f(x) = 1$ for $0 < x < 1$. In such a situation each of $x \varepsilon (0, 1)$ is mode,
If $m = 1$, $n > 1$, $x = 0$ is the mode
If $m > 1$, $n = 1$, $x = 1$ is the mode
If $m > 1, n > 1, x = \frac{m-1}{m+n-2}$ is the mode.

(vii) Characteristic function of beta distribution of first kind is $\frac{1}{B(m,n)}\sum_{j=0}^{\infty}\frac{(it)^j}{j!}B(m+j,n)$.

(viii) The harmonic mean of $\beta_I(m, n)$ is

$$H = \frac{m-1}{(m+n-1)}$$

(ix) If $X \sim \beta_I(m, n)$ and $Y \sim \beta_I(p, q)$ are independent variates such that $p + q = m$, the variate XY is distributed as $\beta_I(p, n + q)$.

Q. 48 Explicate beta distribution of second kind.

Ans. A continuous random variable X with parameters m and n is said to follow beta distribution of second kind if its probability density function is

$$f_X(x) = \left[\begin{array}{ll} \frac{1}{B(m,n)}\frac{x^{m-1}}{(1+x)^{m+n}} & \text{for } \underset{0<x<\infty}{m,n>0} \\ 0 & \text{otherwise} \end{array}\right.$$

It is generally denoted as $\beta_{II}(m, n)$.

The distribution function of beta distribution of second kind is

$$F_X(x) = \left[\begin{array}{ll} \frac{1}{B(m,n)}\int_0^x \frac{u^{m-1}\,du}{(1+u)^{m+n}} & \text{for } \underset{0<x<\infty}{m,n>0} \\ 0 & \text{otherwise} \end{array}\right.$$

Main features of beta distribution of second kind:

(i) Its mean is $\frac{m}{n-1}$

(ii) Its variance is $\frac{m(m+n-1)}{(n-1)^2(n-2)}$

(iii) The harmonic mean of $\beta_{II}(m, n)$ is $\left(\frac{m-1}{n}\right)$.

(iv) Characteristic function of beta distribution of second kind is

$$\frac{1}{\Gamma m\Gamma n}\sum_{k=0}^{\infty}\frac{(it)^k}{k!}\Gamma(m+k)\Gamma(n-k)$$

Note: As a matter of fact, both beta type I and type II distribution are same except in respect of range. Beta type I has range (0, 1) and beta type II has range (0, ∞).

Q. 49 When does a variable follow gamma distribution and what are its properties?

Ans. A positive random variable X is said to follow gamma distribution with parameters n and α if and only if its probability density function is

$$f_X(x;\alpha,n)=\begin{cases}\dfrac{\alpha^n}{\Gamma n}e^{-\alpha x}x^{n-1} & \text{for } \underset{n,\,\alpha>0}{x>0} \text{ and}\\ 0 & \text{otherwise}\end{cases}$$

It is generally denoted by $\gamma(\alpha, n)$ or gam (α, n).

If $\alpha = 0$, the gamma distribution is simply denoted as $\gamma(n)$.

The cumulative distribution of gamma variate X is

$$F_X(x)=\begin{cases}\dfrac{\alpha^n}{\Gamma n}\displaystyle\int_0^x e^{-\alpha u}u^{n-1} & \text{for } x>0\\ 0 & \text{otherwise}\end{cases}$$

It is also sometimes called *incomplete gamma function.*

Properties of gamma distribution:

(i) Mean of gamma distribution is $\dfrac{n}{\alpha}$.

(ii) Variance of gamma variate is $\dfrac{n}{\alpha^2}$.

(iii) If $\alpha < 1$, var. > mean; if $\alpha > 1$, var < mean and if $\alpha = 1$, variance = mean.

(iv) Moment generating function of gamma distribution

$$M_X(t)=\left(\frac{\alpha}{\alpha-t}\right)^n \text{ or } \left(1-\frac{t}{\alpha}\right)^{-n}$$

(v) First four central moments and their relationship with analogous cumulants are:

$$\mu_1=\kappa_1=\frac{n}{\alpha},\ \mu_2=\kappa_2=\frac{n}{\alpha^2},\ \mu_3=\kappa_3=\frac{2n}{\alpha^3},$$

$$\mu_4=\kappa_4+3\kappa_2^2=\frac{\left(6n+3n^2\right)}{\alpha^4}.$$

(vi) Pearson's coefficients of skewness are, $\beta_1 = \dfrac{4}{n}$ or $\gamma_1=\sqrt{\beta_1}=\dfrac{2}{\sqrt{n}}$

(vii) Pearson's coefficients of Kurtosis are, $\beta_2 = 3+\dfrac{6}{n}$, or $\gamma_2=\beta_2-3=\dfrac{6}{n}$.

Since $\gamma_1 > 0$, it ratifies that the gamma frequency curve is positively skewed. Further $\beta_2 > 3$ or $\gamma_2 > 0$ clearly indicate that gamma distribution is leptokurtic.

(viii) Characteristic function of gamma distribution is $\left(\dfrac{\alpha}{\alpha-it}\right)^n$ or $\left(1-\dfrac{it}{\alpha}\right)^{-n}$.

(ix) If $X_1, X_2, \ldots X_k$ and k i.i.d. $\gamma(\alpha, n_i)$ variates for $i = 1, 2, \ldots, k$, the sum of the variates, $X_1 + X_2 + \ldots + X_k$ is also a gamma variate with parameters α and $\sum n_i$, *i.e.*, $\gamma(\alpha, \sum n_i)$. It is known as *additive* or *reproductive* property of gamma variate.

(x) As $n \to \infty$, gamma distribution tends to normal distribution. It is called the *limiting form* of gamma distribution.

(xi) If $X \sim \gamma(\alpha, n)$, the mode of gamma distribution, is $\dfrac{n-1}{\alpha}$ for $n > 1$. Again if, $n < 1$, the mode is 0.

(xii) If in gam (α, n), $n = 1$, the gamma distribution is same as exponential distribution.

(xiii) If $X \sim \gamma(\alpha, n), E\left(\sqrt{X}\right)=\dfrac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma\alpha\ \Gamma n}$.

(xiv) If X and Y are two gamma variates distributed as $\gamma(n_1)$ and $\gamma(n_2)$ respectively, then

(a) the variable $(X + Y) \sim \gamma(n_1 + n_2)$.

(b) the variable $\dfrac{X}{X+Y}$ is distributed as $\beta_I(n_1, n_2)$.

(c) the variate $\dfrac{X}{Y}$ is distributed as $\beta_{II}(n_1, n_2)$.

Q. 50 Give a brief description of Logistic distribution.

Ans. The logistic distribution of a continuous variable X is given in the form of cumulative distribution function (c.d.f.) which is

$$F_X(x;\alpha,\beta)=\frac{1}{1+e^{-\frac{(x-\alpha)}{\beta}}}$$

for $-\infty<\alpha<\infty;\ \beta>0$.

The model tolerance levels in bioassays follows logistic distribution.

Properties of logistic distribution

(i) Its mean is α.

(ii) Its variance is $\frac{\pi^2\beta^2}{3}$.

(iii) Logistic distribution curve is symmetrical.

Q. 51 Give concisely Pareto distribution.

Ans. A continuous random variable X which has probability density function

$$f_X(x;x_0,\theta)=\frac{\theta}{x_0}\left(\frac{x_0}{x}\right)^{\theta+1}$$

for $x>0;\ \theta>0$

is said to follow Pareto distribution.

The income of people exceeding a certain limit x_0 follows Pareto distribution. It has found its application in modelling problems.

Properties of Pareto distribution

(i) Mean of Pareto distribution is $\frac{\theta x_0}{\theta-1}$ for $\theta>1$.

(ii) Variance of Pareto distribution is

$$\frac{\theta x_0^2}{\theta-2}-\left(\frac{\theta x_0}{\theta-1}\right)^2 \text{ for } \theta>2.$$

(iii) Moment generating function of Pareto distribution does not exist.

Q. 52 Give briefly Weibull distribution.

Ans. Weibull distribution was discovered by Swedish physicist Wallodi Weibull in 1939. A continuous random variable X is said to follow Weibull distribution if its probability density function

$$f_X(x;\alpha,\beta)=\frac{\alpha}{\beta^\alpha}x^{\alpha-1}e^{(-x/\beta)^\alpha}$$

for $x>0,\ \alpha,\ \beta>0$.

This distribution has two parameters α and β. It has application in reliability theory. Corrosion, weight loss of an alloy, tensile strength of a metal follow Weibull distribution.

Properties of Weibull distribution:

(i) Weibull distribution has mean $\beta\Gamma\left(1+\frac{1}{\alpha}\right)$.

(ii) Weibull distribution has variance

$$\beta^2\left[\Gamma\left(1+\frac{2}{\alpha}\right)-\left\{\Gamma\left(1+\frac{1}{\alpha}\right)\right\}^2\right].$$

Q. 53 Give the concept of circular distributions.

Ans. When a distribution is absolutely continuous with respect to lebesgue measure on the circumference of a unit circle and it can be represented by a density function $f(x)$ where the random variable X refers to an angle measured from a chosen direction satisfying,

$$f(x)\geq 0$$

subject to,

$$\int_0^\pi f(x)dx=1\ ;\ 0\leq x\leq 2\pi.$$

Such families of distributions are known as circular distributions.

Q. 54 Give a brief account of the Pearsonian system of distributions.

Ans. Karl Pearson gave various types of distributions through differential equation:

A density function $f_X(x)$ which satisfies the differential equation with constants a, b_0, b_1 and b_2,

$$\frac{1}{f_X(x)}\frac{d}{dx}f_X(x)=\frac{x-a}{b_0+b_1x+b_2x^2}$$

is said to belong to Pearsonian system of distribution density function. A number of the distributions studied so far belong to the Pearsonian system of distributions. For instance, gamma distribution belong to the Pearsonian type III distribution. Normal distribution belongs to the Pearsonian zero type distribution.

Q. 55 What do you understand by sampling distribution?

Ans. Sampling distribution describes the manner in which a statistic or a function of statistics, which is/are a function(s) of the random sample variate values $x_1, x_2, ..., x_n$, will vary from one sample to another of the same size.

Some popular and useful sampling distributions are χ^2, *t*, *z* and *F*.

Q. 56 Explicate the Chi-square distribution and give its properties.

Ans. The chi-square distribution was first discovered by Helmert in 1876 and later independently by Karl Pearson in 1900.

If *X* is *N* (0, 1) variate, then X^2 is known as the Chi-square variate. If $X \sim N(\mu, \sigma^2)$, then the standard normal deviate $Z = \left(\frac{x-\mu}{\sigma}\right) \sim N(0, 1)$ and Z^2 is distributed a Chi-square (χ^2) with 1 degree of freedom (d.f.). If $X_1, X_2, ..., x_n$ are *n* independent variates distributed as $N(\mu_i, \sigma_i^2)$, then

$$\sum_{i=1}^{n} Z_i^2 = \sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma_i}\right)^2$$

is distributed as Chi-square with *n* d.f. The probability density function of the Chi-square distribution is,

$$f(\chi^2) = \frac{1}{2^{\frac{n}{2}}\Gamma\frac{n}{2}} e^{-\chi^2/2}(\chi^2)^{\frac{n}{2}-1}$$

$$0 \le \chi^2 < \infty.$$

The Chi-square can be expressed in terms of sample variance (s^2) also. If $x_1, x_2, ..., x_n$ is a random sample[1] of size *n* from a normal population, the quantity $\frac{ks^2}{\sigma^2}$ is distributed as Chi-square with *k* d.f. where $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ and $k = n - 1$. σ^2 is the population variance of the population from which the sample has been drawn. The probability density function of Chi-square distribution is

$$f(\chi^2) = \frac{1}{2^{k/2}\,\Gamma\, k/2} e^{-\chi^2/2} \cdot (\chi^2)^{k/2-1}$$

$$0 \le \chi^2 \le \infty.$$

Chi-square is gamma $\left(\frac{1}{2}, \frac{k}{2}\right)$ or $\chi^2 \sim \text{gam}\left(\frac{1}{2}, \frac{k}{2}\right)$.

Chi-square distribution has only one parameter *n* or *k* as the case may be, which stands for the degrees of freedom of χ^2.

Properties of the Chi-square distribution

Properties have been expressed taking the first p.d.f. of χ^2 having *n* d.f.:

(i) Mean of the Chi-square distribution is *n*.

(ii) Variance of the Chi-square distribution is 2*n*.

(iii) Mode of the Chi-square distribution curve lies at the point $\chi^2 = n - 2$.

(iv) Moment generating of the Chi-square distribution is $(1-2t)^{-n/2}$.

(v) Characteristic function of the Chi-square distribution is $(1-2it)^{-n/2}$.

(vi) First four central moments and their relation with cumulants are:

$$\mu_1 = \kappa_1 = n, \mu_2 = \kappa_2 = 2n, \mu_3 = \kappa_3 = 8n,$$

$$\mu_4 = \kappa_4 + 3\kappa_2^2 = 48n + 12n^2$$

(vii) Measure of skewness, $\beta_1 = \frac{8}{n}$ or $\gamma_1 = \sqrt{\beta_1} = 2\sqrt{\frac{2}{n}}$.

(viii) Measure of Kurtosis, $\beta_2 = 3 + \frac{12}{n}$ or $\gamma_2 = \beta_2 - 3 = \frac{12}{n}$.

The values of β_1 and β_2 lead us to conclude that the χ^2-distribution curve has positive skewness since $\beta_1 > 0$, χ^2-distribution curve is leptokurtic as $\beta_2 > 3$.

(ix) Pearson's coefficient of skewness, $S_k = \frac{\text{mean} - \text{mode}}{\text{S.D.}}$ is $\left(\frac{2}{n}\right)^{1/2}$. This also leads to the conclusion that Chi-square distribution curve has positive skewness.

(x) The Chi-square distribution curve with 1 or 2. d.f. is hyperbolic in shape whereas for $n > 2$, the curve is unsymmetrical bell-shaped.

(xi) Recurrence relation between r^{th} and $r - 1^{th}$ raw moment of the Chi-square distribution is,

$$\mu'_r = (n + 2r - 2)\mu'_{r-1}$$

(xii) Recurrence relation between central (absolute) moments of the Chi-square distribution is,

$$\mu_{r+1} = 2r(\mu_r + n\mu_{r-1}) \text{ for } r \geq 1.$$

(xiii) If $X_1, X_2, ..., X_k$ are k independently distributed Chi-square variates such that $X_i \sim \chi^2_{ni}$ (for $i = 1, 2, ..., k$) $\sum_{i=1}^{k} X_i$ is also a Chi-square variate with $\sum n_i$ d.f. This is known as *additive* or *reproductive property of Chi-square.*

(xiv) If X and Y are two independently distributed Chi-square variates such that $X \sim \chi^2_{n_1}$ and $Y \sim \chi^2_{n_2}$ the variate $(X - Y) \sim \chi^2_{n_1 - n_2}$ for $n_1 > n_2$.

(xv) If $X \sim \chi^2_{n_1}, X + Y \sim x_{n_1+n_2}$, then $Y \sim \chi^2_{n_2}$.

(xvi) If $X \sim \chi^2_{n_1}, Y \sim \chi^2_{n_2}$ and $(X + Y) \sim \chi^2_{n_1+n_2}$, the Chi-square variates X and Y are independent.

(xvii) If n is large, then $\lim_{n\to\infty} \chi^2_n \to N(n, 2n)$. This is known as the limiting property of the Chi-square.

(xviii) Points of inflexion of the Chi-square distribution curve are

$$x = (n-2) \pm \{2(n-2)\}^{1/2}$$

(xix) Points of inflexion of the Chi-square distribution curve are equidistant from its mode.

(xx) If $X \sim \chi^2_{n_1}$ and $Y \sim \chi^2_{n_2}$ are independent Chi-square variates, the distribution of the quotient (X/Y) is beta distribution of type II, *i.e.*, $X/Y \sim \beta_{II}\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$.

(xxi) If $X \sim \chi^2_{n_1}$ and $Y \sim \chi^2_{n_2}$ are two independent Chi-square variates, the distribution of the quotient $X/(X + Y)$ is beta distribution of type I, *i.e.*, $\frac{X}{X+Y} \sim \beta_I\left(\frac{n_1}{2}, \frac{n_2}{2}\right)$.

(xxii) If all $X_i \sim N(0, 1)$ for $i = 1, 2, ..., n$, $\Sigma X_i^2 \sim \chi^2_n$. But if X_i's are distributed normally with unit variance and non-zero means, *i.e.*, $X_i \sim N(\mu_i, 1)$. The distribution of ΣX_i^2 is known as *non-central Chi-square* (χ'^2) with non-centrality parameter λ where $\lambda = \frac{1}{2}\Sigma\mu_i^2$. We denote $\chi'^2 = \chi'^2_{(n,\lambda)}$.

(xxiii) The Chi-square distribution is used to test whether a hypothetical value σ_0^2 of the population variance is true or not.

(xxiv) χ^2 is used to make a *test* of *goodness of-fit.*

(xxv) χ^2 is used to test the *independence* of *attributes.*

(xxvi) χ^2 is used to test the validity of a hypothetical ratios.

(xxvii) χ^2 is used to test the homogeneity of several population variances.

(xxviii) χ^2 is used to test the equality of several population correlation coefficients.

Q. 57 What statistic is known as Pearson's Chi-square statistic?

Ans. In a frequency distribution of large sample having k classes such that the frequency of the i^{th}-class is f_i, the statistic $\sum_{i=1}^{k} \frac{(f_i - np_i)^2}{np_i}$ is distributed as Chi-square with $(k - 1)$ d.f. where np_i is the expected frequency corresponding to the observed frequency f_i and p_i is the probability that a sample observation x_i belong to the i^{th} - class. Also $\Sigma f_i = n$.

Q. 58 What is a Chi-distribution?

Ans. The positive square root of χ_n^2 is called the Chi-distribution, *i.e.*, $+\sqrt{\chi_n^2} = \chi_n$. The mean of Chi-distribution is $\sqrt{n}$.

Q. 59 Define and discuss student's t-distribution.

Ans. Student's t is defined as the deviation of sample mean from its population mean expressed in terms of standard error.

The credit of t-distribution goes to W.S. Gosset who published it in 1908 in research paper entitled, "The probable error of the mean". Professor R.A. Fisher defined t as the ratio of a normal variate $X \sim N(0, 1)$ and the square root $\sqrt{Y/v}$, where $Y \sim \chi_n^2$ and v is the degree of freedom of χ^2, *i.e.*, $t = X \Big/ \sqrt{\frac{Y}{v}}$. The two approaches result into the same t-distribution. Student's t is extremely used in the theory and application of statistics.

Let $x_1, x_2, ..., x_n$ be a random sample drawn from a normal distribution having mean μ and standard deviation σ (unknown). The statistic $\sqrt{n}(\bar{x} - \mu)/s$ is distributed as student's $- t$ with $(n - 1)$ d.f.[2], where $\bar{x}$ is the sample mean and s, the sample standard deviation. Thus, student's t-statistic is,

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

where, $$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

The probability density function of student's t with v $(= n - 1)$ d.f. is

$$f_v(t) = \frac{1}{\sqrt{v}\,\beta\left(\frac{1}{2}, \frac{v}{2}\right)\left(1 + \frac{t^2}{v}\right)^{(v+1)/2}}$$

$$= \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v}\sqrt{\pi}\,\Gamma\frac{v}{2}}\left(1 + \frac{t^2}{v}\right)^{-(v+1)/2}$$

$$-\infty < t < \infty$$

since $\beta\left(\frac{1}{2}, \frac{v}{2}\right) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\Gamma\frac{v}{2}\sqrt{\pi}}$ and $\Gamma\frac{1}{2} = \sqrt{\pi}$

Properties of student's t-distribution:

(i) t-distribution is symmetrical about the origin since $f(t) = f(-t)$.

(ii) All moments of odd order are zero, *i.e.*, $\mu'_{2r+1} = 0$.

(iii) The mean of t-distribution is zero.

(iv) The variance of t-distribution is $\frac{n}{n-2}$ for $n > 2$.

(v) The third moment $\mu'_3 = 0$ also $\mu_3 = 0$.

(vi) The fourth central moment

$$\mu_4 = \frac{3n^2}{(n-2)(n-4)} \text{ for } n > 4.$$

(vii) Measure of skewness, $\beta_1 = 0$.

(viii) Measure of Kurtosis, $\beta_2 = \frac{3(n-2)}{(n-4)}$ for $n > 4$.

This reveals that t-distribution is symmetrical about mean and is leptokurtic because $\beta_2 > 3$ as

$$\left(\frac{n-2}{n-4}\right) > 1. \text{ Also } \lim_{n\to\infty} \beta_2 = \lim_{n\to\infty} \frac{3\left(1 - \frac{2}{n}\right)}{\left(1 - \frac{4}{n}\right)} = 3.$$

The limiting value of $\beta_2 = 3$. Hence, this shows that for large n, t_n is approximately $N(0, 1)$. In practice $n > 30$ is considered as large. Some workers take $n \geq 50$ as large.

(ix) Mode of t-distribution lies at the origin $t = 0$.

(x) Points of inflexion of t-distribution are

$$\pm\left(\frac{n}{n+2}\right)^{1/2}$$

(xi) Moment generating function of t-distribution does not exist.

(xii) Recurrence formula for even order moments of t-distribution is:

$$\mu_{2r} = \frac{n(2r-1)}{(n-2r)}\mu_{2r-2} \text{ for } n > 2r$$

(xiii) the maximum height of distribution curve is

$$\frac{1}{\sqrt{n-1}\,\beta\left(\frac{1}{2}, \frac{n-1}{2}\right)} \text{ at the point } t = 0.$$

(xiv) If $n = 2$, the probability density function reduces to $\frac{1}{\pi(1+t^2)}$ which is the standard Cauchy distribution. Hence, t with 1 d.f. reduces to Cauchy distribution.

(xv) Probability tables for t-distribution can be prepared for various d.f. and different value of $t = t_0$. Fisher and Yates prepared tables for $P_F = P(t > t_0)$. William Gosset prepared tables for $P_S = P(t \leq t_0)$. By little manipulation it can be shown that $P_F = 2(1 - P_S)$. t-tables are prepared for various d.f. for direct usage in statistical inference which are extremely useful and extensively used.

(xvi) t-distribution is used to test whether the sample mean conforms to a specified value of population mean or not.

(xvii) t-distribution provides the facility of testing the equality of two population means based on sample means.

(xviii) The significance of correlation coefficient and regression coefficient is tested by student-t.

(xix) Significance of partial correlation coefficient is also tested by student-t.

(xx) If $X \sim N(\delta, 1)$ and $Y \sim \chi^2$ with n d.f. are two independent variates, the distribution of the statistic

$$t' = X/\sqrt{Y/n}$$

is known as non-central t variate and is said to follow non-central t distribution with n d.f. and non-centrality parameter δ. Non-central t distribution is seldom needed to develop the power function of certain tests concerning normal population.

Q. 60 Define Fisher's z-statistics and discuss z-distribution appropriately.

Ans. Sir R.A. Fisher in 1925 defined a statistic which is based on the ratio of two sample variances. Suppose the variances of two random samples based on sizes n_1 and n_2 drawn from two normal populations are s_1^2 and s_2^2 respectively. Fisher's z-statistic is defined as

$$z = \frac{1}{2}\log_e\left(\frac{s_1^2}{s_2^2}\right)$$

or $$\frac{s_1^2}{s_2^2} = e^{2z}$$

Putting $$\frac{s_1^2}{s_2^2} = F,$$

$$F = e^{2z}.$$

The letter F is the first letter of Fisher's name as a mark of respect given to him by G.W. Snedecor. Probability density function of Fisher's z-distribution is,

$$f_F(z) = \frac{2\nu_1^{\nu_1/2}\,\nu_2^{\nu_2/2}}{\beta\left(\frac{\nu_1}{2}, \frac{\nu_2}{2}\right)}\frac{e^{\nu_1 z}}{\left(\nu_2 + \nu_1 e^{2z}\right)^{\frac{\nu_1+\nu_2}{2}}}$$

$$-\infty < z < \infty$$

where $\nu_1 = (n_1 - 1)$, $\nu_2 = (n_2 - 1)$

ν_1 and ν_2 are called the degrees of freedom of Fisher's z-distribution.

[1] Random sample has been discussed in Chapter 9.

[2] Degrees of freedom has been defined in Chapter 10.

Properties of Fisher's z-distribution:

(i) It is highly skew distribution.

(ii) z-distribution is a family of distributions for different values of v_1 and v_2.

(iii) The mean of z-distribution is $\frac{1}{2}\left(\frac{1}{\nu_2}-\frac{1}{\nu_1}\right)$.

(iv) The variance of z-distribution is

$$\frac{1}{2}\left(\frac{1}{\nu_1}+\frac{1}{\nu_2}+\frac{1}{\nu_1^2}+\frac{1}{\nu_2^2}\right)$$

(v) The third moment,

$$\mu_3=\frac{1}{2}\left\{\left(\frac{1}{\nu_2^2}-\frac{1}{\nu_1^2}\right)+\ \frac{1}{2}\left(\frac{1}{\nu_2^3}-\frac{1}{\nu_1^3}\right)\right\}.$$

(vi) The fourth moment,

$$\mu_4=\left(\frac{1}{\nu_2^3}+\frac{1}{\nu_1^3}\right)+3\left(\frac{1}{\nu_2^4}+\frac{1}{\nu_1^4}\right)+3\mu_2^2$$

(vii) Moment generating function of z-distribution is,

$$M_z(t)=\left(\frac{\nu_2}{\nu_1}\right)^{t/2}\frac{\Gamma\left(\frac{\nu_1+t}{2}\right)\Gamma\left(\frac{\nu_2-t}{2}\right)}{\Gamma\frac{\nu_1}{2}\Gamma\frac{\nu_2}{2}}$$

(viii) Characteristic function of z-distribution is,

$$\phi_z(t)=\left(\frac{\nu_2}{\nu_1}\right)^{it/2}\frac{\Gamma\left(\frac{\nu_1+it}{2}\right)\Gamma\left(\frac{\nu_2-it}{2}\right)}{\Gamma\frac{\nu_1}{2}\Gamma\frac{\nu_2}{2}}$$

(ix) z-distribution tends to normal with mean $\frac{1}{2}\left(\frac{1}{\nu_2}-\frac{1}{\nu_1}\right)$ and variance $\frac{1}{2}\left(\frac{1}{\nu_1}+\frac{1}{\nu_2}\right)$ when ν_1 and ν_2 are large.

Q. 58 What is Snedecor's F and F-distribution?

Ans. G.W. Snedecor founded F-statistics as the ratio of two sample variances and prepared F-tables in 1934. Suppose there are two samples of sizes n_1 and n_2 from normal populations $N\left(\mu_1,\sigma_1^2\right)$ and $N\left(\mu_2,\sigma_2^2\right)$ respectively. Suppose s_1^2, s_2^2 are the sample variances respectively.

We know,

$$\frac{\nu_1 s_1^2}{\sigma_1^2}\sim\chi_1^2 \text{ where } \nu_1=(n_1-1)$$

and $$\frac{\nu_2 s_2^2}{\sigma_2^2}\sim\chi_2^2 \text{ where } \nu_2=(n_2-1)$$

The ratio,

$$\frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2}=\frac{\chi_1^2/\nu_1}{\chi_2^2/\nu_2}$$

$$=F_{\nu_1,\nu_2}$$

If we have to test $\sigma_1^2=\sigma_2^2$, under this hypothesis

$$\frac{s_1^2}{s_2^2}=F_{\nu_1,\nu_2}$$

From the above expression, it is evident that the ratio of two independent Chi-squares *vis-a-vis* the ratio of two independent variances is distributed as F. ν_1 and ν_2 are called the d.f. of F-distribution. As a norm larger sample variance is taken in the numerator. The probability density function of F-distribution is

$$f_{\nu_1,\nu_2}(F)=\frac{\left(\frac{\nu_1}{\nu_2}\right)^{\nu_1/2}}{\beta\left(\frac{\nu_1}{2},\frac{\nu_2}{2}\right)}\frac{F^{\frac{\nu_1}{2}-1}}{\left(1+\frac{\nu_1}{\nu_2}F\right)^{\frac{\nu_1+\nu_2}{2}}}$$

$$0\le F<\infty$$

F-distribution has two parameters ν_1 and ν_2.

Properties of F-distribution:

(i) F-distribution extends along abscissa from 0 to ∞.

(ii) F-distribution curve wholly lies in the first quadrant.

(iii) Shape of the curve depends on the d.f. ν_1 and ν_2.

(iv) F-distribution curve is a positive skew curve and is highly positively skewed when ν_2 is small ($\nu_2 < 5$).

(v) The curve is unimodal and its mode is at the point $F = \nu_2 (\nu_1 - 2)/\nu_1 (\nu_2 + 2)$ for ($\nu_1 \geq 3$). The mode is always less than unity.

(vi) Mean of F-distribution is $\nu_2/(\nu_2 - 2)$ for $\nu_2 \geq 3$.

(vii) Variance of F-distribution is

$$\frac{2\nu_2^2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-2)^2(\nu_2-4)} \text{ for } \nu_2 > 4$$

(viii) Moment generating function of F-distribution does not exist.

(ix) There exists a very useful relation between F-values having interchanged d.f., *i.e.*,

$$F_{\alpha;(\nu_1,\nu_2)} = \frac{1}{F_{1-\alpha;(\nu_2,\nu_1)}}$$

or $$P\left\{F_{\nu_1,\nu_2} \geq c\right\} = P\left\{F_{\nu_2,\nu_1} \leq \frac{1}{c}\right\}$$

This is known as the reciprocal property of F-distribution.

(x) If X_1 and X_2 are two variables having the density function $f(x) = e^{-x}$ ($0 < x$ ∞), the variate X_1/X_2 follows F-distribution.

(xi) If $X \sim F$ (ν_1, ν_2), the variable $\nu_1 X/(\nu_2 + \nu_1 X)$ is distributed as beta distribution of type I, *i.e.*, $\beta_I\left(\frac{\nu_1}{2},\frac{\nu_2}{2}\right)$ and the variable $\nu_1\ F/\nu_2$ is distributed as $\beta_{II}\left(\frac{\nu_1}{2},\frac{\nu_2}{2}\right)$.

(xii) If $\nu_1 = 1$ and $\nu_2 \to \infty, \sqrt{F_{1,\infty}} \sim N(0,1)$.

(xiii) If $\nu_1 = 1$ and $\nu_2 \to \infty, F_{1,\infty} = \chi_1^2$

(xiv) If t has ν d.f. and F has $(1, \nu)$ d.f., $t_\nu^2 = F_{1,\nu}$

(xv) If t has ∞ d.f., $\chi_1^2 = t_\infty^2$.

(xvi) The two points of inflexion of the F-distribution are equidistant from its mode.

(xvii) F-distribution is used to test the equality of two population variances.

(xviii) F-distribution is entirely used in analysis of variance.

(xix) F-distribution is applied to test the equality of several regression coefficients.

(xx) F-distribution is used to test the equality of several population means.

(xxi) If χ_1^2 is a non-central χ^2 with ν_1 d.f. and non-centrality parameter λ and χ_2^2 is a central χ^2 with ν_2 d.f., the distribution of $\frac{\chi_1^2/\nu_1}{\chi_2^2/\nu_2}$ is F' which is known as non-central F.

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s) in the blanks:

1. A discrete variable can take a ______ number of values within its range.
2. A continuous variable can take any value within its ________.
3. If a fair coin is tossed three times, the probability function p (x) of the number of heads x is ________.
4. A fair coin is tossed three times. If x denote the number of heads and y denotes the number of head runs of two or three heads, the probability function of the variable $Z = x + y$ is __________.

5. If a discrete random variable has the probability function as,

x:	0	1	2	3	4	5	6	7	8
$p(x)$:	k	$2k$	$3k$	$5k$	$5k$	$4k$	$3k$	$2k$	k

then the value of k is equal to _________ and $E(x)$ = _________.

6. If the probability density function of a variable x is defined as,

$$f(x) = cx(2-x) \qquad 0 \le x < 2$$

then the value of c _________.

7. If p.d.f. of a variable x is defined as,

$$f(x)=\frac{3}{4}x(2-x), \qquad 0 \le x < 2$$

then the median of the distribution is ______.

8. The probability density function $f(x)$ cannot exceed _______.

9. If a random variable X possesses the following function.

x:	+3	+2	+1	0	− 1	− 2	− 3
$p(x)$:	0.1	0.2	$3k$	k	$2k$	0	0.1

then

(a) the value of k is _______.

(b) the mean of X is ________.

(c) the variance X is _______.

10. If $p(x)=\dfrac{x}{12}$ for x = 1, 2, 3, 4, 5, 6

= 0 otherwise.

then

(a) $P(x = 2 \text{ or } 3)$ = ________

(b) $P\left\{\left(\frac{1}{2}<x<\frac{5}{2}\right)x>1\right\}$ = ________

11. The function $f(x) = x^2; -1 < x < 1$

= 0 otherwise

is a possible _________.

12. If $f(x)$ is the p.d.f. of x; $a \le x \le b$ then

(a) the mean μ_1 is equal to _______.

(b) the logarithm of the geometric mean, $\log G$ = __________.

(c) the harmonic mean of $x, \dfrac{1}{H}$ = _______.

(d) the expected value of x^2, *i.e.*, $E(x^2)$ __________.

(e) the variance of x, *i.e.*, $V(x)$ = _________.

(f) the mean deviation about mean μ, *i.e.*, M.D. = _________.

(g) the median of the frequency distribution, M = _________.

(h) i^{th} quartile of the frequency distribution for = 1, 2, 3, *i.e.*, Q_i = _________.

(i) i^{th} decile of the frequency distribution for i = 1, 2, ... 9, *i.e.*, D_i = __________.

(j) i^{th} percentile of the frequency distribution for i = 1, 2, ..., 99, *i.e.*, P_i = ________.

13. If $f(x)$ is a p.d.f. of a variable X defined as,

$$\begin{aligned} f(x) &= cx,\ 1 \le x \le 2 \\ &= c,\ 2 \le x \le 3 \\ &= 0 \text{ otherwise} \end{aligned}$$

then the value of c is _______.

14. If the exponential distribution is given as,

$$f(x) = e^{-x},\ 0 \le x \le \infty$$

then

(a) the mean of the distribution is _______.

(b) the variance of the distribution _______.

(c) the third moment of the distribution is ________.

(d) Pearson's cosntant β_1 = ______.

15. If the probability density function of a variable x is given as,

$$f(x)=\frac{x}{a^2}e^{-x^2/2a^2},\ 0 \le x < \infty$$

then the variance of the distribution is _________.

16. If a random variable X has the p.d.f.

$$\begin{aligned} f(x) &= 3x,\ 0 < x < 1 \\ &= 0 \text{ otherwise} \end{aligned}$$

then the p.d.f. of $y = 4x + 3$ is _________.

17. If x has a rectangular distribution

$$f(x)=\frac{1}{4}, -2 \le x \le 2.$$

then p.d.f. $g(y)$ of a variable $y = \sin x$ is ____________.

18. The distribution of a random variable x in the range [0, 2] is defined by

$$f(x) \neq x^3, 0 < x \leq 1$$

$$= (2 - x)^3, 1 < x \leq 2$$

then the mean of the distribution $f(x)$ is _________.

19. If ce^{-ax} is a p.d.f. for $0 < x < \infty$, the value of c is equal to _________.

20. The mean of binomial distribution $b(n, p)$ is _________ and its variance is ________.

21. The mean of the binomial distribution is __________ than its variance.

22. If the mean and variance of binomial distribution are 2 and 1 respectively, the $P(x \leq 1)$ is equal to _________.

23. If on an average the rain falls on ten days in a month (30 days), the probability that the rain will fall on two days of the week is ___________.

24. In a police control room there are on an average 3 calls per 10 minute interval. The probability of receiving 4 calls in a 10 minute interval is __________.

25. If the probability of a rocket hitting the target is $\frac{1}{2}$ and the rockets are being launched one after the other, then the probability that the 10th launch being the 5th hit on the target is _________.

26. Shifting of origin does not affect the ______ distribution.

27. Mean and variance of geometric distribution are _________.

28. Under shifting of time origin in geometric distribution, the p.d.f. remain the same. This property is known as _________.

29. The mode of the geometric distribution.

$$f(x) = \left(\frac{1}{2}\right)^x$$

for $x = 1, 2, 3, \ldots$ is _______.

30. In case of geometric variate x, the variance of x is always ________ mean of x.

31. Negative binomial distribution $nb(r, p)$ reduces to geometric distribution when __________.

32. Polya's distribution reduces to geometric distribution when _________.

33. In hypergeometric distribution, the probability of successive draws (trails) are _________.

34. The probability of a success changes from trial to trial in _________.

35. The number of trials in hypergeometric distribution is _________.

36. Hypergeometric distribution reduces to binomial distribution when _________.

37. If the rectangular distribution, $f(x) = \frac{1}{b-a}, a \leq x \leq b$, the mean and the median of the distribution are _________.

38. Rectangle distribution is _________.

39. Mode of the rectangular distribution _______.

40. Mean deviation about mean in case of rectangular distribution is _______.

41. Variance of rectangular distribution is equal to __________.

42. Moment generating function for rectangular distribution _________.

43. If the random variable x has the rectangular distribution with p.d.f., $f(x) = \frac{1}{\theta}, 0 < x \leq \theta$, then

(a) the mean of the distribution is _______.

(b) the variance of the distribution is ______.

(c) the third central moment is equal to __________.

(d) the fourth central moment is ________.

(e) the coefficient of skewness is ________.

(f) The coefficient of Kurtosis is _______.

44. The number of Parameter(s) involved in exponential p.d.f. is/are ______.

45. The tails of the exponential distribution curve are ______.

46. The exponential distribution is ______ peaked than normal.

47. If $x \sim$ Expo (λ), variable $y = e^{-\lambda x}$ has ________ distribution.

48. The variance of $L(\lambda)$ is ______ the variance of Expo (λ).

49. The distribution curve of double exponential distribution in respect of its bulginess is ______.

50. Semi-interquartile range of double exponential distribution is ______.

51. If $X \sim N(\mu, \sigma)$, the standard normal deviate is distributed as ________.

52. The maximum height of the normal curve lies at the point ________.

53. The p.d.f at the point of inflexion of the normal curve is ______.

54. If $X_1, X_2, ..., X_n$ are i.i.d normal variates with mean μ and variance σ^2, the variable $\sum_{i=1}^{n} X_i$ is distributed as ______.

55. If $X_1, X_2, ..., X_n$ are i.i.d $N(\mu, \sigma^2)$, the variable $\bar{x}$ is distributed as ______

56. The normal distribution curve is

(a) ________ (b) ________ (c) ______

57. For a normal distribution, quartile deviation, mean deviation and standard deviation are in the ratio ______.

58. For a normal distribution $N(\mu, \sigma^2)$, the mean deviation from mean is ______.

59. For a normal frequency, the odd order moment μ_{2r+1} are equal to ______.

60. The moment generating function of the normal distribution $N(\mu, \sigma^2)$ is ________.

61. The characteristic function of the normal distribution $N(\mu, \sigma^2)$ is ________.

62. If $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$, the variable $X + Y$ is distributed as ______.

63. If $X \sim N(12.5, 3.5^2)$ and $Y \sim N(8.5, 2.5^2)$, the variable $X + Y$ is distributed as ______.

64. If the income X (rupees) of people is distributed normally with mean $\mu = 500$ and S.D. $\sigma = 200$, the percentage of people having the income:

Given,

z:	0.50	1.00	1.25
$\phi(z)$	0.19146	0.19146	0.39435
z:	1.50	2.25	2.50
$\phi(z)$	0.43319	0.48778	0.49379

(a) more than Rs. 1000 is ________.

(b) less than Rs. 250 is ________.

(c) between Rs. 400 and 600 is ________.

(d) between Rs. 200 and 400 is ________.

(e) between Rs. 700 and Rs. 950 is ________.

65. In problem number 64, if the total number of workers in a factory is 500, the number of employees having income.

(a) more than Rs. 1000 is ________.

(b) less than Rs. 250 is ________.

(c) between Rs. 400 and 600 is ________.

(d) between Rs. 200 and 400 is ________.

(e) between Rs. 700 and Rs. 950 is ______.

66. The mean of lognormal distribution is ________.

67. The variance of lognormal distribution is ________.

68. The right tail of lognormal distribution is ________.

69. Log normal distribution curve is ______ peaked than normal curve.

70. If $X_1 \sim \log\text{–}N(\mu_1, \sigma_1^2)$ and $X_2 \sim \log\text{–}N(\mu_2, \sigma_2^2)$, the distribution of X_1/X_2 is ______.

71. If $X_1 \sim \log\text{–}N(\mu_1, \sigma_1^2)$ and $X_2 \sim \log\text{–}N(\mu_2, \sigma_2^2)$ the distribution of $(X_1 - X_2)$ is ________.

72. If $X_1, X_2 ..., X_n$ are n observations on log normal variate $X \sim f(x; \mu, \sigma^2)$, the geometric mean $G = (x_1, x_2 ..., x_n)^{1/n}$ has ________ distribution.

73. The probability density function of a Cauchy variate X with parameters 2 and 5 is _______.

74. Mean of Cauchy distribution is _________.

75. Variance of Cauchy distribution is _______.

76. Moment generating function of Cauchy distribution is _______.

77. If $X \sim C$ (5, 4), the maximum of Cauchy distribution lies at the point _________.

78. ________ moments of Cauchy variate exist.

79. If $X \sim C$ (2, 1), the mode of the Cauchy distribution lies at the point __________.

80. If $X_1, X_2 ..., X_n$ are i.i.d. $C(\alpha, \beta)$, their mean, $\overline{X}$ is distributed as __________.

81. If X and Y are standard normal variates, the variable X/Y has the distribution_________.

82. If X and Y are i.i.d. variates and X/Y is C (0, 1), X and Y _________ necessarily normal.

83. If $X \sim \beta_I(\alpha, \beta)$, the variate X follows U (0, 1) for the values of α and β as ________.

84. Mean of distribution $\beta_I(\alpha, \beta)$ is ________.

85. Mean of distribution $\beta_{II}(\alpha, \beta)$ is ________.

86. The shape of beta distribution curve depends on the values of ________.

87. If a random variable $X \sim \gamma(n, \alpha)$, the relation between mean and variance is ________.

88. As regards the position of tails, gamma distribution curve has __________.

89. As regards the peakedness, gamma distribution curve is __________.

90. If $X \sim \gamma(n_1)$ and $Y \sim \gamma(n_2)$, the variable $X + Y$ is distributed as ___________.

91. If $X \sim \gamma(n_1)$ and $Y \sim \gamma(n_2)$, the variable $X/(X + Y)$ is distributed as ___________.

92. The cumulative distribution function of gamma distribution is also known as _______.

93. If X follows logistic distribution with parameter α and β, the variance of the distribution is independent of _________.

94. The tails of the logistic distribution curve are __________.

95. Pareto distribution has application in ___________.

96. Weibull distribution has application in ________.

97. The variable like _______ follows Weibull distribution.

98. If X is a binomial variate with parameters $n = 25$, and $p = 0.2$. The probability $P\{x < \mu_x - 2\,\sigma_x\}$ is __________.

99. If X is a binomial variate with parameters n and p, the distribution of the variable $Y = (n - X)$ is a __________ variate with parameters ______.

100. If a variable X follows Poisson distribution with $P(x = 1) = P(x = 2)$, the probability $P(x = 1 \text{ or } 2)$ is _________.

101. If X is a Poisson variate with $P(x = 1) = P(x = 2)$, the mean of the Poisson variate is equal to __________.

102. The normal curve is symmetrical about mean 50 and 5 per cent values are greater than 70. The standard S.D. of the distribution is _______ [Given that ϕ (1.64) = 0.05].

103. The standard deviation of a Poisson variate is 2, the mean of the Poisson variate is _______.

104. The shape of Chi-square distribution curve with d.f. 1 is __________.

105. If the Chi-square distribution has 10 d.f., the relation between μ'_3 and μ'_2 is _________.

106. If the Chi-square has 8 d.f., the relation between μ_3, μ_2 and μ_1 is __________.

107. In a way the Chi-square distribution is equivalent to __________.

108. If χ_1^2 and χ_2^2 are two independent Chi-square variates with d.f. n_1 and n_2, respectively, then

(a) the distribution of χ_1^2/χ_2^2 is ________.

(b) the distribution of $\dfrac{\chi_1^2/n_1}{\chi_2^2/n_2}$ is ________.

(c) the distribution of $\left(\chi_1^2 + \chi_2^2\right)$ is ______.

(d) the distribution of $\dfrac{\chi_1^2}{\chi_1^2 + \chi_2^2}$ is ______.

(e) the distribution of $\left(\chi_1^2 - \chi_2^2\right)$ is ______.

109. The mean of the Chi-square distribution is ______ of its variance.

110. The third moment of the chi-square distribution is ______ of its variance.

111. Pearson's coefficient of skewness for Chi-square distribution curve is ______.

112. The Chi-square distribution curve for d.f. 3 or more is always ______.

113. If the d.f. n for Chi-square are large, the Chi-square distribution tends to ______.

114. The positive square root of χ_n^2 follow ______ distribution.

115. The mean of the Chi-square distribution with n d.f. is ______.

116. If the variates X_i $(i = 1, 2, ..., n)$ are distributed as $N(\mu, 1)$, the variate $\sum X_i^2$ is distributed as ______.

117. If $x_1, x_2, ..., x_n$ is a random sample from a population $N(\mu, \sigma^2)$, the distribution of $\dfrac{\Sigma(x_i - \bar{x})^2}{\sigma^2}$ is ______.

118. If $X_1, X_2, ..., X_n$ are n i.i.d. expo (λ) variates, the distribution of $2\lambda \Sigma X_i$ is ______.

119. Pearson's Chi-square statistic for K classes is ______.

120. t_n-distribution tends to normal if ______.

121. t-distribution curve in respect of tails is always ______.

122. Measure of Kurtosis for t-distribution curve cannot be defined if ______.

123. t-distribution curve as regards its peak is always ______.

124. Maximum height of t-distribution curve is ______.

125. t-distribution with 1 d.f. reduces to ______.

126. Equality of two population means can be tested by ______.

127. Significance of Pearson's correlation coefficient can be tested by ______.

128. Independence of two attributes can be tested by ______.

129. Whether a given frequency distribution follows an specified distribution or not can be tested by ______.

130. The population variance σ^2 has a hypothetical value 25 or not can be tested by ______.

131. If $X \sim N(0, 1)$ and $Y \sim \chi_n^2$, the statistic $\dfrac{\sqrt{n}X}{\sqrt{Y}}$ is distributed as ______.

132. The ratio of two sample variances is distributed as ______.

133. The relation between Fisher's z and Snedecor's F is ______.

134. The mean of z-distribution is ______.

135. Mode of F-distribution is always ______ unity.

136. Moment generating function of F-distribution ______.

137. Reciprocal property of F-distribution leads to the relation ______.

138. For distributions F_{ν_1,ν_2} and $\chi_{\nu_1}^2$ and $\nu_2 \to \infty$, the relation between F and χ^2 is ______.

139. If the d.f. for χ^2 are large, the χ^2-distribution tends to ______ distribution.

140. If s^2 is the variance of a sample of size n from a population $N(\mu, \sigma^2)$, the statistic $\dfrac{(n-1)s^2}{\sigma^2}$ is distributed as ______.

141. If x_1 and x_2 be a random sample from a population $N(1, 1)$, the variate $\dfrac{x_1 - x_2}{\sqrt{2}}$ is distributed as ______.

142. Beta type I and beta type II distributions are same excepting in respect of ________.

143. Characteristic function is not useful to workout the moments in case of ________ distribution.

144. If a variate $X \sim \gamma(\alpha, 1)$, the distribution of X is same at that of ________ distribution.

145. If a variate $Y \sim b\ (y;\ n,\ p)$, variate Y/n is known as ________ variate.

146. A variable X representing the angles and satisfying the condition $\int_0^{\pi} f(x)\,dx = 1$ for $f(x) \geq 0$ and $0 \leq x \leq \pi$ follows ________ distribution.

147. The parameters of a multinomial distribution having k classes and n observations are ________.

148. The variable $y = -2 \log x$ for $X \sim U(0, 1)$ is distributed as Chi-square with ________ d.f.

149. The variable $Y = -2 \sum_{i=1}^{n} \log X_i$, where X_i are i.i.d. $U(0, 1)$, follows ________ distribution.

150. A distribution having always mean and standard deviation equal is ________.

SECTION-C

Multiple Choice Questions

Select one correct alternative out of the given ones

Q. 1 Two random variables X and Y are said to be independent if:

(a) $E(XY) = 1$

(b) $E(XY) = 0$

(c) $E(XY) = E(X)\,E(Y)$

(d) $E(XY) =$ any constant value

Q. 2 If X and Y are two random variables such that their expectations exist and $P(x \leq y) = 1$, then

(a) $E(X) \leq E(Y)$

(b) $E(X) \geq E(Y)$

(c) $E(X) = E(Y)$

(d) none of the above

Q. 3 If X and Y two independent variables and their expected values are $\overline{X}$ and $\overline{Y}$ respectively, then

(a) $E\{(X-\overline{X})(Y-\overline{Y})\} = 0$

(b) $E\{(X-\overline{X})(Y-\overline{Y})\} = 1$

(c) $E\{(X-\overline{X})(Y-\overline{Y})\} = C$ (constant)

(d) all the above

Q. 4 If X is a random variable with its mean $\overline{X}$, the expression $E(X-\overline{X})^2$ represents:

(a) the variance of X

(b) second central moment

(c) both (a) and (b)

(d) none of (a) and (b)

Q. 5 If X and Y are two random variables, then

(a) $E\{(XY)^2\} = E(X^2)\,E(Y^2)$

(b) $E\{(XY)^2\} = E(X^2 Y^2)$

(c) $E\{(XY)^2\} \geq E(X^2)\,E(Y^2)$

(d) $E\{(XY)^2\} \leq E(X^2)\,E(Y^2)$

Q. 6 The outcomes of tossing a coin three times are a variable of the type:

(a) continuous random variable

(b) discrete random variable

(c) neither discrete nor continuous random variable

(d) discrete as well as continuous random variable

Q. 7 The height of persons in a country is a random variable of the type:

(a) continuous r.v.

(b) discrete r.v.

(c) neither discrete nor continuous r.v.

(d) continuous as well as discrete r.v.

Q. 8 If X and Y are two random variables with means $\overline{X}$ and $\overline{Y}$ respectively, then the expression

$$E\left[\left(X-\overline{X}\right)\left(Y-\overline{Y}\right)\right]$$

is called:

(a) variance of X
(b) variance of Y
(c) cov (X, Y)
(d) moments of X and Y

Q. 9 If X is a random variable, $E\,(e^{tx})$ is known as:

(a) characteristic function
(b) moment generating function
(c) probability generating function
(d) all the above

Q. 10 If X is a random variable, the $E\,(t^x)$ is known as:

(a) characteristic function
(b) moment generating function
(c) probability generating function
(d) the x^{th} moment

Q. 11 If X is a random variable with mean μ, the $E\,(X-\mu)^r$ is called:

(a) variance
(b) r^{th} raw moment
(c) r^{th} central moment
(d) none of the above

Q. 12 If X is a random variable, $E\,(e^{itx})$ is known as:

(a) characteristic function
(b) moment generating function
(c) probability generating function
(d) all the above

Q. 13 If $X_1, X_2, \ldots, X_n$ be a sequence of mutually independent random variables where X_i can take only positive integral values and

$$S_m = \sum_{i=1}^{m} X_i\,(m \le n), S_n = \sum_{i=1}^{n} X_i, E(X) = \mu > 0$$

then

(a) $E\left(\dfrac{S_m}{s_n}\right)=1$

(b) $E\left(\dfrac{S_m}{s_n}\right)=0$

(c) $E\left(\dfrac{S_m}{S_n}\right)=\dfrac{m}{n}$

(d) $E\left(\dfrac{S_m}{S_n}\right)=\infty$

Q. 14 If X is a random variable which can take only non-negative values, then

(a) $E\,(X^2) = [E\,(X)]^2$
(b) $E\,(X^2) \ge [E\,(X)]^2$
(c) $E\,(X^2) \le [E\,(X)]^2$
(d) none of the above

Q. 15 If X is a random variable having its p.d.f. $f(x)$, the $E\,(X)$ is called:

(a) arithmetic mean
(b) geometric mean
(c) harmonic mean
(d) first quartile

Q. 16 If X is a random variable and $f\,(x)$ is its p.d.f., $E\left(\dfrac{1}{X}\right)$ is used to find:

(a) arithmetic mean
(b) harmonic mean
(c) geometric mean
(d) first central moment

Q. 17 If X is a random variable and its p.d.f. is $f\,(x)$, $E\,(\log x)$ represents:

(a) arithmetic mean
(b) geometric mean
(c) harmonic mean
(d) logarithmic mean

Q. 18 If X and Y are two random variables, the covariance between the variables $aX + b$ and $cY + d$ in terms of COV (X, Y) is:

(a) COV $(aX + b, cY + d)$ = COV (X, Y)
(b) COV $(aX + b, cY + d) = abcd \times$ COV (X, Y)
(c) COV $(aX + b, cY + d) = ac$ COV $(X, Y) + bd$
(d) COV $(aX + b, cY + d) = ac$ COV (X, Y)

Q. 19 If X, Y and Z are three random variables, then

(a) COV $(X + Y, Z)$ = COV (X, Z) + COV (Y, Z)
(b) COV $(X + Y, Z)$ = COV (X, Z, YZ)
(c) COV $(X + Y, Z)$ = COV (X, Y, Z)
(d) COV $(X + Y, Z) = 0$

Q. 20 If X is a random variable and r is an integer, then $E(X^r)$ represents:

(a) r^{th} central moment
(b) r^{th} factorial moment
(c) r^{th} raw moment
(d) none of the above

Q. 21 For Bernoulli distribution with probability p of a success and q of a failure, the relation between mean and variance that holds is:

(a) mean < variance
(b) mean > variance
(c) mean = variance
(d) mean ≤ variance

Q. 22 The outcomes of an experiment classified as success A or $\overline{A}$ failure will follow a Bernoulli distribution iff:

(a) $P(A) = \frac{1}{2}$
(b) $P(A) = 0$
(c) $P(A) = 1$
(d) $P(A)$ remains constant in all trials

Q. 23 The mean and variance of a binomial distribution are 8 and 4, respectively. Then, $P(X = 1)$ is equal to:

(a) $\frac{1}{2^{12}}$
(b) $\frac{1}{2^4}$
(c) $\frac{1}{2^6}$
(d) $\frac{1}{2^8}$

Q. 24 If for a binomial distribution, $b(n, p)$, $n = 4$ and also $P(X = 2) = 3P(X = 3)$, the value of p is:

(a) $\frac{9}{11}$
(b) 1
(c) $\frac{1}{3}$
(d) none of the above

Q. 25 If for a binomial distribution $b(n, p)$, mean $= 4$, variance $= \frac{4}{3}$, the probability, $P(X \geq 5)$ is equal to:

(a) $\left(\frac{2}{3}\right)^6$
(b) $\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)$
(c) $\left(\frac{1}{3}\right)^6$
(d) $4\left(\frac{2}{3}\right)^6$

Q. 26 An experiment succeeds twice as often as it fails. The chance that in the next six trials, there shall be at least four successes is:

(a) $\frac{240}{729}$
(b) $\frac{489}{729}$
(c) $\frac{496}{729}$
(d) none of the above

Q. 27 A manufacturer produces switches and experiences that 2 per cent switches are defective. The probability that in a box of 50 switches, there are at most two defective is:

(a) $2.5\, e^{-1}$
(b) e^{-1}
(c) $2\, e^{-1}$
(d) none of the above

Q. 28 If $X \sim b\left(3, \frac{1}{2}\right)$ and $Y \sim b\left(5, \frac{1}{2}\right)$, the probability of $P(X + Y = 3)$ is:
(a) 7/16
(b) 7/32
(c) 11/16
(d) none of the above

Q. 29 If X and Y are two Poisson variates such $X \sim P(1)$ and $Y \sim P(2)$, the probability, $P(X + Y < 3)$ is:
(a) e^{-3}
(b) $3e^{-3}$
(c) $4e^{-3}$
(d) $8.5e^{-3}$

Q. 30 If X is a binomial variate with its mean $\mu = 4$ and third moment $\mu_3 = 4.8$, the value of Pearson's constant β_1 is:
(a) 2/3
(b) 5/6
(c) 0
(d) none of the above

Q. 31 If $X \sim b(n, p)$, the distribution of $Y = (n - X)$ is:
(a) $b(n, 1)$
(b) $b(n, x)$
(c) $b(n, p)$
(d) $b(n, q)$

Q. 32 A family of parametric distribution in which mean is equal to variance is:
(a) binomial distribution
(b) gamma distribution
(c) normal distribution
(d) Poisson distribution

Q. 33 A family of parametric distributions in which mean is always greater than its variance is:
(a) binomial distribution
(b) geometric distribution
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 34 A family of parametric distributions having mean <, =, > variance is:
(a) gamma distribution
(b) exponential distribution
(c) logistic distribution
(d) all the above

Q. 35 The family of parametric distributions which has mean always less than variance is:
(a) beta distribution
(b) lognormal distribution
(c) Weibull distribution
(d) negative binomial distribution

Q. 36 The family of parametric distributions, for which the mean and variance does not exist, is:
(a) Polya's distribution
(b) Cauchy distribution
(c) negative binomial distribution
(d) normal distribution

Q. 37 A family of parametric distributions which has mean always less than its variance is:
(a) negative binomial distribution
(b) Polya's distribution
(c) beta distribution of second kind
(d) hypergeometric distribution

Q. 38 The family of parametric distributions for which moment generating function does not exist is:
(a) student t-distribution
(b) Pareto distribution
(c) F-distribution
(d) all the above

Q. 39 The distribution possessing the memoryless property is:
(a) gamma distribution
(b) geometric distribution
(c) hypergeometric distribution
(d) all the above

Q. 40 The distribution in which the probability at each successive draw varies is:
(a) hypergeometric distribution
(b) geometric distribution
(c) binomial distribution
(d) discrete uniform distribution

Q. 41 In hypergeometric distribution, H.G. (N, k, n), if $N \to \infty, \frac{k}{N} \to p$, the hypergeometric distribution reduces to:
(a) binomial distribution

(b) geometric distribution
(c) normal distribution
(d) none of the above

Q. 42 Negative binomial distribution, $nb\ (x;\ r,\ p)$ for $r = 1$ reduces to:
(a) binomial distribution
(b) Poisson distribution
(c) hypergeometric distribution
(d) geometric distribution

Q. 43 The distribution for which the mode does not exist is:
(a) normal distribution
(b) t-distribution
(c) continuous rectangular distribution
(d) F-distribution

Q. 44 For a normal curve, the Q.D., M.D. and S.D. are in the ratio
(a) 5 : 6 : 7
(b) 10 : 12 :15
(c) 2 : 3 : 4
(d) none of the above

Q. 45 The mode of the geometric distribution $\left(\frac{1}{2}\right)^X$ for $X = 1, 2, \ldots$ is:
(a) 1
(b) 0
(c) 1/2
(d) does not exist

Q. 46 The moment generating function of Bernouli distribution is:
(a) $(q + pe^t)^n$
(b) $(q + pe^t)^{-n}$
(c) $(q + pe^t)$
(d) $(q + pe^{-t})$

Q. 47 If $X \sim N(\mu, \sigma^2)$, the points of inflexion of normal distribution curve are:
(a) $\pm \mu$
(b) $\mu \pm \sigma$
(c) $\sigma \pm \mu$
(d) $\pm \sigma$

Q. 48 If $X \sim N(\mu, \sigma^2)$, the maximum probability at the point of inflexion of normal distribution is:
(a) $\frac{1}{\sqrt{2\pi}} e^{1/2}$
(b) $\frac{1}{\sqrt{2\pi}} e^{-1/2}$
(c) $\frac{1}{\sigma\sqrt{2\pi}} e^{-1/2}$
(d) $\frac{1}{\sqrt{2\pi}}$

Q. 49 An approximate relation between Q.D. and S.D. of normal distribution is:
(a) 5 Q.D. = 4 S.D.
(b) 4 Q.D. = 5 S.D.
(c) 2 Q.D. = 3 S.D.
(d) 3 Q.D. = 2 S.D.

Q. 50 An approximate relation between M.D. about mean and S.D. of a normal distribution is:
(a) 5 M.D. = 4 S.D.
(b) 4 M.D. = 5 S.D.
(c) 3 M.D. = 3 S.D.
(d) 3 M.D. = 2 S.D.

Q. 51 Pearson's constants for a normal distribution with mean μ and variance σ^2 are:
(a) $\beta_1 = 3, \beta_2 = 0, \gamma_1 = 0, \gamma_2 = -3$
(b) $\beta_1 = 0, \beta_2 = 3, \gamma_1 = 0, \gamma_2 = 0$
(c) $\beta_1 = 0, \beta_2 = 0, \gamma_1 = 0, \gamma_2 = 3$
(d) $\beta_1 = 0, \beta_2 = 3, \gamma_1 = 0, \gamma_2 = 3$

Q. 52 The area under the standard normal curve beyond the lines $z = \pm 1.96$ is:
(a) 95 per cent
(b) 90 per cent
(c) 5 per cent
(d) 10 per cent

Q. 53 X is a binomial variate with parameters n and p. If $n = 1$, the distribution of X reduces to:
(a) Poisson distribution
(b) binomial distribution itself
(c) Bernoulli distribution
(d) discrete uniform distribution

Q. 54 If a random variable X has the following probability distribution:

x:	-1	-2	1	2
Prob:	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{3}$

then the expected value of X is:

(a) 3/2
(b) 1/6
(c) 1/2
(d) none of the above

Q. 55 A box contain 12 items out of which 4 are defective. A person selects 6 items from the box. The expected number of defective items out of his selected items is:

(a) 2
(b) 3
(c) 3/2
(d) none of the above

Q. 56 If X is a normal variate with mean 20 and variance 64, the probability that X lies between 12 and 32 is:

(a) 0.4332
(b) 0.1189
(c) 0.7475
(d) 0.5

$$\begin{bmatrix} \text{Given:} & Z: & -1.0 & 1.5 \\ & \phi(z): & 0.3143 & 0.4332 \end{bmatrix}$$

Q. 57 For the normal variate X in Q. No. 56, the percentage of items between 24 and 44 is:

(a) 30.72
(b) 19.27
(c) 69.02
(d) 30.98

$$\begin{bmatrix} \text{Given:} & Z: & 0.5 & 3.0 \\ & \phi(z): & 0.1915 & 0.4987 \end{bmatrix}$$

Q. 58 If Z is a standard normal deviate, the proportion of items lying between $Z = -0.5$ and $Z = -3.0$ is:

(a) 0.4987
(b) 0.1915
(c) 0.3072
(d) 0.3098

Q. 59 If X is a normal variate representing the income in Rs. per day with mean = 50 and S.D. = 10. If the number of workers in a factory is 1200, then the number of workers having income more than Rs. 62.00 per day is:

(a) 462
(b) 138
(c) 738
(d) none of the above

[Given: $z = 1.2$, $\phi(z) = 0.3849$]

Q. 60 Assuming the distribution of diameters of shafts normal with mean = 5 and S.D. = 0.05. The tolerance limit of shafts is 4.90 to 5.10 cm. In a consignment of 200 shafts, the number of shafts out of tolerance limits is:

(a) 5
(b) 191
(c) 9
(d) 20

[Given: $z = 2.0$, $\phi(z) = 0.4772$]

Q. 61 Assuming that the height of students is distributed as $N(\mu, \sigma^2)$. Out of a large number of students, 5 per cent are above 72 inches and 10 per cent are below 60 inches. the mean and S.D. of the normal distribution are:

(a) $\mu = 0, \sigma = 1$
(b) $\mu = 65, \sigma = 5$
(c) $\mu = 66, \sigma = 4$
(d) $\mu = 65, \sigma = 4$

$$\begin{bmatrix} \text{Given: } \phi(z_1) = 0.45, & z_1 = 1.64 \\ \phi(z_2) = 0.40, & z_2 = 1.28 \end{bmatrix}$$

Q. 62 Probability mass function for a binomial distribution with usual notations is:

(a) $\binom{n}{X} p^n q^{n-X}$

(b) $\binom{n}{X} p^n q^X$

(c) $\binom{n}{X} p^{n-X} q^X$

(d) none of the above.

Q. 63 With usual notations, the probability of hypergeometric variate X is given as:

(a) $\binom{k}{X}\binom{N-k}{n-X}\Big/\binom{N}{X}$

(b) $\binom{n}{k}\binom{N-k}{n-X}\Big/\binom{N}{n}$

(c) $\binom{k}{X}\binom{N-k}{n-X}\Big/\binom{N}{n}$

(d) $\binom{n}{X}\binom{N-k}{n-X}\Big/\binom{N}{n}$

Q. 64 The probability mass function for the negative binomial distribution with parameters r and p is:

(a) $\binom{X+r-1}{r-1}p^r q^X$

(b) $\binom{-r}{X}(-1)^X p^r q^X$

(c) $\binom{-r}{X}p^r(-q)^X$

(d) all the above

Q. 65 Negative binomial distribution reduces to Polya's distribution if we substitute:

(a) $r=\frac{1}{\beta}, p=\frac{1}{1+\beta\mu}$

(b) $r=\beta, p=1+\beta\mu$

(c) $r=\frac{1}{\beta}, p=1+\beta\mu$

(d) all the above

Q. 66 In the Polya's distribution

$$p(X)=\binom{-r}{X}\left(\frac{1}{1+\beta\mu}\right)^{1/\beta}\left(\frac{\beta\mu}{1+\beta\mu}\right)^X$$

if we put $\beta = 1$, Polya's distribution reduces to:

(a) negative binomial distribution
(b) geometric distribution
(c) Poisson distribution
(d) none of the above

Q. 67 If $X \sim N$ (5, 1), the probability density function for the normal variate X is:

(a) $\frac{1}{5\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-1}{5}\right)^2}$

(b) $\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X-1}{5}\right)^2}$

(c) $\frac{1}{5\sqrt{2\pi}}e^{-\frac{1}{2}X^2}$

(d) $\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}(X-5)^2}$

Q. 68 If $X \sim N(8, 64)$, the standard normal deviate Z will be:

(a) $Z=\frac{X-64}{8}$

(b) $Z=\frac{X-8}{64}$

(c) $Z=\frac{X-8}{8}$

(d) $Z=\frac{8-X}{8}$

Q. 69 The characteristic function of the normal distribution of a normal variate $X \sim N$ (μ, σ^2) is:

(a) $e^{\mu t-\frac{1}{2}\sigma^2 t^2}$

(b) $e^{-i\mu t+\frac{1}{2}\sigma^2 t^2}$

(c) both (a) and (b)
(d) none of (a) and (b)

Q. 70 The probability density function for beta type II distribution with parameters $\alpha, \beta > 0$ is:

(a) $\frac{X^{\alpha-1}}{(1+X)^{\alpha+\beta}}$ for $X > 0$

(b) $\frac{1}{B(\alpha,\beta)}\cdot\frac{X^{\beta-1}}{(1+X)^{\alpha+\beta}}$ for $0 \le X \le 1$

(c) $\frac{1}{B(\alpha,\beta)} \cdot \frac{X^{\alpha-1}}{(1+X)^{\alpha+\beta}}$ for $0 \le X \le \infty$

(d) $\frac{1}{B(\alpha,\beta)} \cdot \frac{X^{\alpha-1}}{(1-X)^{\alpha+\beta}}$ for $0 < X < \infty$

Q. 71 The probability density function for beta distribution of first kind with parameters $m, n > 0$ is:

(a) $\frac{1}{B(m,n)} X^{m-1}(1+X)^{n-1}; 0 < X < 1$

(b) $\frac{1}{B(n,m)} X^{m-1}(1-X)^{n-1}; 0 < X < 1$

(c) $\frac{1}{B(m,n)} X^{m-1}x^{n}; 0 < X < 1$

(d) $\frac{1}{B(m,n)} X^{m-1}(1-X)^{n-1}; 0 < X < 1$

Q. 72 If X is a Poisson variate with parameter μ, the moment generating function of Poisson variate is:

(a) $e^{\mu t} - 1$

(b) $e^{\mu(e^{t}-1)}$

(c) $e^{\mu(e^{it}-1)}$

(d) $e^{i\mu(e^{t}-1)}$

Q. 73 The characteristic function of the binomial distribution for the binomial variate $X \sim b\,(n, p)$ is:

(a) $(q + pe^{it})$

(b) $(p + qe^{it})^n$

(c) $(p + qe^{t})^n$

(d) $(q + pe^{it})^n$

Q. 74 If $X \sim$ Expo (5), the probability density function of X is:

(a) $5e^{-5X}$ for $X > 0$

(b) e^{-5X} for $X > 0$

(c) $5e^{-X}$ for $X > 0$

(d) $\frac{1}{5}e^{-5x}$ for $X > 0$

Q. 75 The characteristic function of Laplace exponential distribution $L\,(\lambda)$ is:

(a) $e^{\mu t}\frac{\lambda^2}{\lambda^2 + t^2}$

(b) $e^{i\mu t}\frac{\lambda^2}{\lambda^2 - t^2}$

(c) $e^{i\mu t}\frac{\lambda^2}{\lambda^2 + t^2}$

(d) $e^{\mu t}\frac{\lambda^2}{(\lambda^2 - t^2)}$

Q. 76 The probability density function for Laplace variate $X \sim L\,(\mu, \lambda)$ is:

(a) $\frac{1}{2}e^{-\lambda|X-\mu|}$

(b) $\frac{1}{2}\mu\, e^{-\lambda|X-\mu|}$

(c) $\frac{1}{2}\lambda\, e^{\lambda|X-\mu|}$

(d) $\frac{1}{2}\lambda\, e^{-\lambda|X-\mu|}$

Q. 77 Laplace distribution curve in respect to tails is:

(a) not skewed
(b) positive skewed
(c) negatively skewed
(d) not definite

Q. 78 Laplace distribution curve with regard to peakedness is:

(a) more peaked than normal
(b) less peaked than normal
(c) adequately peaked
(d) depends on the values of its parameters

Q. 79 The moment generating function for geometric distribution with parameter p is:

(a) $p\,(1 - qe^{t})$
(b) $p\,(1 - qe^{it})$
(c) $p/(1 - qe^{it})$
(d) $p/(1 - qe^{t})$

Q. 80 The distribution function of a continuous uniform distribution of a variable X lying in the interval (a, b) is:

(a) $\dfrac{1}{b-a}$

(b) $\dfrac{X-a}{b-a}$

(c) $\dfrac{b-a}{X-a}$

(d) $\dfrac{X-b}{b-a}$

Q. 81 If $X \sim C\,(-2, 3)$, the probability density function of the variate X is:

(a) $\dfrac{1}{3\pi\left\{1-\left(\dfrac{X+2}{3}\right)^2\right\}}$ for $-\infty < X \le \infty$

(b) $\dfrac{1}{\pi\left\{1+\left(\dfrac{X+2}{3}\right)^2\right\}}$ for $-\infty < X \le \infty$

(c) $\dfrac{1}{3\pi\left\{1+\left(\dfrac{X+2}{3}\right)^2\right\}}$ for $-\infty < X < \infty$

(d) all the above

Q. 82 The characteristic function of the Cauchy distribution $X \sim C\,(\alpha, \beta)$ is:

(a) $e^{i\alpha t - \beta t}$

(b) $e^{\alpha t - i\beta t}$

(c) $e^{i\,(\alpha t - \beta t)}$

(d) $e^{i\alpha t - \beta\,|\,t|}$

Q. 83 If $X_1, X_2, \ldots, X_n$ are n i.i.d. $C\,(\alpha, \beta)$ variates, the mean of X_i's is distributed as:

(a) $C\left(\dfrac{\alpha}{n}, \dfrac{\beta}{n}\right)$

(b) $C\left(\alpha, \dfrac{\beta}{\sqrt{n}}\right)$

(c) $C\,(\alpha, \beta)$

(d) $C\,(n\,\alpha, n\,\beta)$

Q. 84 The probability density function of a random variable X distributed as $\gamma\,(n)$ is:

(a) $\dfrac{1}{\Gamma n} X^{n-1} e^{-X}$

(b) $\Gamma n\, X^{n-1} e^{X}$

(c) $\dfrac{1}{\Gamma n} (1-X)^{n-1} e^{-X}$

(d) $\dfrac{1}{\Gamma n} X^{n-1} e^{-1/X}$

Q. 85 The characteristic function of the distribution $\gamma\,(\alpha, n)$ is:

(a) $\left(1-\dfrac{it}{\alpha}\right)^{-n}$

(b) $\left(\dfrac{\alpha}{\alpha - it}\right)^{n}$

(c) $\left(\dfrac{1}{1-\dfrac{it}{\alpha}}\right)^{n}$

(d) any of the above

Q. 86 If X and Y are two gamma variate $\gamma\,(n_1)$ and $\gamma\,(n_2)$, the distribution of $\dfrac{X}{Y}$ is:

(a) $\beta_I\,(n_1, n_2)$

(b) F_{n_1, n_2}

(c) $\beta_{II}\,(n_1, n_2)$

(d) $\gamma\,(n_1 + n_2)$

Q. 87 The characteristic function of beta distribution of first kind, *i.e.*, $\beta_I\,(\alpha, \beta)$ is:

(a) $\dfrac{1}{B(\alpha,\beta)} \displaystyle\sum_{j=0}^{\infty} (it)^j B(\alpha + j; \beta)$

(b) $\dfrac{1}{B(\alpha,\beta)} \displaystyle\sum_{j=0}^{\infty} \dfrac{(it)^j}{j!} B(\alpha, \beta + j)$

(c) $\frac{1}{B(\alpha,\beta)}\sum_{j=0}^{\infty}\frac{t^j}{j!}(\alpha+j,\beta)$

(d) $\frac{1}{B(\alpha,\beta)}\sum_{j=0}^{\infty}\frac{(it)^j}{j!}(\alpha+j,\beta)$

Q. 88 If X is a r.v., the probability density function of the variable $\log_e x \sim N(\mu, \sigma^2)$ is:

(a) $\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2\sigma^2}(\log_e x-\mu)^2}$

(b) $\frac{1}{X\sqrt{2\pi}}e^{-\frac{1}{2\sigma^2}(\log_e x-\mu)^2}$

(c) $\frac{1}{\sigma X\sqrt{2\pi}}e^{-\frac{1}{2\sigma^2}(\log_e x-\mu)^2}$

(d) any of the above

Q. 89 If $\log_e x \sim N(\mu_1,\sigma_1^2)$ and $\log_e y \sim N(\mu_2, \sigma_2^2)$, the variable ($\log_e x - \log_e y$) is distributed as:

(a) $N(\mu_1-\mu_2,\sigma_1^2-\sigma_2^2)$

(b) $N(\mu_1-\mu_2,\sigma_1^2+\sigma_2^2)$

(c) $N(\mu_1,\sigma_1^2)-N(\mu_2,\sigma_2^2)$

(d) none of the above

Q. 90 The variance of the logistic distribution function given as $1/[1+e^{-\frac{(X-\alpha)}{\beta}}]$ is:

(a) $\frac{\pi\beta^2}{\alpha^2}$

(b) $\frac{\pi^2\alpha^2}{\beta^2}$

(c) $\frac{\pi^2\beta^2}{\alpha^2}$

(d) $\frac{\pi^2\beta^2}{3}$

Q. 91 The mean of the Pareto distribution $f(X; X_0, \theta)$ is:

(a) $\frac{X_0}{\theta-1}$ for $\theta > 1$

(b) $\frac{\theta X_0}{\theta-1}$ for $\theta > 1$

(c) $\frac{\theta}{X_0-1}$ for $X_0 > 1$

(d) none of the above

Q. 92 The variance of the Pareto distribution $f(x; X_0, \theta)$ is:

(a) $\frac{\theta X_0^2}{\theta-2}$ for $\theta > 2$

(b) $\left(\frac{\theta X_0}{\theta-1}\right)^2$ for $\theta > 1$

(c) $\frac{\theta X_0^2}{\theta-2}-\left(\frac{\theta X_0}{\theta-1}\right)^2$ for $\theta > 2$

(d) $\frac{\theta X_0}{\theta-2}-\left(\frac{\theta X_0}{\theta-1}\right)^2$ for $\theta > 2$

Q. 93 The mean of Weibull distribution with parameters α, $\beta > 0$ is:

(a) $\beta\Gamma\left(1+\frac{1}{\alpha}\right)$

(b) $\alpha\Gamma\left(1+\frac{1}{\alpha}\right)$

(c) $\alpha\Gamma\left(1+\frac{1}{\beta}\right)$

(d) $\Gamma\left(\alpha+\frac{1}{\beta}\right)$

Q. 94 The variance of the Weibull distribution will parameters α, β is given as:

(a) $\beta^2\,\Gamma\left(1+\frac{2}{\alpha}\right)-\beta^2\left(\Gamma\left(1+\frac{1}{\alpha}\right)\right)^2$

(b) $\beta^2\left[\Gamma\left(1+\frac{2}{\alpha}\right)-\left(\Gamma\left(1+\frac{1}{\alpha}\right)\right)^2\right]$

(c) $\beta^2\Gamma\left(1+\frac{2}{\alpha}\right)-\left(\beta\Gamma\left(1+\frac{1}{\alpha}\right)\right)^2$

(d) all the above

Q. 95 Student's t-distribution was given by:
(a) G.W. Snedecor
(b) R.A. Fisher
(c) W.S. Gosset
(d) none of the above

Q. 96 Student's t-distribution curve is symmetrical about mean, it means that:
(a) odd order moments are zero
(b) even order moments are zero
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 97 If $X \sim N(0, 1)$ and $Y \sim \chi^2/n$, the distribution of the variate $X/\sqrt{Y}$ follows:
(a) Cauchy's distribution
(b) Fisher's t-distribution
(c) student's t-distribution
(d) none of the above

Q. 98 The p.d.f. of student's t-distribution based on the random sample $X_1, X_2, \ldots, X_n$ from a population $N(\mu, \sigma^2)$ is:

(a) $\dfrac{1}{B\left(\frac{1}{2},\frac{n-1}{2}\right)}\left(1+\dfrac{t^2}{n-1}\right)^{-n/2}$

(b) $\dfrac{1}{\sqrt{n-1}\,B\left(\frac{1}{2},\frac{n-1}{2}\right)}\left(1+\dfrac{t^2}{n}\right)^{-\frac{n+1}{2}}$

(c) $\dfrac{1}{\sqrt{n-1}\,B\left(\frac{1}{2},\frac{n}{2}\right)}\left(1+\dfrac{t^2}{n-1}\right)^{-\frac{n}{2}}$

(d) $\dfrac{1}{\sqrt{n-1}\,B\left(\frac{1}{2},\frac{n-1}{2}\right)}\left(1+\dfrac{t^2}{n-1}\right)^{-\frac{n}{2}}$

Q. 99 The degrees of freedom for students-t based on a random sample of size n is:
(a) $n-1$
(b) n
(c) $(n-2)$
(d) $\dfrac{n-1}{2}$

Q. 100 If the sample size $n = 2$, the student's t-distribution reduces to:
(a) normal distribution
(b) F-distribution
(c) Cauchy distribution
(d) none of the above

Q. 101 If n, the sample size is larger than 30, the student's t-distribution tends to:
(a) normal distribution
(b) F-distribution
(c) Cauchy distribution
(d) Chi-square distribution

Q. 102 The points of inflexion of t-distribution are:

(a) $\pm\sqrt{\dfrac{n}{n+1}}$

(b) $\pm\left(\dfrac{n}{n-2}\right)^{1/2}$

(c) $\pm\left(\dfrac{n}{n+2}\right)^{1/2}$

(d) $\pm\sqrt{\dfrac{n+2}{n}}$

Q. 103 Maximum height of the student's t-distribution curve at the point $t = 0$ is:

(a) $\dfrac{1}{B\left(\frac{1}{2},\frac{n-1}{2}\right)}$

(b) $\dfrac{1}{\sqrt{n-1}\,B\left(\dfrac{1}{2},\dfrac{n-1}{2}\right)}$

(c) $\dfrac{1}{\sqrt{n-1}\,B\left(\dfrac{1}{2},\dfrac{n}{2}\right)}$.

(d) $\sqrt{n-1}\,B\left(\dfrac{1}{2},\dfrac{n-1}{2}\right)$

Q. 104 For $n > 4$ and $n < 30$, the t-distribution curve with regard to peakedness is:

(a) mesokurtic
(b) platykurtic
(c) leptokurtic
(d) bimodal

Q. 105 t-distribution is used to test:

(a) the validity of a postulated value of population mean
(b) to test the significance of sample correlation coefficient
(c) to test the equality of two population means
(d) all the above

Q. 106 The value of statistic t to test a hypothetical value 20 of population mean from a sample of size 10 having its mean = 18.5 and variance = 1.21 is:

(a) -3.71
(b) -11.16
(c) 3.71
(d) -4.31

Q. 107 A random sample of 17 items from a heap of machine parts gives a mean of 42 and S.D. = 6.25. The value of statistic t to test the hypothesis that the population mean = 38 is:

(a) 2.64
(b) 6.6
(c) 2.56
(d) none of the above

Q. 108 If $Z_1, Z_2, \ldots, Z_n$ are n i.i.d. variates, the distribution of $\sum\limits_{i=1}^{n} Z_i^2$ is:

(a) student-t^2
(b) χ^2 with n d.f.
(c) χ^2 with $(n-1)$ d.f.
(d) all the above

Q. 109 The probability density function of the sum of squares of independent n normal variates $N(0, 1)$ is:

(a) $\dfrac{1}{2^{n/2}\Gamma\dfrac{n-1}{2}} e^{-\chi^2/2}\left(\chi^2\right)^{n-1}$

(b) $\dfrac{1}{2^{n/2}\Gamma\dfrac{n}{2}} e^{-\chi^2/2}\left(\chi^2\right)^{\frac{n-1}{2}}$

(c) $\dfrac{1}{2^{n/2}\Gamma\dfrac{n}{2}} e^{-\chi^2/2}\left(\chi^2\right)^{\frac{n}{2}-1}$

(d) $\dfrac{1}{2^{n}\Gamma\dfrac{n}{2}} e^{-\chi^2}\left(\chi^2\right)^{\frac{n}{2}-1}$

Q. 110 The relation between the mean and variance of χ^2 with n d.f. is:

(a) mean = 2 variance
(b) 2 mean = variance
(c) mean = variance
(d) none of the above

Q. 111 Chi-square distribution curve in respect of symmetry is:

(a) negatively skew
(b) symmetrical
(c) positively skew
(d) any of the above

Q. 112 Chi-square distribution curve with regard to bulginess is:

(a) mesokurtic
(b) leptokurtic
(c) platykurtic
(d) not definite

Q. 113 Moment generating function of the Chi-square distribution is:

(a) $(1-2it)^{n/2}$

(b) $(1-2t)^{n/2}$
(c) $(1-2it)^{-n/2}$
(d) $(1-2t)^{-n/2}$

Q. 114 Mode of the Chi-square distribution with n d.f. lies at the point:
(a) $\chi^2 = m - 1$
(b) $\chi^2 = n$
(c) $\chi^2 = n - 2$
(d) $\chi^2 = 1/(n-2)$

Q. 115 The points of inflexion of the Chi-square distribution curve lie at the points:
(a) $(n-2) \pm (n-2)^{1/2}$
(b) $(n-2) \pm \{2(n-2)\}^{1/2}$
(c) $(n-2) \pm 2(n-2)^{1/2}$
(d) $\left\{\dfrac{n}{2(n-2)}\right\}^{1/2}$

Q. 116 If X and Y are distributed as χ^2 with d.f. n_1 and n_2, respectively, the distribution of the variate X/Y is:
(a) $\beta_I\left(\dfrac{n_1}{2}, \dfrac{n_2}{2}\right)$
(b) $\beta_{II}\left(\dfrac{n_1}{2}, \dfrac{n_2}{2}\right)$
(c) χ^2 with d.f. $(n_1 - n_2)$
(d) none of the above

Q. 117 If $X \sim \chi^2_{n_1}$ and $Y \sim \chi^2_{n_2}$, the distribution of the variate $(X - Y)$ is:
(a) $\beta_I\left(\dfrac{n_1}{2}, \dfrac{n_2}{2}\right)$
(b) $\beta_{II}\left(\dfrac{n_1}{2}, \dfrac{n_2}{2}\right)$
(c) χ^2 with $(n_1 - n_2)$ d.f.
(d) all the above

Q. 118 If X and Y are both Chi-square variate with d.f. n_1 and n_2 respectively, the distribution of the variate $X/(X+Y)$ is:
(a) $\beta_I\left(\dfrac{n_1}{2}, \dfrac{n_2}{2}\right)$
(b) $\beta_{II}\left(\dfrac{n_1}{2}, \dfrac{n_2}{2}\right)$
(c) χ^2 with $\left(\dfrac{n_1 - n_2}{2}\right)$d.f.
(d) none of the above

Q. 119 Pearson's Chi-square with usual notations f_i, p_i and n for k classes in a frequency distribution is:
(a) $\chi^2 = \displaystyle\sum_{i=1}^{k} \frac{(f_i - np_i)^2}{np_i}$
(b) $\chi^2 = \displaystyle\sum_{i=1}^{k} \frac{f_i^2}{np_i} - n$
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 120 In throwing a fair die 90 times, the number of times, the spots 1, 2, ..., 6 appeared upside were as follows:

No. of spots:	1	2	3	4	5	6
Frequency:	12	16	14	20	18	10

The value of Pearson's Chi-square for the given frequency distribution is:
(a) 0.31
(b) 4.66
(c) 1.20
(d) none of the above

Q. 121 The variate $\sqrt{\chi^2_n}$ will be distributed as:
(a) Fisher's t with n d.f.
(b) Gamma distribution
(c) exponential distribution
(d) Chi-distribution

Q. 122 If $X_i \sim \chi^2_{n_i}$ for $i = 1, 2, ..., n$, the distribution of the variate $\sum_{i=1}^{n} X_i$ is:
(a) normal distribution
(b) Chi-distribution
(c) χ^2 distribution with $\Sigma\, n_i$ d.f.
(d) none of the above

Q. 123 Chi-square distribution is useful to test the:

(a) independence of attributes

(b) equality of several population correlation coefficients

(c) equality of several population variances

(d) all the above

Q. 124 Chi-square distribution is used for the test of:

(a) goodness of fit

(b) hypothetical value of population variance

(c) both (a) and (b)

(d) neither (a) nor (b)

Q. 125 The shape of Chi-square distribution curve for χ^2 with d.f. 1 or 2 is:

(a) a parabola

(b) a hyperbola

(c) a J-shaped curve

(d) a bell-shaped curve

Q. 126 The Chi-square distribution is said to be a non-central chi-square if:

(a) If each of X_i is not distributed as $N(0, 1)$ in ΣX_i^2 for $i = 1, 2, \ldots, n$.

(b) If the variate $X_i \sim N(\mu_i, 1)$ for $i = 1, 2, \ldots, n$.

(c) both (a) and (b)

(d) none of (a) and (b)

Q. 127 If each of $X_i \sim N(\mu_i, 1)$, for $i = 1, 2, \ldots, k$, the non-centrality parameter λ for non-central Chi-square distribution is:

(a) $\lambda = \sum_{i=1}^{k} \mu_i$

(b) $\lambda = \sum_{i=1}^{k} \mu_i^2$

(c) $\lambda = \frac{1}{n} \sum_{i=1}^{k} \mu_i^2$

(d) $\lambda = \frac{1}{2} \sum_{i=1}^{k} \mu_i^2$

Q. 128 If the d.f. n for the Chi-square distribution tend to infinity, the chi-square distribution tends to:

(a) Fisher's t-distribution with n d.f.

(b) normal distribution with mean n and variance $2n$

(c) both (a) and (b)

(d) none of (a) and (b)

Q. 129 The relation between statistics t and χ^2 is:

(a) $t_1^2 = \chi_\infty^2$

(b) $t_n^2 = \chi_1^2$

(c) $t_\infty^2 = \chi_1^2$

(d) $t_1^2 = \chi_1^2$

Q. 130 Fisher's z-statistic in terms of two sample variances is expressed as:

(a) $z = \log_{10}\left(\frac{s_1^2}{s_2^2}\right)$

(b) $z = \frac{1}{2}\log_{10}\left(\frac{s_1^2}{s_2^2}\right)$

(c) $z = \log_e\left(\frac{s_1^2}{s_2^2}\right)$

(d) $z = \frac{1}{2}\log_e\left(\frac{s_1^2}{s_2^2}\right)$

Q. 131 The probability density function of Fisher's z with d.f. k_1 and k_2 is:

(a) $$\frac{2k_1^{k_1/2}k_2^{k_2/2}}{B\left(\frac{k_1}{2}, \frac{k_2}{2}\right)} \frac{e^{k_1 z}}{\left(k_2 + k_1 e^{2z}\right)^{\frac{k_1+k_2}{2}}}$$

(b) $$\frac{2k_1^{k_1/2}k_2^{k_2/2}}{B\left(\frac{k_1}{2}, \frac{k_2}{2}\right)} \frac{e^{k_1 z}}{\left(k_1 + k_2 e^{2z}\right)^{\frac{k_1+k_2}{2}}}$$

(c) $$\frac{2k_1^{k_2/2}k_2^{k_1/2}}{B\left(\frac{k_1}{2}, \frac{k_2}{2}\right)} \frac{e^{k_2 z}}{\left(k_2 + k_1 e^{2z}\right)^{\frac{k_1+k_2}{2}}}$$

(d) none of the above

Q. 132 Moment generating function of Fisher's z-distribution with d.f. k_1 and k_2 is:

(a) $\left(\frac{k_2}{k_1}\right)^{t/2} \frac{\Gamma\left(\frac{k_1+t}{2}\right)\Gamma\left(\frac{k_2+t}{2}\right)}{\Gamma\frac{k_1}{2}\Gamma\left(\frac{k_2-t}{2}\right)}$

(b) $\left(\frac{k_2}{k_1}\right)^{t/2} \frac{\Gamma\left(\frac{k_1+t}{2}\right)\Gamma\left(\frac{k_2-t}{2}\right)}{\Gamma\frac{k_1}{2}\Gamma\frac{k_2}{2}}$

(c) $\left(\frac{k_2}{k_1}\right)^{t} \frac{\Gamma\left(\frac{k_1+k_2 t}{2}\right)}{\Gamma\frac{k_1}{2}\Gamma\frac{k_2}{2}}$

(d) $\left(\frac{k_2}{k_1}\right)^{t/2} \frac{\Gamma\left(\frac{k_1+t}{2}\right)\Gamma\left(\frac{k_2+t}{2}\right)}{\Gamma\left(k_1+\frac{k_2}{2}\right)}$

Q. 133 z-distribution with d.f. k_1 and k_2 has mean equal to:

(a) $\frac{1}{2}\left(\frac{1}{k_1}+\frac{1}{k_2}\right)$

(b) $\frac{1}{2}(k_1+k_2)$

(c) $\frac{1}{2}\left(\frac{1}{k_2}-\frac{1}{k_1}\right)$

(d) $2\left(\frac{1}{k_2}-\frac{1}{k_1}\right)$

Q. 134 The variance of Fisher's z-distribution with d.f. k_1 and k_2 is:

(a) $\frac{1}{2}\left(\frac{1}{k_1}+\frac{1}{k_2}-\frac{1}{k_1^2}\right)$

(b) $\frac{1}{2}\left(\frac{1}{k_1}+\frac{1}{k_1^2}-\frac{1}{k_2}-\frac{1}{k_2^2}\right)$

(c) $\frac{1}{4}\left(\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_1^2}+\frac{1}{k_2^2}\right)$

(d) $\frac{1}{2}\left(\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_1^2}+\frac{1}{k_2^2}\right)$

Q. 135 When d.f. k_1 and k_2 are large, the z-distribution tends to:
(a) normal distribution
(b) F-distribution
(c) χ^2-distribution
(d) none of the above

Q. 136 Fisher's z is closely related to:
(a) Helmert χ^2
(b) Snedecor's F
(c) Fisher's t
(d) all the above

Q. 137 The relation between Snedecor's F and Fisher's z is:

(a) $z=\frac{1}{2}\log_e(F)$

(b) $F=e^{2z}$
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 138 F-distribution was invented by:
(a) R.A. Fisher
(b) G.W. Snedecor
(c) W.S. Gosset
(d) all the above

Q. 139 The variate F with usual notations is defined as:

(a) $F=\frac{\chi_1^2/\nu_1}{\chi_2^2/\nu_2}$

(b) $F=\frac{s_1^2}{s_2^2}$

(c) e^{2z}
(d) all the above

Q. 140 The parameters of F_{ν_1,ν_2}-distribution are:

(a) ν_1 and ν_2
(b) s_1^2 and s_2^2

(c) χ_1^2 and χ_2^2

(d) 0 and ∞

Q. 141 The range of F-variate is:

(a) $-\infty$ to ∞

(b) 0 to 1

(c) 0 to ∞

(d) $-\infty$ to 0

Q. 142 The probability density function of F with usual notations is:

(a) $$\frac{(\nu_2/\nu_1)^{\nu_2/2}}{B\left(\frac{\nu_1}{2},\frac{\nu_2}{2}\right)}\frac{F^{\nu_1/2-1}}{\left(1+\frac{\nu_1}{\nu_2}F\right)^{\frac{\nu_1+\nu_2}{2}}}$$

(b) $$\frac{(\nu_1/\nu_2)^{\nu_2/2}}{B\left(\frac{\nu_2}{2},\frac{\nu_1}{2}\right)}\frac{F^{\nu_1/2-1}}{\left(1+\frac{\nu_1}{\nu_2}F\right)^{\nu_1+\nu_2}}$$

(c) $$\frac{(\nu_1/\nu_2)^{\nu_1/2}}{B\left(\frac{\nu_2}{2},\frac{\nu_1}{2}\right)}\frac{F^{\nu_1/2-1}}{\left(1+\frac{\nu_2}{\nu_1}F\right)^{\nu_1+\frac{\nu_2}{2}}}$$

(d) none of the above

Q. 143 F-distribution curve in respect of tails is:

(a) negative skew

(b) positive skew

(c) symmetrical

(d) any of the above

Q. 144 F_{ν_1,ν_2} distribution curve becomes highly positive skew when:

(a) ν_1 is less than 5

(b) ν_2 is less than 5

(c) any of ν_1 and ν_2 is less than 5

(d) ν_2 is greater than 5

Q. 145 The mode of F-distribution curve for ν_1, $\nu_2 \geq 3$ lies at the point:

(a) $F=\frac{\nu_2(\nu_1-2)}{\nu_1(\nu_2+2)}$

(b) $F=\frac{\nu_1(\nu_2-2)}{\nu_2(\nu_1+2)}$

(c) $F=\frac{\nu_1(\nu_2-2)}{\nu_2(\nu_1-2)}$

(d) $F=\frac{(\nu_1+2)\nu_1}{(\nu_2+2)\nu_2}$

Q. 146 Mean of the F-distribution with d.f. ν_1 and ν_2 for $\nu_2 \geq 3$ is:

(a) $\frac{\nu_2}{\nu_1-2}$

(b) $\frac{\nu_1}{\nu_2-2}$

(c) $\frac{\nu_1}{\nu_1-2}$

(d) $\frac{\nu_2}{\nu_2-2}$

Q. 147 Second central moment of the F_{ν_1,ν_2} – distribution is given by the formula:

(a) $\frac{2\nu_2^2(\nu_1+\nu_2-2)}{\nu_2(\nu_2-2)^2(\nu_2-4)}$

(b) $\frac{2(\nu_1+\nu_2-2)\nu_2^2}{\nu_2(\nu_2-2)(\nu_2-4)}$

(c) $\frac{2\nu_2(\nu_1+\nu_2-2)}{\nu_2(\nu_2-2)(\nu_2-4)}$

(d) $\frac{2\nu_2^2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-2)^2(\nu_2-4)}$

Q. 148 Moment generating function of F-distribution is:

(a) $\left(1+\frac{\nu_1}{\nu_2}e^{tF}\right)^{\nu_2-1}$

(b) $\left(1+\frac{\nu_2}{\nu_1}Fe^t\right)^{\frac{\nu_1+\nu_2}{2}-1}$

(c) $\left(1+\frac{\nu_2}{\nu_1}e^{tF}\right)^{\nu_1/2-1}$

(d) does not exist

Q. 149 The reciprocal property of F_{ν_1,ν_2} – distribution can be expressed as:

(a) $F_{1-\alpha;\nu_2,\nu_1} = \frac{1}{F_{\alpha;\nu_1,\nu_2}}$

(b) $P\left(F_{\nu_1,\nu_2} \geq C\right) = P\left(F_{\nu_2,\nu_1} \leq \frac{1}{C}\right)$

(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 150 If $X \sim F(m, n)$, the variable $\frac{nX}{n+mX}$ follows the distribution:

(a) $\beta_{II}(m,n)$

(b) $\beta_{II}\left(\frac{m}{2},\frac{n}{2}\right)$

(c) $\beta_{I}\left(\frac{m}{2},\frac{n}{2}\right)$

(d) $\beta_I(m, n)$

Q. 151 If $X \sim F(m, n)$, the variable $\frac{m}{n}X$ is distributed as:

(a) $\beta_{II}(m, n)$

(b) $\beta_{II}\left(\frac{m}{2},\frac{n}{2}\right)$

(c) $\beta_{I}\left(\frac{m}{2},\frac{n}{2}\right)$

(d) $\beta_I(m, n)$

Q. 152 The relation between student's-t and F-distribution is:

(a) $F_{1,1} = t_n^2$

(b) $F_{n,1} = t_1^2$

(c) $t_\infty^2 = F_{1,n}$

(d) none of the above

Q. 153 The non-central F-distribution is defined as:
(a) The ratio of two non-central χ^2
(b) The ratio of a central-χ^2 and a non-central χ^2 divided by their corresponding d.f.
(c) The ratio of a central-χ^2 and a non-central χ^2
(d) all the above

Q. 154 F-distribution is applied for:
(a) testing the equality of two population variances
(b) for testing the equality of two or more population means
(c) for testing the equality of several regression coefficients
(d) all the above

Q. 155 The distribution having the m.g.f. $\frac{1}{\left(3-2e^t\right)}$ can be identified as:
(a) negative binomial distribution
(b) geometric distribution
(c) exponential distribution
(d) none of the above

Q. 156 The given probability function,

$$f(X) = \frac{1}{2^X} \text{ for } X = 1,2,3,\ldots$$

represents:
(a) Bernoulli distribution
(b) Poisson distribution
(c) Geometric distribution
(d) All the above

Q. 157 The characteristic function of a distribution is given as:

$$e^{i\mu t}\left(1+\frac{t^2}{\lambda^2}\right)^{-1}$$

By identicality, the distribution for which it stands is:

(a) exponential distribution
(b) Poisson distribution
(c) Cauchy distribution
(d) double exponential distribution

Q. 158 Normal distribution was invented by:

(a) Laplace
(b) De-Moivre
(c) Gauss
(d) all the above

Q. 159 If a variate $X \sim \beta_I (m, n)$ where $m < 1$ and $n < 1$, the beta distribution is:

(a) bimodal with its mode at the points $X = 0$ and $X = 1$
(b) unimodal with its mode at the point $X = 1$.
(c) unimodal with its mode at the point $X = 0$
(d) mode does not exist

Q. 160 If a variate $X \sim \beta_I (m, n)$ where $m > 1, n > 1$, the mode lies at the point:

(a) $\dfrac{m}{m+n-2}$

(b) $\dfrac{m-1}{m+n-1}$

(c) $\dfrac{m-1}{m+n-2}$

(d) $\dfrac{m}{m+n}$

Q. 161 If a beta variate $X \sim \beta_I (m, n)$ where $m = 1$ and $n > 1$ the mode lies at the point:

(a) $X = 1$
(b) $X = 0$
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 162 If a beta variate $X \sim \beta_I (m, n)$ where $m = 1$, $n = 1$, the mode lies at the point(s):

(a) $X = 1$
(b) $X = 0$
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 163 The abbreviation i.i.d stands for:

(a) independent and identically distributed
(b) identically and independently distributed
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 164 If a variate $X \sim \gamma\,(\alpha, 1)$, the p.d.f. of X is same as that of:

(a) Chi-square distribution
(b) exponential distribution
(c) normal distribution
(d) Weibull distribution

Q. 165 If the moment generating function of a distribution is $(q + pe^t)^n$, the variance of the distribution is:

(a) $2n$
(b) pq
(c) npq
(d) pq/n

Q. 166 The distribution for which the moment generating function does not exist but moments exits is:

(a) Pareto distribution
(b) t-distribution
(c) F-distribution
(d) all the above

Q. 167 The distribution of which the characteristic function is not useful in finding the moments is:

(a) negative binomial distribution
(b) hypergeometric distribution
(c) geometric distribution
(d) none of the above

Q. 168 Pearson's coefficient of skewness for the negative binomial distribution

$$\binom{-r}{X} p^r (-q)^X \text{ is:}$$

(a) $\dfrac{(p+q)^2}{rpq}$

(b) $\dfrac{(P+Q)^2}{rpq}$ where $p = \dfrac{1}{Q}, q = \dfrac{P}{Q}$

(c) $\dfrac{(P+Q)^2}{PQ}$

(d) $\dfrac{(P+Q)^2}{rPQ}$

Q. 169 The name of the distribution having the characteristic function $(e^{itb} - e^{ita})/it\,(b-a)$ is:

(a) continuous rectangular distribution
(b) discrete rectangular distribution
(c) normal distribution
(d) none of the above

Q. 170 If $X \sim b\,(n_1, p_1)$ and $X_2 \sim b\,(n_2, p_2)$ the sum of the variates $(X_1 + X_2)$ is distributed as:

(a) hypergeometric distribution
(b) binomial distribution
(c) Poisson distribution
(d) none of the above

Q. 171 If X_1 and X_2 are two independent Poisson variates with parameters λ_1 and λ_2 respectively, the variable $(X_1 + X_2)$ follows:

(a) binomial distribution with parameters $(\lambda_1 + \lambda_2)$
(b) Poisson distribution with parameter $(\lambda_1 + \lambda_2)$
(c) either of (a) and (b)
(d) neither of (a) and (b)

Q. 172 The skewness of a binomial distribution will be zero if:

(a) $p < \dfrac{1}{2}$

(b) $p > \dfrac{1}{2}$

(c) $p = \dfrac{1}{2}$

(d) $p < q$

Q. 173 If a variable $Y \sim b\,(Y;\, n,\, p)$ the variable Y/n is called:

(a) a relative variate
(b) a Pseudo variate
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 174 Binomial distribution tends to Poisson distribution when:

(a) $n \to \infty, p \to 0$ and $np = \mu$ (finite)

(b) $n \to \infty, p \to \dfrac{1}{2}$ and $np \to \mu$ (finite)

(c) $n \to 0, p \to 0$ and $np \to 0$

(d) $n \to 15, p \to 0$ and $np \to 0$

Q. 175 If for a normal distribution, $Q_1 = 54.52$ and $Q_3 = 78.86$, the median of the distribution is:

(a) 12.17
(b) 39.43
(c) 66.69
(d) none of the above

Q. 176 A discrete random variable has probability mass function

$p\,(x) = kq^x\,p;\ p + q = 1;\ x = 2, 3, 4, \ldots$

The value k should be equal to,

(a) $1/q^2$
(b) $1/p$
(c) $1/q$
(d) $1/pq$

Q. 177 If a discrete random variable takes on four values − 1, 0, 3, 4 with probabilities 1/6, k, 1/4 and $1-6k$, where k is a constant, then the value of k is:

(a) 1/3
(b) 2/9
(c) 1/12
(d) 5/24

Q. 178 Let X be a continuous random variable with probability density function,

$$\begin{aligned} f\,(x) &= kx;\ 0 \le x \le 1 \\ &= k;\ 1 \le x \le 2 \\ &= 0;\ \text{otherwise} \end{aligned}$$

The value of k is equal to:

(a) 1/4
(b) 2/3
(c) 2/5
(d) 3/4

Q. 179 For the distribution function of a random variable X, $F(5) - F(2)$ is equal to:

(a) $p(2 < x < 5)$
(b) $p(2 \le x < 5)$
(c) $p(2 \le x \le 5)$
(d) $p(2 < x \le 5)$

Q. 180 If a continuous random variable X has probability density function,

$$f(x) = \frac{1}{3}; \quad -1 \le x \le 0$$
$$= \frac{2}{3}; \quad 0 \le x \le 1,$$

then $E(X^2)$ is equal to:

(a) 1/9
(b) 2/3
(c) 5/12
(d) 1/3

Q. 181 If binomial random variable has mean = 4 and variance = 3, then its third central moment μ_3 is:

(a) 1/2
(b) 5/2
(c) 3/2
(d) 7/4

Q. 182 A Poisson random variable has $\mu_4 = 2$, the value of its mean is:

(a) 1/3
(b) 2/3
(c) 1/4
(d) 3/4

Q. 183 A negative binomial variate has probability mass function,

$$f(x) = \binom{n+x-1}{x} q^x\ p^n; x = 0, 1, 2, \ldots$$

with its mean = 2 and variance = 3. The value of p is equal to

(a) 2/3
(b) 1/3
(c) 1/4
(d) 3/4

Q. 184 A random variable has uniform distribution over the interval [− 1, 3]. This distribution has variance equal to:

(a) 8/5
(b) 4/3
(c) 13/4
(d) 9/2

Q. 185 For an exponential distribution with probability density function,

$$f(x) = \frac{1}{2} e^{-x/2}; x \ge 0,$$

its mean and variance are:

(a) $\left(\frac{1}{2}, 2\right)$

(b) $\left(2, \frac{1}{4}\right)$

(c) $\left(\frac{1}{2}, \frac{1}{4}\right)$

(d) (2, 4)

Q. 186 If a variable x has probability density function,

$$f(x) = \frac{1}{\Gamma\alpha\,\beta^\alpha} x^{\alpha-1} e^{-x/\beta}; x \ge 0$$

then its variance is:

(a) $\alpha\beta^2$
(b) $\alpha^2\beta$
(c) $\alpha^2\beta^2$
(d) α/β^2

Q. 187 A normal random variable has mean = 2 and variance = 4. Its fourth central moment μ_4 will be:

(a) 16
(b) 64
(c) 80
(d) 48

Q. 188 If a random variable X has mean 3 and standard deviation 5, then, the variance of the variable $Y = 2X - 5$ is,

(a) 25
(b) 45
(c) 100
(d) 50

Q. 189 The moment generating function of a random variable X is,

$$M_x(t)=\frac{2}{5}+\frac{1}{3}e^{2t}+\frac{4}{15}e^{3t}.$$

The expected value of X is,

(a) 22/15

(b) 9/5

(c) 17/15

(d) 11/5

Q. 190 A random variable X is distributed as $F_{(4,\,7)}$, the mode of the distribution is:

(a) 7/6

(b) 21/10

(c) 7/18

(d) 8/21

Q. 191 If X_1 and X_2 are two independent χ^2-variates, which of following has also χ^2-distribution?

(a) $X_1/(X_1+X_2)$

(b) X_1+X_2

(c) X_1/X_2

(d) X_2/X_1

Q. 192 If X_1 and X_2 are two random variables having the same probability density function $f(x)=e^{-x}$ where $x>0$, the variable X_1/X_2 follows:

(a) χ^2-distribution

(b) t-distribution

(c) F-distribution

(d) β_I-distribution

Q. 193 The distribution χ_1^2 is equivalent to the distribution:

(a) F_1,∞

(b) $F_1,0$

(c) $F_\infty,1$

(d) $F_{1,1}$

Q. 194 If X_1 and X_2 are two independent χ^2-variates with n_1 and n_2 d.f. respectively, then the variable X_1/X_2 is distributed as:

(a) $F_{(n_1,n_2)}$

(b) $F_{(n_1/2,\,n_2/2)}$

(c) $\gamma(n_1/n_2)$

(d) $\beta_{II(n_1/2,\,n_2/2)}$

Q. 195 If X_1 and X_2 are two gamma variates distributed as $\gamma(n_1)$ and $\gamma(n_2)$ respectively, which of the following has $\beta_I(n_1, n_2)$ distribution?

(a) $X_1/(X_1+X_2)$

(b) X_1+X_2

(c) X_1/X_2

(d) X_1-X_2

Q. 196 If X_1 and X_2 have $\gamma(n_1)$ and $\gamma(n_2)$ distributions respectively, then the variable X_1/X_2 is distributed as:

(a) $\beta_I(n_1,n_2)$

(b) $\beta_{II}(n_1,n_2)$

(c) $F(n_1,n_2)$

(d) none of the above

Q. 197 Let X has F-distribution with (n_1, n_2) d.f. The distribution of $1/X$ will be:

(a) t-distribution with n_2 d.f.

(b) F-distribution with $\left(\frac{1}{n_1},\frac{1}{n_2}\right)$ d.f.

(c) F-distribution with (n_2, n_1) d.f.

(d) χ^2 distribution with n_1 d.f.

Q. 198 Let $X \sim F(n_1, n_2)$ then the variable $Y=1\Big/\left(1+\frac{n_1}{n_2}X\right)$ follows the distribution:

(a) $\beta_I\left(\frac{n_2}{2},\frac{n_1}{2}\right)$

(b) $\beta_{II}\left(\frac{n_2}{2},\frac{n_1}{2}\right)$

(c) $F\left(\frac{1}{n_1}, \frac{1}{n_2}\right)$

(d) $F_{(n_2, n_1)}$

Q. 199 Let $X \sim N(\mu, \sigma^2)$, then the central moments of odd order are:
(a) one
(b) zero
(c) infinite
(d) positive

Q. 200 Let X be a random variable $U(0, 1)$, then the variable $y = -2 \log X$ follows:
(a) Log-normal distribution
(b) Gamma distribution
(c) chi-square distribution
(d) exponential distribution

Q. 201 The number of parameters in a multinomial distribution having k classes and n observations is:
(a) $n + 1$
(b) $k + 1$
(c) $n - k$
(d) $n + k$

Q. 202 The variable $y = -2 \log x$, where x is distributed as $U(0, 1)$, follows:
(a) F-distribution
(b) t-distribution
(c) χ^2-distribution
(d) exponential distribution

Q. 203 The distribution type of the variable $y = -2 \sum_{i=1}^{n} \log X_i$ is same as that of the variable:
(a) $-2 \log X_i$
(b) $2 \log \left(\prod_{i=1}^{n} X_i\right)^{-1}$
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 204 The variable $Y = -2 \sum_{i=1}^{n} \log X_i$, where all X_i are i.i.d. $U(0, 1)$, follows:
(a) χ^2-distribution with n d.f.
(b) χ^2-distribution with $2n$ d.f.
(c) log-normal distribution
(d) circular distribution

Q. 205 A variable X with moment generating function $M_X(t) = \left(\frac{2}{3} + \frac{1}{3} e^t\right)$ is distributed with mean and variance as:
(a) mean $= \frac{2}{3}$, variance $= \frac{2}{9}$
(b) mean $= \frac{1}{3}$, variance $= \frac{2}{9}$
(c) mean $= \frac{1}{3}$, variance $= \frac{2}{3}$
(d) mean $= \frac{2}{3}$, variance $= \frac{1}{9}$

Q. 206 If a distribution has moment generating function $M_X(t) = \left(2 - e^t\right)^{-3}$, then the distribution is:
(a) geometric distribution
(b) hypergeometric distribution
(c) binomial distribution
(d) negative binomial distribution

Q. 207 If X is a standard normal variate, then $\frac{1}{2} X^2$ is a gamma variate with parameters:
(a) $1, \frac{1}{2}$
(b) $\frac{1}{2}, 1$
(c) $\frac{1}{2}, \frac{1}{2}$
(d) 1, 1

Q. 208 If the moment generating function of a random variable X is $\left(\frac{1}{3} + \frac{2}{3} e^t\right)$, then X is a:

(a) Bernoulli variate
(b) Poisson variate
(c) binomial variate
(d) negative binomial variate

Q. 209 If a variable X has the p.d.f., $f(x)\ \frac{1}{4}\cdot xe^{-x/2}$ for $x > 0$, then the variable X is distributed as:
(a) gamma variate
(b) chi-square variate
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 210 If a variable X has the p.d.f., $f(x)\ \frac{1}{4}\cdot xe^{-x/2}$ for $0 \le x < \infty$, then the distribution has mean and variance as:
(a) mean = 2, variance = 4
(b) mean = 1/2, variance = 1/4
(c) mean = 4, variance =2
(d) mean = 4, variance = 8

ANSWERS

SECTION-B

(1) finite (2) domain (3) $\begin{array}{cccccc} x: & 0 & 1 & 2 & 3 \\ p(x): & \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{array}$

(4) $\begin{array}{cccccc} z: & 0 & 1 & 2 & 3 & 4 \\ p(z): & \frac{1}{8} & \frac{3}{8} & \frac{1}{8} & \frac{2}{8} & \frac{1}{8} \end{array}$ (5) $\frac{1}{26}$ and $E(x) = 3.96$

(6) $\frac{3}{4}$ (7) 1 (8) unity (9) (a) 0.1 (b) 0.5 (c) 2.85

(10) (a) $\frac{5}{12}$ (b) $\frac{2}{11}$ (11) p.d.f (12) (a) $E(X) = \int_a^b xf(x)dx$ (b) $\log G = \int_a^b \log xf(x)dx$ (c) $\frac{1}{H} = \int_a^b \frac{1}{x} f(x)dx$ (d) $E(X^2) = \int_a^b x^2 f(x)dx$ (e) $\mu_2 = \int_a^b x^2 f(x)dx - \left\{\int_a^b xf(x)dx\right\}^2$ (f) M.D. $= \int_a^b |x-\mu| f(x)dx$ (g) $\int_a^M f(x)dx = \int_M^b f(x)dx = \frac{1}{2}$ (h) $\int_a^{Q_i} f(x) = \frac{i}{4}$ (i) $\int_a^{D_i} f(x)dx = \frac{i}{10}$ (j) $\int_a^{P_i} f(x)dx = \frac{i}{100}$ (13) 0.4 (14) (a) 1 (b) 1 (c) 2 (d) 4 (15) 1 (16) $\frac{3}{16}(y-3)$ (17) $\frac{1}{4\sqrt{1-y^2}}$ (18) $\frac{1}{2}$ (19) a (20) np; npq (21) greater than (22) $\frac{5}{16}$ (23) $\frac{224}{729}$ (24) $\frac{27}{8}e^{-3}$ (25) $\frac{63}{512}$ (26) geometric (27) related (28) lack of memory (29) 1 (30) greater than (31) $r = 1$ (32) $\beta = 1$ (33) dependent (34) hypergeometric distribution (35) fixed (36) $N \to \infty$ and $\frac{k}{N} \to p$ (37) equal (38) symmetrical (39) does not exist (40) $\frac{b-a}{4}$ (41) $\frac{(b-a)^2}{12}$ (42) exist (43) (a) $\frac{\theta}{2}$ (b) $\frac{\theta^2}{12}$ (c) 0 (d) $\frac{\theta^4}{80}$ (e) 0 (f) $\frac{9}{5}$ (44) one (45) symmetrical (46) more (47) U (0, 1) (48) twice (49) leptokurtic (50) $\frac{1}{\lambda}\log 2$ (51) N (0, 1) (52) $X = \mu$ (53) $\frac{1}{\sigma\sqrt{2\pi}}e^{-1/2}$ (54) $N(n\mu, n\sigma^2)$ (55) $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$ (56) (a) unimodal (b) not skewed (c) Mesokurtic (57) 10 : 12 : 15 (58) $\sigma\sqrt{\frac{2}{\pi}}$ or $\frac{4}{5}\sigma$ (59) zero (60) $e^{\mu t + \frac{1}{2}\sigma^2 t^2}$ (61) $e^{i\mu t - \frac{1}{2}\sigma^2 t^2}$ (62) $N\left(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2\right)$ (63) N (21.0, 18.5) (64) (a) 0.62% (b) 10.56% (c) 38.29% (d) 24.17% (e) 29.63% (65) (a) 3 (b) 53

(c) 191 (d) 121 (e) 148 (66) $e^{\mu+\frac{1}{2}\sigma^2}$ (67) $e^{2\mu+\sigma^2}e^{\sigma^2-1}$ (68) elongated (69) more (70) lognormal

(71) $\log-N\left(\mu_1-\mu_2,\sigma_1^2+\sigma_2^2\right)$

(72) $\log-N\left(\mu,\dfrac{\sigma^2}{n}\right)$

(73) $1\Big/\left[5\pi\left\{1+\left(\dfrac{X-2}{5}\right)^2\right\}\right]$

(74) not existing (75) not existing (76) non-existing (77) $X = 5$ (78) No (79) $X = 2$ (80) C (α, β) (81) C (0, 1) (82) are not (83) $\alpha = 1$, $\beta = 1$ (84) $\dfrac{\alpha}{\alpha+\beta}$ (85) $\dfrac{\alpha}{\beta-1}$ (86) α and β (87) mean = α × variance (88) positive skewness (89) leptokurtic (90) $\gamma\ (n_1 + n_2)$ (91) $\beta_I\ (n_1, n_2)$ (92) incomplete gamma function (93) α (94) symmetrical (95) modelling problem (96) reliability theory (97) corrosion (98) $(0.8)^{25}$ (99) binomial; n and q (100) $4e^{-2}$ (101) 2 (102) $\sigma = 12.2$ (103) 4 (104) hyperbolic (105) $\mu_3' = 14\mu_2'$ (106) $\mu_3 = 4(\mu_2 + 8\mu_1)$ (107) gamma distribution (108) (a) $\beta_2\left(\dfrac{n_1}{2},\dfrac{n_2}{2}\right)$(b) F-distribution with d.f. (n_1, n_2) (c) χ^2 will d.f. $(n_1 + n_2)$ (d) $\beta_1\left(\dfrac{n_1}{2},\dfrac{n_2}{2}\right)$(e) χ^2 with $(n_1 - n_2)$ d.f. (109) half (110) four times (111) $\left(\dfrac{2}{n}\right)^{1/2}$ (112) leptokurtic (113) normal distribution (114) Chi (115) n (116) non-central chi-square (117) Chi-square with $(n - 1)$ d.f. (118) Chi-square with $2n$ d.f. (119) $\displaystyle\sum_{i=1}^{k}\frac{(f_i-np_i)^2}{np_i}$ (120) $n \to \infty$ (121) symmetric (122) $n \le 4$ (123) leptokurtic (124) $1/\sqrt{n-1}\,B\left(\dfrac{1}{2},\dfrac{n-1}{2}\right)$(125) Cauchy distribution (126) t-test (127) t-test (128) χ^2-test (129) χ^2-test (130) χ^2-test (131) Fisher's-t (132) Snedecor's-F (133) $F = e^{2z}$ (134) $\dfrac{1}{2}\left(\dfrac{1}{\nu_2}-\dfrac{1}{\nu_1}\right)$ (135) less than (136) does not exist (137) $F_{\alpha;\nu_1,\nu_2} = \dfrac{1}{F_{1-\alpha;\nu_2,\nu_1}}$

(138) $\nu_1 F = \chi^2$ (139) normal (140) χ^2 with $(n - 1)$ d.f. (141) N(0, 1) (142) range (143) hypergeometric (144) exponential (145) Pseudo binomial (146) circular distribution (147) n, p_1, p_2, ..., p_k (148) n (149) chi-square (150) exponential distribution

SECTION-C

(1) c (2) a (3) a (4) c (5) d (6) b
(7) a (8) c (9) b (10) c (11) c (12) a
(13) c (14) c (15) a (16) b (17) b (18) d
(19) a (20) c (21) b (22) d (23) a (24) c
(25) d (26) c (27) a (28) b (29) d (30) b
(31) d (32) d (33) c (34) d (35) d (36) b
(37) a (38) d (39) b (40) a (41) a (42) d
(43) c (44) b (45) a (46) c (47) b (48) c
(49) d (50) a (51) b (52) c (53) c (54) b
(55) a (56) c (57) a (58) c (59) b (60) c
(61) d (62) d (63) c (64) d (65) a (66) b
(67) d (68) c (69) d (70) c (71) d (72) b
(73) d (74) a (75) c (76) d (77) a (78) a
(79) d (80) b (81) c (82) d (83) c (84) a
(85) d (86) c (87) d (88) c (89) b (90) d
(91) b (92) c (93) a (94) d (95) c (96) a
(97) b (98) d (99) a (100) c (101) a (102) c
(103) b (104) c (105) d (106) d (107) a (108) b
(109) c (110) b (111) c (112) b (113) d (114) c
(115) b (116) b (117) c (118) a (119) c (120) b
(121) d (122) c (123) d (124) c (125) b (126) c
(127) d (128) b (129) c (130) d (131) a (132) b
(133) c (134) d (135) a (136) b (137) c (138) b
(139) d (140) a (141) c (142) d (143) b (144) b
(145) a (146) d (147) d (148) d (149) c (150) c
(151) b (152) d (153) b (154) d (155) b (156) c
(157) d (158) d (159) a (160) c (161) b (162) d
(163) a (164) b (165) c (166) d (167) b (168) d
(169) a (170) b (171) b (172) c (173) c (174) a

(175) c	(176) a	(177) c	(178) b	(179) c	(180) d
(181) c	(182) b	(183) a	(184) b	(185) d	(186) a
(187) d	(188) c	(189) a	(190) c	(191) b	(192) c
(193) a	(194) d	(195) a	(196) b	(197) c	(198) a
(199) b	(200) c	(201) b	(202) c	(203) c	(204) b
(205) b	(206) d	(207) a	(208) a	(209) c	(210) d

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Chapter **8**

Bivariate Random Variable and Distributions

SECTION-A

Short Essay Type Questions

Q. 1 What do you understand by bivariate random variable?

Ans. In many situations, a scientist has to measure more than one factor or character at a time on the same item or some adjutant items. These measurements may have different units of measurements and moreover the range or the domain of the variables may also be different. If two variables are studied simultaneously in respect of their distribution, then they are known as bivariate random variable. Two or more variables studied concurrently with regard to their distributions are known in general *multivariate studies*. The variables may be discrete or continuous or the *mixture* of the two. But in this chapter we have dealt with the situations when both the variables are either discrete or continuous. Just like univariate variables, the two variables have also various distributions. Their properties have also been studied which behave differently than univariates because of the impact of one variable over the other. The variables should be defined over the same sample space. In this situation the variables can be jointly discrete or jointly continuous. Here, we are dealing with two-dimensional spaces or planes.

Q. 2 Define bivariate discrete random variable and give an illustration.

Ans. A random variable (X, Y) is said to be a two-dimensional discrete random variable if it can take only a countable number of points (x, y) in a two-dimensional space. The random variable (X, Y) is also said to be *joint discrete random variable.*

As for illustration, let us consider an experiment of tossing a coin three times and take the variable X as the number of tails in first tossing and Y the number of heads in three tosses. In this problem our bivariate random variable is the ordered pair (X, Y). Also both the variables X and Y are discrete. The possible outcomes are:

HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.

The pairs of variate values (x, y) are,

(x, y) : (0, 3), (0, 2), (0, 2), (0, 1), (1, 2), (1, 1), (1, 1), (1, 0).

Q. 3 Define bivariate continuous random variable.

Ans. A two-dimensional random variable (X, Y) is called a bivariate continuous random variable if there exists a function $f(x, y) \geq 0$ such that for $-\infty < x, y < \infty$, the distribution function $F(x, y)$ of (X, Y) is given as

$$F(x,y)=\int_{-\infty}^{x}\int_{-\infty}^{y}f(u,v)\,du\,dv.$$

The function $f(x, y)$ is called the joint probability density function of (X, Y).

As an example, the age X of husband and age Y of wife at marriage when treated jointly, they represent the bivariate continuous random variable.

Q. 4 Explain joint distribution function. What are its properties?

Ans. Let X and Y be two random variables defined over the same probability space. The joint *cumulative distribution function* (c.d.f) of X and Y is denoted by $F_{X,Y}(x, y)$ and is defined as the $P(X \le x, Y \le y)$ for all (x, y) in the $X - Y$ plane. Its domain is just the $X - Y$ plane. Cumulative distribution function is often known as *distribution function* only.

Properties of joint distribution function:

(i) $\lim\limits_{\substack{x\to\infty\\ y\to\infty}} F(x,y)=F(\infty,\infty)=1.$

(ii) $\lim\limits_{x\to-\infty} F(x,y)=F(-\infty,y)=0$ for all y.

(iii) $\lim\limits_{y\to-\infty} F(x,y)=F(x,-\infty)=0$ for all x.

(iv) $F(x,y)$ is right continuous in each argument,

i.e., $\lim\limits_{h\to 0} F(x+h,y)=\lim\limits_{h\to 0}(x,y+h)=F(x,y).$

(v) $P(x_1\le x\le x_2, y_1<y\le y_2)=F(x_2,y_2)$

$-F(x_1,y_2)-F(x_2,y_1)+F(x_1,y_1)\ge 0.$

(vi) $0\le F(x,y)\le 1.$

Q. 5 Discuss joint discrete density (probability mass) function.

Ans. The joint density function of X and Y is said to be discrete if there exists a non-negative function $p(x, y)$ such that it vanishes everywhere except at a finite or countably infinite number of mass points. The joint probability distribution describes the relationship between x and y and assigns probabilities to all possible outcomes (x, y). Thus, $p(x, y) = P(X = x, Y = y)$ for all x, y in the $X - Y$ plane. $p(x, y)$ is called the joint probability mass function (p.m.f.). Some writers use $f(x, y)$ in place of $p(x, y) \cdot p(x, y)$ holds the following properties:

(i) $p(x_i, y_i)\ge 0$ for all i, j

(ii) $F(x,y)=\sum\limits_{x_i\le x}\sum\limits_{y_i\le y}p(x_i,y_i)$

(iii) $\sum\limits_{i}\sum\limits_{j}p(x_i,y_i)=1$ for $i,j=1,2,\ldots$

Q. 6 Explain joint probability density function of the continuous random variables X and Y.

Ans. A function $f_{X,Y}(x.y)$ is a continuous function of x, y which gives the joint probability distribution of X and Y such that the probability of the variates falling within the infinitesimal rectangular region bounded by the lines $x\pm\frac{1}{2}dx$ and $y\pm\frac{1}{2}dy$ is expressed as $f_{X,Y}(x, y)\, dx, dy$. This function is called the joint probability density function (p.d.f.). Symbolically,

$$P\left(x-\frac{1}{2}dx\le X\le x+\frac{1}{2}dx, y-\frac{1}{2}dy\le Y\le y+\frac{1}{2}dy\right)$$
$$=f_{X,Y}(x,y)\,dx\,dy$$

Often the suffix X, Y to f is omitted.

Q. 7 Define marginal probability functions.

Ans. If X and Y are joint discrete random variables with probability function $p(x, y)$, the individual distribution of either X or Y is called the marginal distributions of X or Y and/or denoted as $p_X(x)$, and $p_Y(y)$, respectively. Thus,

$$p_X(x)=P(X=x)=\sum_y p(x,y)=\sum_y p(X=x,Y=y)$$

Similarly,

$$p_Y(y)=P(Y=y)=\sum_x p(x,y)=\sum_x p(X=x,Y=y)$$

Q. 8 Define marginal probability density function.

Ans. If X and Y are jointly continuous random variables and their joint p.d.f. is $f_{X,Y}(x, y)$, the marginal probability density function of the variable X is given as:

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$$

Similarly, the marginal probability density function of the variable Y is given as:

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx$$

Q. 9 What do you understand by conditional random variable?

Ans. If (X, Y) is a bivariate random variable, the consideration of the variable Y for given value of X is known as the conditional variable Y for given X and is denoted as $Y|X = x$ or $Y|x$.

Similarly, the conditional X for given Y can be defined and denoted as $X|Y = y$ or $X|y$.

Q. 10 Define conditional probability mass function.

Ans. Let X and Y be two discrete random variables with their p.m.f. $p_{X,\ Y}(x,\ y)$. The conditional probability mass function of Y given X usually denoted as $p_{Y|x}(y|x)$ or $p(y|x)$ is defined as the ratio of the joint probability mass function to the marginal probability of X, *i.e.*,

$$p_{Y|x}(y|x) = \frac{p_{X,Y}(x,y)}{p_X(x)}$$

Similarly, the conditional probability mass function of X given Y can be given as,

$$p_{X|y}(x|y) = \frac{p_{X,Y}(x,y)}{p_Y(y)}$$

Also, the conditional discrete distribution function,

$$F_{Y|x}(y|x) = \sum_{y_j \le y} p_{Y|x}(y_j|x) = P(Y \le y|X = x)$$

and

$$F_{X|y}(x|y) = \sum_{x_i \le x} p_{X|y}(x_i|y) = P(X \le x|Y = y).$$

Note: The suffix X, Y or $Y \mid x$ or $X \mid y$ with p or F is often not indicated.

Q. 11 Define conditional probability density function.

Ans. If X and Y are two bivariate continuous random variables and their joint p.d.f. is $f_{X,\ Y}(x,\ y)$, the conditional probability density function of Y given $X = x$ is given as,

$$f_{Y|x}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)} \text{ for } f_X(x) > 0$$

where $f_X\ (x)$ is the marginal p.d.f. of X and is undefined at points where $f_X\ (x) = 0$.

Similarly, the conditional p.d.f. of X given $Y = y$ is given as,

$$f_{X|y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} \text{ for } f_Y(y) > 0$$

where $f_Y\ (y)$ is the marginal p.d.f. of Y and is undefined at the points where $f_Y\ (y) = 0$.

Q. 12 Explain independence of two random variables.

Ans. If X and Y are two random variables and their joint p.d.f. is $f_{X,\ Y}(x,\ y)$, the variables X and Y are independent if and only if (*iff*),

$$f_{X,Y}(x,y) = f_Y(y) \cdot f_X(x)$$

for all values, x, y of X and Y.

In terms of conditional distributions,

$$f_{Y|x}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)} = \frac{f_X(x) \cdot f_Y(y)}{f_X(x)} = f_Y(y)$$

Similarly,

$$f_{X|y}(x|y) = f_X(x).$$

Also,

$$F_{X,Y}(x,y) = F_X(x)\,F_Y(y).$$

The definition of independence remains as such for discrete case except that one has to call $f_{X,\ Y}(x,\ y)$ as joint probability mass function and $f_Y\ (y)$ and $f_X\ (x)$ as marginal probability mass function. Also, instead of f for density function, one may use p for mass functions, *i.e.*, one may use $p_{X,\ Y}(x,\ y)$, $p_X\ (x)$ and $p_Y\ (y)$ instead of $f_{X,\ Y}(x,\ y)$, $f_X\ (x)$ and $f_Y\ (y)$ respectively.

Q. 13 What do you understand by conditional expectation?

Ans. Let X and Y be two random variables having joint p.d.f. $f_{X,Y}(x, y)$ and their marginal p.d.f's be $f_X(x)$ and $f_Y(y)$. The conditional expectation of $Y \mid X = x$ is given as,

$$E(Y|x) = \int_{-\infty}^{\infty} y \frac{f_{X,Y}(x,y)}{f_X(x)} dy \text{ where } f_X(x) > 0$$

Similarly, the conditional expectation of $X \mid Y = y$ is given as,

$$E(X|y) = \int_{-\infty}^{\infty} x \frac{f_{X,Y}(x,y)}{f_Y(y)} dx \text{ for } f_Y(y) > 0.$$

For discrete random variables X and Y, the conditional expectation of Y given X is given as,

$$E(Y|x) = \sum_j y_j \frac{p_{ij}}{p_i} \text{ for } p_i > 0$$

$$= \sum_j y_j \, p(j|i)$$

Similarly, $E(X|y) = \sum_i x_i \dfrac{p_{ij}}{p_j}$ for $p_j > 0$

where, $p_{ij} = P(X = x_i, Y = y_j)$,

$$p_i = P(X = x_i), p_j = P(Y = y_j)$$

Q. 14 Give properties of conditional expectation.

Ans.

(i) $E\{E(Y|x)\} = E(Y)$

(ii) $E(XY|x) = XE(Y|x)$

(iii) $E(XY) = E\{X\,E(Y|x)\}$

(iv) IF X and Y are independent, then $E(XY) = E(X)\,E(Y)$. But the converse is not true.

(v) $E(Y \mid X = x)$ is called the regression curve of Y on X.

(vi) $E(X \mid Y = x)$ is called the regression curve of X on Y.

(vii) The conditional expectation of a function $g(x, y)$ of X, Y is,

$$E[g(x,y)|X = x] = \int_{-\infty}^{\infty} g(x,y) f_{Y|x}(y|x)\,dy$$

when X, Y are continuous.

or,

$$E[g(x,y)|Y = y] = \int_{-\infty}^{\infty} g(x,y) f_{X|y}(x|y)\,dx$$

when X, Y are continuous.

Also, $E[g(x,y)|X = x] = \sum_j g(x, y_j) \dfrac{p_{ij}}{p_i}$

when X, Y are discrete.

$$E[g(x,y)|Y = y] = \sum_i g(x_i, y) \frac{p_{ij}}{p_j}$$

when X, Y are discrete. If X and Y are independent, then

$$E[\phi_1(x)\phi_2(y)] = E[\phi_1(x)]E[\phi_2(y)]$$

Q. 15 Give conditional variance.

Ans. The conditional variance of a variable Y given $X = x$ is denoted by $V(Y|X = x)$ and is defined as,

$$V(Y|X = x) = E(Y^2|X = x) - [E(Y|X = x)]^2$$

Q. 16 Express variance of a variable X in terms of conditional variances.

Ans. If X and Y are two random variables having their joint p.d.f. $f(x, y)$, then the variance of X in terms of conditional variance is,

$$V(X) = E[V(X|y)] + V[E(X|y)]$$

Q. 17 Discuss covariance between two variables X and Y. What are its properties?

Ans. Covariance between two variables X and Y is defined,

$$\begin{aligned} \text{cov}(X, Y) &= E[\{X - E(x)\}\{Y - E(y)\}] \\ &= E[(X - \mu_X)(Y - \mu_Y)] \\ &= E(XY) - \mu_X \mu_Y = \sigma_{XY} \end{aligned}$$

Properties of covariance:

(i) Covariance between two variables is a measure of strength and direction of association between two variables.

(ii) Covariance tends to measure the linear relationship of X and Y.

(iii) If the variates X and Y are independent, the cov (X, Y) is zero.

(iv) The magnitude of cov (X, Y) does not indicate much towards the linear relationship between X and Y as it is subject to the variances of X and Y.

(v) If cov $(X, Y) = 0$, it means that the variables X and Y are uncorrelated but not necessarily independent.

Q. 18 Define conditional covariance.

Ans. If X, Y, and Z are three random variables, the covariance between X and Y given $Z = z$ is,

$$\operatorname{cov}(X,Y|z) = E\left[\{(X - E(X|z)\}\right.$$
$$\left.\{Y - E(Y|z)\}|z\right]$$

Q. 19 Express covariance between two variables for a given third variable in terms of conditional covariances.

Ans. If X, Y, and Z are three random variables, the covariance between X and Y for a given value of Z in terms of conditional covariances is,

$$\operatorname{cov}(X,Y) = E\left[\operatorname{cov}(X,Y|z)\right] + \operatorname{cov}\left[E(X|z)\ E(Y|z)\right]$$

Q. 20 Define joint moment generating function for bivariate random variables.

Ans. If X and Y are two random variables having the joint p.d.f. $f(x,y)$, then their joint moment generating function (m.g.f.) $M_{X,Y}(t_1, t_2)$ is defined as,

$$M_{X,Y}(t_1,t_2) = E\left(e^{t_1x+t_2y}\right)$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{t_1x+t_2y} f(x,y)\,dx\,dy$$

Q. 21 Define joint raw moments for bivariate distribution.

Ans. The $(r, s)^{\text{th}}$ moment of the variables X and Y about the origin is called the raw moment of bivariate distribution and is given as,

$$\mu'_{r,s} = E\left(X^r Y^s\right)$$

Also $\quad \mu'_{r,0} = E\left(X^r\right)$ and $\mu'_{0,s} = E\left(Y^s\right)$

for $r, s = 0, 1, 2, 3, \ldots$

Q. 22 Define joint central moments for the bivariate distributions.

Ans. The $(r, s)^{\text{th}}$ moment about the means μ_X, μ_Y of (X, Y) is called the central moment of bivariate distribution and is obtained as,

$$\mu_{r,s} = E\left\{(X-\mu_X)^r(Y-\mu_Y)^s\right\}$$

for $r, s = 0, 1, 2, \ldots$

Also,

$$\mu_{1,0} = E(X-\mu_X) = 0;\ \mu_{0,1} = E(Y-\mu_Y) = 0;$$
$$\mu_{2,0} = E\left\{(X-\mu_X)^2\right\} = \sigma_X^2;$$
$$\mu_{0,2} = E\left\{(Y-\mu_Y)^2\right\} = \sigma_Y^2$$
$$\mu_{11} = E\left\{(X-\mu_X)(Y-\mu_Y)\right\} = \operatorname{cov}(X,Y)$$

Q. 23 Explicate bivariate normal distribution and its properties.

Ans. The bivariate continuous random variables (X, Y) with population means μ_X and μ_Y, population variances σ_X^2 and σ_Y^2 respectively and constant ρ, which is known as the population correlation coefficient between X and Y, are said to follow *bivariate normal distribution* if their probability density function is:

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \times$$
$$e^{-\frac{1}{2(1-\rho^2)}\left\{\left(\frac{x-\mu_X}{\sigma_X}\right)^2 - 2\rho\left(\frac{x-\mu_X}{\sigma_X}\right)\left(\frac{y-\mu_Y}{\sigma_Y}\right) + \left(\frac{y-\mu_Y}{\sigma_Y}\right)^2\right\}}$$

for $-\infty < \mu_X, \mu_Y < \infty, \sigma_X^2, \sigma_Y^2 > 0$ and $0 \le \rho \le 1$.

Bivariate normal distribution is sometimes named after its inventors as *Gaussian, Laplace-Gauss* and *Bravais*.

Bivariate normal distribution has five parameters namely, $\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2$ and ρ.

If we put $\dfrac{x-\mu_X}{\sigma_X} = u$ and $\dfrac{y-\mu_Y}{\sigma_Y} = v$, then

$$g(u,v)=\frac{1}{2\pi\left(1-\rho^2\right)}e^{-\frac{1}{2\left(1-\rho^2\right)}\left(u^2-2\rho uv+v^2\right)}$$

This is known as the standardised bivariate normal distribution with parameters (0, 0, 1, 1, ρ).

Latest definition: A random vector (*X*, *Y*) is said to possess bivariate normal distribution if for any two constants *a* and *b*, Z = *aX* + *bY* is a normal variate.

Bivariate normal distribution is denoted as BVN $\left(\mu_X,\mu_Y,\sigma_X^2,\sigma_Y^2,\rho\right)$.

Properties of bivariate normal distribution:

(i) Moment generating function of bivariate normal distribution. If (*X*, *Y*) ~ BVN ($\mu_X,\mu_Y,\sigma_X^2,\sigma_Y^2,\rho)$ is given as,

$$M_{X,Y}(t_1,t_2)\doteq E\left(e^{t_1x+t_2y}\right)$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{t_1x+t_2y}f(x,y)\,dx\,dy$$

$$=e^{t_1\mu_X+t_2\mu_Y+1/2\left(t_1^2\sigma_X^2+t_2^2\sigma_Y^2+2\rho t_1t_2\sigma_X\sigma_Y\right)}$$

If (*X*, *Y*) ~ BVN (0, 0, 1, 1, ρ), then

$$M_{X,Y}(t_1,t_2)=e^{1/2\left(t_1^2+t_2^2+2\rho t_1t_2\right)}$$

If *X* and *Y* are independent, *i.e.*, if ρ = 0, then

$$M_{X,Y}(t_1,t_2)=M_X(t_1)\,M_Y(t_2)$$

(*r*, *s*)th moment about the origin is,

$$\mu'_{r,s}=\text{coeff. of }\frac{t_1^rt_2^s}{r!s!}\text{ in }M_{X,Y}(t_1,t_2)$$

$$\mu'_{1,0}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}xf(x,y)\,dx\,dy$$

$$=\int_{-\infty}^{\infty}xf_X(x)\,dx$$

$$=\mu_X$$

Similarly, $\mu'_{01}=\mu_Y$

(1, 1)th moment about mean is,

$$\mu_{11}=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-\mu_X)(y-\mu_Y)f(x,y)\,dx\,dy$$

$$=\text{cov }(X,Y)$$

$$\mu_{2,0}=\int_{-\infty}^{\infty}(x-\mu_X)^2f_X(x)\,dx$$

Similarly, $\mu_{0,2}=\int_{-\infty}^{\infty}(y-\mu_Y)^2f_Y(y)\,dy.$

Also, the correlation coefficient between *X* and *Y*,

$$\rho_{XY}=\frac{\mu_{11}}{\sqrt{\mu_{2,0}\,\mu_{0,2}}}$$

(ii) Marginal distribution *X* if (*X*, *Y*) ~ BVN (μ_X, $\mu_Y,\sigma_X,\sigma_Y,\rho)$ is given as,

$$f_Y(y)=\frac{1}{\sqrt{2\pi}\,\sigma_X}e^{-\frac{1}{2}\left(\frac{x-\mu_Y}{\sigma_X}\right)^2}$$

Similarly, the marginal distribution of *Y* is,

$$f_Y(y)=\frac{1}{\sqrt{2\pi}\,\sigma_Y}e^{-\frac{1}{2}\left(\frac{y-\mu_Y}{\sigma_Y}\right)^2}$$

Marginal density describes the probability density of one variable ignoring the other.

(iii) Conditional distribution of *Y* given *X* for the BVN $\left(\mu_X,\mu_Y,\sigma_X^2,\sigma_Y^2,\rho\right)$ distribution is,

$$f_{Y|x}(y|x)=\frac{1}{\sqrt{2\pi}\,\sigma_Y\sqrt{1-\rho^2}}$$

$$e^{-\frac{1}{2\left(1-\rho^2\right)\sigma_Y^2}\left\{(y-\mu_Y)-\rho\frac{\sigma_Y}{\sigma_Y}(x-\mu_X)\right\}^2}$$

for $-\infty<y\leq\infty$.

The right hand expression is the probability density function of the univariate normal distribution of *Y* with mean $\mu_Y+\rho\frac{\sigma_Y}{\sigma_X}(x-\mu_X)$ and variance $\sigma_Y^2\left(1-\rho^2\right)$. In other words, the mean of the conditional distribution of Y given X,

$$E(Y|x) = \mu_Y + \rho\frac{\rho_Y}{\sigma_X}(x - \mu_X)$$

and $\quad V(Y|x) = \sigma_Y^2(1-\rho^2)$

Similarly, the conditional distribution of X given Y is,

$$f_{X|y}(x|y) = \frac{1}{\sqrt{2\pi}\,\sigma_X\sqrt{1-\rho^2}}$$

$$e^{-\frac{1}{2(1-\rho^2)\sigma_X^2}\left\{(x-\mu_X)-\rho\frac{\sigma_X}{\sigma_Y}(y-\mu_Y)\right\}^2}$$

The conditional distribution of X given Y has mean,

$$E(X|y) = \mu_X + \rho\frac{\sigma_X}{\sigma_Y}(y - \mu_Y)$$

and variance, $V(X|y) = \sigma_X^2(1-\rho^2)$

The conditional density of a variate describes the probability of a variable when the value of the other variable is known.

(iv) If $(X, Y) \sim N(0, 0, 1, 1, \rho)$, the correlation between X^2 and Y^2 is ρ^2.

(v) The surface $z = f(x, y)$ is called *normal correlation surface* or *bivariate normal density surface*.

(vi) For bivariate normal distributions, elliptical contours can be used for densities in $X - Y$ plane. The exponent of e in bivariate normal function describes the densities in $X - Y$ plane.

(vii) If (X, Y) are bivariate normal variates with joint p.d.f. $f(x, y)$, the $E(Y \mid x)$ is a function of x and is defined as the regression of Y on X. Similarly $E(X \mid y)$ is a function of y and is defined as the regression of X on Y.

(viii) If the regression of Y on X is linear, it is not necessary that the regression of X on Y is also linear.

(ix) If the regression of X and Y is linear, it does not imply that the regression of Y on X is linear.

(x) If $(X, Y) \sim$ BVN $\left(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho\right)$, the distribution of the variable,

$$Z = \frac{(X-\mu_X)/\sigma_X}{(Y-\mu_Y)/\sigma_Y}$$

follows Couchy distribution with parameters ρ and $\sqrt{1-\rho^2}$, *i.e.*, $Z \sim C\left(\rho, \sqrt{1-\rho^2}\right)$ and the modal value of Z is ρ.
Again if,

$$Z = \frac{X-\mu_X}{Y-\mu_Y}.$$

then $Z \sim C\left(\rho\frac{\sigma_X}{\sigma_Y}, \frac{\sigma_X}{\sigma_Y}\sqrt{1-\rho^2}\right)$ and its modal value is $\rho\frac{\sigma_X}{\sigma_Y}$.

(xi) In case of bivariate random variable (X, Y) it is possible that each of the marginal distribution of X and Y is normal but their joint distribution is not bivariate normal.

(xii) If (X, Y) follows biviariate normal distribution, then X and Y are independent only if the correlation between them is zero, *i.e.*, $\rho = 0$.

(xiii) For the standard BVN distribution (0, 0, 1, 1, ρ), the recurrence relation between moments is,

$$\mu_{r,s} = (r+s-1)\rho\mu_{r-1,s-1} + (r-1)(s-1) \times \left(1-\rho^2\right)\mu_{r-2,s-2}$$

(xiv) If in the given BVN distribution with parameters $\left(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho\right)$, $\sigma_X = \sigma_Y$ and $\rho = 0$ the density function is known as *circular normal*.

(xv) If in the BVN $\left(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho\right)$ distribution, $\sigma_X \neq \sigma_Y$ and $\rho = 0$ the bivariate normal distribution is known as *elliptical* normal distribution.

(xvi) For a standardised bivariate normal distribution, *i.e.*, $(U, V) \sim \text{BVN}(0, 0, 1, 1, \rho)$, the values of various moments are as follows:

$$\mu_{0,0}=1;\quad \mu_{1,1}=\rho;\quad \mu_{2,2}=1+2\rho^2;$$

$$\mu_{3,3}=3\rho\left(3+2\rho^2\right)$$

$$\mu_{1,2}=\mu_{2,1}=0;\ \mu_{1,3}=\mu_{3,1}=3\rho;$$

$$\mu_{1,4}=\mu_{4,1}=0;\ \mu_{2,3}=\mu_{3,2}=0;$$

$$\mu_{1,5}=\mu_{5,1}=5\rho;\ \mu_{2,4}=\mu_{4,2}=3\left(1+4\rho^2\right)$$

(xvii) If (X_i, Y_i) are BVN $\left(\mu_1,\mu_2,\sigma_1^2,\sigma_2^2,\rho\right)$ for i = 1, 2 , ..., n, then $\left(\bar{X},\bar{Y}\right)$ is distributed as

$$\text{BVN}\left(\mu_1,\mu_2,\frac{\sigma_1^2}{n},\frac{\sigma_2^2}{n},\rho\right).$$

Q. 24 Consider an experiment of tossing a coin thrice. Also take the variate X as the number of tails appearing first time and Y the number of heads appearing in three tosses, *i.e.*, X can take values 0, 1 and Y can take values 0, 1, 2, 3. In this experiment, the bivariate values (x, y) for the sample space

HHH, HHT, HTH, HTT, THH, THT, TTH, TTT are (0, 3), (0, 2), (0, 2), (0, 1), (1, 2), (1, 1), (1, 1), (1, 0). Let p be the constant probability of occurrence of each sample point. The probabilities for each (x, y) values can be presented in the following two way Table.

Y X	*0*	*1*	*2*	*3*	*Total*
0	0	p	$2p$	p	$4p$
1	p	$2p$	p	0	$4p$
Total	p	$3p$	$3p$	p	$8p$

For bivariate distribution given in the above table find the values of (i) $E(X)$, (ii) $E(Y)$, (iii) $E(X + Y)$, (iv) *cov* (X, Y) (v) var $(Y \mid X = 1)$, (vi) $V(X \mid Y = 2)$ (vii) $P(X \mid Y = 2)$ (viii) $P(Y \mid X = 1)$, (ix) $P(X = 1, Y = 2)$, (x) $P(Y = 3 \mid X = 0)$ (xi) $P(X = 1 \mid Y = 1)$.

Ans. We know that the sum of probabilities is equal to 1. Hence, $8p = 1$ or $p=\frac{1}{8}$

(i) $E(X)=0\times 4p+1\times 4p=4p=4\times\frac{1}{8}=\frac{1}{2}$

(ii) $E(Y)=0\times p+1\times 3p+2\times 3p+3\times p=$

$12p=\frac{12}{8}=\frac{3}{2}$

(iii) $E(X+Y)=E(X)+E(Y)=\frac{1}{2}+\frac{3}{2}=2.0$

(iv) $\text{cov}(X,Y)=E(XY)-E(X)\,E(Y)$

$$E(XY)=\sum_{i;j}\left(x_iy_j\right)p_{ij}$$

$$=(0\times 0)0+(0\times 1)\,p+(0\times 2)2p+$$
$$(0\times 3)\,p+(1\times 0)\,p+(1\times 1)2p+$$
$$(1\times 2)\,p+(1\times 3)0$$

$$=4p=\frac{1}{2}$$

$$\therefore\ \text{cov}(X,Y)=\frac{1}{2}-\frac{1}{2}\times\frac{3}{2}=-\frac{1}{4}$$

(v) $V(Y/X=1)=E\left(Y^2/X=1\right)-\left\{E(Y/X=1)\right\}^2$

$$E\left(Y^2/X=1\right)=\sum_y\frac{y^2p(1,y)}{P(x=1)}$$

$$=\frac{0\times p+1\times 2p+4\times p+9\times 0}{4p}$$

$$=\frac{3}{2}$$

$$E(Y|X=1)=\frac{\sum_y yp(1,y)}{P(X=1)}$$

$$=\frac{0\times p+1\times 2p+2\times p+3\times 0}{4p}$$

$$=1$$

$$V(Y/X=1)=\frac{3}{2}-(1)^2=\frac{1}{2}$$

(vi) $V(X/Y=2)=E\left(X^2|Y=2\right)$

$$-\{E(X|Y=2)\}^2$$

$$E\left(X^2|Y=2\right)=\frac{\sum_x x^2p(x,2)}{P(Y=2)}$$

$$=\frac{0\times 2p+1\times p}{3\times p}=\frac{1}{3}$$

$$E(X|Y=2)=\frac{\sum_x x\,p(x,2)}{P(Y=2)}$$

$$=\frac{0\times 2p+1\times p}{3\times p}=\frac{1}{3}$$

$$V(X/Y=2)=\frac{1}{3}-\left(\frac{1}{3}\right)^2=\frac{2}{9}$$

(vii) $P(X|Y=2)=\dfrac{\sum_x p(x,2)}{P(Y=2)}$

$$=\frac{2p+p}{3p}=1$$

(viii) $P(Y|X=1)=\dfrac{\sum_y p(1,y)}{P(X=1)}$

$$=\frac{p+2p+p+0}{4p}=1$$

(ix) $P(X=1|Y=2)=\dfrac{P(1,2)}{P(Y=2)}=\dfrac{p}{3p}=\dfrac{1}{3}$

(x) $P(Y=3|X=0)=\dfrac{P(0,3)}{P(X=0)}=\dfrac{p}{4p}=\dfrac{1}{4}$

(xi) $P(X=1|Y=1)=\dfrac{P(1,1)}{P(Y=1)}=\dfrac{2p}{3p}=\dfrac{2}{3}$

Q. 25 Given the joint distribution of (X, Y) as BVN $(3, 4, 16, 25, 0.8)$, find (i) $P(5 < X < 9 \mid Y = 6)$ (ii) $P(-3 < Y < 3 \mid X = 5)$.

Z:	0.3	1.0	1.97	3.0
Given ϕ (z):	0.11791	0.34134	0.47558	0.49865

Ans. (i) We know that the conditional distribution of X given Y has mean $\mu_X+\rho\dfrac{\sigma_X}{\sigma_Y}(y-\mu_Y)$ and variance $\sigma_X^2\left(1-\rho^2\right)$.

$$\therefore \quad E(X|Y+6)=3+0.8\times\frac{4}{5}(6-4)=4.28$$

and $\quad V(X|y)=16\left(1-0.8^2\right)=5.76$

Using conditional mean and variance,

$$P(5<X<9|Y=6)$$

$$=P\left(\frac{5-4.28}{2.4}<\frac{X-4.28}{2.4}<\frac{9-4.28}{2.4}\right)$$

$$=P(0.3<Z<1.97)$$

$$=\phi(1.97)-\phi(0.3)$$

$$=0.47558-0.11791$$

$$=0.35767$$

(ii) The conditional distribution of Y given X has mean $\mu_Y+\rho\dfrac{\sigma_Y}{\sigma_X}(x-\mu_X)$ and variance $\sigma_Y^2\left(1-\rho^2\right)$.

$$\therefore \quad E(Y|X=5)=4+0.8\times\frac{5}{4}(5-3)=6.0$$

$$V(Y|X=5)=25\left[1-(0.8)^2\right]=9.0$$

Using conditional mean and variance,

$$P(-3<Y<3|X=5)=P\left(\frac{-3-6}{3}<\frac{Y-6}{3}<\frac{3-6}{3}\right)$$

$$=P(-3<Z<-1)$$

$$=\phi|(-3)|-\phi|(-1)|$$

$$= 0.49865 - 0.34134$$

$$= 0.15731$$

Q. 26 If $(X, Y) \sim$ BVN $(3, 6, 16, 25, \rho)$ and $P(1 < X < 5 \mid Y = 6) = 0.724$ then find the values of ρ.

[Given: $\phi(1.09) = 0.36214$]

Ans. $E(X|Y=6) = \mu_X + \rho\frac{\sigma_X}{\sigma_Y}(y-\mu_Y)$

$$= 3 + \rho\cdot\frac{4}{5}(6-6) = 3.0$$

$$V(X|Y=6) = \sigma_X^2\left(1-\rho^2\right)$$

$$= 16\left(1-\rho^2\right)$$

$$P(1 < X < 5|Y=6)$$

$$= P\left(\frac{1-3}{4\sqrt{1-\rho^2}} < \frac{X-3}{4\sqrt{1-\rho^2}} < \frac{5-3}{4\sqrt{1-\rho^2}}\right)$$

$$= P\left(\frac{-2}{4\sqrt{1-\rho^2}} < Z < \frac{2}{4\sqrt{1-\rho^2}}\right)$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-1/2\sqrt{1-\rho^2}}^{+1/2\sqrt{1-\rho^2}} e^{-1/2z^2}dz = 0.724$$

$$= \frac{2}{\sqrt{2\pi}}\int_{0}^{+1/2\sqrt{1-\rho^2}} e^{-1/2z^2}dz = 0.724$$

$$= \frac{1}{\sqrt{2\pi}}\int_{0}^{+1/2\sqrt{1-\rho^2}} e^{-1/2z^2}dz = 0.362$$

Using the area under the standard normal curve,

$$\phi(1.09) = 0.362$$

$$\therefore \quad 1.09 = \frac{1}{2\sqrt{1-\rho^2}}$$

or $$\rho = 0.89.$$

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s) in the blanks:

1. The distribution of two or more variables taken simultaneously is a _______ problem.
2. The multivariate random variables should be defined over the same _______.
3. The marks X and Y secured by examinees in Physics and Chemistry will follow _______ normal distribution.
4. Joint cumulative distribution function, $F_{X,Y}(x, y) =$ _______.
5. The cumulative distribution function $F(x, y)$ lies between _______ and _______.
6. $P(a \le X \le b, c \le Y \le d)$ in terms of cumulative distribution function is equal to _______.
7. Joint probability mass function $p(x, y) =$ _______.
8. The relation between joint cumulative distribution function $F(x, y)$ and joint probability mass function $p(x, y)$ is _______.
9. The value of the expression $\sum_{\text{all } i}\sum_{\text{all } j} p\left(x_i, y_j\right)$ is always _______.
10. If $p(x, y)$ is the joint probability mass function, the marginal probability function $p_X(x)$ is equal to _______.
11. Let $p(x, y)$, $p_X(x)$ and $p_y(y)$ be the joint and marginal probability mass functions of the random variables X and Y. The conditional probability mass function $p_{Y|x}(y|x) =$ _______.
12. The conditional discrete distribution function $F_{X|y}(x|y)$ is equal to _______.
13. Let the joint probability density function of two random variables X and Y is $f(x, y)$. The

marginal probability density function $f_Y(y)$ = ________.

14. The conditional probability density function $f(Y \mid x)$ of Y given $X = x$ can be found out by the formula ________.

15. E $(Y \mid x)$ is called the ________ of Y on X.

16. The expression for conditional variance of a variable Y for a given value of $X = x$ is ________.

17. The variance of a variable X in terms of the conditional variance of Y given $X = x$ is ________.

18. If two variables X and Y are independent, then $E(XY)$ = ________.

19. If X and Y are two independent variables, then $f_{X,Y}(x, y)$ = ________.

20. If X and Y are two independent variables, the conditional distribution of X given $Y = y$, *i.e.*, $f_{X|y}(x \mid y)$ = ________.

21. If X, Y and Z are three random variable, the covariance between X, Y for a given value of Z, *i.e.*, cov $(X, Y \mid Z)$ = ________.

22. If X, Y and Z are three random variates, the covariance between X and Y in terms of conditional covariance, *i.e.*, cov (X, Y) = ________.

23. If two variables X, Y with joint p.d.f. $f_{X,Y}(x, y)$ are independent, the joint moment generating function $M_{X,Y}(t_1, t_2)$ = ________.

24. Given a joint bivariate normal distribution of X, Y as BVN $\left(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho\right)$, the marginal distribution $f_X(x)$ = ________.

25. For conditional distribution $f_{X|Y}(x|y)$, the mean = $E(X| y)$ = ________.

26. If $X, Y \sim$ BVN $(0, 0, 1, 1, \rho)$, the correlation coefficient between X^2 and Y^2 is equal to ________.

27. If (X, Y) are distributed as BVN $(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho)$, the variable $Z = \dfrac{X - \mu_X}{Y - \mu_Y}$ follows ________.

28. If the regression of Y on X is linear, it is ________ that the regression of X on Y is linear.

29. For a bivariate normal distribution $f(x, y)$, the surface $z = f(x, y)$ is called ________.

30. For a bivariate normal density surface of the joint random variables X and Y is given as ________.

31. $(r, s)^{th}$ moment of the joint random variables having the joint p.d.f. $f(x, y)$ are given as ________.

32. Two bivariate normally distributed variables X and Y are independent if they are ________.

33. If $f(x, y) = 4xy$, for $0 < x < 1$, $0 < y < 1$, then
    (i) $E(Y|x)$ = ________.
    (ii) $V(Y|x)$ = ________.
    (iii) $E(XY|x)$ = ________.

34. If $\rho = 0$, the joint density of a bivariate normal distribution is equal to the product of ________.

35. If X and Y are two independent random variables, $E(Y/X = x)$ ________ on x.

36. If $f(x, y) = 2(x + y)$ for $0 < x, y < 1$, then
    (i) marginal probability function of x = ________.
    (ii) $E(Y/X = x)$ ________.

37. $P(X > Y) = 1$ implies that $E(X)$ ________ $E(Y)$.

38. If the covariance between X and Y is zero, it mean that X and Y are ________.

39. If X and Y are independent variables, then cov (X, Y) = ________.

40. If the variables (X, Y) follows BVN (1, 2, 4, 9, 0.5) distribution, the conditional distribution of $(X \mid y = 5)$ has mean ________ and variance ________.

SECTION-C

Multiple Choice Questions

Select the correct alternative out of the given ones:

Q. 1 If (*X*, *Y*) is a bivariate discrete random variable, the number of values which (*X*, *Y*) can taken in the $X - Y$ plane is:

(a) infinite
(b) finite
(c) any number of values
(d) none of the above

Q. 2 $(1, 1)^{th}$ moment μ_{11} of the bivariate distribution is called:

(a) Var (*X*, *Y*)
(b) Var (*X*) · Var (*Y*)
(c) cov (*X*, *Y*)
(d) correlation between *X*,*Y*

Q. 3 Variance of *X* in a bivariate distribution of *X* and *Y* in terms of moments is represented by:

(a) μ_{20}
(b) μ_{02}
(c) μ_{11}
(d) μ_{00}

Q. 4 In rolling of two distinct dice at a time, the variable *X* is defined as the number greater than 2 and the variable *Y* as the sum of numbers of two dices is less than 10. These bivariates (*X*, *Y*) are:

(a) continuous type
(b) discrete type
(c) continuous and discrete both
(d) neither continuous nor discrete

Q. 5 The heights of fathers and their sons form bivariable variables which are:

(a) continuous variables
(b) discrete variables
(c) pseudo variables
(d) none of the above

Q. 6 If a BVN distribution with parameter $(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho)$ is such that $\sigma_X = \sigma_Y$, $\rho = 0$, the distribution is known as:

(a) uniform normal
(b) rectangular normal
(c) elliptical normal
(d) circular normal

Q. 7 If in a BVN $(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho)$ distribution, $\sigma_X \neq \sigma_Y$, $\rho = 0$, the distribution is named as:

(a) symmetrical normal
(b) uniform normal
(c) elliptical normal
(d) circular normal

Q. 8 If in a bivariate normal distribution of the variables *X* and *Y*, $\rho_{XY} = 0$, it implies that *X* and *Y* are

(a) uncorrelated but not independent
(b) uncorrelated and independent
(c) independent but not uncorrelated
(d) correlated and dependent

Q. 9 Bivariate normal distribution is also named as:

(a) Bravais distribution
(b) Laplace-Gauss distribution
(c) Gaussian distribution
(d) all the above

Q. 10 Joint distribution function of (*X*, *Y*) is equivalent to the probability:

(a) $P\,(X = x, Y = y)$
(b) $P(X \leq x, Y \leq y)$
(c) $P(X \leq x, Y = y)$
(d) $P(X \geq x, Y \geq y)$

Q. 11 Joint cumulative distribution function $F\,(x, y)$ lies within the limits:

(a) -1 and 1
(b) -1 and 0
(c) $-\infty$ and 0
(d) 0 and 1

Q. 12 For the joint p.d.f. $f(x, y)$, the marginal distribution of Y given $X = x$ is given as:

(a) $\sum_{\text{all } x} f(x,y)$

(b) $\int_{-\infty}^{\infty} f(x,y)\,dx$

(c) $\int_{-\infty}^{\infty} f(x,y)\,dx\,dy$

(d) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx$

Q. 13 If X and Y are independent, the cumulative distribution $F_{X,Y}(x, y)$ is equal to:

(a) $F_X(x)\,F_Y(y)$

(b) $P(X \le x)\,P(Y \le y)$

(c) both (a) and (b)

(d) neither (a) nor (b)

Q. 14 If X and Y are two independent variables, then

(a) $E(XY) = E(X)\,E(Y)$

(b) $\text{cov}(X, Y) = 0$

(c) $\rho_{X,Y} = 0$

(d) all the above

Q. 15 $E(Y \mid X = x)$ is called the:

(a) regression curve of X on Y

(b) regression curve of Y on X

(c) both (a) and (b)

(d) neither (a) and (b)

Q. 16 If the joint distribution of the variables X and Y is BVN (0, 0, 1, 1, ρ), the correlation coefficient between X^2 and Y^2 is equal to:

(a) 1

(b) -1

(c) ρ^2

(d) 0

Q. 17 If $f(x, y)$ is a binormal density function, then the surface $Z = f(x, y)$ is called:

(a) normal correlation surface

(b) bivariate normal density surface

(c) neither of (a) nor (b)

(d) both (a) and (b)

Q. 18 $(r, s)^{\text{th}}$ moment in a bivariate normal distribution $M_{X,Y}(t_1, t_2)$ can be found as the coefficient of:

(a) $\dfrac{t_1^r t_2^s}{r_s}$

(b) $\dfrac{t_1^r t_2^s}{\Gamma r\,\Gamma s}$

(c) $\dfrac{t_1^r t_2^s}{r!\,s!}$

(d) $t_1^r t_2^s$

Q. 19 The correlation coefficient ρ between two variables X_1 and X_2 for a bivariate population in terms of moments is:

(a) $\dfrac{\mu_{22}}{\sqrt{\mu_{20}\,\mu_{02}}}$

(b) $\dfrac{\mu_{11}}{\sqrt{\mu_{20}\,\mu_{02}}}$

(c) $\dfrac{\mu_{12}}{\sqrt{\mu_{11}\,\mu_{22}}}$

(d) $\dfrac{\mu_{11}}{\sqrt{\mu_{20}\,\mu_{02}}}$

Q. 20 If X and Y follows BVN $(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho)$ distribution, the conditional variance $(X/Y = y)$ is distributed as:

(a) $N\left[\mu_X + \rho\dfrac{\sigma_X}{\sigma_Y}(y - \mu_Y), \sigma_X^2\left(1 - \rho^2\right)\right]$

(b) $N\left[\mu_X + \rho\dfrac{\sigma_Y}{\sigma_X}(x - \mu_X), \sigma_X^2(1 - \rho)\right]$

(c) both (a) and (b)

(d) neither (a) nor (b)

Q. 21 If the variable (X, Y) is BVN $(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho)$, the variable $Z = \dfrac{(x - \mu_X)/\sigma_X}{(Y - \mu_Y)/\sigma_Y}$ follows:

(a) uniform continuous distribution with parameters [ρ, $(1-\rho^2)$]
(b) exponential distribution with parameters [1, ρ]
(c) Cauchy distribution, with parameters $\left[\rho, \sqrt{1-\rho^2}\right]$
(d) none of the above

Q. 22 The conditional distribution of a discrete variable Y given $X = x$ can be expressed as

(a) $F_{Y|x}(y|x) = P(Y \le y|X = x)$

(b) $F_{Y|x}(y|x) = P(Y = y|X = x)$

(c) $F_{Y|x}(y|x) = \dfrac{P(Y \le y|X \le x)}{P(X = x)}$

(d) $F_{Y|x}(y|x) = \dfrac{P(X = x|Y = y)}{P(X = x)}$

Q. 23 The conditional p.d.f. of X given $Y = y$ for a joint density $f_{X,Y}(x, y)$ can be found by the formula:

(a) $F_{X|y}(x|y) = \dfrac{f_{X,Y}(x,y)}{f_X(x)}$

(b) $F_{X|y}(x|y) = f_{Y/X}(y) \cdot f_{X,Y}(x,y)$

(c) $F_{X|y}(x|y) = \dfrac{f_{X,Y}(x,y)}{f_Y(y)}$

(d) none of the above

Q. 24 Given the joint p.m.f. $p_{X,Y}(x, y)$, the conditional p.m.f of Y given $X = x$ is given by the relation:

(a) $P_{Y/x}(y/x) = \dfrac{p_{X,Y}(x,y)}{P_X(x)}$

(b) $P_{Y/x}(y/x) = p_X(x)/p_Y(y)$

(c) $P_{Y/x}(y/x) = p_Y(y)/p_X(x)$

(d) $P_{Y|x}(y/x) = \dfrac{p_{X,Y}(x,y)}{p_Y(y)\,p_X(x)}$

Q. 25 If E $(Y | x)$ is the conditional expectation of Y given $X = x$, the $E(XY)$ in terms of conditional expectation can be expressed as:

(a) $E(XY) = E(X)E(Y/x)$

(b) $E(XY) = E(Y)E(Y/x)$

(c) $E(XY) = XE(Y/x)$

(d) $E(XY) = E\left[XE(Y/x)\right]$

Q. 26 For any two continuous variables X and Y, if a variable Z which is a linear combination of X and Y follows normal distribution, then X and Y jointly follow:
(a) jointly discrete distribution
(b) jointly continuous distribution
(c) bivariate normal distribution
(d) circular normal distribution

Q. 27 Conditional variance of a variable X for given $Y = y$ in terms of conditional expectation can be expressed as:

(a) $V(X|Y = y) = E\left(Y^2|X = x\right) - [E(X|$ $Y = y)^2]$

(b) $V(X|Y = y) = [E(Y|X = x)]^2 - [E(X^2$ $|Y = y)]$

(c) $V(X|Y = y) = E\left(X^2|X = x\right) - [E(X|$ $Y = y)]^2$

(d) $V(X|Y = y) = E\left(X^2|Y = y\right) - [E(X|$ $|Y = y)]^2$

Q. 28 If the moment generating function $M_{X,Y}$ (t_1, t_2) is $e^{1/2\left(t_1^2+t_2^2\right)}$, then the variable X follows:
(a) standard normal bivariate distribution
(b) χ^2-distribution with 1 d.f.
(c) Cauchy distribution
(d) none of the above

Q. 29 Probability of the event $P(a_1 \le X \le a_2, b_1 \le$ $Y \le b_2)$ in terms of joint cumulative distribution function $F(x, y)$ is:

(a) $F(a_1,b_1)-F(a_1,b_2)-F(a_2,b_1)$
$-F(a_2,b_2)$

(b) $F(a_1,b_1)+F(a_2,b_2)-F(a_1,b_2)$
$-F(a_2,b_1)$

(c) $F(a_1,b_2)-F(a_1,b_1)-F(a_1,b_2)$
$+F(a_2,b_1)$

(d) $F(a_1,b_2)+F(a_2,b_2)+F(a_1,b_2)$
$+F(a_2,b_1)$

Q. 30. The cumulative distribution function F (x, y) of two-dimensional random variables X and Y in terms of probability is equivalent to:

(a) $F(x,y)=P(0<X\le x,0<Y\le y)$

(b) $F(x,y)=P(-\infty<X\le\infty,-\infty<Y\le\infty)$

(c) $F(x,y)=P(-\infty<X\le x,-\infty<Y\le y)$

(d) none of the above

Q. 31 The relation of cumulative distribution function with joint p.d.f. f (x, y) of two-dimensional random variables X and Y is:

(a) $F(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(x,y)\,dx\,dy$

(b) $F(x,y)=\int_{0}^{\infty}\int_{0}^{\infty}F(x,y)\,dx\,dy$

(c) $F(x,y)=\int_{-\infty}^{x}\int_{-\infty}^{x}f(x,y)\,dx\,dy$

(d) $F(x,y)=\int_{-\infty}^{x}\int_{-\infty}^{y}f(x,y)\,dx\,dy$

Q. 32 If F (x, y) is a non-decreasing cumulative distribution of two-dimensional random variables X and Y, then F (x, y) holds the relation:

(a) $F(-\infty, y) = F(x, -\infty) = 0, F(\infty, \infty) = 1$

(b) $F(-\infty, y) = F(x, -\infty) = 1, F(\infty, \infty) = 1$

(c) $F(-\infty, y) = F(x, -\infty) = F(\infty, \infty) = 0$

(d) none of the above

Q. 33 If the joint p.d.f. of two random variables X and Y is defined as,

$$f(x,y)=x+y \text{ for } 0\le x, y\le 1$$
$$= 0 \text{ otherwise}$$

then the marginal distribution of X is:

(a) $f_X(x)=x+\dfrac{1}{4}$

(b) $f_X(x)=x+\dfrac{1}{2}$

(c) $f_X(x) = (x + y + 1)$

(d) none of the above

Q. 34 A player rolls a fair die. He gets – 1 point if the die turns up with 1 to 3 spots and get 1 point if the die turns up with 4 to 6 spots. In this way a variable X has two values – 1 and 1. Also one gets 0 point if the die turns up with 1 spot and 1 point if the die turns up with 6 spots. In this way a second variable Y takes two values 0 and 1. The joint probability distribution is given in the following table.

Y X	0	1	$p_X(x)$
–1	1/6	2/6	1/2
1	2/6	1/6	1/2
$p_Y(y)$	1/2	1/2	1

For the given joint distribution, the expected value of X is:

(a) $E(X)=0$

(b) $E(X)=\dfrac{1}{2}$

(c) $E(X)=1$

(d) $E(X)=\dfrac{1}{4}$

Q. 35 For the problem given in Q. No. 34, the expected value of Y is:

(a) $E(Y) = 0$

(b) $E(Y) = \frac{1}{2}$

(c) $E(Y) = 1$

(d) $E(Y) = 1/4$

Q. 36 For the joint distribution given in Q. No. 34, the variance of X is:

(a) $V(X) = 0$

(b) $V(X) = 1/2$

(c) $V(X) = 1$

(d) $V(X) = \frac{1}{4}$

Q. 37 For the bivariate distribution given in Q. No. 34, the variance of Y is:

(a) $V(Y) = 0$

(b) $V(Y) = 1/2$

(c) $V(Y) = 1$

(d) $V(Y) = \frac{1}{4}$

Q. 38 For the joint distribution given in Q. No. 34, the covariance between X and Y is:

(a) cov $(X, Y) = 0$

(b) cov $(X, Y) = 1/6$

(c) cov $(X, Y) = -1/6$

(d) cov $(X, Y) = -1$

Q. 39 For the given joint distribution in Q. No. 34, the correlation between X and Y is:

(a) $\rho_{XY} = -1/3$

(b) $\rho_{XY} = -1/6$

(c) $\rho_{XY} = -2/3$

(d) $\rho_{XY} = 1/3$

Q. 40 For the problem given in Q. 34, the condition variance of X given $Y = 0$ is:

(a) $V(X/Y=0) = 1/3$

(b) $V(X/Y=0) = 1$

(c) $V(X/Y=0) = \frac{8}{9}$

(d) none of the above

Q. 41 For the discrete bivariate distribution given in Q. No. 34, the conditional variance of Y given $X = 1$ is:

(a) $V(Y/X=1) = \frac{1}{9}$

(b) $B(Y/X=1) = \frac{2}{3}$

(c) $V(Y/X=1) = \frac{2}{9}$

(d) none of the above

Q. 42 Following is the joint probability density function of the BVN distribution of X and Y.

$$f(x,y) = ce^{-\frac{8}{27}\left\{(x-7)^2+4(y+5)^2-2(x-7)(y+5)\right\}}$$

The parameters of BVN distribution are:

(a) $\mu_x = 7, \mu_y = -5, \sigma_x^2 = 36, \sigma_y^2 = 9,\ \rho = 0.5$

(b) $\mu_x = -7, \mu_y = -5, \sigma_x^2 = 6, \sigma_y^2 = 3, \rho = 1$

(c) $\mu_x = 7, \mu_y = 5, \sigma_x^2 = 36, \sigma_y^2 = 3, \rho = 0.5$

(d) none of the above

Q. 43 For the joint density function given in Q. No. 42, the value of the constant c is:

(a) $\frac{1}{18\sqrt{3}\pi}$

(b) $\frac{\sqrt{3}}{54\pi}$

(c) 0.0102

(d) all the above

Q. 44 Given the BVN (1, 2, 9, 16, 0.5) distribution, $P(X \geq 3)$ is:

[Given: ϕ (0.67) = 0.24537]

(a) 0.74537

(b) 0.25463

(c) 0.24537

(d) none of the above

Q. 45 Given the BVN (1, 2, 9, 16, 0.5) distribution, $P(X > 2/y = 2)$ is:

[Given: ϕ (0.38) = 0.15]

(a) 0.65
(b) 0.85
(c) 0.35
(d) 0.15

Q. 46 For a BVN (0, 0, 1, 1, 0.5) distribution, (4, 2)th moment is:

(a) $\mu_{4,2} = \frac{15}{4}$
(b) $\mu_{4,2} = 0$
(c) $\mu_{4,2} = \frac{15}{2}$
(d) $\mu_{4,2} = 6$

Q. 47 For a BVN (0, 0, 1, 1, ρ) distribution, (3, 1)th moment is:

(a) $\mu_{3,1} = 1 + 3\rho$
(b) $\mu_{3,1} = 3(1 - \rho^2)$
(c) $\mu_{3,1} = 3\rho$
(d) none of the above

Q. 48 If the regression of Y on X is linear, the regression of X on Y is:

(a) linear
(b) not necessarily linear
(c) always curvilinear
(d) any of the above

Q. 49 For the discrete variables X and Y, the joint probability p_{ij} is expressed as:

(a) $p_{ij} = P(X = x_i, Y > y_i)$
(b) $p_{ij} = P(X \le x_i, Y \le y_j)$
(c) $p_{ij} = P(X \ge x_i, Y \ge y_j)$
(d) $p_{ij} = P(X = x_i, Y = y_i)$

Q. 50 If $f(x, y) = 3 - x - y$ for $0 \le x, y \le 1$, the marginal distribution of X is:

(a) $f_X(x) = 3 - x$
(b) $f_X(x) = \frac{5}{2} - x$
(c) $f_X(x) = 5 - \frac{x^2}{2}$
(d) none of the above

Q. 51 If the joint p.d.f. of X and Y, $f(x, y) = 3 - x - y$ for $0 \le x \le 1, 0 \le y \le 1$ the marginal distribution of Y is:

(a) $f_Y(y) = \frac{5}{2} - y$
(b) $f_Y(y) = y - \frac{5}{2}$
(c) $f_Y(y) = 3 - y$
(d) $f_Y(y) = 3$

Q. 52 Given the joint p.d.f., $f(x, y) = 3 - x - y$ for $0 \le x, y \le 1$ the means of X and Y are:

(a) $\mu'_{1,0} = \mu'_{0,1} = \frac{15}{6}$
(b) $\mu'_{1,0} = \mu'_{0,1} = 3$
(c) $\mu'_{1,0} = \mu'_{0,1} = \frac{11}{12}$
(d) $\mu'_{1,0} = \mu'_{0,1} = 0$

Q. 53 For the joint p.d.f, $f(x, y) = 3 - x - y$ for $0 \le x, y \le 1$, the variances of X and Y are:

(a) $\mu'_{2,0} = \mu'_{0,2} = \frac{7}{12}$
(b) $\mu'_{2,0} = \mu'_{0,2} = \frac{168}{288}$
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 54 The (1, 1)th central moment for the joint p.d.f, $f(x, y) = 3 - x - y$ for $0 \le x, y \le 1$ is:

(a) $\mu_{1,1} = \frac{-1}{4}$
(b) $\mu_{1,1} = 1/12$
(c) $\mu_{1,1} = 0$
(d) none of the above

Q. 55 Given the joint p.d.f. of X and $Y = 3 - x - y$ for $0 \le x, y \le 1$, the correlation coefficient between X and Y is:

(a) $\rho_{X,Y} = \frac{-1}{2}$

(b) $\rho_{X,Y} = \frac{3}{7}$

(c) $\rho_{X,Y} = \frac{-1}{4}$

(d) $\rho_{X,Y} = \frac{-3}{7}$

Q. 56 The conditional distribution of X given Y for the joint p.d.f. $f(x, y) = 3 - x - y$ for $0 \le x, y \le 1$ is:

(a) $f(x|y) = \frac{3-x-y}{3/2-x}$

(b) $f(x|y) = \frac{\frac{5}{2}-y}{3-x-y}$

(c) $f(x|y) = \frac{3-x-y}{\frac{5}{2}-y}$

(d) none of the above

Q. 57 The conditional distribution of Y given X for the joint p.d.f. $f(x, y) = 3 - x - y$ for $0 \le x, y \le 1$ is:

(a) $f(y|x) = \frac{3-x-y}{\frac{5}{2}-y}$

(b) $f(y|x) = \frac{3-x-y}{\frac{5}{2}-x}$

(c) $f(y|x) = \frac{\frac{5}{2}-y}{\frac{5}{2}-x}$

(d) all the above

Q. 58 If the density function of bivariates X and Y is given as:

$$f(x, y) = 3xy \text{ for } 0 \le x \le 1$$
$$0 \le y \le 1,$$

the marginal distribution of X is:

(a) $f_X(x) = 3x$

(b) $f_X(x) = \frac{3}{2}x$

(c) $f_X(x) = \frac{3}{4}x$

(d) none of the above

Q. 59 For $f(x, y)$ given in Q. No. 58, the marginal distribution of Y is:

(a) $f_Y(y) = 3$

(b) $f_Y(y) = \frac{3x}{2}$

(c) $f_y(y) = \frac{3xy}{2}$

(d) $f_Y(y) = \frac{3}{2}y$

Q. 60 For $f(x, y)$ given in Q. No. 58, the mean of X is:

(a) $\mu'_{1,0} = \frac{3}{4}$

(b) $\mu'_{1,0} = \frac{1}{2}$

(c) $\mu'_{1,0} = 1$

(d) $\mu_{1,0} = 0$

Q. 61 For the BVN density $f(x, y)$ in Q. No. 58, the conditional distribution of Y given $X = x$ is:

(a) $f(y/x) = \frac{3}{2}y$

(b) $f(y/x) = 3y$

(c) $f(y/x) = 2y$

(d) $f(y/x) = y$

Q. 62 (2, 2)th moment of the BVN density given in Q. No. 58 is:

(a) $\mu_{2,2} = \frac{1}{48}$

(b) $\mu_{2,2} = \frac{1}{144}$

(c) $\mu_{2,2} = \frac{1}{4}$

(d) none of the above

Q. 63 The covariance between X, Y for the joint density function given in Q. No. 58 is:

(a) $\mu_{11} = \frac{1}{48}$

(b) $\mu_{11} = \frac{1}{144}$

(c) $\mu_{11} = \frac{1}{4}$

(d) none of the above

Q. 64 The variance of X for the joint p.d.f, $f(x, y)$ given in Q. No. 58 is

(a) $\mu_{2,0} = \frac{1}{4}$

(b) $\mu_{2,0} = \frac{1}{16}$

(c) $\mu_{2,0} = \frac{1}{8}$

(d) any of the above

Q. 65 The correlation coefficient between the variables X and Y having the joint density, $f(x, y) = 3xy$ for $0 \le x \le 1, 0 \le y \le 1$ is:

(a) $\rho_{X,Y} = \frac{1}{3}$

(b) $\rho_{X,Y} = \frac{16}{48}$

(c) $\rho_{X,Y} = \frac{2}{6}$

(d) all the above

Q. 66 Given the joint probability mass function of X and Y be $f(x, y) = \frac{x+y}{21}$; $x = 1, 2, 3$; $y = 1, 2$ the $P(x = 3)$ is equal to:

(a) 3/7

(b) 1/9

(c) 4/9

(d) 4/7

Q. 67 Given the joint probability density function of X and Y as,

$$f(x, y) = 4xy; 0 \le x \le 1, 0 \le y \le 1$$
$$= 0; \text{otherwise.}$$

$P\left(0 < x < \frac{1}{2}; \frac{1}{2} \le y \le 1\right)$ is equal to

(a) 1/4

(b) 5/16

(c) 3/16

(d) 3/8

Q. 68 If (X, Y) are bivariate $N(0, 0, 1, 1, \rho)$, then the variables $(x + y)$ and $(x - y)$ are:

(a) correlated with $\rho = \frac{1}{2}$

(b) independently distributed

(c) negatively correlated

(d) none of the above

Q. 69 Let (X, Y) be jointly distributed with density function,

$$f(x, y) = \begin{cases} e^{-x-y}; & 0 < x < \infty, 0 < y < \infty \\ 0 & ; \text{ otherwise} \end{cases}$$

Then X and Y are:

(a) independent

(b) both having the mean unity

(c) both having the variance unity

(d) all the above.

Q. 70 Let $f(x, y) = 1; -x < y < x, 0 < x < 1$
$= 0$; otherwise

Then, the marginal density function of X is:

(a) $2x$

(b) 1

(c) $\frac{1}{2}x$

(d) $2y$

ANSWERS

SECTION-B

(1) multivariate (2) sample space (3) bivariate (4) $P(X \le x, Y \le y)$ (5) 0 and 1 (6) $F(a, c) + F(b, d) - F(a, d) - F(b, c)$ (7) $P(X = x, Y = y)$ (8) $F(x, y) = \sum_{x_i \le x} \sum_{y_i \le y} p(x_i, y_i)$ (9) 1 (10) $\sum_{y} p(x, y)$ (11) $p(x, y)/p_X(x)$ (12) $\sum_{x_i \le x} p_{X|y}(x_i|y)$ (13) $\int_{-\infty}^{\infty} f(x, y)\,dx$ (14) $f_{X,Y}(x, y)/f_X(x)$ (15) regression curve (16) $V(Y \mid X = x) = E(Y^2|X = x) - [E(Y|X = x)]^2$ (17) $V(X) = E(V(X \mid y)] + V[E(X \mid y)]$ (18) $E(X)\,E(Y)$ (19) $f_X(x)f_Y(y)$ (20) $f_X(x)$ (21) $E[\{X - E(X|Z)\}\{Y - E(Y|Z)\}|Z]$ (22) $E[\text{cov}(X, Y|z)] + \text{cov}[E(X|Z)E(Y|z)]$ (23) $M_X(t_1)M_Y(t_2)$ (24) $\frac{1}{\sqrt{2\pi}\sigma_X} e^{-\frac{1}{2\sigma_X^2}(x-\mu_X)^2}$ (25) $\mu_X + \rho\frac{\sigma_X}{\sigma_Y}(y - \mu_Y)$ (26) ρ^2 (27) Cauchy distribution (28) not necessary (29) normal correlation surface (30) $z = f(x, y)$ (31) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^r y^s f(x, y)\,dx\,dy$ (32) uncorrelated

(33) (i) 2/3 $\left[\text{Hint: } f_{Y|X}(y|x) = \frac{4xy}{2x} = 2y, E(Y|x) = \int_0^1 y \cdot 2y\,dy = \frac{2}{3}\right]$

(ii) 1/9 $\left[\text{Hint: } E(Y^2|x) = \int_0^1 y^2 \cdot 2y\,dy = \frac{1}{2}, V(Y|x) = \frac{1}{2} - \left(\frac{2}{3}\right)^2 = \frac{1}{9}\right]$

(iii) 2x/3 $\left[E(XY|x) = xE(Y|x) = \frac{2}{3}x\right]$.

(34) marginal densities (35) does not depend

(36) (i) $2x + 1\left[\text{Hint: } f_x(x) = \int_0^1 2(x + y)\,dy = 2x(y)_0^1 + 2\left(\frac{y^2}{2}\right)_0^1 = 2x + 1\right]$

(ii) $\frac{3x + 2}{3(2x + 1)}$

(37) is greater than (38) are not linearly related (39) zero (40) 2 and 3.

SECTION-C

(1) b (2) c (3) a (4) b (5) a (6) d
(7) c (8) b (9) d (10) b (11) d (12) b
(13) c (14) d (15) b (16) c (17) d (18) c
(19) c (20) a (21) c (22) c (23) c (24) a
(25) d (26) c (27) d (28) a (29) b (30) c
(31) d (32) a (33) b

(34) a $\left[\text{Hint: } E(X) = (-1) \times \frac{1}{2} + 1 \times \frac{1}{2} = 0\right]$

(35) b $\left[\text{Hint: } E(Y) = 0 \times \frac{1}{2} + 1 \times \frac{1}{2} = \frac{1}{2}\right]$

(36) c $\left[\text{Hint: } V(X) = (-1)^2 \times \frac{1}{2} + (1)^2 \times \frac{1}{2} - 0 = 1\right]$

(37) d $\left[\text{Hint: } V(Y) = 0 \times \frac{1}{2} + (1)^2 \times \frac{1}{2} - \frac{1}{4} = \frac{1}{4}\right]$

(38) c [Hint: $\text{cov}(X, Y) = E(XY) - E(Y)E(X)$

$$= E(XY) = 0 \times (-1)\frac{1}{6} + 0 \times (1) \times \frac{2}{6} + 1 \times 1 \times \left(\frac{-2}{6}\right) + 1 \times 1 \times \frac{1}{6} = \frac{-1}{6}$$

(39) a $\left[\text{Hint: } P_{XY} = \frac{-1/6}{\sqrt{1 \times 1/4}} = -\frac{1}{3}\right.$

(40) c $\left[\text{Hint: } E\left(X^2/Y=0\right)=\dfrac{(-1)^2\cdot\frac{1}{6}+(1)^2\cdot 2/6}{1/2}\right.$

$$=1$$

$$E(X/Y=0)=\frac{-1\times\frac{1}{6}+1\times 2/6}{1/2}=\frac{1}{3},$$

$$\left.V(X/Y=0)=1-\frac{1}{9}=\frac{8}{9}\right]$$

(41) c $\left[\text{Hint: } E\left(Y^2/X=1\right)=\dfrac{0\times\frac{2}{6}+(1)^2\times\frac{1}{6}}{1/2}=\dfrac{1}{3},\right.$

$$E(Y/X=1)=\frac{0\times\frac{2}{6}+1\times\frac{1}{6}}{1/2}=\frac{1}{3},$$

$$\left.V(Y/X=1)=\frac{1}{3}-\frac{1}{9}=\frac{2}{9}\right]$$

(42) a (43) d

(44) b $\left[\text{Hint: } p(X\geq 3)=P\left(\dfrac{X-1}{3}\geq\dfrac{3-1}{3}\right)\right.$

$$\left.=0.5-0.24537=0.25463\right]$$

(45) c[Hint: $\mu(X/y)=1+0.5\times\frac{3}{4}(2-2)=1,$

$$V(X/y)=9\left(1-0.5^2\right)=6.75,$$

$$P\left(\frac{x-1}{\sqrt{6.75}}>\frac{2-1}{\sqrt{6.75}}\right)=P(Z\geq 0.385)$$

$$=0.5-0.15=0.35]$$

(46) d (47) c (48) b (49) d (50) b (51) a
(52) c (53) c (54) a (55) d (56) c (57) b
(58) b (59) d (60) b (61) c (62) d (63) a
(64) b (65) d (66) a (67) c (68) b (69) d
(70) a

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Chapter **9**

Sampling Methods

SECTION-A

Short Essay Type Questions

Q. 1 What is the need of sampling as compared to complete enumeration?

Ans. Sampling is a part of our day-to-day life which we use advertantly or inadvertantly. For instance, a housewife takes one or two grains of rice from the cooking pan and decides whether the rice is cooked or not. A quality controller takes a few items and decides whether the lot is in accordance with the desired specifications or not. A pathologist takes a few drops of blood and tests for any change in blood of the whole body than normal. In all these situations, sampling is inevitable and gives satisfactory results.

Even in those cases where complete enumeration is possible, it is not preferred due to the facts that it is much more time consuming and expensive, requires more skilled and technical personnel, more errors are caused due to greater volume of work, measurement errors, etc.

Complete enumeration is used only for various censuses or in case of small populations.

Q. 2 What do you understand by a population in statistical sense?

Ans. Population is a group of items, units or subjects which is under reference of study. Population may consist of finite or infinite number of units. Population is termed as *universe* by a number of statisticians and scientists.

The inhabitants of a region, number of wheat fields in a state or district, fruit plants in a city, insects in a field, persons suffering from tuberculosis in a city, lepers in India, workers in a factory, students in an university, etc., are a few examples of finite populations. All real numbers, all stars in the sky are examples of infinite populations. Generally, the population consists of a large number of animates and inanimates. Moreover, the units or subjects constituting the population may vary from survey to survey in the same region or sphere of activity depending upon the aims and objectives of the survey.

In brief, one should very well keep in mind that statistical population is not only the human population which is usually conceived in literary sense. It is generally a group or collection of items specified by certain characteristics or defined under certain restrictions.

Q. 3 Name different types of populations or universe and give their description summarily.

Ans. Population can be classified into four categories namely, (i) Finite population (ii) Infinite population (iii) Real population (iv) Hypothetical population.

(i) *Finite population.* If the number of items or units constituting the population is fixed and limited, it is known as finite population. For instance, the workers in a factory, students in a college, etc. This usually consists of existing items.

(ii) *Infinite population.* If the population consists of an infinite number of items, it is called an infinite population. For example, the population of all real numbers lying between 5 and 10, the population of stars in the sky, etc.

(iii) *Real population.* A population consisting of the items which are all present physically is termed as real population.

(iv) *Hypothetical population.* The population consists of the results of repeated trials is named as hypothetical population. The tossing of a coin repeatedly results into a hypothetical population of heads and tails, rolling of a die again and again gives rise to a hypothetical population of numbers from 1 to 6, etc.

Q. 4 In what situations sampling is inevitable?

Ans. Sampling is inevitable in the following situations:

(i) When population is infinite
(ii) When the item or unit is destroyed under investigation.
(iii) When the results are required in a short time.
(iv) When resources for survey are limited particularly in respect of money and trained persons.
(v) When area of survey is wide.

Q. 5 What is a sample?

Ans. Sample is a part or fraction of a population selected on some basis. Sample consists of a few items of a population. In principle a sample should be such that it is a true representative of the population. Usually a random sample is selected. If the population is reasonably homogeneous, a simple random sample is most preferred one. But the moment one starts identifying sampling units on the basis of their characteristics, it gives rise to different sampling methods.

Q. 6 What is meant by sampling method?

Ans. By sampling method we mean the manner or scheme through which the required number of units are selected in a sample from a population.

Q. 7 Mention in brief the objective of sampling.

Ans. The foremost purpose of sampling is to gather maximum information about the population under consideration at minimum cost, time and human power. This is best achieved when the sample contains all the properties of the population.

Q. 8 Define sampling unit and give its two examples.

Ans. The constituents of a population which are the individuals to be sampled from the population and cannot be further subdivided for the purpose of sampling at a time are called *sampling units.* For instance, to know the average income per family, the head of the family is a sampling unit. To know the average yield of wheat, each farm owner's field of wheat is a sampling unit.

Q. 9 What is meant by sampling frame?

Ans. For adopting any sampling procedure it is essential to have a list or a map identifying each sampling unit by a number. Such a list or map is called sampling frame.

A list of voters, a list of households, a list of technical persons, areas in a map marked by number for soil surveys, a list of villages in a district, a list of farmer's fields, etc., are a few examples of sampling frame.

Q. 10 Distinguish between complete enumeration and sampling study.

Ans. In complete enumeration, each and every unit of the population is studied and results are based on all units of the population. Whereas, in sampling study only a selected number of units are studied and results based on the data of these units are supposed to yield information about the whole population.

Q. 11 What do you understand by random sampling?

Ans. When equal probability of selection is attached to each sampling unit at each draw, the selection procedure is known as random sampling.

Suppose, there are N units in the population, then the probability of selection of each unit is $1/N$. Also when each subsequent selection is independent of the previous selection, the selection procedure is known as simple random sampling (srs).

Q. 12 Write a short note on the importance of random sample.

Ans. The sampling distribution of the statistic $\overline{X}$ assuming random sampling from a population N (μ, σ) can be derived mathematically from the properties of random samples based on purely mathematical considerations. It can be shown that the sampling distribution of $\overline{X}$ is exactly $N\left(\mu, \sigma/\sqrt{n}\right)$. This result can be verified by empirical sampling experiments. So a random sample saves the labour of conducting empirical sampling experiments.

All the more, estimates obtained from random samples have highly desirable properties.

Q. 13 Delineate the principles of sampling methods.

Ans. There are four principles of sampling methods as described below:

(i) *Principle of statistical regularity* – This principle ensures that the items selected in a moderately large sample at random from a population on the average possess the characteristics of the population units.

(ii) *Principle of inertia of large numbers* – This law states that other things being same, as the sample size increases, the results tend to be more reliable and accurate.

(iii) *Principles of validity* – By the validity of a sampling design we mean that the sampling method should be such that it enables us to obtain valid estimates and tests about the parameters of the population. Probability sampling fulfils this requirement.

(iv) *Principle of optimisation* – This principle is related to efficiency and cost of a design. It consists of achieving a given level of efficiency at minimum cost or attaining maximum possible efficiency at fixed cost.

Q. 14 Expound sampling and non-sampling errors.

Ans. *Sampling error* – Mostly the population parameters are estimates through samples. In spite of the fact that one may use the best sampling method and all care is being taken in conducting the survey and obtaining the estimates for various characteristics of the population, there is always some discrepancy between the estimates and population values obtained by census studies of the sample population in the same manner. Such resulting discrepancies are termed as sampling errors. The sampling errors cannot be completely eliminated but may be minimised by choosing a proper sample of adequate size and adopting suitable method of estimation.

Non-sampling errors – Errors other than sampling errors in a survey are called non-sampling errors. The errors usually arise due to faulty planning, defective schedules or questionnaires, incompleteness and inaccuracy of returns, non-response, compiling errors, etc. These errors can be minimised by employing efficient investigators and supervisory staff, full coverage, better management, etc. Non-sampling errors are likely to be more widespread in complete enumeration than in a sample survey.

Q. 15 What is purposive (subjective or judgement) sampling?

Ans. In purposive sampling, the selection of units entirely depends on the choice of the investigator. This type of sampling is adopted when it is not possible to adopt any random procedure for selection of sampling units. For instance, a sample of patients suffering from Tuberculosis (TB) has to be drawn. Since, it is not possible to ascertain a population of TB patients, the persons turning up to TB sanitorium and having TB are selected in the sample. Such a method of sampling is known as *purposive, subjective* or *judgement sampling*. In this sampling procedure, there is no involvement of probability. That is why, it is called *subjective sampling*.

Purposive or judgement sampling is not preferred because it is not possible to determine the frequency distribution of the estimates obtained by this procedure and thus sampling error cannot be objectively determined.

Purposive sampling is highly prone to investigator's biases. Hence, purposive or judgement sampling is seldom used.

Q. 16 Define a parameter.

Ans. Any population constant is called a *parameter*. Out of various parameters, mean and variance are largely used besides correlation coefficient, regression coefficient, etc. In a distribution, by parameter one means those population constants which appear in probability density (mass) function.

Q. 17 What is meant by an estimator?

Ans. An estimator is a rule or a function of variates for estimating the population parameters. It is expressed as a function of sample variates. An estimator is itself a random variable and can have any value within its domain. For instance, the estimator for mean $\sum_{i=1}^{n} x_i/n$, which depends on the sample values x_i of a variable X. Similarly, the estimator of variance is $\sum_{i=1}^{n}(x_i - \bar{x})^2/(n-1)$.

Q. 18 How do you distinguish between estimate and estimator?

Ans. A particular value of an estimator from a fixed set of values of a random sample is known as estimate. An estimate stands for the value of a parameter. For example, sample mean $\bar{x}$ is an estimate of population mean μ and sample variance s^2 is an estimate of population variance σ^2. Whereas the functions for $\bar{x}$ and s^2 are estimators.

Q. 19 Define the term statistic.

Ans. A statistic is a function of observable random variables and does not involve any unknown parameter. An statistic is also a random variable but is not necessarily an estimator of a parameter. For example student's $-t\left[t = \frac{\sqrt{n}(\bar{x}-\mu)}{s}\right]$ is a statistic.

Q. 20 Give different types of sampling schemes and describe them in brief.

Ans. There are two types of sampling schemes namely (i) unrestricted random sampling (ii) restricted random sampling.

(i) *Unrestricted random sampling.* In this type of sampling, each and every unit of the population has equal chance of being included in the sample. Simple random sampling is an example of unrestricted sampling.

(ii) *Restricted sampling.* If an investigator has any idea about the heterogeneity of sampling units, the population is divided into homogeneous groups and sample is drawn independently from each group. Such a process of sampling is known as restricted sampling. Stratified sampling, systematic sampling, multistage sampling, etc., are covered under the category of restricted sampling.

Q. 21 Differentiate between simple random sampling with replacement and without replacement.

Ans. If the units are selected or drawn one by one in such a way that a unit drawn at a time is replaced back to the population before the subsequent draw, it is known as *simple random sampling with replacement* (srswr). In this type of sampling from a population of size N, the probability of selection of a unit at each draw remains $1/N$. In srswr, a unit can be included more than once in a sample. Therefore, if the required sample size is n, the effective sample size is sometimes less than n due to the inclusion of one or more units more than once.

With the idea that effective sample size should be adhered to, the simple random sampling without replacement (srswor) is adopted. In this method a unit selected once is not included in the population at any subsequent draw. Hence, the probability of drawing a unit from a population of N units at r^{th} draw is $1/(N - r + 1)$.

In simple random sampling, the probability of selection of any sample of size n from a population consisting of N units remains the same, $1\Big/\binom{N}{n}$, *i.e.*, Here $\binom{N}{n}$ is the number of all possible samples.

Q. 22 Which factors are responsible for the size of a sample?

Ans. The size of a sample depends upon the following factors:

(i) The purpose for which the sample is drawn.

(ii) The heterogeneity of the sampling units in the population. More is the heterogeneity, larger is the size of the sample.

(iii) Resources available for the study in terms of time and money.

(iv) Number of technical persons and/or equipment available.

(v) Precision of estimates required is an important factor in determining the size of a sample. For greater precision usually a large sample is preferred.

Q. 23 How can sample size be determined mathematically?

Ans. The mathematical formula for estimating the sample size is:

$$n = \frac{(U_R \hat{s}_x)^2}{d^2}$$

where, d - the extent of difference which is desired to be detected.

U_R - value of statistic at the reliability level R. The value of U_R is substituted from probability distribution table. If the data are selected from a normal population, then for 95 per cent reliability level, U_R = 1.96

$\hat{s}_x$ - the standard deviation of x which is substituted from some previous study or experience.

Q. 24 Give the formula for sample mean.

Ans. If n sample observation are $x_1, x_2, ..., x_n$, the formula for sample mean is,

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

$$= \frac{1}{n} \sum_{i=1}^{n} x_i$$

Q. 25 What formula is used to calculate the sample variance?

Ans. If $x_1, x_2, ..., x_n$ are the observations of n sample units and $\bar{x}$ is its mean, the formula for variance is,

$$s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$= \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n-1}\left\{\sum_{i=1}^{n} x_i^2 - \frac{(\Sigma_i x_i)^2}{n}\right\}$$

Q. 26 How do you determine the sample mean of proportions in a dichotomous population?

Ans. Let P be the proportion of units in category C_1 and $Q = (1 - P)$ is the proportion of units belonging to the category C_2. The estimate p of P, based on a sample of size n, can be obtained by the formula, $p = \frac{n_1}{n}$ where n_1 is the number of units in the sample belonging to the category C_1. Also $q = \frac{n_2}{n}$ where $n_2 = n - n_1$. Hence $p = 1 - q$.

Suppose the observations are coded as 1 if the unit belongs to C_1 and 0 if the unit belongs to C_2. Thus, the sample mean,

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{n_1}{n} = p$$

or $$n_1 = np = n\bar{x}.$$

Q. 27 How do you calculate the sample variance for proportions when the sample is drawn from a dichotomous population?

Ans. With usual notations, the sample variance for proportions can be calculated by the formula,

$$s_p^2 = \frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n-1}(np - np^2)$$

$$= \frac{np}{n-1}(1-p)$$

$$= \frac{npq}{n-1}$$

Q. 28 Define standard error of an estimator and discuss it briefly.

Ans. *Definition*: Standard error is the standard deviation of the sampling distribution of an estimator.

From a population of N units, $\binom{N}{n}$ samples of size n can possibly be drawn from the population. If the sampling units are distinct, each sample will give more or less a different estimate of a parameter. In this way, the estimates themselves will follow a distribution. The standard deviation of the estimates obtained from different possible samples is called standard error (S.E.). May it be the standard error of the sample mean $\bar{x}$, the standard deviation s, etc. Mostly, the standard error of mean $\bar{x}$ is calculated. Lesser is the value of the standard error, more reliable is the estimate.

Q. 29 What is the advantage of considering standard error instead of standard deviation?

Ans. Standard error is not much influenced by the extreme values present in the population. Moreover it withholds all the virtues of standard deviation. All the more, the reciprocal of standard error is an index of precision of an estimator.

Q. 30 What formula is used to find the standard error of mean?

Ans. The formula for the standard error of sample mean $\bar{x}$ based on a sample of size n drawn from a population of N units is,

$$\text{S.E.}(\bar{x}) = s_{\bar{x}} = \sqrt{\left(\frac{1}{n} - \frac{1}{N}\right)s^2}.$$

where s^2 is the sample variance.

If N is large, the quantity $\frac{1}{N}$ is negligible. Hence,

$$\text{S.E.}(\bar{x}) = s_{\bar{x}} = \sqrt{\frac{s^2}{n}}$$

$$= \frac{s}{\sqrt{n}}$$

Q. 31 Express the quantity S^2.

Ans. S^2 is a quantity which is used in estimation of variance. It is slightly different from population variance $\sigma^2 \cdot S^2$ is expressed as,

$$S^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \mu)^2$$

$$= \frac{1}{N-1}\left(\sum_{i=1}^{N} X_i^2 - N\mu^2\right)$$

Q. 32 Express standard error of sample mean in terms of S^2.

Ans. Standard error of sample mean in terms of S^2 is,

$$\text{S.E.}(\bar{x}) = \sqrt{\left(\frac{N-n}{N}\right)\frac{S^2}{n}}$$

Q. 33 What is finite population correction and sampling fraction?

Ans. The quantity $\frac{N-n}{N}$ or $\left(1 - \frac{n}{N}\right)$ is known as finite population correction (f.p.c) and it tends to unity as $N \to \infty$, *i.e.*, N is large. The quantity $\frac{n}{N}$ is known as sampling fraction. $\frac{n}{N}$ is taken as negligible if its value is 0.05 or less.

Q. 34 How can one estimate population total and its variance?

Ans. The population total for a variate X can be estimated by the formula,

$$\hat{X} = N\bar{x}$$

where $\bar{x}$ is the sample mean based on n sample observations and N is the total number of units in the population.

The variance of $\hat{X}$ in terms of S^2 is obtained by the formula,

$$V(\hat{X}) = N^2 V(\bar{x})$$

$$= N^2 \frac{N-n}{N}\frac{S^2}{n}$$

$$= \frac{N(N-n)}{n}S^2$$

and its estimate,

$$v(\bar{x}) = \frac{N(N-n)}{n}s^2$$

If N_1 is the number of units in a class C_1 out of N population units, the variance of the estimate $\hat{N}_1$ is

$$V\left(\hat{N}_1\right) = V(Np)$$

$$= N^2 V(p)$$

$$= N^2 \frac{N-n}{N-1}\frac{PQ}{n}$$

where $P = \frac{N_1}{N}$ and $Q = 1 - P$.

The estimated variance $v\left(\hat{N}_1\right)$ of $V\left(\hat{N}_1\right)$ can be obtained by the formula,

$$v\left(\hat{N}_1\right) = N^2 \frac{N-n}{N-1}\frac{pq}{n}$$

If N is large such that $\frac{1}{N}$ and $\frac{n}{N}$ are negligible,

$$v\left(\hat{N}_1\right) = N^2 \frac{pq}{n}$$

Also the variance of p is estimated by the formula,

$$v(p) = \frac{N-n}{N(n-1)}pq.$$

If $\frac{n}{N}$ is negligible,

$$v(p) = \frac{pq}{n-1}$$

The square root of v (p) will provide standard error of p, *i.e.*,

$$\text{S.E.}(p) = \sqrt{\frac{pq}{n-1}}$$

Q. 35 What do you understand by stratified random sampling?

Ans. Stratified sampling comes under the category of restricted sampling. In this type of sampling method first the whole population is divided into homogeneous groups under certain criterion. These groups are termed as *strata*. Then the sample is drawn randomly from each stratum independently. The estimates are calculated from the data obtained from all the strata.

Information about each individual sampling unit is rarely available. Hence, the strata are formed on some broad basis such as localities in a city, districts in a state, etc.

Q. 36 Highlight the advantages of stratified sampling.

Ans. Different advantages of stratified sampling can be summarised as follows:

(i) If the admissible error is given, small sample is needed which results into a cut of expenditure.

(ii) In case the cost of survey is fixed, there is reduction in error due to stratification.

(iii) Through stratification, it is possible to gather the information or obtain the estimates for each stratum separately and also an estimate for the whole population.

(iv) Stratified sampling is very convenient from organisational point of view.

(v) If need be, different sampling schemes can be used to draw samples from different strata. But it creates many complications and hence it is mostly avoided.

Q. 37 Mention the exigencies of stratified sampling and describe them in brief.

Ans. Various exigencies arise at each step or stage of stratified sampling which are listed below and discussed one by one.

(i) What should be the criterion for stratification?

(ii) What should be number of strata?

(iii) How to fix the points of demarcation between strata?

(iv) What should be the sample size for each stratum?

(v) What sampling procedure be adopted for sampling from each stratum?

(vi) How to find the estimate for each stratum and then to pool them to get the estimates for the population as a whole?

(i) *Criterion for stratification.* The criterion of stratification depends on the objective of the survey. For example, in opinion surveys, stratification according to educational qualifications or economic status may be a good criterion. Due to administrative and organisational convenience, localities or regions are generally taken as strata.

(ii) *Number of strata.* One should take as many strata as are necessary to maintain the homogeneity of strata. More the number of strata, better it is. But often the strata are taken as regions for which information is required separately. (For formulae, see a book on sampling.)

(iii) *Point of demarcation.* As a rule, strata be marked in such a manner that no sampling unit belongs to more than one stratum.

(iv) *Sample size for each stratum.* Since the sample size influences the estimates, it is necessary that the size of the sample for each stratum should be optimum. Determination of sample size for each stratum is known as *allocation problem.* For this, most frequently used approaches are, (a) proportion allocation, and (b) optimum allocation.

(v) *Stratumwise sampling procedure.* Most commonly used procedure is to draw a simple random sample from each stratum independently. This procedure is called *stratified random sampling.* In some cases, other procedures like probability proportional to size may be used.

(vi) *Calculation of estimates.* Different formulae are given ahead to find the estimated values of population constants.

Q. 38 What is equal allocation?

Ans. Under equal allocation samples of equal size are drawn from each stratum.

Q. 39 Discuss proportional allocation in stratified sampling.

Ans. Suppose,

N – Number of units in the population

K – Number of strata

N_j – Number of units in the j^{th} stratum ($j = 1, 2, ..., k$)

n – the sample size.

n_j – number of units to be selected from j^{th} stratum such that $\sum_j n_j = n$.

under proportional allocations,

$$\frac{n_j}{N_j} = \frac{n}{N}$$

or $$n_j = \frac{n}{N} N_j$$

$$= nW_j$$

where $$W_j = \frac{N_j}{N}$$

Thus proportional allocation gives a *self*-weighing sample.

Q. 40 Discuss optimum allocation.

Ans. The formulae for optimum allocation in various strata were derived by Tschuprow in 1923. Later J. Neyman derived them independently in 1934. That is why such an allocation is named as *Neyman* allocation.

There are three situations under which the optimum allocation is considered: (i) When the variance of the stratified sample mean, $V(\bar{x}_{st})$ is minimised, (ii) When the total cost of the survey is fixed, (iii) When the variance of the stratified sample mean is fixed say, $V(\bar{x}_{st}) = V_0$.

Case (i) For a fixed sample size n, the optimum sample size for j^{th} stratum ($j = 1, 2, ..., k$),

$$n_j = n\frac{\Sigma_j W_j S_j^2}{\Sigma_j W_j S_j}$$

where S_j^2 is the variance of j^{th} stratum. If C, C_0 and C_j are the total, overhead and cost per unit of survey in the j^{th} stratum respectively, the optimum value of n_j which minimises the variance of $\bar{x}_{st}$ is,

$$n_j = n\frac{\left(W_j S_j/\sqrt{C_j}\right)}{\left(\Sigma_j \cdot W_j S_j/\sqrt{C_j}\right)}$$

for $j = 1, 2, ..., k$.

The formula reveals that choose a large n_j if,

(a) n is large.

(b) S_j is large *i.e.*, stratum j is more heterogeneous.

(c) C_j is small.

Case (ii) When the cost of the survey is fixed, optimum sample size for j^{th} stratum is,

$$n_j = C_1 \frac{\Sigma_j\left(W_j S_j/\sqrt{C_j}\right)}{\Sigma_j\left(W_j S_j \sqrt{C_j}\right)}$$

for $j = 1, 2, ..., k$

and $C_I = C - C_0$

Case (iii) When the value of $V(\bar{x}_{st})$ is fixed say, V_0, the optimum sample size for j^{th} stratum is,

$$n_j = \frac{\left(\Sigma_j W_j S_j \sqrt{C_j}\right)\left(\Sigma_j W_j S_j/\sqrt{C_j}\right)}{V_0 + \frac{1}{N}\sum_j W_j S_j^2}$$

Q. 41 Give different formulae for mean and variance of stratified sample.

Ans. Different formulae for stratum mean, variances and stratified sample means and variances are:

If x_{ij} is the i^{th} observation in the j^{th} stratum for $i = 1, 2, ..., N_j$ and $j = 1, 2, ..., k$ then,

(i) j^{th} stratum mean, $\bar{X}_j = \frac{1}{N_j}\sum_{i=1}^{N_j} x_{ij}$

(ii) j^{th} stratum sample mean, $\bar{x}_j = \frac{1}{n_j}\sum_{i=1}^{n_j} x_{ij}$

(iii) j^{th} stratum variance,

$$S_j^2 = \frac{1}{N_j - 1}\sum_{i=1}^{N_j}\left(x_{ij} - \bar{X}_j\right)^2$$

(iv) Sample variance of j^{th} stratum,

$$s_j^2 = \frac{1}{n_j - 1}\sum_{i=1}^{n_j}\left(x_{ij} - \bar{x}_j\right)^2$$

(v) An unbiased estimate of population mean under stratified sampling is,

$$\bar{x}_{st} = \frac{1}{N}\sum_{j=1}^{k} N_j \bar{x}_j$$

(vi) Variance of stratified sample mean,

$$V(\bar{x}_{st}) = \frac{1}{N^2}\sum_{j=1}^{k} N_j\left(N_j - n_j\right)\frac{S_j^2}{n_j}$$

$$= \sum_{j=1}^{k} W_j^2\left(1 - \frac{n_j}{N_j}\right)\frac{S_j^2}{n_j}$$

$$= \sum_{j=1}^{k} \frac{W_j^2 S_j^2}{n_j}$$

when $\frac{n_j}{N_j}$ is negligible.

Estimated value of $V(\bar{x}_{st})$ is,

$$v(\bar{x}_{st}) = \sum_{j=1}^{k} \frac{W_j^2 s_j^2}{n_j}$$

(vii) Under proportional allocation,

$$v(\bar{x}_{st}) = \left(1 - \frac{n}{N}\right)\sum_{j=1}^{k} \frac{W_j S_j^2}{n}$$

$$= \sum_{j=1}^{k} \frac{W_j S_j^2}{n}$$

when $\frac{n}{N}$ is negligible.

Estimated value of $V(\bar{x}_{st})$ under proportional allocation is,

$$v(\bar{x}_{st}) = \sum_{j=1}^{k} \frac{W_j s_j^2}{n}$$

(viii) Variance of $\bar{x}_{st}$ under optimum allocation is,

$$V_{\text{Ney}}(\bar{x}_{st}) = \frac{1}{n}\left(\sum_{j=1}^{k} W_j S_j \sqrt{C_j}\right)\left(\sum_{j=1}^{k} \frac{W_j S_j}{\sqrt{C_j}}\right) - \frac{1}{N}\sum_{j=1}^{k} W_j S_j^2$$

If C_j is same for all units, then

$$V_{\text{Ney}}(\bar{x}_{st}) = \frac{1}{n}\left(\sum_{j=1}^{k} W_j S_j\right)^2 - \frac{1}{N}\sum_{j=1}^{k} W_j S_j^2$$

To have the estimated values of variance, replace S_j by s_j.

Q. 42 How are the variances of $\bar{x}_{st}$ under random sampling, proportional allocation and Neyman allocation related with each other?

Ans. The variances of $\bar{x}_{st}$ under random sampling, proportional allocation and Neyman allocation hold the relation,

$$V_{\text{ran}}(\bar{x}_{st}) \geq V_{\text{Prop}}(\bar{x}_{st}) \geq V_{\text{Ney}}(\bar{x}_{st})$$

This shows $V_{\text{ran}}(\bar{x}_{st})$ is largest and $V_{\text{Ney}}(\bar{x}_{st})$ is minimum.

Q. 43 What do you understand by two-way stratification?

Ans. Sometimes the population is stratified according to two factors, e.g., the persons are stratified according to their qualifications and monthly income. In this way, we have a two-way table and units belong to different cells or substrata. Samples are drawn from each substratum (cell) independently. Such a sampling procedure is known as two-way stratified sampling. Two way stratification is also termed as *deep stratification.*

Further two-way stratification is generally more efficient than one way stratification. Still it is seldom used.

Q. 44. Describe the term 'controlled selection'.

Ans. In many situations we know that if the units of different types are not included in the sample, the estimates are most likely upward or downward biased. To avoid this situation, population is divided into various homogeneous strata and a sample is drawn from each stratum to minimise the probabilities of non-preferred samples and is thus termed as *controlled selection.*

Q. 45 Can one make use of varying probabilities of selection in stratified sampling?

Ans. Yes, often the units within a stratum are selected with replacement and with varying probabilities of selection.

Q. 46 What do you understand by systematic sampling?

Ans. When the population units occur in a deck or sequence or line and a sample of size n is to be drawn, the population is divided into n sequential groups and one unit is drawn from each group situated at equal distances.

The selection procedure is such that one unit is drawn randomly from the first group say, j^{th} unit is selected. Then select $(j + k)$, $(j + 2k)$, ..., $\left(j+\overline{n-1}\,k\right)^{\text{th}}$ units from the subsequent groups. Such a selection procedure is known as *linear systematic sampling*. This procedure fails if the population size N is not a multiple of n.

Q. 47 What is the advantages of systematic sampling?

Ans. Some of the principal advantages of systematic sampling are given below:

(i) The method of selection is very simple.

(ii) The method of selection is cheap in terms of time and money.

(iii) The sample is distributed over the whole population and hence all contiguous parts of the population are well represented in the sample.

(iv) It is easy to locate selected units and is very convenient from organisational point of view

Q. 48 What are the disadvantages of systematic sampling?

Ans.

(i) If the variation in the units is periodic, the units at regular intervals are correlated. In this situation the sample becomes highly lopsided and hence the estimates are biased.

(ii) No single reliable formula is available for estimating the standard error of sample mean. A formula is good enough if the population is of the type it has been expected to, otherwise not. This is a great drawback of systematic sampling.

Q. 49 In what situations systematic sampling is preferred over other sampling procedures?

Ans. Systematic sampling is preferably used when the information is to be collected from cards, trees in a forest, houses in blocks, entries in a register which are in a serial order, etc.

Q. 50 What is circular systematic sampling?

Ans. Linear systematic sampling fails if $N \neq nk$. Circular systematic sampling was first used by D.B. Lahiri in 1952. In circular systematic sampling take $\frac{N}{n}$ as rounded to the nearest integer. Select a random number from 1 to N. Suppose the selected number is m. Now select every $(m + jk)^{th}$ unit when $m + jk < N$ and every $(m + jk - N)^{th}$ unit when $m + jk > N$ putting $j = 1, 2, \ldots$, till n units are selected. Such a procedure of selection is known as *circular systematic sampling*.

Q. 51 Give the formulae for mean and variance of systematic sample.

Ans. Let $x_1, x_2, \ldots, x_n$ be the observations on n selected units of systematic sample. The mean of systematic sample,

$$\bar{x}_{sy} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

The variance of $\bar{x}_{sy}$ when $N = nk$ is,

$$V\left(\bar{x}_{sy}\right) = \frac{N-1}{N}S^2 - \frac{k(n-1)}{N}S^2_{wsy}$$

Suppose x_{ij} represents the i^{th} observation in the j^{th} systematic sample when a number from 1 to k is selected.

In the above formula for variance of $\bar{x}_{sy}$, S^2 represents the variance between samples where,

$$S^2 = \frac{1}{N-1}\sum_{j=1}^{k}\sum_{i=1}^{n}\left(x_{ij} - \bar{X}\right)^2$$

and the variance within systematic samples,

$$S^2_{wsy} = \frac{1}{k(n-1)}\sum_{j=1}^{k}\sum_{i=1}^{n}\left(x_{ij} - \bar{x}_j\right)^2$$

Also, $\frac{N-1}{N}S^2 = \sigma^2$ (Population variance)

and $\frac{k(n-1)}{N}S^2_{wsy} = \frac{1}{N}\sum_{j=1}^{k}\sum_{i=1}^{n}\left(x_{ij} - \bar{x}_j\right)^2$

$= \sigma^2_w$ (Variance within the systematic sample)

$$\therefore \quad V\left(\bar{x}_{sy}\right) = \sigma^2 - \sigma^2_w$$

Hence, we conclude that variance of $\bar{x}_{sy}$ is small when the units within a systematic sample are as heterogeneous as possible.

Q. 52 Compare systematic sampling with stratified sampling.

Ans.

(i) Systematic sampling resembles stratified sampling in the sense that groups of k units look like strata but no criterion has been considered in the formation of groups that ensures homogeneity.

(ii) No independent samples are drawn from each group.

(iii) In systematic sampling we have only one sample from the whole population.

The above three points clearly reveal that systematic sampling is quite different from stratified sampling.

Q. 53 Is the systematic sampling superior to simple random sampling and stratified sampling? Comment.

Ans. Nothing definite can be said about the superiority of systematic sampling over simple random sampling and *vice-versa* as it depends more on the structure of the population. One may be better than the other in particular cases. The same statement holds good while comparing systematic sampling with stratified random sampling.

Q. 54 Discuss double sampling in brief.

Ans. Many a time, some initial information necessitated to draw a sample is not available. In that situation such information can be gathered by taking a large sample provided it is not very expensive and time consuming. Then take a sub-sample to estimate the main characters as per the objectives of the survey. Such a sampling process is known as *double sampling*. Double sampling is also called *two-phase sampling*. For example, an investigator want to draw a sample of agricultural holdings with probability proportional to size. But the areas of holdings are not known. Hence, information about each agricultural holding is collected on a large number of farmer's holdings and then a sample is selected with pps for the main survey.

Double sampling is also helpful in stratified sampling to determine the strata sizes if not known. Double sampling is also very helpful in ratio and regression methods of estimation.

Q. 55 Describe cluster or area sampling in nutshell.

Ans. In many situations, the sampling frame for elementary units of the population is not available, moreover it is not easy to prepare it. But the information is available for groups of elements so called *clusters*. For instance, the list of houses may be available but not the persons residing in them, list of individual farms may not be available but the list of villages is generally available. Hence, in these situations, houses or villages are known as clusters and selection has to be made of houses or villages in the sample. Such a sampling procedure is known as *cluster sampling*.

When the entire area containing the population is divided into smaller area or segments, these small areas or segments are taken as sampling units. This procedure is known as area sampling. Clusters or area segments are also known as *primary units*.

In cluster sampling, precaution should be taken that a unit should never belong to more than one cluster. Also each elementary unit of the population should definitely belong to one primary unit.

Q. 56 In what situations the cluster sampling be preferred?

Ans. The cluster sampling is used when:

(i) the sampling frame is not available and it is too expensive and time consuming to prepare it.

(ii) the sampling units are situated distant apart. In this situation selection of elementary units makes the survey very cumbersome. For instance, selection of farmers in a state.

(iii) the elementary units may not be easily identifiable and locatable. For example, the animals of certain species, the migratory populations, etc.

Q. 57 What sampling design is used to select clusters from a population?

Ans. Usually simple random sampling without replacement is used to select clusters or area segments from a population. But any other design can be used.

Q. 58 Give the formulae for estimating the mean of the characteristic X under single stage cluster sampling.

Ans. Let x_{ij} denote the i^{th} observation in the j^{th} cluster. Suppose,

Number of clusters = N
Sample size = n.
Suppose the number of elementary units in the j^{th} cluster = M_j
Total number of elementary units in the population

$$= \sum_{j=1}^{N} M_j = M$$

The mean of j^{th} cluster,

$$\bar{x}_j = \frac{1}{M_j} \sum_i x_{ij}$$

for $i = 1, 2, ..., M_j$ and $j = 1, 2, ..., N$

The population mean,

$$\bar{X}_c = \Sigma_j \Sigma_i x_{ij} / \Sigma_j M_j$$

$$= \frac{1}{M} \sum_j M_j \bar{x}_j$$

An estimate $\bar{x}_c$ of $\overline{X}_c$, which is unbiased but not consistent, is given as,

$$\bar{x}_c = \frac{1}{n}\sum_j M_j \bar{x}_j / \overline{M}$$

where $$\overline{M} = \frac{M}{N}$$

Let S_b^2 and S_w^2 denote the variance between and within clusters respectively. Now the population variance,

$$\sigma^2 = \frac{1}{NM-1}\sum\nolimits_j \sum\nolimits_i \left(x_{ij} - \overline{X}_c\right)^2$$

Also, $$S_b^2 = \frac{1}{N-1}\sum\nolimits_j \left(\overline{X}_j - \overline{X}_c\right)^2$$

and $$S_w^2 = \frac{1}{N(M-1)}\sum\nolimits_j \sum\nolimits_i \left(x_{ij} - \bar{x}_j\right)^2$$

An unbiased estimate of the variance of $\bar{x}_c$ is given as,

$$v(\bar{x}_c) = \frac{N-n}{Nn}s_b^2$$

where, $$s_b^2 = \frac{1}{n-1}\sum\nolimits_j \left(M_j \bar{x}_j - \bar{x}_c\right)^2$$

for $j = 1, 2, ..., n$

Also, an estimate of s_w^2 is,

$$s_w^2 = \frac{1}{n(M-1)}\sum\nolimits_j \sum\nolimits_i \left(x_{ij} - \bar{x}_j\right)^2$$

Since the units within a cluster are more homogeneous within a cluster than those in different clusters, $S_w^2 \le S_b^2$.

Q. 59 Comment on the efficiency of cluster sampling as compared to simple random sampling without replacement.

Ans. The efficiency of cluster sampling as compared to simple random sampling is,

$$E = \frac{S^2}{MS_b^2}$$

where, S^2 - the variance of cluster

M - the cluster size for each cluster.

S_b^2 - the difference of the total variance & within clusters variance.

From the formula it is apparent that the efficiency of the cluster sampling increases:

(i) if the cluster size decreases.

(ii) if the clusters are so formed that the variation within clusters is as large as possible and between clusters is as small as possible.

Q. 60 What are the main differences between cluster sampling and stratified sampling?

Ans. The main differences between cluster sampling and stratified sampling are:

(i) In stratified sample, a sample is drawn from each stratum (cluster) whereas in cluster sampling, a cluster (stratum) is selected as such.

(ii) A heterogeneous cluster is more preferable whereas a homogeneous stratum is always desirable.

Q. 61 What is multi-stage sampling?

Ans. In single-stage cluster sampling it is costly to include every elementary unit of the selected clusters in the survey. Moreover, it appears superfluous when the clusters are homogeneous. Hence, it is better to select a sample from each selected cluster rather than surveying the clusters as a whole. Selection of a sample from each selected cluster is known as sub-sampling. Under such a sampling procedure a sample is drawn in two stages, *i.e.*, in the first stage a sample of clusters is selected, and in the second stage a sample of elementary units is drawn from each selected cluster. This kind of sampling procedure is known as *two-stage sampling*. For example, if a survey is conducted to have an estimate of crop production, one may prefer to use two-stage sampling. Select villages as first stage units and farms in the villages as second stage units.

The selection procedure can be extended to any number of stages. Hence, in general it is known as *multi-stage sampling.*

Q. 62 Give the formulae for estimates of mean and variance in case of two-stage sampling.

Ans. Suppose, the total number of first stage units $= N$.

No. of second stage units in i^{th} first stage unit $= M_i$ for $i = 1, 2, ..., N$.

No. of first stage units selected $= n$.

No. of second stage units selected from the h^{th} first stage unit $= m_h$ for $h = 1, 2, ..., n$.

x_{ij} - observation on the j^{th} second stage unit belonging to i^{th} first stage selected unit.

$\overline{X}_i$ - Mean of the i^{th} first stage unit on the basis of per second stage unit.

$\bar{x}_h$ - The estimated value of $\overline{X}_i$.

$\bar{x}_s$ - Overall sample mean on the basis of each second stage unit when subsampling has been done.

Total number of second stage unit in the sample,

$$m = \sum_{h=1}^{n} m_h$$

i^{th} first stage unit mean,

$$\overline{X}_i = \frac{1}{M_i}\sum_{j=1}^{M_i} x_{ij}$$

for $i = 1, 2, ..., N$

Estimated value of $\overline{X}_i$,

$$\bar{x}_h = \frac{1}{m_h}\sum_{j=1}^{m_h} x_{ij}$$

Population mean per second stage unit,

$$\overline{X} = \sum_{i=1}^{N} M_i \overline{X}_i \Big/ \sum_{i=1}^{N} M_i$$

$$= \sum_{i=1}^{N} M_i \overline{X}_i / N\overline{M}$$

where $\overline{M} = \frac{1}{N}\sum_{i=1}^{N} M_i$

An unbiased estimate of $\overline{X}_s$ is,

$$\bar{x}_s = \frac{1}{n\overline{M}}\sum_{i=1}^{N} M_i \bar{x}_i$$

The estimated variance of $\bar{x}_s$,

$$v(\bar{x}_s) = \left(\frac{1}{n} - \frac{1}{N}\right)s_b^2 + \frac{1}{nN}\sum_{h=1}^{N} \frac{M_i^2}{\overline{M}^2}\left(\frac{1}{m_h} - \frac{1}{M_h}\right)s_{wh}^2$$

where, $$s_b^2 = \frac{1}{n-1}\sum_{h=1}^{n}\left(\frac{M_h}{M}\bar{x}_h - \bar{x}_s\right)^2$$

and $$s_{wh}^2 = \frac{1}{m_h - 1}\sum_{i}\left(x_{ij} - \bar{x}_h\right)^2$$

Q. 63 Comment on two-stage sampling design.

Ans. It has been found that two-stage sampling design is generally less efficient than single stage sampling except when the correlation between elements in the same (first) stage units is negative.

Q. 64 Write a short note on inverse sampling.

Ans. When the character under study rarely exists and the proportion P of units possessing the character is very small, a simple random sample without replacement does not yield satisfactory results. Hence, for getting good estimate of P. Haldane, 1946 and Finney, 1949 suggested the method of inverse sampling. In inverse sampling, the size of the sample 'n' is not fixed but selection process continues until a predecided number of units possessing the rare character or attribute has been selected in the sample.

Suppose N is the number of units in the population and P is the proportion of units possessing the rare character or attribute under study. The number of units possessing the rare character in NP. Now to estimate P, draw a sample with srswor until the sample contains m units having rare character. Let n be the total number of units selected containing m units of interest. In this type of selection, n is a random variable and follows hypergeometric distribution. An unbiased estimate of P is,

$$p = \frac{m-1}{n-1}$$

An unbiased estimate of the variance of p is,

$$s_p^2 = \frac{p(1-p)}{(n-2)}\left(1-\frac{n-1}{N}\right)$$

If $\frac{n-1}{N}$ is negligible,

$$s_p^2 = \frac{p(1-p)}{n-2}$$

Q. 65 Give the idea of sampling with probability proportional to size (pps).

Ans. When the units vary in size according to a measure which can influence our studies, it was considered more appropriate to select items with probabilities proportional to their size. For instance, the villages having large geographical area will be having more area under crops. Hence, a survey meant for estimating the crop production it is better that the village with larger areas be selected with probability proportional to their area.

Thus, a selection procedure in which the units are selected with varying probabilities in proportion to some measure of the size of the sampling units is known as sampling with *probability proportional to size* (pps). Under *pps* sampling larger unit has more probability of inclusion in the sample as compared to the unit of smaller size.

Q. 66 Give the concept of non-response in brief.

Ans. In surveys it is commonly experienced that complete data from the sampling units or respondents is not obtainable for various reasons. For example in an opinion survey, the selected family might have shifted to some other place, selected person might have died. In mailed questionnaire, many respondents do not send their replies. Such a problem of incomplete sample data due to non-availability of information from the respondents is known as the problem of non-response.

Biases are introduced in the estimates due to non-response. Hence, various methods have been evolved to tackle the problem of non-response, but the details are omitted for brevity.

To deal with the problem of non-response, one approach is to consider the population consisting of two strata, one of respondents and the other of non-respondents in a mailed questionnaire survey. A sub-sample is drawn from non-respondents strata and these units are interviewed personally. Two sample data are pooled to get the estimates of the population parameters. This technique was suggested by Hansen and Hurwitz in 1946.

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s) in the blanks:

1. Sampling is __________ in many situations.
2. All sampling units are present in __________ population.
3. In a __________ population all sampling units are imagined.
4. If the number of units in a population are limited, it is known as __________ population.
5. A population consisting of an unlimited number of units is called an __________ population.
6. If all the units of a population are surveyed, it is called __________.
7. The errors other than sampling errors are termed as __________.
8. The discrepancy between a parameter and its estimate due to sampling process is known as __________.
9. Any population constants is called a ________.
10. A function for estimating a parameter is called as __________.
11. A value of the estimator is an __________ of the parameter.

12. Standard deviation of all possible estimates from samples of fixed size is called __________.

13. Formula for standard error of sample mean $\bar{x}$ based on a sample of size n and with standard deviation s is __________.

14. The list of all the items of a population is known as __________.

15. Another name of population is __________.

16. Number of samples of size n that can be drawn out of N population units through simple random sampling without replacement is __________.

17. The probability of selection of any one sample out of $\binom{N}{n}$ samples is __________.

18. Under simple random sampling with replacement, there can be __________ samples of size n out of N population units $(n < N)$.

19. Under simple random sampling with replacement, the same item can occur __________ in the sample.

20. Formula for sample mean of n observations $x_1, x_2, ..., x_n$ is __________.

21. Sample variance of the variate values x_1, x_2, ..., x_n can be calculated by the formula __________.

22. The sampling procedure in which the population is first divided into homogeneous groups and then a sample is drawn from each group is called __________.

23. Stratified sampling is appropriate when population is __________.

24. Stratification is done in respect of certain __________.

25. Deciding the sample size for each stratum is known as __________ problem.

26. If the sample size of each stratum is in proportion to stratum size, it is called __________.

27. Stratified sampling comes under the category of __________ sampling.

28. More heterogeneous is the population, __________ is the sample size.

29. Standard error of mean in terms of S^2 is __________.

30. The quantity $\frac{N-n}{N}$ in usual notation is called __________.

31. The expression $\frac{n}{N}$ is known as __________.

32. When the units in a sample are selected by the investigator at his will, the selection procedure is known as __________ sampling.

33. Judgement sampling is __________ used in practice.

34. For a high precision of estimates, __________ samples is required.

35. Estimator and estimate are __________.

36. If p is the proportion of units in one category out of two categories, the variance of p can be calculated by the formula __________.

37. Sampling fraction $\frac{n}{N}$ is negligible if it is __________.

38. Determination of sample size for each stratum subject to the cost constrained is known as __________ allocation.

39. Optimum allocation is also known as __________ allocation.

40. Estimation of sample size for a stratum subject to the prefixed value of $V(\bar{x}_{st})$ in stratified sampling is called __________ allocation.

41. Sample under proportion allocation is a __________ sample.

42. In stratified random sampling, the variance of $\bar{x}_{st}$ for a fixed total size of sample is minimum if n_j is proportional to __________.

43. With varying cost C_j per unit in stratified random sampling, the variance of $\bar{x}_{st}$ attains the smallest value if n_j is proportional to __________.

44. Variance of $\bar{x}_{st}$ under ________ allocation is least as compared to proportional allocation.

45. $V(\bar{x}_{st})$ under proportional allocation is less than the variance $\bar{x}_{st}$ under ________.

46. $V_{prop}(\bar{x}_{st})$ lies in between $V_{opt}(\bar{x}_{st})$ and ________.

47. The students in a college are awarded grades *A*, *B* and *C*. For estimating the average *IQ* of the college students, ________ will provide good estimate of average *IQ*.

48. Stratified sampling is not preferred when the population is ________.

49. When a simple random sample is drawn from each stratum, it is known as ________.

50. When there is an infinite population, ________ is not possible.

51. When the items are perishable under investigation it is not possible to do ________.

52. When the sample of same size is drawn from each stratum, it is known as ________.

53. Population is divided into substrata or cell under ________.

54. The two-way stratification is generally ________ efficient than the one-way stratification.

55. When stratification is done to minimise the selection of non-preferred samples, it is known as ________.

56. When the population consists of units arranged in a sequence or deck, one would prefer ________.

57. In systematic sampling, all ________ parts of the population are well represented.

58. The main advantage of systematic sampling is that it is ________ and ________.

59. The main disadvantage of systematic sampling is that ________ formula for estimating the standard error of sample mean is available.

60. When the population size *N* is a multiple of sample size *n*, ________ systematic sampling in appropriate.

61. When the population size *N* is not divisible by the sample size '*n*', ________ systematic sampling is plausible.

62. A subsample is drawn under ________ sampling.

63. Double sampling is termed as ________ sampling.

64. Double sampling is sometimes used in ________ sampling.

65. Double sampling is useful in ________ and ________ estimation methods.

66. Cluster sampling is a good sampling technique when ________ is not available.

67. Cluster sampling is useful when sampling units are situated ________.

68. Clusters or area segments are called ________.

69. A cluster consists of a number of ________ units.

70. In cluster sampling, the variance within clusters is ________ between cluster variance.

71. Efficiency of cluster sampling ________ as the cluster size decreases.

72. When an investigator selects firstly big size clusters and then smaller size clusters from bigger clusters and so on, the sampling method is known as ________.

73. The two-stage sampling is better than the single-stage sampling only if the elementary units in the same unit are ________.

74. In inverse sampling, the sample size is ________.

75. If *p* is the estimated proportion of units having a desired attribute in inverse sampling and *m*, the number of units possessing it out of *n* units drawn in the sample, then $p =$ ________.

76. In inverse sampling, the proportion *p* of units having the desired attribute with *n* units drawn has variance equal to ________.

77. If larger units have greater chance of being included in the sample and *vice-versa*, it is known as sampling with ________.

78. Non-availability of information from the respondents is termed as the problem of ________.

79. Sampling is ________ in all kinds of studies.

80. Sampling studies give good result if a sample is a ________ of the population.

81. For a homogeneous population, simple random sampling is ________ than stratified random sampling.

82. Systematic sample ________ be said to give more reliable results than a random sample.

83. Systematic sample ________ be said to give less reliable results as compared to a simple random sample.

84. It ________ be said that the sample mean is always more or less than population mean.

85. The population mean ________ with increase or decrease of sample size.

86. An infinite population has variance 100 and mean 96. If a random sample of 4 units is selected, the standard deviation of the sampling distribution of mean is ________.

87. The population mean is a ________ value.

88. Precision of estimates ________ by proper stratification.

89. Sampling error may arise due to ________ selection of sample.

90. Sampling error is caused by ________ methods of analysis of data.

91. If all the units selected in the sample are not covered, it is a problem of ________.

92. Non-sampling error arise due to ________ of data.

93. Errors committed in presentation of data are categorised as ________ errors.

94. If estimates are close to their respective population parameters, the estimates are called ________.

95. Cluster sampling ordinarily leads to the ________ of precision.

96. Cluster sampling helps to ________ cost of the survey.

97. Larger the cluster size, ________ efficient it is relative to the element as the sampling unit.

98. Two stage sampling is ________ efficient as compared to single stage sampling.

99. A sampling procedure, in which the units are selected with chance of selection in proportion to some measure of their size, is known as ________ sampling.

100. Under *pps* selection, a unit has ________ chance of being included in the sample than a unit smaller to it.

101. Estimates from random samples possess ________ properties.

102. A random sample saves the labour of conducting ________ sampling trials.

103. If a random sample of adequate size truly represents the population, then it is said to follow the principle of ________.

104. The principle of optimisation makes one to attain a desired level of efficiency at ________ cost.

105. A sampling method resulting into reliable estimates for parameters is said to follow the principle of ________.

106. The index of precision of an estimator is indicated by its ________.

107. Attaining maximum efficiency in estimating for a fixed cost is a part of principle of ________.

108. As the sample size increases, the conclusions based on sample values tend to be more ________ and ________.

109. The reciprocal of standard error is an index of ________ of an estimator.

110. The increase in sample size leading to reliability and accuracy of estimates is governed by the principle of ________.

SECTION-C

Multiple Choice Questions

Select the correct alternative out of given ones:

Q. 1 A sample consists of:
(a) all units of the population
(b) 50 per cent units of the population
(c) 5 per cent units of the population
(d) any fraction of the population

Q. 2 Sampling is inevitable in the situation(s):
(a) blood test of a person
(b) when the population is infinite
(c) testing of life of dry battery cells
(d) all the above

Q. 3 The number of possible samples of size n out of N population units without replacement is:
(a) $\binom{N}{n}$
(b) $(N)_n$
(c) n^2
(d) $n\,!$

Q. 4 The number of possible samples of size n from a population of N units with replacement is:
(a) N^2
(b) n^2
(c) ∞
(d) $N\,!$

Q. 5 The number of all possible samples of size two from a population of 4 units as:
(a) 2
(b) 4
(c) 8
(d) 12

Q. 6 Probability of drawing a unit at each selection remains same in:
(a) srswor
(b) srswr
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 7 Probability of selection varies at each subsequent draw in:
(a) sampling without replacement
(b) sampling with replacement
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 8 An unordered sample of size n can occur in:
(a) n ways
(b) $n!$ ways
(c) one way
(d) n^2 ways

Q. 9 Probability of any one sample of size n being drawn out of N units is:
(a) $1/N$
(b) n/N
(c) $1/n\,!$
(d) $1\Big/\binom{N}{n}$

Q. 10 Probability of including a specified unit in a sample of size n selected out of N units is:
(a) $1/n$
(b) $1/N$
(c) n/N
(d) $\frac{N}{n}$

Q. 11 A selection procedure of a sample having no involvement of probability is known as:
(a) purposive sampling
(b) judgement sampling
(c) subjective sampling
(d) all the above

Q. 12 When an investigator wants a sample containing m units which possess a rare attribute, the appropriate sampling procedure is:
(a) srswor
(b) stratified sampling
(c) inverse sampling
(d) all the above

Q. 13 If larger units have greater probability of their inclusion in the sample, it is known as:
(a) selection with replacement
(b) selection with probability proportional to size
(c) selection with constant probability
(d) probability selection

Q. 14 Simple random sample can be drawn with the help of:
(a) random number tables
(b) chit method
(c) roulette wheel
(d) all the above

Q. 15 Sampling frame is a term used for:
(a) a list of random numbers
(b) a list of voters
(c) a list of sampling units of a population
(d) none of the above

Q. 16 In simple random sampling with replacement, the same sampling unit may be included in the sample:
(a) only once
(b) only twice
(c) more than once
(d) none of the above

Q. 17 A population consisting of all the items which are physically present is called:
(a) hypothetical population
(b) real population
(c) infinite population
(d) none of the above

Q. 18 A population consisting of the results of the conceptually repeated trials is known as:
(a) hypothetical population
(b) finite population
(c) infinite population
(d) real population

Q. 19 If the items are destroyed under investigation, we have to go for:
(a) complete enumeration
(b) sampling studies
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 20 The discrepancies between sample estimate and population parameter is termed as:
(a) human error
(b) formula error
(c) non-sampling error
(d) sampling error

Q. 21 The errors in a survey other than sampling errors are called
(a) formula errors
(b) planning error
(c) non-sampling error
(d) none of the above

Q. 22 A function of variates for estimating a parameter is called:
(a) an estimate
(b) an estimator
(c) a frame
(d) a statistic

Q. 23 An estimator can possess:
(a) a fixed value
(b) any value
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 24 An estimate based on a fixed set of values of a sample always possess:
(a) a single value
(b) any value
(c) a value equal to one
(d) all the above

Q. 25 Students-*t* is categorised as:
(a) an estimate
(b) an estimator
(c) a statistic
(d) none of the above

Q. 26 If each and every unit of a population has equal chance of being included in the sample, it is known as:
(a) restricted sampling
(b) purposive sampling
(c) subjective sampling
(d) unrestricted sampling

Q. 27 The most important factor in determining the size of a sample is:
(a) the availability of resources

(b) purpose of the survey
(c) heterogeneity of population
(d) none of the above

Q. 28 If d is the difference to be detected, u_R the value of the statistic at reliability level R and s the S.D., the formula for the sample size n is:

(a) $n = \frac{U_R s}{d^2}$

(b) $n = \left(\frac{U_R s}{d}\right)^2$

(c) $n = \frac{U_R^2 s}{d^2}$

(d) $n = \frac{U_R s^2}{d^2}$

Q. 29 Having sample observations $x_1, x_2, \ldots, x_n$, the formula for variance is:

(a) $s^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2$

(b) $s^2 = \frac{1}{n-1}\left\{\sum x_i^2 - \frac{(\Sigma x_i)^2}{n}\right\}$

(c) $s^2 = \frac{1}{n-1}\left(\sum x_i^2 - n\bar{x}^2\right)$

(d) all the above

Q. 30 If the observations recorded on five sampled items are 3, 4, 5, 6, 7 the sample variance is:
(a) 1
(b) 0
(c) 2
(d) 2.5

Q. 31 If all observations in a set of observations are same, the variance of the set of values is:
(a) zero
(b) one
(c) infinity
(d) not possible to calculate

Q. 32 A sample of size n is drawn from a dichotomous population. If the sample has proportion p of items of category I and proportion q of category II, then the variance of the proportion p is:

(a) $s_p^2 = \frac{1}{n-1} pq$

(b) $s_p^2 = \frac{1}{n} pq$

(c) $s_p^2 = \frac{n}{n-1} pq$

(d) $s_p^2 = \frac{1}{n} p^2 q$

Q. 33 If a sample $x_1, x_2, \ldots, x_n$ from a dichotomous population has n_1 items of type C_1 with proportion p and n_2 items of type C_2 with proportion q. Also,

$x_i = 1$ if $x_i \varepsilon C_1$
$x_i = 0$ if $x_i \varepsilon C_2$

then which of given four relations does not hold good?

(a) $\bar{x} = p$

(b) $q = 1 - p$

(c) $q = n_2/n$

(d) $p = \frac{n}{n_1}$

Q. 34 Formula for standard error of sample mean $\bar{x}$ based on sample of size n having variances s^2, when population consisted of N items, is:

(a) s/n

(b) $s/\sqrt{n-1}$

(c) $s/\sqrt{N-1}$

(d) $s/\sqrt{n}$

Q. 35 Which of following statement is true?
(a) more the standard error, better it is
(b) less the standard error, better it is
(c) standard error is always zero
(d) standard error is always unity

Q. 36 Which of the following statement is not true?
(a) standard error cannot be zero
(b) standard error cannot be 1
(c) standard error can be negative
(d) all the above

Q. 37 If the sample values are 1, 3, 5, 7, 9 the standard error of sample mean is:
(a) $\text{S.E} = \sqrt{2}$
(b) $\text{S.E} = 1/\sqrt{2}$
(c) S.E = 2.0
(d) S.E = 1/2

Q. 38 If we have a sample of size *n* from a population of *N* units, the finite population correction is:
(a) $\dfrac{N-1}{N}$
(b) $\dfrac{n-1}{N}$
(c) $\dfrac{N-n}{N}$
(d) $\dfrac{N-n}{n}$

Q. 39 If *n* units are selected in a sample from *N* population units, the sampling fraction is given as:
(a) $\dfrac{N}{n}$
(b) $1/N$
(c) $1/n$
(d) n/N

Q. 40 As a normal practice, sampling fraction is considered to be negligible if it is:
(a) less than 10 per cent
(b) less than or equal to 5 per cent
(c) more than 5 per cent
(d) more than 10 per cent

Q. 41 Stratified sampling comes under the category of:
(a) unrestricted sampling
(b) subjective sampling
(c) purposive sampling
(d) restricted sampling

Q. 42 Which of the following statements does not hold good in case of stratified sampling?
(a) stratified sampling is convenient
(b) stratified sampling is always good
(c) enables to gather information about different stratum separately
(d) reduces error for fixed cost

Q. 43 Which one problem out of the four is not related to stratified sampling?
(a) fixing the criterion for stratification
(b) fixing the number of strata
(c) fixing the sample size
(d) fixing the points of demarcation between strata

Q. 44 Regarding the number of strata, which statement is true?
(a) lesser the number of strata, better it is
(b) more the number of strata, poorer it is
(c) more the number of strata, better it is
(d) not more than ten items should be there in a stratum

Q. 45 Under equal allocation in stratified sampling, the sample from each stratum is:
(a) proportional to stratum size
(b) of same size from each stratum
(c) in proportion to the per unit cost of survey of the stratum
(d) all the above

Q. 46 Under proportional allocation, the size of the sample from each stratum depends on:
(a) total sample size
(b) size of the stratum
(c) population size
(d) all the above

Q. 47 Under proportional allocation one gets:
(a) an optimum sample
(b) a self-weighing sample
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 48 Formula for optimum sample size was derived by whom and in which year?
(a) Tschuprow in 1923

(b) J. Neyman in 1934
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 49 How many types of optimum allocation are in common use?
(a) one
(b) two
(c) three
(d) four

Q. 50 With usual notations, the formula for optimum sample size n_j for the j^{th} stratum ($j = 1, 2, ..., k$) for fixed total sample size n is:

(a) $n_j = n\, W_j\, S_j \big/ \left(\Sigma_j\, W_j\, S_j^2\right)$

(b) $n_j = W_j\, S_j \big/ \left(\Sigma_j\, S_j\right)$

(c) $n_j = n\, W_j\, S_j^2 \big/ \left(\Sigma_j\, W_j\, S_j^2\right)$

(d) $n_j = n_j\, W_j\, S_j \big/ \left(\Sigma_j\, W_j\, S_j\right)$

Q. 51 If C, C_0 and C_j are the total, overhead and cost of survey per unit in the j^{th} stratum resp., the optimum sample size for j^{th} stratum ($j = 1, 2, ..., k$) with usual notations which minimises the variance of stratified sample mean is:

(a) $n_j = n \dfrac{W_j S_j \big/ \sqrt{C_j}}{\Sigma_j\, W_j\, S_j \big/ \sqrt{C_j}}$

(b) $n_j = n \dfrac{W_j S_j \sqrt{C_j}}{\Sigma_j\, W_j\, S_j / C_j}$

(c) $n_j = n \dfrac{W_j S_j \big/ \sqrt{C_j}}{\Sigma_j\, W_j\, S_j / C_j}$

(d) $n_j = n \dfrac{W_j S_j\, C_j}{\Sigma_j\, W_j\, S_j\, C_j}$

Q. 52 To have minimum $V(\bar{x}_{st})$, one has to choose a large sample n_j using cost per unit of survey provided:

(a) n is fixed, S_j is small and C_j is small.
(b) n is small, S_j is small and C_j is large.
(c) n is large, S_j is large and C_j is large.
(d) n is large, S_j is small and C_j is small.

Q. 53 If the cost C of the survey is fixed and $C_1 = C - C_0$ where C_0 is overhead cost, optimum sample size for j^{th} stratum can be obtained by the formula:

(a) $n_{opt} = C_1 \dfrac{\Sigma_j\, W_j S_j \big/ \sqrt{C_j}}{\Sigma_j\, W_j\, S_j / C_j}$

(b) $n_{opt} = C_1 \dfrac{\Sigma_j\, W_j S_j \big/ \sqrt{C_j}}{\Sigma_j\, W_j\, S_j \big/ \sqrt{C_j}}$

(c) $n_{opt} = C_1 \dfrac{\Sigma_j\, W_j S_j \sqrt{C_j}}{\Sigma_j\, W_j\, S_j \big/ \sqrt{C_j}}$

(d) $n_{opt} = C_1 \dfrac{\Sigma_j\, W_j S_j\, C_j}{\Sigma_j \left(W_j\, S_j / C_j\right)}$

Q. 54 Formula for optimum sample size for j^{th} stratum with usual notations when the $V(\bar{x}_{st})$ is fixed and equal to V_0 is:

(a) $n_j = \dfrac{W_j S_j}{C_j} \dfrac{\Sigma_j\, W_j S_j \sqrt{C_j}}{V_0 + \Sigma_j\, W_j\, S_j^2}$

(b) $n_j = \dfrac{\left(W_j S_j \big/ \sqrt{C_j}\right)\left(\Sigma_j\, W_j S_j\, C_j\right)}{V_0 + \frac{1}{N} \Sigma_j\, W_j\, S_j}$

(c) $n_j = \sum_j \dfrac{W_j S_j}{\sqrt{C_j}} \dfrac{\Sigma_j\, W_j S_j \sqrt{C_j}}{V_0 + \frac{1}{N} \Sigma_j\, W_j\, S_j^2}$

(d) none of the above

Q. 55 With usual notations, the estimate of the variance of the stratified sample mean $\bar{x}_{st}$ is:

(a) $v(\bar{x}_{st}) = \sum_{j=1}^{k} W_j \left(1 - \dfrac{n_j}{N_j}\right) \dfrac{s_j^2}{n_j}$

(b) $v(\bar{x}_{st}) = \sum_{j=1}^{k} W_j^2 \left(\frac{N_j - n_j}{N_j}\right)\frac{s_j^2}{n_j}$

(c) $v(\bar{x}_{st}) = \frac{1}{N}\sum_{j=1}^{k} W_j \left(\frac{N_j - n_j}{N_j}\right)s_j^2$

(d) any of the above

Q. 56 With usual notations, the estimate of the variance of $\bar{x}_{st}$ under proportional allocation is:

(a) $v(\bar{x}_{st}) = \frac{N-n}{Nn}\sum_{j=1}^{k} W_j s_j^2$

(b) $v(\bar{x}_{st}) = \left(1 - \frac{n}{N}\right)\sum_{j=1}^{k} \frac{W_j^2 s_j^2}{n_j}$

(c) $v(\bar{x}_{st}) = \left(1 - \frac{n}{N}\right)\sum_{j=1}^{k} \frac{W_j s_j}{n}$

(d) $v(\bar{x}_{st}) = \left(1 - \frac{n}{N}\right)\sum_{j=1}^{k} \frac{W_j s_j}{n_j}$

Q. 57 With usual notation, the variance of the mean $\bar{x}_{st}$ of stratified sample under Neyman allocation is:

(a) $V_{\text{Ney}}(\bar{x}_{st}) = \frac{1}{n}\left(\sum_j W_j S_j \sqrt{C_j}\right)\left(\sum_j \frac{W_j C_j}{C_j}\right) - \frac{1}{N}\sum_j W_j S_j^2$

(b) $V_{\text{Ney}}(\bar{x}_{st}) = \frac{1}{n}\left(\sum_j \frac{W_j S_j}{\sqrt{C_j}}\right)^2 - \frac{1}{N}\sum_j W_j S_j^2$

(c) $V_{\text{Ney}}(\bar{x}_{st}) = \frac{1}{n}\left(\sum_j W_j S_j \sqrt{C_j}\right)^2$

(d) $V_{\text{Ney}}(\bar{x}_{st}) = \frac{1}{n}\left(\sum_j W_j S_j \sqrt{C_j}\right)\left(\sum_j \frac{W_j S_j}{\sqrt{C_j}}\right) - \frac{1}{N}\sum_j W_j S_j^2$

Q. 58 If the cost per unit of survey is same for all units, then the formula for $V(\bar{x}_{st})$ under Neyman allocation is:

(a) $V_{\text{Ney}}(\bar{x}_{st}) = \frac{1}{n}(\Sigma W_j S_j)^2 - \frac{1}{N}\sum_j W_j S_j^2$

(b) $V_{\text{Ney}}(\bar{x}_{st}) = \frac{1}{N}\left(\sum_j W_j S_j^2\right) - \frac{1}{n}\sum_j W_j S_j$

(c) $V_{\text{Ney}}(\bar{x}_{st}) = \frac{1}{N}\left(\sum_j W_j S_j^2\right) - \frac{1}{n}\sum_j W_j S_j^2$

(d) $V_{\text{Ney}}(\bar{x}_{st}) = \frac{1}{n}\left[\left(\sum_j W_j S_j\right)^2 - \sum_j W_i S_j^2\right]$

Q. 59 Variance of $\bar{x}_{st}$ under random sampling, proportional allocation and optimum allocation hold the correct inequality as:

(a) $V_{\text{ran}}(\bar{x}_{st}) \leq V_{\text{prop}}(\bar{x}_{st}) \leq V_{\text{opt}}(\bar{x}_{st})$

(b) $V_{\text{ran}}(\bar{x}_{st}) \geq V_{\text{opt}}(\bar{x}_{st}) \geq V_{\text{opt}}(\bar{x}_{st})$

(c) $V_{\text{ran}}(\bar{x}_{st}) \geq V_{\text{prop}}(\bar{x}_{st}) \geq V_{\text{opt}}(\bar{x}_{st})$

(d) all the above

Q. 60 Which of the following statement is correct?

(a) two way stratification can also be used

(b) two way stratification is usually better than one way stratification

(c) two way stratification is not much used

(d) all the above

Q. 61 If a sample is drawn from each stratum minimising the probabilities of non-preferred samples, it is known as:
(a) selection with proportional allocation
(b) controlled selection
(c) haphazard selection
(d) none of the above

Q. 62 Systematic sampling means:
(a) selection of n contiguous units
(b) selection of n units situated at equal distances
(c) selection of n largest units
(d) selection of n middle units in a sequence

Q. 63 If the number of population units N is an integral multiple of sampling size n, the systematic sampling is called:
(a) linear systematic sampling
(b) circular systematic sampling
(c) random systematic sampling
(d) all the above

Q. 64 Circular systematic sampling is used when:
(a) N is a multiple of n
(b) N is a whole number
(c) N is not divisible by n
(d) none of the above

Q. 65 Linear and circular systematic sampling methods are equivalent if and only if:
(a) N is a whole number
(b) n is a whole number
(c) $N = n$
(d) none of the above

Q. 66 Which of the following advantage of systematic sampling you approve?
(a) easy selection of sample
(b) economical
(c) spread of sample over the whole population
(d) all the above

Q. 67 Selected units of a systematic sample are:
(a) not easily locateable
(b) easily locateable
(c) not representing the whole population
(d) all the above

Q. 68 In what situation(s) a systematic sample is more preferred than others?
(a) when the data are on cards
(b) when the items are in row
(c) when the items situated at equal distances are uncorrelated
(d) all the above

Q. 69 A systematic sample does not yield good results if:
(a) variation in units is periodic
(b) units at regular intervals are correlated
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 70 Greatest drawback of systematic sampling is that:
(a) one requires a large sample
(b) data are not easily accessible
(c) no single reliable formula for standard error of mean is available
(d) none of the above

Q. 71 Which of the following statement is correct?
(a) systematic sample is superior than stratified random sample
(b) simple random sample is inferior than systematic sample
(c) stratified random sample is better than systematic sample
(d) none of the above

Q. 72 Double sampling is also known as:
(a) two stage sampling
(b) two phase sampling
(c) two directional sampling
(d) all the above

Q. 73 Double sampling has its utility in:
(a) stratified sampling
(b) ratio method of estimation
(c) regression method of estimation
(d) all the above

Q. 74 In which of the following situation(s) cluster sampling is appropriate?
(a) when the units are situated for apart
(b) when sampling frame is not available
(c) when all the elementary units are not easily identifiable.
(d) all the above

Q. 75 What precaution(s) make(s) cluster sampling more efficient?
(a) by taking clusters of small size
(b) choosing clusters having largest within variation
(c) choosing clusters having least variation between clusters
(d) all the above

Q. 76 Which of the following basis distinguishes cluster sampling from stratified sampling?
(a) clusters are preferably heterogeneous whereas strata are taken as homogeneous as possible.
(b) a sample is always drawn from each stratum whereas no sample of elementary units is drawn from clusters.
(c) small size clusters are better whereas there is no such restriction for stratum size
(d) all the above

Q. 77 What distinction exists between cluster sampling and two stage sampling?
(a) in cluster sampling one studies each unit of the selected cluster whereas in two stage sampling one selects a sample of elementary units from each cluster
(b) in two stage sampling one draws a sample in two stages whereas in cluster sampling only a sample of clusters is selected
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 78 In what situation two stage sampling is better than single stage sampling.
(a) when the elements in the same stage are positively correlated
(b) when the elements in the same stage are negatively correlated
(c) when the elements in the same stage are uncorrelated
(d) none of the above

Q. 79 Which of the following statements is true?
(a) all sampling procedures involve sampling with constant probability
(b) there exists sampling procedure in which the units are selected with varying probability
(c) every selection procedure of a sample involves probability
(d) all the above

Q. 80 Non-response in surveys mean:
(a) non-availability of respondents
(b) non-return of questionnaire by the respondents
(c) refusal to give information by the respondents
(d) all the above

Q. 81 Problem of non-response:
(a) has no solution
(b) can be solved
(c) is meaningless
(d) none of the above

Q. 82 Which of the following statements is true?
(a) population mean increases with the increase in sample size
(b) population mean decreases with increase in sample size
(c) population mean decreases with the decrease in sample size
(d) population mean is a constant value

Q. 83 Which of the following statements does not hold good?
(a) an increase in sample size reduces the standard error
(b) an increase in sample size decreases the sampling error
(c) decrease in sample size results in the reduction of population standard deviation
(d) the precision of an estimate depends on sample size

Q. 84 A sample of 25 units from an infinite population with standard deviation 10 results into a total score of 450. The mean of the sampling distribution is:
(a) 45
(b) 50
(c) 18
(d) 1.8

Q. 85 If population variance of an infinite population is σ^2 and a sample of n items is selected from this population, the standard error of sample mean is equal to:

(a) σ^2/n

(b) σ/n

(c) $\sigma/\sqrt{n}$

(d) σ

Q. 86 A sample of 16 items from an infinite population having S.D. = 4, yielded total scores as 160. The standard error of sampling distribution of mean is:

(a) 1
(b) 10
(c) 40
(d) none of the above

Q. 87 A population was divided into clusters and it was found that within cluster variation was less than the variation between clusters. If a sample of units was selected from each cluster, the sampling procedure used was:

(a) multistage sampling
(b) stratified sampling
(c) cluster sampling
(d) systematic sampling

Q. 88 A population is perfectly homogeneous in respect of a characteristic. What size of sample would you prefer?

(a) a large sample
(b) a small sample
(c) a single item
(d) no item

Q. 89 The selected items of a sample resulted into same values pertaining to a character. The variance of the sample is:

(a) 1
(b) 0
(c) ∞
(d) not determinable

Q. 90 A population is divided into clusters and it has been found that all items within a cluster are alike. Which of the following sampling procedures would you adopt?

(a) simple random sampling
(b) cluster sampling
(c) systematic sampling
(d) stratified sampling

Q. 91 A population of N units is divided into K strata. A sample of size n is to be selected. Let N_j the j^{th} stratum size and n_j the sample size from it ($j = 1, 2, ..., k$). Then formula for selection of n_j under proportional allocation is:

(a) $n_j = \frac{N}{n}$

(b) $n_j = \frac{N}{N_j}$

(c) $\frac{n_j}{N_j} = \frac{n}{N}$

(d) $n_j N_j = Nn$

Q. 92 If an investigator selects districts from a state, Panchayat samities from districts and farmers from Panchayat samities, then such a sampling procedure is known as:

(a) two stage sampling
(b) three stage sampling
(c) cluster sampling
(d) stratified sampling

Q. 93 In sampling with probability proportional to size, the units are selected with probability in proportion to:

(a) the size of the unit
(b) the size of the sample
(c) the size of the population
(d) none of the above

Q. 94 In case of inverse sampling, the proportion 'p' of m units of interest contained in a sample of n units is:

(a) m/n

(b) $(m-1)/n$

(c) $(m-1)/(n+1)$

(d) $(m-1)/(n-1)$

Q. 95 If the respondents do not supply the required information, this problem is known as:
(a) the problem of the non-response
(b) non-sampling error
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 96 Supposing that, in cluster sampling s_w^2 represents the variance within the clusters and s_b^2 between clusters. What is the relation between s_w^2 and s_b^2?
(a) $s_w^2 = s_b^2$
(b) $s_w^2 \geq s_b^2$
(c) $s_w^2 \leq s_b^2$
(d) none of the above

Q. 97 Two stage sampling design is more efficient than single stage sampling if the correlation between units in the first stage is:
(a) negative
(b) positive
(c) zero
(d) none of the above

Q. 98 What sampling design is most appropriate for cluster sampling?
(a) simple random sampling without replacement
(b) simple random sampling with replacement
(c) stratified random sampling
(d) quota sampling

Q. 99 Circular systematic sampling was first used by:
(a) W.G. Cochran
(b) M.H. Hansen
(c) D.B. Lahiri
(d) P.C. Mahalanobis

Q. 100 A population consisting of all real numbers is an example of:
(a) an infinite population
(b) a finite population
(c) an imaginary population
(d) none of the above

Q. 101 A random sample of a reasonably large size possessing almost all properties of the population confirms to the principle of:
(a) inertia of large numbers
(b) statistical regularity
(c) optimisation
(d) Newton's first law of inertia

Q. 102 The errors emerging out of faulty planning of surveys are categorised as:
(a) non-sampling errors
(b) non-response errors
(c) sampling errors
(d) absolute error.

Q. 103 If there is a certain number of very high values in a sample, then it is preferable to calculate:
(a) standard deviation
(b) standard error
(c) variance
(d) all the above

Q. 104 Principle of optimisation in sampling methods is related to:
(a) cost and efficiency of sampling designs
(b) validity of estimates
(c) asymptotic properties of estimates
(d) all the above

Q. 105 Which of the following sampling designs will be categorised as non-probability sampling?
(a) haphazard sampling
(b) convenience sampling
(c) judgement sampling
(d) all the above

Q. 106 The discrepancy between estimates and population parameters is known as:
(a) human error
(b) enumeration error
(c) sampling error
(d) formula error

Q. 107 To meet requirement of the principle of validity of sampling methods, one must adopt:
(a) purposive sampling
(b) restricted sampling
(c) probability sampling

(d) any type of sampling.

Q. 108 There are more chances of non-sampling errors than sampling errors in case of:
(a) studies of large samples
(b) complete enumeration
(c) inefficient investigators
(d) all the above

Q. 109 Increase in reliability and accuracy of results from a sampling study with the increase in sample size is known as the principle of:
(a) optimisation
(b) statistical regularity
(c) law of increasing returns
(d) inertia of large numbers

Q. 110 Sampling error can be reduced by:
(a) choosing a proper probability sampling
(b) selecting a sample of adequate size
(c) using a suitable formula for estimation
(d) all the above

Q. 111 The magnitude of the standard error of an estimate is an index of its:
(a) accuracy
(b) precision
(c) efficiency
(d) all the above

Q. 112 For estimating the population proportion P in a class of a population having N units, the variance of the estimator p of P based on sample for size n is:

(a) $\frac{N}{N-1}\cdot\frac{PQ}{n}$

(b) $\frac{N}{N-1}\cdot\frac{PQ}{N}$

(c) $\frac{N-n}{N-1}\cdot\frac{PQ}{n}$

(d) $\frac{N-1}{N-n}\cdot\frac{PQ}{n}$

Q. 113 For estimating the population mean T, let T_1 be the sample mean under srswor and T_2 under srswr. Then:

(a) $\text{var}(T_1) = \text{var}(T_2)$

(b) $\text{var}(T_1) = 1/\text{var}(T_2)$

(c) $\text{var}(T_1) < \text{var}(T_2)$

(d) $\text{var}(T_1) \geq \text{var}(T_2)$

Q. 114 Let the standard error of an estimator T under srswor is more than the standard error of T under stratified randomly sampling. Then T under stratified sampling as compared to T under srswor is:
(a) more reliable
(b) less reliable
(c) equally reliable
(d) not comparable

Q. 115 Stratified sampling belongs to the category of:
(a) judgement sampling
(b) subjective sampling
(c) controlled sampling
(d) non-random sampling

ANSWERS

SECTION-B

(1) necessary or unavoidable (2) real (3) hypothetical (4) finite (5) infinite (6) complete enumeration or census (7) non-sampling error (8) sampling error (9) parameter (10) estimator (11) estimate (12) standard error (13) $s/\sqrt{n}$ (14) sampling frame (15) universe (16) $\binom{N}{n}$ (17) $1/\binom{N}{n}$ (18) infinite (19) more than once (20) $\Sigma_i x_i/n$ (21) $\Sigma_i (x_i-\bar{x})^2/(n-1)$ (22) stratified sampling (23) heterogeneous (24) character or attribute (25) allocation (26) proportional allocation (27) restricted (28) larger (29) $\left(\frac{N-n}{N}\right)\frac{S^2}{n}$ (30) finite population correction (31) sampling fraction (32) purposive or judgement (33) seldom (34) large (35) different

(36) $\frac{n}{n-1}pq$ (37) 5 per cent or less (38) optimum (39) Neyman (40) optimum (41) self-weighing (42) $W_j S_j$ (43) $N_j S_j/\sqrt{C_j}$ (44) Neyman (45) srswor (46) $V_{\text{ran}}(\bar{x}_{st})$ (47) stratified sampling (48) homogeneous (49) stratified random sampling (50) complete enumeration (51) complete enumeration (52) equal allocation (53) two way stratification (54) more (55) controlled selection (56) systematic sampling (57) contiguous (58) simple; cheap (59) no single (60) linear (61) circular (62) double (63) two phase (64) stratified (65) ratio; regression (66) sampling frame (67) far apart (68) primary units (69) elementary (70) $\le$ (71) increases (72) multistage sampling (73) negatively correlated (74) not fixed or a random variable (75) $(m-1)/(n-1)$ (76) $\frac{p(1-p)}{n-2}\left(1-\frac{n-1}{N}\right)$ (77) probability proportional to size (78) non-response (79) not compulsory (80) true representative (81) better (82) cannot (83) can (84) cannot (85) remains same (86) 5 (87) fixed (88) increases (89) improper (90) faulty (91) non-response (92) inadequacy (93) non-sampling (94) reliable (95) loss (96) reduce (97) less (98) less (99) *pps* (100) greater (101) desirable (102) empirical (103) statistical regularity (104) minimum (105) validity (106) standard error (107) optimisation (108) accurate; reliable (109) precision (110) inertia of large numbers.

SECTION-C

(1) d (2) d (3) a (4) c (5) d (6) b
(7) a (8) b (9) d (10) b (11) d (12) c
(13) b (14) d (15) c (16) c (17) b (18) a
(19) b (20) d (21) c (22) b (23) b (24) a
(25) c (26) d (27) c (28) b (29) d (30) d
(31) a (32) c (33) d (34) d (35) b (36) d
(37) a (38) c (39) d (40) b (41) d (42) b
(43) c (44) c (45) b (46) d (47) b (48) c
(49) c (50) d (51) a (52) d (53) b (54) c
(55) b (56) a (57) d (58) a (59) c (60) d
(61) b (62) b (63) a (64) c (65) d (66) d
(67) b (68) d (69) c (70) c (71) d (72) b
(73) d (74) d (75) d (76) d (77) c (78) b
(79) b (80) d (81) b (82) d (83) c (84) c
(85) c (86) a (87) b (88) c (89) b (90) d
(91) c (92) b (93) a (94) d (95) a (96) c
(97) b (98) a (99) c (100) a (101) b (102) a
(103) b (104) a (105) d (106) c (107) c (108) d
(109) d (110) d (111) b (112) c (113) d (114) a
(115) c

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Chapter **10**

Theory of Estimation

SECTION-A

Short Essay Type Questions

Q. 1 What do you understand by estimation?

Ans. There are a few occasions when population is studied as a whole. As a matter of fact, generally a sample is drawn from the population and population constants are determined on the basis of sample values. *Population parameters* are usually those constants which occur in the probability density or mass function or the moments or some other constants of the population like median.

We know that various sampling procedures do exist and also there are many techniques to determine the value of population constants through sample values. *The constant determined through sample observations which stands for population parameter* θ *or a function* $\tau(\theta)$ *of* θ *is called an estimate.* So in general we adopt $\tau(\theta)$ though $\tau(\theta)$ in many cases is equal to θ.

The choice of a technique depends on the type of the estimator *vis-a-vis* estimate and the purpose of study. The goodness of an estimator is governed by certain properties. An estimator possessing the maximum properties will be considered as a good estimator.

In general an estimator T_n will be rated as good if it differs from $\tau(\theta)$ by a small quantity ε, *i.e.*, the sampling distribution of T_n has high degree of concentration about $\tau(\theta)$. At the same time T_n will be considered as best estimator if for any other estimator T_n', the following inequality holds.

$$P\left\{\left|T_n - \tau(\theta)\right| < \varepsilon\right\} \geq P\left\{\left|T_n' - \tau(\theta)\right| < \varepsilon\right\}$$

for all θ.

But holding of this condition is not easily feasible. Hence, one has to look for other properties of the estimator.

So in estimation theory we are concerned with the properties of estimators and methods of estimation. The merits of an estimator are judged by the properties of the distribution of estimates obtained through estimators, *i.e.*, by the properties of the sampling distribution. Further, it is emphasised that estimation is possible only if there is a random sample.

Q. 2 Differentiate between an estimator and an estimate.

Ans. A known function $T = t(X_1, X_2, ..., X_n)$ of the observable variates of a random sample $X_1, X_2, ...,$ X_n whose values are used to obtain the estimate of a parameter θ or a function of θ, is called an estimator. An estimator is itself a random variable. If $x_1, x_2, ...,$ x_n are the values of the random sample $X_1, X_2, ..., X_n$, the value $t(x_1, x_2, ..., x_n)$ of the estimator T_n is known as an estimate of the parameter θ. For example $\frac{1}{n}\sum X_i$ $(i = 1, 2, ..., n)$ is an estimator whereas $\bar{x} = \frac{1}{n}\sum x_i$ is an estimate.

Q. 3 Throw light on the types of estimates.

Ans. With the help of sample observations we find a value which is taken as a value of the parameter θ. This value is termed as *point estimate*. Single estimated value is known as point estimate or simply estimate.

Also, instead of estimating a single value of a parameter from sample values, a range t_1, t_2 of numbers, which constitute an interval, determined with the help of sample values and supposed to include the parameter θ with certain confidence level $\gamma = 1 - \alpha$ is known as confidence interval. t_1 and t_2 $(t_1 < t_2)$ are called the lower and upper limits of the interval estimate.

Q. 4 Name different properties of estimators.

Ans. Different properties of estimators *vis-a-vis* estimates are as follows:

(i) Consistency
(ii) Unbiasedness
(iii) Mean-squared error
(iv) Best asymptotically normal estimator or BAN estimators
(v) Efficiency
(vi) Sufficiency
(vii) Completeness
(viii) Minimum Variance Unbiased estimators
(ix) Admissibility.

Q. 5 Define and discuss consistency of estimators in brief.

Ans. An estimator T_n calculated from n sample variates is said to be a consistent estimator of a parameter θ if for any individual small positive quantity ε and a small positive value η there exists some N such that the following inequality holds, the estimator T_n is said to be consistent, if

$$P\left[\{T_n - \tau(\theta)\} < \varepsilon\right] < 1 - \eta \text{ for } n > N$$

Here T_n converges in probability to τ (θ) or stochastically converging to τ θ. The above definition is known as *simple consistency*. In short, T_n is said to be a consistent estimator of τ (θ) if T_n converges to τ (θ) in probability, *i.e.*,

$$T_n \xrightarrow{p} \tau(\theta)$$

Consistency is a limiting property of estimators and it reveals the behaviour of an estimator T_n as $n \to \infty$. As a simple example, $\Sigma X_i/n, \Sigma X_i/(n-1), \Sigma X_i/(n-2), \ldots$ are all consistent estimators of population mean and the selection of either of them on the basis of consistency is not possible. So one has to look for other properties of T_n.

Q. 6 What is meant by unbiasedness of estimators?

Ans. An estimator T_n is said to be an unbiased estimator of τ (θ) if the expected value of T_n is equal to

$$E(T_n) = \tau(\theta) \text{ for all } \theta.$$

If $E(T_n) \neq \tau(\theta)$, then $E(T_n) - \tau(\theta) = bias.$

In short, an estimator is unbiased if the mean of the sampling distribution of T_n is equal to τ (θ).

Also the difference $E(T_n) - \tau(\theta)$ is called the bias of the estimator T_n. Bias will be positive, if $\tau(\theta) < E(T_n)$ and negative if $\tau(\theta) > E(T_n)$.

Q. 7 Describe the role of mean-squared error in estimation theory.

Ans. If there are more than one unbiased estimators, the problem arises which one to choose out of the class of unbiased estimators. Not only this, one aspires that the sampling variance as well as bias should be minimum. These problems are tackled with the help of mean-squared error (m.s.e). The mean-squared error of an estimator T_n of τ (θ) is given as,

$$\begin{aligned}\text{m.s.e} &= E[T_n - \tau(\theta)]^2 \\ &= E[E_\theta(T_n) - \tau(\theta) + T_n - E_\theta(T_n)]^2 \\ &= E[E_\theta(T_n) - \tau(\theta)]^2 + E[T_n - E_\theta(T_n)]^2\end{aligned}$$

$$= E_\theta\left[(T_n) - \tau(\theta)\right]^2 + E\left[T_n - E_\theta(T_n)\right]^2$$

$$= (bias)^2 + \text{var}(T_n)$$

where $E_\theta(T_n) - \tau(\theta) =$ bias

Mean squared error will be minimum if T_n is an unbiased estimator of τ (θ), *i.e.*, $E_\theta(T_n) = \tau(\theta)$ and when var (T_n) is minimum. It is an impossible task to have an estimator with least mean squared error. Hence, it is our endeavour to search for an estimator with uniformly minimum variance among the class of unbiased estimators which we call uniformly minimum variance unbiased estimator (UMVUE).

By definition, an estimator T_n based on the random sample $X_1, X_2, \ldots, X_n$ is said to be UMVUE of $\tau(\theta)$ if $E_\theta(T_n) = \tau(\theta)$ and var $(T_n) \leq \text{var}(T_n^*)$ where T_n^* is any other unbiased estimator of τ (θ).

Q. 8 Define mean-squared error consistency.

Ans. Suppose $T_1, T_2, \ldots, T_n$ is a sequence of estimators of τ (θ) based on a sample $X_1, X_2, \ldots, X_n$, *i.e.*, $T_n = t(X_1, X_2, \ldots, X_n)$. The sequence of estimators $T_1, T_2, \ldots, T_n$ is said to be mean-squared error consistent estimator of τ (θ) if and only if

$$\lim_{n\to\infty} E_\theta\left\{T_n - \tau(\theta)\right\}^2 = 0$$

for all θ in Θ. Mean-squared error consistency implies that the estimator T_n is unbiased as well as the variance of T_n approaches to zero as n tends to infinity.

Q. 9 State Crammer-Rao inequality for lower bound of variance of an estimator.

Ans. Suppose $X_1, X_2, \ldots, X_n$ is a random sample from $f(x, \theta)$ and $T_n = t(X_1, X_2, \ldots, X_n)$ is an estimator of τ (θ) based on the random sample of size n. Crammer-Rao inequality states that under the regularity conditions.

(i) $\frac{\partial}{\partial\theta}\log f(x;\theta)$ exists for all x and θ.

(ii) $\frac{\partial}{\partial\theta}\int\ldots\int \prod_{i=1}^{n} f(x_i,\theta)\,dx_1\,dx_2\ldots dx_n$

$$= \int\ldots\int \frac{d}{\partial\theta}\prod_{i=1}^{n} f(x_i;\theta)\,dx_1\,dx_2\ldots,dx_n$$

(iii) $\frac{\partial}{\partial\theta}\int\ldots\int t(x_1, x_2, \ldots, x_n)\prod_{i=1}^{n} f(x_i;\theta)$

$$dx_1\,dx_2\ldots dx_n = \int\ldots\int t(x_1, x_2, \ldots, x_n)$$

$$\frac{\partial}{\partial\theta}\prod_{i=1}^{n} f(x_i;\theta)\,dx_1\,dx_2\ldots dx_n$$

(iv) $0 \leq E\left[\left\{\frac{\partial}{\partial\theta}\log f(x_i;\theta)\right\}^2\right] < \infty$ for all θ in Θ

where Θ is the parameter space.

The variance of T_n an estimator of τ (θ) which is differentiable with respect to θ, *i.e.*, τ' (θ) exists, satisfies the inequality,

$$V_\theta(T_n) \geq \frac{\left[\tau'(\theta)\right]^2}{E\left[\frac{\partial}{\partial\theta}\log f(x_i;\theta)\right]^2}$$

$$\geq -\frac{\left[\tau'(\theta)\right]^2}{E\left[\frac{\partial^2}{\partial\theta^2}\log f(x_i;\theta)\right]}$$

The quantity $E\left[\frac{\partial}{\partial\theta}\log f(x_i;\theta)\right]^2$ is called the amount of information about θ in the set of random variables $X_1, X_2, \ldots, X_n$.

If τ (θ) = θ, then τ' (θ) = 1. In this situation,

$$V_\theta(T_n) \geq -\frac{1}{E\left[\frac{\partial}{\partial\theta}\log f(x_i;\theta)\right]^2}$$

The quantity $-1/E\left[\frac{\partial}{\partial\theta}\log f(x_i;\theta)\right]^2$ is called the information limit of T_n. If $X_1, X_2, \ldots, X_n$ are n independent and identically distributed random variables, then

$$V_\theta(T_n) \geq \frac{[\tau'(\theta)]^2}{nE\left[\frac{\partial}{\partial\theta}\log f(x_i;\theta)\right]^2}$$

$$\geq -\frac{[\tau'(\theta)]^2}{nE\left[\frac{\partial^2}{\partial\theta^2}\log f(x_i;\theta)\right]}$$

It is worth noting that Crammer-Rao inequality gives the lower bound of $V_\theta(T_n)$. An estimator which attains this lower bound for all θ in Θ will be called a minimum variance bound (MVB) estimator. Crammer-Rao inequality remains valid even through the random variates $X_1, X_2, ..., X_n$ are discrete. C.R. Rao gave this inequality in 1945, whereas H. Crammer independently gave it in 1946. However it had already been given by Aitken and Silverstone in 1942. Chapman Robins had also given the lower bound for the variance of T_n parallel to Crammer-Rao inequality. Chapman-Robins inequality is considered to be an improvement over Crammer-Rao inequality as it does not require stringent regularity conditions.

A. Bhattacharyya also obtained the lower bound for $V_\theta(T_n)$ in 1946 under some amended regularity conditions of Crammer-Rao inequality.

Q. 10 Define the best asymptotically normal estimators and throw light on its implications.

Ans. Suppose we have a sequence of estimators T_n of various sample sizes say $T_n = t_n(x_1, x_2, ..., x_n)$ where n indicates the size of the sample on which T_n is based. For clarity $T_1 = t_1(X_1)$, $T_2 = t_2(X_1, X_2)$, ..., $T_n = t_n(X_1, X_2, ..., X_n)$. Also we know, the estimator 'sample mean' of population mean remains $\Sigma X_i/n$ (i = 1, 2, ..., n) whatever may be the sample size though for varying n, we get different estimates. But out of these estimates, one would like to select one which is closest to the parameter value. Hence, a sequence of estimators $T_1, T_2, ..., T_n$ of $\tau(\theta)$ will be called best asymptotically normal (BAN) if and only if it satisfies the following conditions:

(i) The distribution of $\sqrt{n}\,[T_n - \tau(\theta)] \to N(0, \sigma)^2$ as $n \to \infty$

(ii) For any small quantity ε,

$$\lim_{n\to\infty} P_\theta[|T_n - \tau(\theta)| > \varepsilon] = 0 \text{ for all } \theta \text{ in } \Theta$$

It is the same condition as for consistency.

(iii) If $\{T_n^*\}$ is any sequence of estimators other than $\{T_n\}$ such that,

$$\sqrt{n}\left[T_n^* - \tau(\theta)\right] \to N\left(0, \sigma^{*2}\right) \text{as } \nu \to \infty,$$

Then $\sigma^{*2} \geq \sigma^2$ for all θ in Θ in an open interval.

It has been observed that reasonably good estimators are usually asymptotically normally distributed.

Since BAN estimators are consistent as well, they are sometimes given another name, *consistent asymptotically normal efficient* (CANE) estimators.

BAN estimators assimilate the limiting property as well as the property of minimum variance.

Q. 11 Express the efficieny of an estimator.

Ans. An unbiased estimator T_n is said to be efficient than any other estimator T_n^* of τ (θ) if and only if,

$$V(T_n) < V\left(T_n^*\right)$$

Also the relative efficiency of T_n as compared to T_n^* is given as,

$$\text{R.E.} = \frac{V\left(T_n^*\right)}{V(T_n)}$$

Crammer gave the term *efficient estimator* to mean a *minimum variance unbiased estimator* (MVUE) or *best unbiased estimator*. Hence, MVU estimator is unbiased and also among the class of unbiased estimators it possesses minimum variance.

A MVU estimator is unique in the sense that

$$V(T_n) = V\left(T_n^*\right) \Rightarrow T_n = T_n^*$$

Q. 12 Expatiate the concept of sufficiency of an estimator.

Ans.

Definition 1. An estimator T_n is said to be sufficient for $\tau(\theta)$ if it provides all the information contained in the sample about the parametric function $\tau(\theta)$.

If we have a random sample $X_1, X_2, ..., X_n$ from a density $f(x; \theta)$, a statistic $S = s(X_1, X_2, ..., X_n)$ (which is itself a random variable) which condenses n random variables $X_1, X_2, ..., X_n$ into a single random variable and provides as much information as the random sample $X_1, X_2, ..., X_n$ could reveal about the probability distribution characterised by $f(x; \theta)$ is known as sufficient statistics. So, the idea is that if we know the sufficient statistics, there is no need of knowing the sample values.

Definition 2. A statistic $S = s(X_1, X_2, ..., X_n)$ is sufficient for the parameter θ of a density $f(x; \theta)$ if the conditional distribution of $X_1, X_2, ..., X_n$ for a given value of $S = s$ is independent of the parameter θ.

From this definition it is clear that one cannot expect to get an information about θ from a distribution that does not contain θ.

Definition 3. This definition is known as Fisher-Neyman factorization theorem. The theorem can be stated as follows:

Let $X_1, X_2, ..., X_n$ be a random sample from a density $f(x; \theta)$. A statistic $S = s(X_1, X_2, ..., X_n)$ is said to be sufficient statistic for θ if the joint probability density function of $X_1, X_2, ..., X_n$ can be factorized as,

$$f_{X_1, X_2, ..., X_n}(x_1, x_2, \ldots, x_n; \theta) = g\{s(x_1, x_2, \ldots, x_n); \theta\}$$
$$\times h(x_1, x_2, \ldots x_n) = g(s; \theta) h(x_1, x_2, \ldots x_n)$$

On the right hand $g(s; \theta)$ is the density function of s (the sufficient statistic) involving the unknown parameter θ and $h(x_1, x_2, ..., x_n)$ is the conditional density function $X_1, X_2, ..., X_n$ for given s and is independent of θ.

Q. 13 Give the idea of minimal sufficient statistics.

Ans. The idea of sufficient statistics is to condense the data to a form that provides maximum information. Hence, a sufficient statistics S will be termed as minimal sufficient statistics if the statistics for the sample $X_1, X_2, ..., X_n$ cannot be condensed any more beyond S without sacrificing the sufficiency.

Q. 14 Define complete statistic and complete parametric families..

Ans. Let $f(x; \theta)$ be a parametric family of univariate or multivariate distributions depending on the vector of parameter θ and $h(x)$ be any statistic independent of θ. If

$$E_l[h(x)] = \int h(x) f(x;\theta) dx$$
$$= 0 \Rightarrow h(x) = 0 \text{ for all } \theta,$$

then $f(x; \theta)$ is said to be complete.

The term 'complete' is appropriate in the sense that there exists no non-zero function which is orthogonal to all members of the family.

A sufficient statistics $S = s(X_1, X_2, ..., X_n)$ is said to be complete, if

$$E_\theta(S) = 0 \Rightarrow s = 0$$

A statistics which is unbiased and complete is necessarily unique if a MVU estimator exists.

Q. 15 State Rao-Blackwell theorem.

Ans. Suppose $X_1, X_2, ..., X_n$ is a random sample from the density $f(x; \theta)$ and $S = s(X_1, X_2, ..., X_n)$ is any sufficient statistic for θ of the density $f(x; \theta)$. Also let $T = t(X_1, X_2, ..., X_n)$ be any other unbiased estimator of $\tau(\theta)$. Also let there be any other statistic T_1, such that $T_1 = E(T/S = s)$. Then,

(i) T_1 is a statistic and is a function of the sufficient statistic S, *i.e.*, $T_1 = t_1(s)$.

(ii) $E_\theta(T_1) = \tau(\theta)$, *i.e.*, T_1 is an unbiased estimator of $\tau(\theta)$.

(iii) $\text{var}_\theta(T_1) \le \text{var}_\theta(T)$ for every θ.

(iv) $\text{var}_\theta(T_1) < \text{var}_\theta(T)$ for some θ.

The theorem can easily be generalised for a set of sufficient statistics $S_1 = s_1(X_1, X_2, ..., X_n)$, $S_2 = s_2(X_1, X_2, ..., X_n)$ ..., $S_k = s(X_1, X_2, ..., X_n)$ and also θ be a vector. The implication of the theorem is that given an unbiased estimator, another unbiased estimator T_1 which is a function of the sufficient statistics can be created which will not have greater variance than that of T.

The concepts of consistency, efficiency and sufficiency are due to R.A. Fisher.

Q. 16 Give invariance property of estimators.

Ans. This property relates to the coding of data. Let $X_1, X_2, ..., X_n$ be a random sample from the density $f(x; \theta)$ and $x_1, x_2, ..., x_n$ be the sample value of $X_1, X_2, ..., X_n$ respectively. Also θ be a vector. Then an estimator $T = t(X_1, X_2, ..., X_n)$ is said to be *location invariant* if and only if $t(x_1+c, x_2+c, \ldots, x_n+c) = t(x_1, x_2, \ldots, x_n)+c$ for all c where c is a constant value.

An estimator $T = t(X_1, X_2, ..., X_n)$ is said to be scale invariant if and only if $t(cx_1, cx_2, ..., cx_n) = ct(x_1, x_2, ..., x_n)$ for all c where c is a non-zero constant.

Q. 17 When do you call an estimator admissible?

Ans. Let $X_1, X_2, ..., X_n$ be a random sample from the density $f(x; \theta)$ and $T_1 = t_1(X_1, X_2, ..., X_n)$ and $T_2 = t_2(X_1, X_2, ..., X_n)$ be any two estimators for θ. The estimator T_1 is said to be a better estimator than T_2 if and only if the risks hold the inequalities,

$$R(t_1, \theta) \le R(t_2, \theta) \text{ for all } \theta \text{ in } \Theta$$

$$R(t_1, \theta) < R(t_2, \theta) \text{ for some } \theta \text{ in } \Theta$$

t_1 will be said to be an admissible estimator if and only if there is no other estimator t_2 better than t_1.

The readers should note that $R(t, \theta)$ is called the risk of taking t as an estimator of θ and is defined as the expected loss (average loss), *i.e.*,

$$R(t, \theta) = E\{l(T, \theta)\}$$

where $T = t(X_1, X_2, ..., X_n)$

$$R(t, \theta) = \int l(T, \theta) f_T(t) dt$$

where $f_T(t)$ is the density of the estimator T. Concept of risk is more akin to decision theoretic approach.

Q. 18 Give the names of various methods of estimation of a parameter.

Ans. Various methods of estimation are listed below:

1. Method of maximum likelihood
2. Least square method
3. Method of moments
4. Method of minimum Chi-square
5. Method of modified minimum Chi-square
6. Bayes' estimation procedure.

Q. 19 Describe the method of maximum likelihood estimation.

Ans. Maximum likelihood principle is due to R.A. Fisher in 1921.

Let $X_1, X_2, ..., X_n$ be n independent observations from $f(x; \theta)$ where θ is a single unknown parameter. The joint probability density function of the sample is called likelihood function (LF) and is written as,

$$\underset{\sim}{L}(x|\theta) = f(x_1, \theta) f(x_2, \theta) \ldots f(x_n, \theta)$$

According to maximum likelihood principle, one should take that value of estimator θ within the admissible range of θ which makes $L(x|\theta)$ maximum. For this, the method of maxima-minima is used. If $L(x|\theta)$ is differentiable *twice, i.e.*, if the first and second derivatives of $L(x|\theta)$ exists, put $L'(x|\theta) = 0$ and solve for θ. Also, for maxima check that $L''(x \mid \theta)$ is negative for a value of θ obtained by $L'(x \mid \theta)$. If so, the solution of $L(x|\theta)$ provides the maximum likelihood estimate of θ. In practice it is better to take logarithm of $L(x|\theta)$ and then differentiate and solve it. This makes the estimation process easier.

If θ is a K dimensional parametric vector, *i.e.*, $\theta = (\theta_1, \theta_2, \ldots, \theta_k)$, then the estimator $(\hat{\theta}_1, \hat{\theta}_2, ..., \hat{\theta}_k)$, which maximise $L(x|\theta_1, \theta_2, \ldots, \theta_k)$, can be obtained by differentiating partially the log $\{L(x \mid \theta_1, \theta_2, ..., \theta_k)$, with respect to $\theta_1, \theta_2, ..., \theta_k$ respectively and equating them to zero. The solution of k equations provides the estimates of $\theta_1, \theta_2, ..., \theta_k$. Notationally,

$$\frac{\partial}{\partial \theta_1}\{\log L(x|\theta_1, \theta_2, \ldots, \theta_k)\} = 0$$

$$\frac{\partial}{\partial\theta_2}\{\log L(x|\theta_1,\theta_2,\ldots,\theta_k)\}=0$$

$$\vdots$$

$$\frac{\partial}{\partial\theta_k}\{\log L(x|\theta_1,\theta_2,\ldots,\theta_k)\}=0$$

In this way we get k equations in k unknowns. These equations are often called the *likelihood equations.* Solving these equations one gets the maximum likelihood estimates of $\theta_1,\theta_2,\ldots,\theta_k$. To show that $\hat{\theta}$, the k-dimensional vector of estimates provides the supremum of $L(x|\theta)$, it is enough to show that the matrix $\left(\frac{\partial^2 \log L}{\partial\theta_i\partial\theta_j}\right)_{\theta=\hat{\theta}}$ is negative definite.

Q. 20 What properties of estimators are being usually held by maximum likelihood estimators?

Ans. In general, the following statements about the properties of maximum likelihood estimators (MLE) can be given:

(i) A MLE is not necessarily unique.
(ii) A MLE is not necessarily unbiased.
(iii) A MLE may not be consistent in rare cases.
(iv) A MLE may not be uniformly minimum variance unbiased estimator (UMVUE).
(v) If a sufficient statistic exists, it is a function of the maximum likelihood estimators.
(vi) If a minimum variance bound (MVB) unbiased estimator exists, maximum likelihood estimator provides it.
(vii) If $T=t(X_1,X_2,\ldots,X_n)$ is a sufficient statistics for a parameter θ and a unique MLE $\hat{\theta}$ of θ exists, $\hat{\theta}$ is a function of t. Also, if any MLE exists, MLE $\hat{\theta}$ can be found which is a function of t.
(viii) If $T=t(X_1,X_2,\ldots,X_n)$ is a MLE of θ and $\tau(\theta)$ is a one to one function of θ, $\tau(t)$ is a MLE of $\tau(\theta)$. This is known as *invariance property* of maximum likelihood estimator.
(ix) Under very general conditions, maximum likelihood estimators are consistent. Huzurbazar in 1948 showed that under regularity conditions, as sample size n tends to infinity, there exists a unique consistent maximum likelihood estimator.
(x) Under certain conditions, it has been observed that ML estimator is consistent but it is generally unbiased.
(xi) When maximum likelihood estimator exists, it is most efficient for large samples under the assumption that the distribution of the estimator tends to normal.

Q. 21 A random sample of size n is drawn from a normal population $N(\mu, \sigma^2)$. Estimate μ and σ^2 by the method of maximum likelihood. Also comment on the properties of maximum likelihood estimators.

Ans. The likelihood function,

$$\log L(x_i|\mu,\sigma^2)=\left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n e^{-\frac{1}{2\sigma^2}\Sigma(x_i-\mu)^2}$$

To estimate μ and σ^2, we take logarithm and differentiate partially w.r.t μ and σ^2 respectively and equate them to zero.

$$\log L=-\frac{n}{2}\log(2\pi)-\frac{n}{2}\log\sigma^2$$

$$-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\frac{\partial\log L}{\partial\mu}=\frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\hat{\mu})=0$$

which implies that $\sum_{i=1}^{n}(x_i-\hat{\mu})=0$ since $\frac{1}{\sigma^2}\neq 0.$

$$\therefore \qquad \sum_{i=1}^{n}x_i-n\hat{\mu}=0$$

or

$$\hat{\mu}=\frac{1}{n}\sum_{i=1}^{n}x_i=\bar{x}$$

Again, $\frac{\partial\log L}{\partial\sigma^2}=\frac{-n}{2\hat{\sigma}^2}+\frac{1}{2\hat{\sigma}^4}\sum(x_i-\hat{\mu})^2=0$

or $$\frac{1}{\hat{\sigma}^2}\sum_{i=1}^{n}(x_i-\bar{x})^2=n$$

or $$\hat{\sigma}^2=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

The ML estimate of μ is sample mean $\bar{x}$ which is *unbiased, consistent* and BAN. The estimate of σ^2 is $\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$ which is not unbiased but consistent. Also $\bar{x}$ is minimum variance bound estimate of μ with variance $\frac{\sigma^2}{n}$. It can be verified that ML estimators of μ and σ^2 are invariant in case of normal distribution. Also the estimators $\bar{x}$ and $\Sigma(x_i-\bar{x})^2/n$ are sufficient for μ and σ^2.

Q. 22 If n_1 trials conducted are of Bernoullian type following binomial distribution, find the maximum likelihood estimate of p.

Ans. We know that probability function of binomial distribution is

$$f(n_1,x_i)=\binom{n_1}{x_i}p^{x_i}(1-p)^{n_1-x_i}$$

for $i = 1, 2, \ldots, n$

The likelihood function,

$$L(x|p)=\prod_{i=1}^{n}\binom{n_1}{x_i}p^{x_i}(1-p)^{n_1-x_i}$$

Taking logarithm of both sides,

$$\log L=\sum_{i=1}^{n}\log\binom{n_1}{x_i}+\sum_{i=1}^{n}x_i\log p$$

$$+\sum_{i=1}^{n}(n_1-x_i)\log(1-p)$$

Differentiating partially w.r.t p and equating to zero.

$$\frac{\partial\log L}{\partial p}=0+\sum x_i\cdot\frac{1}{\hat{p}}-\frac{nn_1-\Sigma x_i}{1-\hat{p}}=0$$

i.e., $$nn_1\hat{p}=\Sigma x_i$$

or $$\hat{p}=\frac{\Sigma x_i}{nn_1}$$

$$=\frac{\bar{x}}{n_1}$$

It is trivial to show that $\bar{x}/n_1$ is a maximum likelihood estimate of p.

Q. 23 If X is a Poisson variate with parameter μ, find the maximum likelihood estimate of μ.

Ans. $$P(x;\mu)=\frac{e^{-\mu}\mu^{x}}{x!}$$

for $x = 0, 1, 2, \ldots, n$.

The likelihood function,

$$L(x_i|\mu)=\prod_{i=1}^{n}\frac{e^{-\mu}\mu^{x_i}}{x_i!}$$

for $i = 1, 2, \ldots, n$

$$\log L=\sum_{i=1}^{n}\log_e e^{-\mu}+\sum_{i=1}^{n}x_i\log\mu-\sum_{i=1}^{n}\log_e(x_i!)$$

$$=-n\mu+\sum_{i=1}^{n}x_i\log\mu-\sum_{i=1}^{n}\log_e(x_i!)$$

$$\frac{\partial\log L}{\partial\mu}=-n+\frac{\Sigma x_i}{\hat{\mu}}-0=0$$

$\therefore$ $$\frac{\Sigma x_i}{\hat{\mu}}=n$$

or $$\hat{\mu}=\frac{\Sigma x_i}{n}=\bar{x}$$

Sample mean is the maximum likelihood estimate of μ.

Q. 24 For gamma distribution $G(x;\alpha,\lambda)$ find the ML estimate of α where λ is taken to be a known constant.

Ans. $$G(x;\alpha,\lambda)=\frac{\alpha^{\lambda}}{\Gamma\lambda}e^{-\alpha x}x^{\lambda-1}\text{ for }\alpha>0$$

$$L(x_i|\alpha,\lambda)=\prod_{i=1}^{n}\frac{\alpha^{\lambda}}{\Gamma\lambda}e^{-\alpha x_i}x_i^{\lambda-1}$$

$$\log L = \sum_{i=1}^{n} \log\frac{\alpha^{\lambda}}{\Gamma\lambda} - \alpha\sum_{i=1}^{n} x_i + (\lambda-1)\sum_{i=1}^{n} \log x_i$$

Differentiating partially w.r.t α, we get

$$\frac{\partial \log L}{\partial\alpha} = \frac{n}{\Gamma\lambda}\frac{\lambda\alpha^{\lambda-1}}{\hat{\alpha}^{\lambda}} - \sum x_i + 0 = 0$$

$$\therefore \qquad \frac{n\lambda}{\hat{\alpha}\Gamma\lambda} - \sum x_i = 0$$

$$\frac{n\lambda}{\hat{\alpha}\Gamma\lambda} = \sum x_i$$

or $$\hat{\alpha} = \frac{n\lambda}{\Gamma\lambda\Sigma x_i} = \frac{\lambda}{\Gamma\lambda\bar{x}}.$$

Q. 25 Find the maximum likelihood estimate of the parameter θ of the distribution,

$$f(x;\theta) = \frac{1}{2}e^{-|x-\theta|}$$

$$-\infty < x < \infty, -\infty < \theta < \infty$$

Ans. The likelihood function for the sample observations $x_1, x_2, \ldots, x_n$ is

$$L = \frac{1}{2}e^{-|x_1-\theta|}\cdot\frac{1}{2}e^{-|x_2-\theta|},\ldots,\frac{1}{2}e^{-|x_n-\theta|}$$

$$L = \left(\frac{1}{2}\right)^{n}\prod_{i=1}^{n} e^{-|x_i-\theta|}$$

$$\log\ L = -n\log 2 - \sum_{i=1}^{n}|x_i-\theta|$$

log L is not differentiable w.r.t θ. However according to ML principle log L has to be maximised. Log L is maximum when $\Sigma|x_i-\theta|$ is minimum. We know that the mean deviation about a constant θ is $\Sigma|x_i-\theta|/n$ which is minimum when θ is the median of the sample.

Therefore, when n is odd,

$$\hat{\theta} = \text{med}(x_1, x_2, \ldots, x_n)$$

when n is even, then

$$Y_{n/2} \le \theta \le Y_{\frac{n}{2}+1}$$

where $Y_1, Y_2, \ldots, Y_n$ are taken to be the ordered statistics.

Note: This example proves that ML estimator is not necessarily unique.

Q. 26 Obtain the ML estimator of α and β of the uniform distribution having the probability density function,

$$f(x;\alpha,\beta) = \frac{1}{\beta-\alpha} \text{ for } \alpha \le x \le b$$
$$= 0 \quad \text{otherwise.}$$

$$L = \prod_{i=1}^{n} f(x_i, \alpha, \beta)$$

$$= \prod_{i=1}^{n} \frac{1}{\beta-\alpha}$$

$$\log L = \sum_{i=1}^{n} \log\frac{1}{(\beta-\alpha)}$$

$$= -n\log(\beta-\alpha)$$

Here, $$\frac{\partial}{\partial\alpha}\log L = \frac{n}{\beta-\alpha} = 0$$

and $$\frac{\partial}{\partial\beta}\log L = \frac{n}{\beta-\alpha} = 0.$$

These likelihood equations fail to estimate α and β. Hence, we have to think in some other direction. Let $Y_1, Y_2, \ldots, Y_n$ be the ordered statistics. Since x lies between α and β, we have

$$\alpha \le Y_1 \le Y_2 \le \ldots \le Y_n \le \beta$$

Therefore,

$$\hat{\alpha} = Y_1 \text{ and } \hat{\beta} = Y_n.$$

Q. 27 Discuss the method of least square estimation.

Ans. The idea of least square estimation emerges from the method of maximum likelihood itself. Consider the ML estimation of μ on the basis of a sample of size n from a normal population $N(\mu, \sigma^2)$

$$f(y;\mu,\alpha^2) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2\sigma^2}(y-\mu)^2}$$

$$L\left(y_i, \mu, \alpha^2\right) = \left(\frac{1}{\sqrt{2\pi\,\sigma^2}}\right)^n e^{-\frac{1}{2\sigma^2}\sum\limits_{i=1}^{n}(y_i-\mu)^2}$$

$$\log L = -\frac{n}{2}\log_e\left(2\pi\,\sigma^2\right) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(y_i-\mu\right)^2$$

Maximising log L implies that $\Sigma\left(y_i-\mu\right)^2$ must be minimised, *i.e.*, sum of squares, $\Sigma\left(y_i-\mu\right)^2$ must be least square.

The method of least square estimation is mostly used in estimating the parameters of linear function. This very idea can be translated by considering μ itself a linear function of certain parameters β_j *(j = 1, 2, ..., k) i.e.*

$$\mu = \sum_{j=1}^{k} x_j\,\beta_j$$

where x_j's are some known coefficients of β_j's forming a linear function of β_j. In this situation, estimation of β_j's is in the offing.

For estimating β_j's., we have to minimise,

$$\sum_{j=1}^{k}\left(y_i - \sum_{j=1}^{k} x_j\,\beta_j\right)^2$$

with respect to β_j.

Q. 28 Describe the least square method of estimation of the parameters of simple linear model.

Ans. Consider the simple linear model,

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

Let there be n paired observations $(x_1, y_1), (x_2, y_2)$, ..., (x_n, y_n). In the above model Y is called the dependent variable and X, the independent variable and ε is the random error. For the i^{th} pair of observation

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

or

$$\varepsilon_i = \left(y_i - \beta_0 - \beta_1 x_i\right)$$

ε_i may be positive or negative. To avoid the problem of sign of error, square both sides and also take the sum over all pairs. Thus,

$$\sum_{i=1}^{n}\varepsilon_i^2 = \sum_{i=1}^{n}\left(y_i - \beta_0 - \beta_1 x_i\right)^2$$

To get the best estimate of β_0 and β_1, one has to minimise $\Sigma\varepsilon_i^2$, which amounts to minimising $\Sigma\left(y_i - \beta_0 - \beta_1 x_i\right)^2$. Suppose $\Sigma\,\varepsilon_i^2 = Q$..

The values of β_0 and β_1 which minimise Q are known as *least square estimates*. To estimate β_0 and β_1, differentiate Q partially w.r.t. β_0 and β_1 respectively and equate to zero i.e.

$$\frac{\partial Q}{\partial\beta_0} = 0 \text{ and } \frac{\partial Q}{\partial\beta_1} = 0$$

This gives,

$$\Sigma\,y_i = n\hat{\beta}_0 + \hat{\beta}_1\,\Sigma\,x_i$$

and

$$\Sigma\,x_i\,y_i = \hat{\beta}_0\,\Sigma\,x_i + \hat{\beta}_1\,\Sigma\,x_i^2$$

where $\hat{\beta}_0$ and $\hat{\beta}_1$ are the estimated values of β_0 and β_1 respectively.

The two equations are known as *normal equations*. Solving these equations we obtain,

$$\hat{\beta}_0 = \bar{y} - b\bar{x}$$

$$\hat{\beta}_1 = \frac{\Sigma\,x_i\,y_i - \dfrac{(\Sigma\,x_i)(\Sigma\,y_i)}{n}}{\Sigma\,x_i^2 - \dfrac{(\Sigma x_i)^2}{n}}$$

Note: Method of least square estimation can be extended to multiple linear models and curvilinear models. This has been dealt with in the sequel.

Q. 29 Throw light on the properties of least square estimates.

Ans. Least square (LS) estimates are not so popular without reason. They possess some marvellous properties which are as follows:

(i) Least square estimators are unbiased in case of linear models.

(ii) $\hat{\beta}_0$ and $\hat{\beta}_1$ are the uniformly minimum variance estimators (UMVUE).

(iii) The LS estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ do not possess the asymptotically normal property.

(iv) The least square method is a device to obtain *best linear unbiased estimators* (BLUEs) under linear set up.

Q. 30 Define best linear unbiased estimator.

Ans. It has been known that sample mean is an unbiased estimate of population mean in case of finite as well as infinite populations. Also the sample mean is a linear function $a_1 x_1 + a_2 x_2 + \ldots + a_n x_n$ of the sample values $x_1, x_2, \ldots, x_n$ subject to the condition $\Sigma a_i = 1$ for $i = 1, 2, \ldots, n$. Linear functions of the sample values with known a_i's represented as $\Sigma a_i x_i$, such that $\Sigma a_i = 1$, are called *unbiased linear estimators.*

If one is to select an unbiased linear estimator from amongst the class of such estimators, some criteria have to be fixed. One widely accepted criterion to select one linear unbiased estimator out of all available linear unbiased estimators is to select one which has the smallest possible variance. Such a linear estimator is called *minimum variance linear unbiased estimator.* It is generally called the *best linear unbiased estimator* (BLUE). The adjective 'best' refers to minimum variance.

If an unbiased quadratic estimator of the variance σ^2 exists and it possesses the minimum variance out of the class of available unbiased quadratic estimators, it is known as *minimum variance quadratic estimator.*

Q. 31 State Gauss-Markov theorem.

Ans. Least square theory was put forth by Gauss in 1809 and minimum variance approach to the estimators of β_i's was propounded by Markov in 1900. Since determining of minimum variance linear unbiased estimator involves both the concepts, the theorem is known as *Gauss-Markov theorem.* It can be stated as follows:

Let y_i $(i = 1, 2, \ldots, n)$ be n independent variables with mean $\sum_{j=1}^{k} \beta_j x_{ji}$ for $n > k$ and all y_i have same variance σ^2 where x_{ji} are known and linearly independent vectors. The minimum variance linear unbiased estimators of the regression coefficients β_j are b_j $(j = 1, 2, \ldots, k)$.

Under the terms and conditions imposed above, the minimum variance linear unbiased estimators of the regression coefficients are identically the same as the least square estimators.

The combination of the above two statements is known as Gauss-Markov theorem.

Q. 32 Give in brief the method of moments for estimating the population parameters.

Ans. It is the oldest method of estimating the parameters of a distribution. It was first put forward by Karl Pearson in 1894. The method of moments consists of equating the sample moments to the corresponding moments of distribution, which are the functions of the unknown parameters. Here, we equate as many sample moments as there are unknown parameters. Solving the simultaneous equations, one obtains the estimates of the moments of the population in terms of sample variates.

We know,

$$\mu'_r = E(X^r)$$

$$\mu_r = E(X - \mu)^r$$

Also, $$\mu'_1 = E(X) = \mu_1$$

Let $X_1, X_2, \ldots, X_n$ be a random sample from a population $f(x; \theta_1, \theta_2, \ldots, \theta_k)$. Then the r^{th} sample moment

$$\mu_r = \frac{1}{n}\sum_i (x_i - \bar{x})^r$$

Also, $$\mu'_1 = E(X)$$

$$= \frac{1}{n}\sum_i x_i$$

$$= \bar{x}$$

Now to obtain second and higher order moments, equate

$$\mu_r = m_r \, (i = 1, 2, 3, \ldots, k)$$

Solving the above $(k - 1)$ equations, one gets the estimates of the population moments, which are the moments of the population distribution.

Q. 33 Write the properties of the estimates obtained by the method of moments.

Ans.

(i) Under fairly general conditions, the estimates obtained by the method of moments will have asymptotically normal distribution for large n.

(ii) The mean of the distribution of estimate will differ from the true value of the parameter by a quantity of order $\frac{1}{n}$.

(iii) The variance of the distribution of estimate will be of the type c^2/n.

(iv) In general, the deviation estimators obtained by the method of moments are less efficient than the maximum likelihood estimators. In particular cases, they are equivalent.

Q. 34 Estimate the parameters μ and σ^2 of the normal distribution by the method of moments.

Ans. Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal population $N(\mu, \sigma^2)$.

We know, $m_1' = \mu_1' = \mu = \overline{X}$

$$\mu_2 = \mu_2' - (\mu_1')^2$$

$$= \frac{1}{n}\sum X_i^2 - \overline{X}^2$$

for $i = 1, 2, \ldots, n$.

$$= \frac{1}{n}\left(\sum X_i^2 - n\overline{X}^2\right)$$

$$= \frac{1}{n}\sum \left(X_i - \overline{X}\right)^2$$

Therefore, $\overline{X}$ is an unbiased estimator of μ whereas $\Sigma\left(X_i - \overline{X}\right)^2/n$ is not an unbiased estimator for σ^2.

Q. 35 Estimate the parameter λ of the exponential distribution, $f(x;\lambda) = \lambda e^{-\lambda x}$ for $0 \le x \le \infty$, by the method of moments.

Ans. $m_1' = \mu_1' = E(X)$

$$= \int_0^\infty x\lambda\, x^{-\lambda x}dx$$

which is a gamma integral. Hence,

$$m_1' = \mu_1' = \frac{\lambda\Gamma 2}{\lambda^2}$$

$$= \frac{1}{\lambda}$$

But $m_1' = \bar{x}$. Therefore, the estimated value of λ is $\frac{1}{\bar{x}}$.

Q. 36 Find the estimate of the lone parameter λ of the Poisson distribution $\frac{e^{-\lambda}\lambda^x}{x!}$, by the method of moments.

Ans. Let $X_1, X_2, \ldots, X_n$ be a random sample from a Poisson distribution $P(x; \lambda)$. We know in case of Poisson distribution, its mean and variance are equal. The mean,

$$m_1' = \mu_1' = E(X)$$

$$= \sum_{x=0}^{n} x\frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \lambda\sum_{x=0}^{n} e^{-\lambda}\frac{\lambda^{x-1}}{(x-1)!}$$

$$= \lambda = \bar{x}$$

Thus, the estimate of the parameter λ by the method of moments is the sample mean $\bar{x}$.

Q. 37 How can one estimate the parameters of a distribution by the method of minimum Chi-square?

Ans. The method of minimum Chi-square makes use of the Pearson's Chi-square statistic. This method can be used in case of discrete distributions or for grouped data from a continuous distribution.

Let $f_1, f_2, \ldots, f_k$ be the observed frequencies in K groups or classes and unknown probabilities that f_i elements belong to the i^{th} group or class be p_i ($i = 1$, 2, ..., k). p's are the functions of the unknown parameters $\theta_1, \theta_2, \ldots, \theta_m$. Thus $p_i = p_i(\underset{\sim}{\theta})$ where $\underset{\sim}{\theta} = (\theta_1, \theta_2, \ldots, \theta_m)$. Suppose the total sample size is n. Therefore, $\Sigma f_i = n$. The expected frequencies are $np_1(\underset{\sim}{\theta}), np_2(\underset{\sim}{\theta}), \ldots, np_k(\underset{\sim}{\theta})$. We know, Pearsonian Chi-square statistic is,

$$\chi^2 = \sum_{i=1}^{k} \frac{\left[f_i - np_i(\underset{\sim}{\theta})\right]^2}{np_i(\underset{\sim}{\theta})}$$

$$= \sum_{i=1}^{k} \frac{f_i^2}{np_i(\underset{\sim}{\theta})} - n$$

Under the method of minimum Chi-square one has to choose $(\theta_1, \theta_2, \ldots, \theta_m)$ which minimises χ^2. This will be minimum when $np_i(\theta)$ is as close as possible to f_i. So to obtain the estimates of θ_i's, partially differentiate χ^2-statistic w.r.t. $\theta_i\,(i = 1, 2, \ldots, m)$ successively and equate them to zero. Also check that the second derivatives are non-negative, *i.e.*,

$$\frac{\partial \chi^2}{\partial \theta_i} = 0 \text{ for } i = 1, 2, \ldots, m$$

and
$$\frac{\partial^2 \chi^2}{\partial \theta_i^2} \geq 0$$

$\frac{\partial \chi^2}{\partial \theta_i} = 0$ provides m simultaneous equations in m unknowns. Solving these m-equations for m unknown parameters, one gets the estimated values of $\theta_1, \theta_2, \ldots, \theta_m$ respectively.

Q. 38 What modification in minimum-χ^2 method of estimation gives rise to the method of modified minimum Chi-square?

Ans. Expected frequency $np_i\,(\theta)$ in the denominator of χ^2-statistic causes certain difficulties. Hence, a modification has been suggested which makes the process of differentiation easier. The modified Chi-square statistic is,

$$\chi^2 = \sum_{i=1}^{k} \frac{\left[np_i(\underset{\sim}{\theta}) - f_i\right]^2}{f_i}$$

$$= \sum_{i=1}^{k} \frac{n^2 p_i^2(\underset{\sim}{\theta})}{f_i} - n$$

Rest of the procedure of modified minimum Chi-square remains same as that of minimum Chi-square.

Q. 39 Compare the method of minimum Chi-square with maximum likelihood estimation.

Ans.

(i) It is trivial to prove that for large n, the estimates obtained by the method of minimum Chi-square are almost equal to the estimates obtained by maximum likelihood method.

(ii) J. Berkson in 1955 showed that in particular cases, the mean square error of estimators under minimum χ^2 is smaller as compared to the mean square error of ML estimators.

Q. 40 What properties the minimum Chi-square estimators hold?

Ans.

(i) The minimum Chi-square estimators are consistent.

(ii) Minimum χ^2 estimators are asymptotically normal.

(iii) Minimum χ^2 estimators are efficient.

(iv) Minimum χ^2 estimators are not necessarily unbiased.

Q. 41 Comment on the use of minimum χ^2 estimation method.

Ans. Minimum χ^2 is rarely used in practice. It is used only when it is difficult to solve the simultaneous equations obtained under maximum likelihood estimation method.

Q. 42 Write down the motif behind the Bayesian theory of estimation.

Ans. In planning of any investigation or experiment some prior information is available on the statistical properties of the variable to be studied. For instance, the type of distribution, the range of the parameters involved, etc. Heretofore, the methods of estimation did not make use of any such information. But Sir Thomas Bayes propounded the theory which makes use of the prior information to get the estimates of population parameters. The basic difference between non-Bayesians and Bayesians is that non-Bayesians consider the parameter of a population a fixed quantity, whereas Bayesians regard the parameter of a distribution a random variable.

The most important thing in the Bayesian approach is the specification of a distribution on the parametric space, which has been named as prior distribution. The specification of the prior distribution is mostly based on pragmatic grounds, *i.e.*, it is based on some previous experiment, investigation study or knowledge.

The Bayesian theory has always been a matter of controversy. Bayesians believe that it is generally very effective, whereas non-Bayesians consider it irrelevant.

Q. 43 Present the general structure of Bayes' estimators.

Ans. In the process of estimation so far we have a density $f(x; \theta)$ for each $\theta \varepsilon \Theta$, (the parametric space) from which a random sample $X_1, X_2, ..., X_n$ is drawn. Here we denote $f(x \mid \theta)$ for the conditional density of X for given value of parameter $\Theta = \theta$. Also suppose that $g(\theta)$ is a known prior distribution of θ. The density $f(x; \theta)$ in which X and Θ both are random variables and can be given as,

$$f(x;\theta) = f(x|\theta)g(\theta)$$

Using the Bayes' probability rule,

$$f(\theta|x) = \frac{f(x|\theta)g(\theta)}{\int f(x|\theta)g(\theta)d\theta}$$

The Bayesian point estimator of a parametric function $\tau(\theta)$ of θ can be obtained by the expression,

$$E[\tau(\theta)] = \int \tau(\theta) f(\theta|x) d\theta$$

$$\hat{\tau}(\theta) = \frac{\int \tau(\theta) f(x|\theta) g(\theta) d\theta}{\int f(x|\theta) g(\theta) d\theta}$$

Bayes estimator of $\tau(\theta)$ given above is under squared error loss function.

Q. 44 Give an account of the properties of Bayes estimators.

Ans.

(i) Bayes estimator is always a function of minimal sufficient statistics.

(ii) For large n, Bayes estimators always tend to maximum likelihood estimators, whatever may be the prior density $g(\theta)$.

(iii) In many cases Bayes estimators are asymptotically consistent.

(iv) Properties of Bayes estimators are generally given in terms of minimum risk, mean squared error, etc., which are not discussed here.

Q. 45 Define minimax estimator.

Ans. An estimator of which the maximum risk is less than or equal to maximum risk of any other estimator is said to be a minimax estimator. Notionally T_1 is a minimax estimator if for any other estimator T_2,

$$\sup_{\theta} R(T_1, \theta) \le \sup_{\theta} R(T_2, \theta)$$

Q. 46 Find Bayes estimator of the parameter p of a binomial distribution with x successes out of n given that the prior distribution of p is a beta distribution with parameters α and β.

Ans. The mass function,

$$f(x|p) = \binom{n}{x} p^x (1-p)^{n-x}$$

and $$g(p) = \frac{1}{B(\alpha,\beta)} p^{\alpha-1}(1-p)^{\beta-1}$$

Thus, $$f(x;p) = f(x|p)g(p)$$

Again,

$$f(p|x)=\frac{\binom{n}{x}p^x(1-p)^{n-x}p^{\alpha-1}(1-p)^{\beta-1}}{B(\alpha,\beta)}$$

$$\hat{p}=\frac{\int p\binom{n}{x}p^x(1-p)^{n-x}p^{\alpha-1}(1-p)^{\beta-1}dp}{\int\binom{n}{x}p^x(1-p)^{n-x}p^{\alpha-1}(1-p)^{\beta-1}dp}$$

$$=\frac{\int p^{x+\alpha+1-1}(1-p)^{n+\beta-x-1}dp}{\int p^{x+\alpha-1}(1-p)^{n+\beta-x-1}dp}$$

$$=\frac{B(x+\alpha+1,n+\beta-x)}{B(x+\alpha,n+\beta-x)}$$

$$=\frac{\dfrac{\Gamma(x+\alpha+1)\Gamma(n+\beta-x)}{\Gamma(\alpha+\beta+n+1)}}{\dfrac{\Gamma(x+\alpha)\Gamma(n+\beta-x)}{\Gamma(\alpha+\beta+n)}}$$

$$\therefore\ \hat{p}=\frac{x+\alpha}{n+\alpha+\beta}$$

Q. 47 Find Bayes estimator of the single parameter λ of the Poisson distribution when it is known that the prior distribution of λ is a gamma distribution.

Ans. Let there be a random sample $X_1, X_2, \ldots, X_n$.

As per question,

$$f(x;\lambda)=\frac{e^{-\lambda}\lambda^x}{x!}\ \text{for } x=0,1,2,\ldots,n.$$

$$g(\lambda)=\frac{\alpha^r}{\Gamma r}\lambda^{r-1}e^{-\alpha\lambda}\ \text{for } 0\le\lambda\le\infty$$

We know,

$$f(x;\lambda)=f(x|\lambda)g(\lambda)$$

$$=\frac{e^{-\lambda}\lambda^x}{x!}\cdot\frac{\alpha^r}{\Gamma r}\lambda^{r-1}e^{-\alpha\lambda}$$

$$\mathrm{E}(\lambda)=\frac{\int_0^\infty\lambda\frac{e^{-\lambda}\lambda^x}{x!}\lambda^{r-1}e^{-\alpha\lambda}d\lambda}{\int_0^\infty\frac{e^{-\lambda}\cdot\lambda^x}{x!}\lambda^{r-1}e^{-\alpha\lambda}d\lambda}$$

$$=\frac{\int_0^\infty e^{-(1+\alpha)\lambda}\lambda^{(x+r+1)-1}d\lambda}{\int_0^\infty e^{-(1+\alpha)\lambda}\lambda^{x+r-1}d\lambda}$$

$$=\frac{\Gamma(x+r+1)/(1+\alpha)^{x+r+1}}{\Gamma(x+r)/(1+\alpha)^{x+r}}$$

$$\hat{\lambda}=\frac{x+r}{1+\alpha}$$

Q. 48 Define Pitman's estimator for location parameter of a distribution.

Ans. Let $X_1, X_2, \ldots, X_n$ be a random sample from a density $f(x;\theta)$ where θ is a location parameter of the distribution. Then the Pitman estimator $\hat{\theta}=t\,(X_1, X_2, \ldots, X_n)$ *is given by the formula,*

$$t=\frac{\int\theta\prod_{i=1}^{n}f(x_i;\theta)d\theta}{\int\prod_{i=1}^{n}f(x_i;\theta)d\theta}$$

$$=\frac{\int\theta L(x|\theta)d\theta}{\int L(x|\theta)d\theta}$$

Q. 49 Give the properties of Pitman estimator for location which it usually holds.

Ans.

(i) The Pitman estimator for location has smallest mean squared error.

(ii) A Pitman estimator is a function of sufficient statistics.

(iii) Pitman estimator is a minimax estimator on the real line.

Q. 50 Give the expression for Pitman estimator for scale parameter of a distribution.

Ans. Let $X_1, X_2, \ldots, X_n$ be a random sample from a density $f(x;\theta)$ where θ is a scale parameter such that $\theta > 0$. Also $f(x;\theta)$ exists for $x > 0$ and otherwise zero.

Then the Pitman estimator $\hat{\theta}=t\,(X_1, X_2, \ldots, X_n)$ for scale parameter can be obtained by the expression:

$$\hat{\theta} = \frac{\int \frac{1}{\theta^2} \prod_{i=1}^{n} f(x_i;\theta)\, d\theta}{\int \frac{1}{\theta^3} \prod_{i=1}^{n} f(x_i;\theta)\, d\theta}$$

$$= \frac{\int \frac{1}{\theta^2} L(x|\theta)\, d\theta}{\int \frac{1}{\theta^3} L(x|\theta)\, d\theta}$$

Pitman estimator for scale is also a function of sufficient statistics.

Q. 51 Find the Pitman estimator of the mean μ of a normal population $N(\mu, 1)$.

Ans. Let $X_1, X_2, ..., X_n$ be a random sample from the density $f(x; \mu, 1)$.

We know,

$$f(x; \mu, 1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-\mu)^2}$$

Pitman estimator,

$$\hat{\mu} = \frac{\int \mu \left(\frac{1}{2\pi}\right)^{n/2} e^{-\frac{1}{2}\sum(x_i-\mu)^2}\, d\mu}{\int \left(\frac{1}{2\pi}\right)^{n/2} e^{-\frac{1}{2}\sum(x_i-\mu)^2}\, d\mu}$$

$$= \frac{\int \mu\, e^{-\frac{1}{2}\left\{\sum x_i^2 + n\mu^2 - 2\mu \sum x_i\right\}}\, d\mu}{\int e^{-\frac{1}{2}\left\{\sum x_i^2 + n\mu^2 - 2\mu \sum x_i\right\}}\, d\mu}$$

$$= \frac{\int \mu\, e^{\left(-\frac{n}{2}\mu^2 + \mu \sum x_i\right)}\, d\mu}{\int e^{\left(-\frac{n}{2}\mu^2 + \mu \sum x_i\right)}\, d\mu}$$

$$= \bar{x}$$

Q. 52 Explain the general procedure for parametric interval estimation on the basis of sample drawn from continuous distributions.

Ans. Let $X_1, X_2, ..., X_n$ be a random sample $T_1 = t_1(X_1, X_2, ..., X_n)$ and $T_2 = t_2(X_1, X_2, ..., X_n)$ be any two statistics such that $T_1 < T_2$, then the confidence limits for a parametric function $\tau(\theta)$ of θ can be obtained by the relations,

$$P_\theta\{T_1 > \tau|\theta\} = \alpha_1$$

and $$P_\theta\{T_2 > \tau|\theta\} = \alpha_2$$

where α_1 and α_2 are independent of θ and are the areas of critical region on the left and right tails respectively of the probability density curve. Also $\alpha_1 + \alpha_2 = \alpha$. The confidence interval is given as,

$$P_\theta\{T_1 \le \tau(\theta) \le T_2\} = 1 - \alpha \text{ for } \theta \varepsilon \Theta$$

where $(1 - \alpha)$ is called the *confidence coefficient* and is generally given as $(1 - \alpha)\, 100 = \gamma$ per cent.

Also the confidence interval is $(t_2 - t_1)$.

Thus, γ per cent confidence limits are t_1 and t_2.

Q. 53 Find 95 per cent confidence limit for population mean μ of the population $N(\mu, \sigma^2)$ in case when σ^2 is known.

Ans. If $\bar{x}$ is the sample mean and $Z_{\alpha/2}$ be the critical value of the standard normal deviate Z such that 0.025 area is on the left of the lower limit and 0.025 area under the normal curve to the right of the upper limit, *i.e.*,

$$\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \ge -Z_{\alpha/2}$$

and $$\frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \le Z_{\alpha/2}$$

From the first inequality we get

$$\mu \le \bar{x} + Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

and from second,

$$\mu \ge \bar{x} - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Combining the two inequalities, we can write,

$$\bar{x} - Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

Thus, the confidence limits for μ are $\bar{x} \mp Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$.

Also, the confidence interval for μ is

$$\left(\bar{x}+Z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right)-\left(\bar{x}-Z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}\right)=2Z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}.$$

When the population standard deviation σ is not known, then to find the confidence limits for μ, replace $Z_{\alpha/2}$ by $t_{\alpha/2}$ and σ by its estimate s. In this situation the limits for μ are $\bar{x} \mp t_{\alpha/2}\frac{s}{\sqrt{n}}$ and confidence interval is $2t_{\alpha/2}\cdot\frac{s}{\sqrt{n}}$.

Q. 54 Give the inequality for finding out the confidence limits for the variance σ^2 of normal population, $N(\mu, \sigma^2)$ when its mean μ is unknown.

Ans. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$ population. Also let s^2 be the sample variance. We know that $(n-1)s^2/\sigma^2$ is distributed as χ^2 with $(n-1)$ degrees of freedom. Then we have to find out two values of χ^2 say χ_1^2 and χ_2^2 such that

$$P\left\{\chi_1^2 \le \frac{(n-1)s^2}{\sigma^2} \le \chi_2^2\right\}=\gamma$$

The values of χ_1^2 and χ_2^2 are chosen in such way that,

$$P\left\{\frac{(n-1)s^2}{\sigma^2}<\chi_1^2\right\}=P\left\{\frac{(n-1)s^2}{\sigma^2}>\chi^2\right\}$$

$$=(1-\gamma)/2=\alpha/2$$

Such a confidence interval is referred to as *equal tails confidence interval*. Thus,

$$P\left\{\frac{(n-1)s^2}{\chi_2^2}\le\sigma^2\le\frac{(n-1)s^2}{\chi_1^2}\right\}=\gamma$$

for $\chi_1^2 \le \chi_2^2$

For $\gamma = 0.95$, $\chi_1^2=\chi_{0.975}^2$ and $\chi_2^2=\chi_{0.025}^2$. Thus, the confidence limits with $(1-\alpha)\,100=\gamma$ confidence coefficient are,

$$\binom{\text{Lower}}{\text{limit}}L=\frac{(n-1)s^2}{\chi^2_{(1+\gamma)/2,n-1}}$$

and

$$\binom{\text{Upper}}{\text{limit}}U=\frac{(n-1)s^2}{\chi^2_{(1-\gamma)/2,n-1}}$$

If someone is interested to find out the confidence limits for the standard deviation σ, one way is to find out the confidence limits for σ^2 and take their under roots. So the confidence limits for σ are, $L_1=\sqrt{L}$ and $U_1=\sqrt{U}$. This is not an exact method but it gives quite satisfactory results.

Q. 55 Give the formula for obtaining confidence limits for the difference between the means of two normal populations.

Ans. Let the two normal populations be $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$. Confidence limits for $(\mu_1-\mu_2)$ with confidence coefficient $(1-\alpha)\,100=\gamma$ per cent, when two samples $X_{11}, X_{12}, \ldots, X_{1n_1}$ and $X_{21}, X_{22}, \ldots, X_{2n_2}$ have been drawn from the two populations, respectively, in case when $\sigma_1^2=\sigma_2^2$ and are unknown, can be obtained by the formula,

$$(\bar{x}_1-\bar{x}_2)\mp t_{(1+\gamma)/2,(n_1+n_2-2)}\cdot s_{(\bar{x}_1-\bar{x}_2)}$$

where,

$$s_{(\bar{x}_1-\bar{x}_2)}=\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$

and

$$s_p^2=\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{(n_1+n_2-2)}$$

In case when $\sigma_1^2\ne\sigma_2^2$, confidence limits for $(\mu_1-\mu_2)$ by Cochran's approximation are:

$$(\bar{x}_1-\bar{x}_2)\mp t^*\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

where, $$t^* = \frac{\frac{s_1^2}{n_1} \cdot t_{\frac{1-\gamma}{2}, n_1-1} + \frac{s_2^2}{n_2} \cdot t_{\frac{1-\gamma}{2}, n_2-1}}{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Please note that s_1^2 and s_2^2 are the sample variances.

Q. 56 How will you find the confidence limits for the mean of difference of paired observations?

Ans. Let n pairs of observation on the variables X and Y be (X_1, Y_1), (X_2, Y_2),..., (X_n, Y_n). In this situation X and Y are said to be correlated. Also suppose $X - Y = d$. The confidence limits for the population mean difference μ_D with confidence coefficient γ can be calculated by the formula,

$$\bar{d} \mp t_{\frac{1+\gamma}{2}, (n-1)} \cdot s_{\bar{d}}$$

where, $$s_{\bar{d}}^2 = \frac{1}{(n-1)}\left[\sum d_i^2 - \frac{(\Sigma d_i)^2}{n}\right]$$

for $i = 1, 2, ..., n$.

Q. 57 Obtain the confidence limits for the ratio of variances of two normal populations.

Ans. Adopting the usual notations, confidence limits for the ratio σ_1^2/σ_2^2 of variances of two normal populations with confidence coefficient γ can be given as,

$$L = \frac{s_1^2}{s_2^2} F_{\frac{1+\gamma}{2}(n_1-1, n_2-1)}$$

$$U = \frac{s_1^2}{s_2^2} F_{\frac{1-\gamma}{2}(n_1-1, n_2-1)}$$

Q. 58 Clarify the concept of shortest confidence interval.

Ans. We have two statistics T_1 and T_2 which are functions of sample variates X_1, X_2, ..., X_n from a density $f(x; \theta)$ such that the area to the left of T_1 is α_1 and to the right of T_2 is α_2, both $\alpha_1, \alpha_2 > 0$ in such a way that $\alpha_1 + \alpha_2 = \alpha$. There can be an innumerable number of values of α_1 and α_2 which satisfy the condition $\alpha_1 + \alpha_2 = \alpha$ leading to an innumerable number of confidence intervals. Now the problem arises which one to choose out of countless number of confidence intervals. One widely accepted criterion is the width of the confidence interval. An interval with smaller width is better than the other having the same confidence coefficient, *i.e.*,

$$T_2 - T_1 \le T_2' - T_1' \text{ for all } \theta \varepsilon \Theta$$

Hence, a confidence interval $(T_2 - T_1)$ which is uniformly shorter than any other confidence interval $(T_2' - T_1')$ for θ in Θ is known as the *shortest confidence interval* and is most preferred one.

It is worth pointing out that in most of the problems it is not possible to construct confidence interval of shortest width for a given confidence coefficient $1 - \alpha$. Hence, equal tail confidence intervals are generally worked out.

Q. 59 What do you understand by confidence region?

Ans. Sometimes an investigator aspires to extend the idea of interval estimation to more than one parameters simultaneously. For instance, one may wish to estimate the parameters μ and σ^2 of normal distribution simultaneously by some confidence region in a plane.

Suppose θ_1 and θ_2 are two parameters for which the confidence region has to be worked out. Let (t_1, t_2) and (m_1, m_2) be the statistics which are to constitute a simultaneous interval leading to the confidence region of confidence coefficient $(1 - \alpha)$ in such a way that

$$P(t_1 \le \theta_1 \le t_2 \text{ and } m_1 \le \theta \le m_2) = 1 - \alpha.$$

Practically, it is hardly possible.

Another approach is to find two statistics ω_1 and ω_2 such that,

$$P(\omega_1 \le \theta_1^2 + \theta_2^2 \le \omega_2) = 1 - \alpha.$$

It is also not a satisfactory proposition.

Q. 60 What do you understand by one-sided confidence interval?

Ans. Let $X_1, X_2, ..., X_n$ be a random sample from the density $f(x; \theta)$ and $T_1 = t_1(X_1, X_2, ..., X_n)$ be any statistic such that

$$P(t_1 \le \theta) = 1 - \alpha \text{ for all } \theta$$

Then t_1 is known as the lower confidence bound for the parameter θ with confidence coefficient $(1 - \alpha)$. The interval $\{t_1, \theta\}$ is termed as the *lower one-sided confidence interval* for θ with confidence coefficient $(1 - \alpha)$.

Similarly, any other statistic $T_2 = t_2(X_1, X_2, ..., X_n)$ for which

$$P(t_2 \ge \theta) = 1 - \alpha \text{ for all } \theta$$

is called the upper confidence bound for θ with confidence coefficient $(1 - \alpha)$. The interval $(-\infty, T_2)$ is known as *upper one-sided confidence interval* with confidence coefficient $(1 - \alpha)$.

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s) in the blanks:

1. A sample constant representing a population parameter is known as __________.
2. An estimator T_n which is most concentrated about a parameter θ is the __________ estimator.
3. Estimation is __________ if we have a purposive sample.
4. Estimation is possible only in case of a __________.
5. An estimator is itself a __________.
6. A value of an estimator is called an __________.
7. If $X_1, X_2, ..., X_n$ be a random sample, the expression $\Sigma X_i/n$ is an __________.
8. A single value of an estimator for a population parameter θ is called its __________ estimate.
9. If an estimator T_n converges in probability to the parameteric function $\tau(\theta)$, T_n is said to be a __________ estimator.
10. $\Sigma X_i/n$ for $i = 1, 2, ..., n$ is a __________ estimator of population mean.
11. If the expected value of an estimator T_n is equal to the value of the parameter θ, T_n is said to be an __________ estimator of θ.
12. The difference between the expected value of an estimator and the value of the corresponding parameter is known as __________.
13. If T_n is an estimator of a parametric function $\tau(\theta)$, the mean square error of T_n is equal to __________.
14. For the mean square error to be minimum, bias should be __________.
15. If an unbiased estimator T_n is such that for any other unbiased estimator $\tau_n^*, v(T_n) \le v(T_n^*), T_n$ is a __________.
16. If $T_n = t_n(X_1, X_2, ..., X_n)$, an estimator of $\tau(\theta)$, is such that $\lim_{n \to \infty}[T_n - \tau(\theta)]^2 = 0, T_n$ is said to be __________ consistent.
17. For discrete variable Crammer-Rao inequality __________.
18. If T_n is an estimator of a parameter θ of the density $f(x; \theta)$ the quantity

$$E\left[\frac{\partial}{\partial\theta}\log f(x;\theta)\right]^2 \text{ is called the } \_\_\_\_\_\_\_\_\_\_.$$

19. An estimator of $v_\theta(T_n)$ which attains lower bound for all θ is known as __________.
20. An inequality parallel to Crammer-Rao inequality was also given by __________.
21. Crammer and Rao gave the inequality for lower bound of $V_\theta(T_n)$ in __________ years.

22. A. Bhattacharyya inequality provides ________ of $V_\theta(T_n)$.

23. Chapman-Robins inequality is ________ than Crammer-Rao inequality.

24. Crammer gave the inequality for lower bound of $V_\theta(T_n)$ in ________.

25. C.R. Rao give the theorem for lower bound of $V_\theta(T_n)$ in ________.

26. ________ proved the theorem for lower bound of $V_\theta(T_n)$ earlier than Crammer-Rao but not published.

27. Another name of BAN estimator is ________.

28. BAN estimators possess ________ property.

29. BAN estimators have ________ variance.

30. An estimator is efficient if its variance is ________ than the variance of any other estimator.

31. Relative efficiency of an estimator T_n as compared to an estimator is T_n' given as ________.

32. If T_n and T_n^* are two estimators such that $V(T_n) = V(T_n^*)$, then ________.

33. If a statistic $T = t(X_1, X_2, \ldots, X_n)$ provides as much information as the random sample $X_1, X_2, \ldots, X_n$ could provide, then T is a ________.

34. If $T = t(X_1, X_2, \ldots, X_n)$ is a sufficient statistic for a parameter θ of the density $f(x; \theta)$ then the conditional distribution of $X_1, X_2, \ldots, X_n$ is ________ of θ for any value of $T = t$.

35. If $S = s(X_1, (X_2, \ldots, (X_n)$ is a sufficient statistic for θ of density $f(x; \theta)$ and $f(x_i; \theta)$ for $i = 1, 2, \ldots, n$ can be factorised as $g(s, \theta)$ $h(x)$, then $s(X_1, X_2, \ldots, X_n)$ is a ________.

36. A sufficient statistic $S = s(X_1, X_2, \ldots, X_n)$ is called a ________ statistic if it cannot be condensed any more without sacrificing the criterion of sufficiency.

37. If $f(x; \theta)$ is a family of distributions and $h(x)$ is any statistic such that $E[h(x)] = 0$, then $f(x, \theta)$ is called ________.

38. An unbiased and complete statistic is compulsorily ________.

39. An unbiased and complete statistic is a ________ estimator provided MVUE exists.

40. ________ theorem states that given an unbiased estimator T, another unbiased estimator T_1, which is a function of sufficient statistics, can be formed which will not have larger variance than that of T for all parameters θ.

41. An estimator whose risk is not greater than the risk of any other estimator T of a parameter θ is called an ________ estimator.

42. Expected loss in choosing an estimator T of a parameter θ is called its ________.

43. The joint probability density function of sample variates is called ________.

44. A value of a parameter θ which maximises the likelihood function is known as ________ estimate of θ.

45. A maximum likelihood estimate is not necessarily ________.

46. If a MVB unbiased estimator exists, ________ estimator provides it.

47. A maximum likelihood estimator may be consistent but not necessarily ________.

48. If a random sample $X_1, X_2, \ldots, X_n$ is drawn from a population $N(\mu, \sigma^2)$, the maximum likelihood estimate of μ is ________.

49. If a random sample $x_1, x_2, \ldots, x_n$ is selected from a normal population $N(\mu, \sigma^2)$, the maximum likelihood estimate of σ^2 is ________.

50. Maximum likelihood estimator $\Sigma(X_i - \overline{X})^2/n$

of the variance σ^2 of a normal density $f\left(x;\mu,\sigma^2\right)$ is a __________.

51. If $\bar{x}$ is a maximum likelihood estimator of μ of a normal population, $\bar{x}$ is a __________.

52. For a gam (x, α, λ) distribution with λ known, the maximum likelihood estimate of α is __________.

53. Maximum likelihood estimate of the parameter θ of the distribution $f(x, \theta) = \frac{1}{2}e^{-|x-\theta|}$ is __________.

54. For a rectangular distribution $1/(\beta-\alpha)$, the maximum likelihood estimates of α and β are __________ and __________ respectively.

55. Least square method is a device to obtain a __________.

56. Linear estimators obtained by the method of least squares are __________.

57. Linear estimators obtained by the method of least squares are __________ estimators.

58. Determining of minimum variance linear unbiased estimator is due to __________.

59. Method of moments for estimating the parameters of a distribution was given by __________ in __________.

60. The estimators obtained by the method of moments under general conditions are __________ for large sample size.

61. The estimators obtained by the method of moments are __________ efficient than maximum likelihood estimators.

62. The estimator of σ^2 based on a random sample $X_1, X_2, ..., X_n$ from a population N (μ, σ^2) by the method of moments is __________.

63. The estimate of λ of a Poisson distribution P (x; λ) based on a sample size n by the method of moments is __________.

64. The estimate of the parameter λ of the exponential distribution $\lambda e^{-\lambda x}$ by the method of moments is __________.

65. The estimation of a parameter by the method of minimum Chi-square utilises __________ statistic.

66. Statistics for modified minimum Chi-square differs from minimum Chi-square in respect of __________ in the statistic.

67. The estimators obtained by the method of minimum Chi-square and maximum likelihood estimators are __________.

68 In particular cases minimum Chi-square estimators have lesser mean square error than ML estimators was proved by __________ in __________.

69. Minimum Chi-square estimators are __________.

70. Minimum Chi-square estimators are not necessarily __________.

71. Under Bayesian approach, the parameter of a distribution is a __________.

72. Non-Bayesian always consider a parameter as a __________.

73. In Bayesian approach, an investigator has always to specify __________.

74. Prior distribution is based on some __________.

75. The density function f (x; θ) under Bayesian approach with known prior distribution $g(\theta)$ is always expressed as __________.

76. Formula for Bayes estimator with usual notations can be expressed as __________.

77. Bayes estimator is always a function of __________ statistics.

78. For large samples, Bayes estimators tend to __________ estimators.

79. Bayes estimators in some cases are __________.

80. If two estimators T_1 and T_2 are such that $\sup_{\theta} R(T_1, \theta), \leq \sup_{\theta} R(T_2, \theta)$ then T_1 is said to be a __________ estimator.

81. Bayes estimator of parameter p of the binomial distribution given that the prior distribution of p is beta distribution with density f (x; α, β) is __________.

82. If a variable X follows $P(x; \lambda)$ distribution and the prior distribution of λ is $G(x; n, \alpha)$ the Bayes estimator of λ is __________.

83. Formula for Pitman estimator of the parameter θ of a density $f(x; \theta)$ is given by the formula __________.

84 Pitman estimator possesses __________ mean square error.

85. Pitman estimator is a function of __________.

86. Pitman estimator is a __________ estimator.

87. Pitman estimator for location is also __________.

88. Pitman estimator for μ of a normal population $N(\mu, 1)$ is __________.

89. Interval estimate is determined in terms of __________.

90. An interval estimate with __________ interval is best.

91. Confidence interval is specified by the __________ limits.

92. When confidence interval for two or more parameters is estimated simultaneously, then __________ is delimited.

93. Consistency ensures that the difference between the estimator T_n and the parametric function $\tau(\theta)$ __________ as n increases indefinitely.

94. The best estimator implies that the distribution of an estimator be __________ around the true parameter.

95. If t is a consistent estimator of θ, then t^2 is a __________ estimator of θ^2.

96. For the Poisson distribution $P(x; \lambda)$, $1/\bar{x}$ is a __________ estimator of λ.

97. If mean of the sampling distribution of an estimator T_n is equal to population mean, it is a __________ estimator.

98. If $E(T_n) > \theta$, the parameter value T_n is said to be __________.

99. An unbiased estimator is not necessarily __________.

100. If $x_1, x_2, \ldots, x_n$ is a random sample from a normal population, $\Sigma(x_i - \bar{x})^2/n$ is not a __________ estimator of σ^2.

101. An estimator with smaller variance than that of another estimator is __________.

102. If $X_1, X_2, \ldots, X_n$ is a random sample from an infinite population and S^2 is defined as $\Sigma(X_i - \bar{X})^2/n, \frac{n}{n-1}S^2$ is an __________ estimator of population variance σ^2.

103. Sample standard deviation is a __________ and __________ estimator of population standard deviation.

104. If T_n is a sufficient estimator of θ of density $f(x; \theta), \frac{\partial}{\partial\theta}\log L$ is a function of __________ and __________ only.

105. Let $X_1, X_2, \ldots, X_n$ be a random sample from a density $f(x; \theta)$. If $S = s(X_1, X_2, \ldots, X_n)$ is a complete sufficient statistic and $T' = t(s)$, a function of S, is an unbiased estimator of $\tau(\theta)$, T' is an __________ of $\tau(\theta)$. [*Lehmann-Scheffe theorem.*]

106. Completeness confers __________ property of an estimator.

107. The credit of factorisation theorem for sufficiency goes to __________.

108. __________ estimator may not be unique.

109. If $X_1, X_2, \ldots, X_n$ is a random sample from the uniform distribution $f(x; \theta) = \frac{1}{\theta}, 0 < x \le \theta, Y = \max(X_1, X_2, \ldots, x_n)$ is a __________ estimator of θ.

110. If X is a Poisson $(x; \lambda)$, the sufficient statistic for λ is __________.

111. If we have a random sample of size n from a population $N(\mu, \sigma^2)$, then sample mean is __________ efficient than sample median.

112. Let there be a sample of size n from a normal population with mean μ and variance σ^2. The efficiency of median relative to the mean is __________.

113. Suppose a sample of size n is drawn randomly from a normal density $f(x;\ \mu,\ \sigma^2)$. The estimate of mean deviation about mean is a __________ estimate of σ.

114. Consistent estimators are not necessary __________.

115. An unbiased estimator need not be __________.

116. If a sufficient estimator exists, it is function of the __________ estimator.

117. An unbiased estimator of the reciprocal of the parameter of a binomial distribution is __________.

118. If a function $f(t)$ of the sufficient statistics $T = t(X_1, X_2, \ldots, X_n)$ is unbiased for $\tau(\theta)$ and is also unique, this is the __________ estimate.

119. If a maximum likelihood estimator exists, it is __________ among the class of such estimators.

120. If the mean $\bar{x}$ of a sample drawn from a normal population is a maximum likelihood estimator, $\bar{x}$ is a __________ estimator of population mean.

121. Sample mean is an __________ and __________ estimate of population mean.

122. In case of sampling from normal population, $N(\mu, \sigma^2)$, median is an __________ estimator of μ.

123. The variance of median tends to zero as n, the sample size tends to infinity, hence median is a __________ estimator of μ.

124. If T_1 and T_2 are two MVU estimators for $T(\theta)$, then __________.

125. If X_1, X_2, X_3 is a random sample from a population $N(\mu,\ \sigma^2)$ and $T_1 = (X_1 + X_2 + X_3)/3$ and $T_2 = 2X_1 + X_2 - 2X_3$ are two estimators for μ, T_1 is __________ than T_2.

SECTION-C

Multiple Choice Question

Select the correct alternative out of given ones:

Q. 1 Parameters are those constants which occur in:

(a) Samples
(b) probability density function
(c) a formula
(d) none of the above

Q. 2 Estimation of parameters in all scientific investigations is of:

(a) prime importance
(b) secondary importance
(c) no use
(d) deceptive nature

Q. 3 Estimate and estimator are:

(a) synonyms
(b) different
(c) related to population
(d) none of the above

Q. 4 An estimator is considered to be the best if its distribution is:

(a) Continuous
(b) discrete
(c) Concentrated about the true parameter value
(d) normal

Q. 5 An estimator T_n based on a sample of size n is considered to be the best estimator of θ if:

(a) $P\{|T_n - \theta| < \varepsilon\} \geq P\{|T_n^* - \theta| < \varepsilon\}$

(b) $P\{|T_n - \theta| > \varepsilon\} \geq P\{|T_n^* - \theta| > \varepsilon\}$

(c) $P\{|T_n - \theta| < \varepsilon\} = P\{|T_n^* - \theta| < \varepsilon\}$

for all θ

(d) none of the above

Q. 6 An estimator of a parametric function $\tau(\theta)$ is said to be the best if it possesses:

(a) any two properties of a good estimator

(b) at least three properties of a good estimator

(c) all the properties of a good estimator

(d) all the above

Q. 7 The type of estimates are:

(a) point estimate

(b) interval estimates

(c) estimation of confidence region

(d) all the above

Q. 8 If an estimator T_n of population parameter θ converges in probability to θ as n tends to infinity is said to be:

(a) sufficient

(b) efficient

(c) consistent

(d) unbiased

Q. 9 The estimator $\Sigma X/n$ of population mean is:

(a) an unbiased estimator

(b) a consistent estimator

(c) both (a) and (b)

(d) neither (a) nor (b)

Q. 10 If $X_1, X_2, \ldots, X_n$ is a random sample from a population $N(0, \sigma^2)$, the sufficient statistic for σ^2 is:

(a) ΣX_i

(b) ΣX_i^2

(c) $(\Sigma X_i)^2$

(d) none of the above

Q. 11 If $x_1, x_2, \ldots, x_n$ be a random sample from a $N(\mu, \sigma^2)$ population, the sufficient statistic for μ is:

(a) $\Sigma(x_i - \bar{x})$

(b) $\bar{x}/n$

(c) Σx_i

(d) $\Sigma(x_i - \bar{x})^2$

Q. 12 Factorisation theorem for sufficiency is known as:

(a) Rao-Blackwell theorem

(b) Crammer-Rao theorem

(c) Chapman-Robins theorem

(d) Fisher-Neyman theorem

Q. 13 Consistency can specifically be named as:

(a) simple consistency

(b) mean-squared consistency

(c) simple consistency and mean squared consistency both

(d) all the above

Q. 14 If the expected value of an estimator is not equal to its parametric function $\tau(\theta)$, it is said to be a:

(a) unbiased estimator

(b) biased estimator

(c) consistent estimator

(d) none of the above

Q. 15 Bias of an estimator can be:

(a) positive

(b) negative

(c) either positive or negative

(d) always zero

Q. 16 If $X_1, X_2, \ldots, X_n$ be a random sample from an infinite population where $S^2 = \frac{1}{n}\sum_i (X_i - \bar{X})^2$, the unbiased estimator for the population variance σ^2 is:

(a) $\frac{1}{n-1}S^2$

(b) $\frac{1}{n}S^2$

(c) $\frac{n-1}{n}S^2$

(d) $\frac{n}{n-1}S^2$

Q. 17 If $X_1, X_2, \ldots, X_n$ is a random sample from an infinite population, an estimator for the population variance σ^2 such as:

(a) $\frac{1}{n}\sum\left(X_i - \overline{X}\right)^2$ is an unbiased estimator of σ^2

(b) $\frac{1}{n}\sum\left(X_i - \overline{X}\right)^2$ is a biased estimator of σ^2

(c) $\sum\left(X_i - \overline{X}\right)^2$ is an unbiased estimator of σ^2

(d) none of the above

Q. 18 If $x_1, x_2, \ldots, x_n$ be the values of random sample from a normal population $N\left(\mu, \sigma^2\right)$, $s^2 = \frac{1}{n-1}\sum\left(x_i - \bar{x}\right)^2$ is a:

(a) unbiased estimator of σ^2
(b) sufficient statistics for σ^2
(c) mean squared error consistent estimator of σ^2
(d) all the above

Q. 19 Simple consistency of an estimator T_n of τ (θ) means:

(a) $P_\theta\left\{\left|T_n - \tau(\theta)\right| > \varepsilon\right\} = 1$

(b) $\lim\limits_{n\to\infty} P_\theta\left\{\left|T_n - \tau(\theta)\right| < \varepsilon\right\} = 1$

(c) $\lim\limits_{n\to\infty} P_\theta\left\{\left|T_n - \tau(\theta)\right| < \varepsilon\right\} = 0$

(d) all the above

Q. 20 A sequence of estimators $T_1, T_2, \ldots, T_n$ of τ (θ) is said to be a best asymptotically normal estimator if it satisfies the condition:

(a) $\sqrt{n}\left[T_n - \tau(\theta)\right] \sim N\left(0, \sigma^2\right)$ as $n \to \infty$

(b) T_n is consistent

(c) T_n has minimum variance as compared to the variance of any other estimator T_n^*.

(d) all the above

Q. 21 If $X_1, X_2, \ldots, X_n$ is a random sample from a population $N(\mu, \sigma^2)$, the estimator $\frac{\sum X_i}{n}$ is:

(a) a BAN estimator for μ
(b) a consistent estimator for μ
(c) a unbiased estimator of μ
(d) all the above

Q. 22 Which of the estimators are BAN estimators?

(a) ML estimators
(b) Minimum Chi-square estimators
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 23 Crammer-Rao inequality is based on:

(a) stringent conditions
(b) mild conditions
(c) no conditions
(d) none of the above

Q. 24 Regularity conditions of Crammer-Rao inequality are related to:

(a) integrability of functions
(b) differentiability of functions
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 25 Crammer-Rao inequality with regard to the variance of an estimator provides:

(a) upper bound on the variance
(b) lower bound on the variance
(c) asymptotic variance of an estimator
(d) none of the above

Q. 26 Crammer-Rao inequality is valid in case of:

(a) continuous variables
(b) discrete variables
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 27 Crammer-Rao inequality was given by them:

(a) jointly
(b) in different years
(c) in the same year
(d) none of the above

Q. 28 The denominator in the Crammer-Rao inequality is known as:

(a) information limit

(b) lower bound of the variance
(c) upper bound of the variance
(d) all the above

Q. 29 The inequality for the lower bound of the variance of an estimator which is not based on stringent regularity conditions was given by:

(a) Aitken and Silverstone
(b) Neyman-Person
(c) Chapman-Robins
(d) none of the above

Q. 30 If $X_1, X_2, \ldots, X_n$ are *i.i.d* variates from a density $f(x; \theta)$, then the Crammer-Rao bound on the variance of an estimator T_n of $\tau(\theta)$ is given by the equality:

(a) $V_\theta(T_n) \geq \dfrac{[\tau(\theta)]^2}{nE\left[\dfrac{\partial}{\partial\theta}\log f(x;\theta)\right]^2}$

(b) $V_\theta(T_n) \geq \dfrac{[\tau'(\theta)]^2}{nE\left[\dfrac{\partial^2}{\partial\theta^2}\log f(x;\theta)\right]}$

(c) $V_\theta(T_n) \geq -\dfrac{[\tau'(\theta)]^2}{E\left[\dfrac{\partial^2}{\partial\theta^2}\log f(x;\theta)\right]}$

(d) $V_\theta(T_n) \geq -\dfrac{[\tau'(\theta)]^2}{nE\left[\dfrac{\partial^2}{\partial\theta^2}\log f(x;\theta)\right]}$

Q. 31 The lower bound for the variance of an estimator T_n under amended regularity conditions of Crammer-Rao was given by:

(a) R.A. Fisher
(b) A. Bhattacharyya
(c) Silverstone
(d) all the above

Q. 32 Another name of best asymptotically normal estimator is:

(a) minimum variance unbiased estimator
(b) best linear unbiased estimator
(c) consistent asymptotically normal efficient estimator
(d) all the above

Q. 33 Mean squared error of an estimator T_n of $\tau(\theta)$ is expressed as:

(a) bias + $\text{var}_\theta(T_n)$
(b) $[\text{bias} + \text{var}_\theta(T_n)]^2$
(c) $(\text{bias})^2 + [\text{var}_\theta(T_n)]^2$
(d) $(\text{bias})^2 + \text{var}_\theta(T_n)$

Q. 34 Mean squared error of an estimator T_n of $\tau(\theta)$ is minimum only if

(a) bias and $\text{var}_\theta(T_n)$ both are zero.
(b) bias is zero and $\text{var}_\theta(T_n)$ is minimum
(c) bias is minimum and $\text{var}_\theta(T_n)$ is zero
(d) none of the above

Q. 35 Mean squared consistency of an estimator T_n of $\tau(\theta)$ implies that:

(a) T_n is biased but has minimum variance
(b) T_n is unbiased but has minimum variance
(c) T_n is unbiased and $\text{var}_\theta(T_n)$ tends to zero as sample size n tends to infinity
(d) T_n and $\text{var}_\theta(T_n)$ tends to zero as sample size n tends to infinity

Q. 36 If T_n and T_n^* are two unbiased estimators of $\tau(\theta)$ based on the random sample $X_1, X_2, \ldots, X_n$, then T_n is said to be UMVUE if and only if:

(a) $V(T_n) \geq V(T_n^*)$

(b) $V(T_n) \leq V(T_n^*)$

(c) $V(T_n) = V(T_n^*)$

(d) $V(T_n) = V(T_n^*) = 1$

Q. 37 An estimator T_n of $\tau(\theta)$ is said to be more efficient than any other estimator T_n^* of $\tau(\theta)$ if and only if

(a) $\text{var}\left(T_n\right) < \text{var}\left(T_n^*\right)$

(b) $\dfrac{\text{var}\left(T_n\right)}{\text{var}\left(T_n^*\right)} < 1$

(c) $\dfrac{V\left(T_n^*\right)}{V\left(T_n\right)} > 1$

(d) all the above

Q. 38 A minimum variance unbiased estimator T_n is said to be unique if for any other estimator T_n^*,

(a) $\text{var}\left(T_n\right) = \text{var}\left(T_n^*\right)$

(b) $\text{var}\left(T_n\right) \leq \text{var}\left(T_n^*\right)$

(c) both (a) and (b)

(d) neither (a) nor (b)

Q. 39 An estimator T_n is said to be a sufficient statistic for a parametric function τ (θ) if it contained all the information which is contained in the

(a) population

(b) parametric function τ (θ)

(c) sample

(d) none of the above

Q. 40 If $X_1, X_2, \ldots, X_n$ is a random sample from a density $f(x; \theta)$ for $a < x < b$ where a and b are independent of θ, then any statistic $\hat{\theta}$ will be called a sufficient statistic if the joint probability density function g $(x_1, x_2, \ldots, x_n;$ $\theta)$ of $X_1, X_2, \ldots, X_n$ can be expressed as:

(a) $g_1\left(x_i; \hat{\theta}\right) g_2\left(\hat{\theta}\right)$

(b) $g_1\left(\hat{\theta}, \theta\right) g_2\left(x_1, x_2, \ldots, x_n\right)$

(c) $g_1\left(\hat{\theta}, \theta\right) g_2\left(x_1, x_2, \ldots, x_n; \theta\right)$

(d) none of the above

Q. 41 Let $X_1, X_2, \ldots, X_n$ be a random sample from a density f $(x; \theta)$. A statistic $S = s(x_1, x_2, \ldots, x_n)$ is said to be a sufficient statistics if the conditional distribution of $x_1, x_2, \ldots, x_n$ given $S = s$ is:

(a) independent of s

(b) dependent of θ

(c) independent of θ

(d) all the above

Q. 42 Let $x_1, x_2, \ldots, x_n$ be a random sample from a Bernoulli population $p^x\,(1-p)^{n-x}$. A sufficient statistics for p is:

(a) Σx_i

(b) Πx_i

(c) $\text{Max}\left(x_1, x_2, \ldots, x_n\right)$

(d) $\text{Min}\left(x_1, x_2, \ldots, x_n\right)$

Q. 43 Rao-Blackwell theorem enables us to obtain minimum variance unbiased estimator through:

(a) unbiased estimators

(b) complete statistics

(c) efficient statistics

(d) sufficient statistics

Q. 44 A sufficient statistics is minimal if and only if it is a:

(a) minimum sufficient statistics in a sequence of sufficient statistics.

(b) a function of every other sufficient statistics

(c) a function of UMVU estimators

(d) all the above

Q. 45 A sufficient statistic $S = s\left(x_1, x_2, \ldots, x_n\right)$ is said to be complete for a parameter θ if:

(a) $E_\theta(S) = 0 \Rightarrow S = 0$

(b) $E_\theta(S) = 1 \Rightarrow S = 1$

(c) either (a) or (b)

(d) neither (a) nor (b)

Q. 46 If f $(x; \theta)$ is any family of parametric distribution and $g(x)$ be any function independent of θ, then $g(x)$ is a complete family if:

(a) $V_\theta\,[g\,(x)] = 0$
(b) $E\,[g\,(x)] = 0$
(c) $g\,(x;\,\theta) = g\,(x)\cdot g\,(\theta)$
(d) none of the above

Q. 47 An estimator $T_1 = t_1(x_1, x_2, \ldots, x_n)$ for θ is said to be admissible if for any other estimator $T_2 = t_2(x_1, x_2, \ldots, x_n)$ for θ, the relation is of the type:
(a) $R(t_1,\theta) \geq R(t_2,\theta)$
(b) $R\,(t_1,\theta) = R\,(t_2,\theta)$
(c) $R\,(t_1,\theta) \leq R\,(t_2,\theta)$
(d) none of the above

Q. 48 Let X be a random sample of size one from a normal population with mean 0 and variance σ^2. Then the sufficient statistics for σ^2 is:
(a) $|\,X\,|$
(b) X
(c) X^2
(d) none of the above

Q. 49 If T_1 and T_2 are two most efficient estimators with the same variance S^2 and the correlation between them is ρ, the variance of $(T_1 + T_2)/2$ is equal to:
(a) S^2
(b) $\rho\, S^2$
(c) $(1+\rho)S^2/4$
(d) $(1+\rho)S^2/2$

Q. 50 If the sample mean $\bar{x}$ is an estimate of population mean μ, then $\bar{x}$ is:
(a) unbiased and efficient
(b) unbiased and inefficient
(c) biased and efficient
(d) biased and inefficient

Q. 51 Sample standard deviation as an estimate of population standard deviation is:
(a) unbiased and efficient
(b) unbiased and inefficient
(c) biased and efficient
(d) biased and inefficient

$$\left[\text{Hint: var}\,(\bar{x}) = \frac{\sigma^2}{n} \text{ and var(med)} = \frac{\pi\,\sigma^2}{2n}\right]$$

Q. 52 Efficieny of sample mean as compared to median as an estimate of the mean of a normal population is:
(a) 64 per cent
(b) 157 per cent
(c) 317 per cent
(d) 31.5 per cent

Q. 53 If t is a consistent estimator of θ, then:
(a) t is also a consistent estimator of θ^2
(b) t^2 is also a consistent estimator of θ
(c) t^2 is also a consistent estimator θ^2
(d) none of the above

Q. 54 If T_n is a consistent estimator of θ, then e^{T_n} is a:
(a) unbiased estimator of e^θ
(b) consistent estimator of e^θ
(c) MVU estimator of e^θ
(d) none of the above

Q. 55 If $X_1, X_2, \ldots, X_n$ is a random sample from a population $p^x\,(1-p)^{n-x}$ for $x = 0, 1$ and $0 < p < 1$, the sufficient statistics for p is:
(a) $\sum_{i=1}^{n} X_i$
(b) $\prod_{i=1}^{n} X_i$
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 56 If $X_1, X_2, \ldots, X_n$ is a random sample from the population having the density function,

$$f(x;\theta) = \frac{1}{\sqrt{2\pi\,\theta}}\,e^{-\frac{1}{2}\frac{x^2}{\theta}},$$

then the maximum likelihood estimator for θ is:

(a) $\sqrt{\Sigma X_i^2/n}$

(b) $\Sigma X_i^2/n$

(c) $\sqrt{\Sigma X_i^2/n}$

(d) $\Sigma X_i^2/\sqrt{n}$

Q. 57 If $X_1, X_2, \ldots, X_n$ is a random sample from a population

$$\frac{1}{\theta\sqrt{2\pi}} e^{-x^2/2\theta^2},$$

the maximum likelihood for θ is:

(a) $\Sigma X_i/n$

(b) $\Sigma X_i^2/n$

(c) $\sqrt{\Sigma_i X_i^2}/n$

(d) $\sqrt{\Sigma X_i^2/n}$

Q. 58 The diameter of cylindrical rods is assumed to be normally distributed with a variance of 0.04 cm. A sample of 25 rods has a mean diameter of 4.5 cm. 95% confidence limits for population mean are:

(a) 4.5 ∓ 0.004

(b) 4.5 ∓ 0.0016

(c) 4.5 ∓ 0.078

(d) 4.5 ∓ 0.2

Q. 59 The monthly expenditure in excess of Rs. 2000 of the families in a locality is approximately negative exponentially distributed. A random sample of four families' per month expenditure was reported as Rs. 3000, Rs. 3500, Rs. 4500 and Rs. 5000. An estimate of the average expenditure for the given distribution of the locality is:

(a) 2000

(b) 3000

(c) 4000

(d) none of the above

Q. 60 If $T = t(X_1, X_2, \ldots, X_n)$ is a sufficient statistics for a parameter θ and an unique MLE $\hat{\theta}$ for θ exists, then

(a) $\hat{\theta} = t(X_1, X_2, \ldots, X_n)$

(b) $\hat{\theta}$ is a function of t

(c) $\hat{\theta}$ is independent of t

(d) none of the above

Q. 61 If $T = t(X_1, X_2, \ldots, X_n)$ is a MLE of θ and $T(\theta)$ is a one to one function of θ, then

(a) $T(t)$ is a MVU estimator of $T(\theta)$.

(b) $T(t)$ is a unbiased estimate of $T(\theta)$

(c) $T(t)$ is a MLE of $T(\theta)$

(d) all the above

Q. 62 In estimating the parameters of a linear function, most commonly used method of estimation is:

(a) maximum likelihood method

(b) least square method

(c) method of minimum Chi-square

(d) method of moments

Q. 63 The set of equations obtained in the process of least square estimation are called:

(a) normal equations

(b) intrinsic equations

(c) simultaneous equations

(d) all the above

Q. 64 Least square estimators of the parameters of a linear model are:

(a) unbiased

(b) BLUE

(c) UMVU

(d) all the above

Q. 65 Least square estimators of the parameters of a linear model are not:

(a) necessarily consistent

(b) scale invariant

(c) asymptotically normal

(d) all the above

Q. 66 Least square theory was propounded by whom and in which year?

(a) Gauss in 1809

(b) Markov in 1900

(c) Fisher in 1920
(d) none of the above

Q. 67 The minimum variance approach was put forth by whom and in which year?
(a) Gauss in 1809
(b) Markov in 1900
(c) Fisher in 1920
(d) all the above

Q. 68 The credit of inventing the method of moments for estimating the parameters goes to:
(a) R.A. Fisher
(b) J. Neyman
(c) Laplace
(d) Karl Pearson

Q. 69. By the method of moments one can estimate:
(a) all constants of a population
(b) only mean and variance of a distribution
(c) all moments of a population distribution
(d) all the above

Q. 70 Generally the estimators obtained by the method of moments as compared to ML estimators are:
(a) less efficient
(b) more efficient
(c) equally efficient
(d) none of the above

Q. 71 Method of minimum Chi-square for the estimation of parameters utilises:
(a) Chi-square distribution function
(b) Pearson's Chi-square statistic
(c) Contingency table
(d) All the above

Q. 72 Modified minimum Chi-square differs from minimum Chi-square in respect of:
(a) numerator of Chi-square statistic
(b) denominator of Chi-square statistic
(c) basic approach
(d) none of the above

Q. 73 Mean square error of estimators obtained by the method of minimum Chi-square is:
(a) less than ML estimators
(b) more than ML estimators
(c) equal to ML estimators
(d) none of the above

Q. 74 Minimum Chi-square estimators are:
(a) consistent
(b) asymptotically normal
(c) efficient
(d) all the above

Q. 75 Minimum Chi-square estimators are not necessarily:
(a) efficient
(b) consistent
(c) unbiased
(d) all the above

Q. 76 Main feature of Bayes' approach in the estimation of parameter is:
(a) to consider the parameter a random variable
(b) specification of the prior distribution
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 77 The probability density function $f(x;\theta)$ in which X and Θ both are random variables can be expressed for given value of $\Theta = \theta$ and prior density $g(\theta)$ as:

(a) $f(x;\theta) = f(x|\theta)g(\theta)$

(b) $f(x;\theta) = f(x|\theta)/g(\theta)$

(c) $f(x;\theta) = g(\theta)/f(x|\theta)$

(d) none of the above

Q. 78 Bayes' approach is:
(a) universally accepted
(b) a matter of controversy
(c) irrelevant
(d) none of the above

Q. 79 Bayes estimator $\hat{\tau}(\theta)$ of a parametric function $\tau(\theta)$ of θ can be obtained by the formula with usual notations as:

(a) $\hat{\tau}(\theta) = \dfrac{\int \tau(\theta) f(x|\theta) d\theta}{\int f(x|\theta) g(\theta) d\theta}$

(b) $\hat{\tau}(\theta) = \dfrac{\int \tau(\theta) f(x|\theta) f(\theta) g(\theta) d\theta}{\int g(\theta) d\theta}$

(c) $\hat{\tau}(\theta) = \dfrac{\int \tau(\theta) g(\theta) d\theta}{\int g(\theta) f(x|\theta) d\theta}$

(d) $\hat{\tau}(\theta) = \dfrac{\int \tau(\theta) f(x|\theta) g(\theta) d\theta}{\int f(x|\theta) g(\theta) d\theta}$

Q. 80 Which of the following statements is true?

(a) Bayes estimator is always a function of minimal sufficient statistics
(b) Bayes estimators are most efficient
(c) Bayes estimators are always asymptotically normal
(d) None of the above

Q. 81 Which of the following statement is not correct?

(a) Bayes estimators in many cases are asymptotically consistent
(b) For large n, estimators tend to ML estimators irrespective of prior density $g(\theta)$
(c) Properties of Bayes estimators are given in terms of maximum risk
(d) Goodness of a Bayes estimator is measured in terms of mean squared error loss

Q. 82 Pitman's estimator for location parameters are generally:

(a) unbiased
(b) consistent
(c) a function of sufficient statistics
(d) none of the above

Q. 83 Pitman estimator for location usually possess:

(a) smallest mean square error
(b) asymptotic property
(c) a property of complete statistic
(d) all the above

Q. 84 Formula for Pitman estimator for the scale parameter θ, where $L(x_i|\theta)$ is the likelihood function for $i = 1, 2, ..., n$ is:

(a) $\hat{\theta} = \dfrac{\int \hat{\theta}\, L(x_i|\theta) d\theta}{\int \theta^3 L(x_i|\theta) d\theta}$

(b) $\hat{\theta} = \dfrac{\int \frac{1}{\theta^2} L(x_i|\theta) d\theta}{\int \frac{1}{\theta^3} L(x_i|\theta) d\theta}$

(c) $\hat{\theta} = \dfrac{\int \frac{1}{\theta^2} L(x_i|\theta) d\theta}{\int \frac{1}{\theta} L(x_i|\theta) d\theta}$

(d) none of the above

Q. 85 For a fixed confidence coefficient $(1 - \alpha)$, the most preferred confidence interval for the parameter θ is one:

(a) with shortest width
(b) with largest width
(c) with an average width
(d) none of the above

Q. 86 The most pragmatic approach for determining $(1 - \alpha)$ per cent confidence interval is to find out:

(a) zero width confidence interval
(b) equal tail confidence coefficient interval
(c) a confidence interval such that the combined area of both the tails is equal to α
(d) none of the above

Q. 87 Sample median as an estimator of population mean is always:

(a) unbiased
(b) efficient
(c) sufficient
(d) none of the above

Q. 88 Confidence region tentamounts to estimation of:

(a) Confidence interval for a parameter of a distribution
(b) confidence interval for two or more parameters of a population distribution
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 89 A family receives 1, 2 and 3 wrong telephone calls on three randomly selected days. Assuming that the wrong calls follow Poisson distribution, the estimate of the number of wrong calls in 6 days is:

(a) 6
(b) 12
(c) 36
(d) none of the above

Q. 90 Given the probability statement that,

$$P(4.35 \le \theta \le 15.67) = 0.90$$

which of the following statement is correct in respect of given probability statement?

(a) 4.35 and 15.67 are 90 per cent confidence limits

(b) The confidence interval is 11.32

(c) Probability that μ lies in the interval (4, 35, 15.67) is 0.90

(d) all the above

Q. 91 If t_n is a sufficient statistic for θ based on the sample $x_1, x_2, \ldots, x_n$, the function

$\frac{\partial}{\partial\theta} \mathrm{Log}\, L$ is a function of:

(a) θ only

(b) t_n only

(c) t_n and θ only

(d) none of the above

Q. 92 The concepts of consistency, efficiency and sufficiency are due to:

(a) J. Neyman

(b) R.A. Fisher

(c) C.R. Rao

(d) J. Berkson

Q. 93 If $X_1, X_2, \ldots, X_n$ is a random sample from any distribution having K^{th} moment $E(X^k)$, the consistent estimator for $E(X^k)$ is:

(a) $\sum_{i=1}^{n} X_i^k$

(b) $\frac{1}{n}\sum_{i=1}^{n} X_i^k$

(c) both (a) and (b)

(d) none of (a) and (b)

Q. 94 If $\bar{x}$ is a sample mean from the binomial distribution b (1, p), than

(a) $\bar{x}$ is a sufficient statistics for p

(b) $\bar{x}$ is an efficient statistics for p

(c) both (a) and (b)

(d) none of (a) and (b)

Q. 95 Formula for obtaining 95% confidence limits for the mean μ of a normal population $N(\mu, \sigma^2)$ with known σ are:

(a) $-1.96 \le \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < 1.96$

(b) $P\left(-Z_{\alpha/2} \le \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} \le Z_{\alpha/2}\right) = 0.95$

(c) $\bar{x} \mp 1.96\frac{\sigma}{\sqrt{n}}$

(d) all the above

Q. 96 If the density function of a variable x is,

$$f(x;\theta) = \theta\, e^{-\theta x}$$

for $0 < x < \infty$,

95% central confidence limits for large sample n are

(a) $\left(1 \pm \frac{1.96}{\sqrt{n}}\right)\bar{x}$

(b) $\left(1 \pm \frac{1.96\bar{x}}{\sqrt{n}}\right)\Big/\bar{x}$

(c) $\left(\frac{1 \pm 1.96}{\sqrt{n}}\right)\Big/\bar{x}$

(d) none of the above

Q. 97 Formula for the confidence interval with (1 − α) confidence coefficient for the variance of the normal distribution $N(\mu, \sigma^2)$, when μ is known, is given as:

(a) $P\left[\chi^2_{1-\alpha/2} \le \frac{ns^2}{\sigma^2} \le \chi^2_{\alpha/2}\right] = 1 - \alpha$

(b) $P\left[\frac{ns^2}{\chi^2_{\alpha/2}} \le \sigma^2 \le \frac{ns^2}{\chi^2_{1-\alpha/2}}\right] = 1 - \alpha$

(c) both (a) and (b)

(d) neither (a) nor (b)

Q. 98 Formula for 95% confidence limits for the variance of population $N(\mu, \sigma^2)$, when μ is unknown, is:

(a) $\frac{ns^2}{\chi^2_{(n-1)(0.025)}} \le \sigma^2 \le \frac{ns^2}{\chi^2_{(n-1)(0.975)}}$

(b) $\frac{ns^2}{\chi^2_{0.025}} \le \sigma^2 \le \frac{ns^2}{\chi^2_{0.975}}$

(c) both (a) and (b)

(d) neither (a) and (b)

Q. 99 A random sample of 16 housewives has an average body weight of 52 kg and an standard deviation of 3.6 kg. 99% central confidence limits for body weight in general are: [Given: $t_{15,\,0.01} = 2.95$]

(a) (54.66; 49.345)

(b) (52.66; 51.34)

(c) (55.28; 48.72)

(d) none of the above

Q. 100 Two samples from two normal populations having equal variances of size 10 and 12 have means 12 and 10 and variances 2 and 5 respectively. 95% confidence limits for the difference between two population means are: [Given: $t_{0.05,\,20} = 2.086$]

(a) (−1.57; 5.43)

(b) (0.214; 3.786)

(c) (0.477; 3.523)

(d) none of the above

Q. 101 Confidence limits for the population mean difference can be found out only when:

(a) the observations on two variable are paired

(b) when the two variables are correlated

(c) both (a) and (b)

(d) neither (a) nor (b)

Q. 102 Formula for the confidence interval for the ratio of variances of two normal population involves:

(a) χ^2-distribution

(b) F-distribution

(c) t-distribution

(d) none of the above

Q. 103 Consistency of an estimator is a:

(a) large sample property only

(b) small sample property only

(c) property not related to sample size

(d) property applicable to any sample size

Q. 104 In sampling from a population having mean μ and variance σ^2, the estimator $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$ be the sequence of estimators of μ based on samples of size n, then $\bar{X}_n$ is:

(a) BAN estimator of μ

(b) mean-squared error consistent estimator of μ

(c) unbiased estimator of μ

(d) all the above

Q. 105 For the exponential distribution,

$$f(x;\theta) = \frac{1}{\theta} e^{-x/\theta}; x > 0, \theta > 0,$$

the estimator $\Sigma X_i/n$, based on a sample of size n, is an unbiased estimator of:

(a) 1/θ

(b) $1/\theta^2$

(c) θ

(d) θ^2

Q. 106 For an estimator to be consistent, the unbiasedness of the estimator is:

(a) necessary

(b) sufficient

(c) necessary as well as sufficient

(d) neither necessary nor sufficient

Q. 107 For the sample mean to be an unbiased estimator of population mean, the condition of normality of population is:

(a) necessary but not sufficient

(b) necessary as well as sufficient

(c) sufficient but not necessary

(d) none of the above

Q. 108 For the distribution,

$$f(x;\theta) = \frac{1}{\theta}; 0 \le x \le \theta$$

a sufficient estimator for θ, based on a sample $X_1, X_2, \ldots, X_n$ is,

(a) $\Sigma X_i/n$

(b) $\sqrt{\Sigma X_i^2}$

(c) $\max(X_1, X_2, \ldots, X_n)$

(d) $\min(X_1, X_2, \ldots, X_n)$

Q. 109 A confidence interval of confidence coefficient $(1 - \alpha)$ is best which has:
(a) smallest width
(b) vastest width
(c) upper and lower limits equidistant from the parameter
(d) one-sided confidence interval.

Q. 110 Let $X_1, X_2, \ldots, X_n$ be a sample from density $N(\theta, \theta^2)$, then the statistic $T = \left(\Sigma X_i, \Sigma X_i^2\right)$ for θ is:
(a) sufficient and complete
(b) sufficient but not complete
(c) not sufficient but complete
(d) neither sufficient nor complete

Q. 111 The maximum likelihood estimators are necessarily:
(a) unbiased
(b) sufficient
(c) most efficient
(d) unique

Q. 112 Least square estimators under linear set up are:
(a) unbiased
(b) UMVUE's
(c) BLUE's
(d) all the above

Q. 113 For a random sample from a Poisson population $P(\lambda)$, the maximum likelihood estimate of λ is:
(a) median
(b) mode
(c) geometric mean
(d) mean

Q. 114 For a random sample $(x_1, x_2, \ldots, x_n)$ from a population $N(\mu, \sigma^2)$, the maximum likelihood estimator of σ^2 is:

(a) $\frac{1}{n}\sum_i \left(X_i - \overline{X}\right)^2$

(b) $\frac{1}{n-1}\sum_i \left(X_i - \overline{X}\right)^2$

(c) $\frac{1}{n}\sum_i (X_i - \mu)^2$

(d) $\frac{1}{n-1}\sum_i (X_i - \mu)^2$.

Q. 115 If the variance of an estimator attains the Crammer-Rao lower bound, the estimator is:
(a) most efficient
(b) sufficient
(c) consistent
(d) admissible

ANSWERS

SECTION-B

(1) estimate (2) best (3) not possible (4) random sample (5) random variable (6) estimate (7) estimator (8) point (9) consistent (10) consistent (11) unbiased (12) bias (13) (bias)^2 + var (T_n) (14) zero (15) UMVUE (16) mean-squared error (17) holds (18) information (19) MVB (20) Aitken and Silverstone (21) different (22) lower bound (23) better (24) 1946 (25) 1945 (26) Aitken (27) CANE (28) limiting (29) minimum (30) less (31) $V(T_n')/V(T_n)$ (32) $T_n = T_n^*$ (33) sufficient statistic (34) independent (35) sufficient statistic (36) minimal sufficient (37) complete (38) unique (39) MVU (40) Rao-Blackwell (41) admissible (42) risk (43) likelihood function (44) maximum likelihood (45) unbiased/consistent/unique (46) maximum likelihood (47) unbiased (48) $\bar{x}$ (49) $\Sigma(x_i - \bar{x})^2/n$ (50) sufficient statistic (51) sufficient statistic (52) $\lambda/\Gamma\lambda\,\bar{x}$ (53)

med $(x_1, x_2, \ldots, x_n)$ (54) smallest X; largest X (55) BLUE (56) unbiased (57) UMVU (58) Gauss-Markov (59) Karl Pearson; 1894 (60) asymptotically normal (61) less (62) $\Sigma(X_i - \overline{X})^2/n$ (63) sample mean (64) $1/\bar{x}$ (65) Pearson's Chi-square (66) denominator (67) almost equal (68) J. Berkson; 1955 (69) consistent/asymptotically normal/efficient (70) unbiased (71) random variable (72) fixed quantity (73) prior distribution (74) previous knowledge (75) $f(x/\theta)\,g(\theta)$ (76) $\int \tau(\theta) f(x|\theta) g(\theta) \Big/ \int f(x/\theta) g(\theta)\, d\theta$ (77) minimal sufficient (78) maximum likelihood (79) asymptotically normal (80) minimax (81) $(x+\alpha)/\ (n+\alpha+\beta)$ (82) $(x+r)/(1+\alpha)$ (83) $\int \theta L(x|\theta) d\theta / \int L(x|\theta)\, d\theta$ (84) smallest (85) sufficient statistic (86) minimax (87) location invariant (88) $\overline{X}_n$ (89) confidence coefficient (90) shortest (91) upper and lower (92) confidence region (93) decreases (94) concentrated (95) consistent (96) consistent (97) unbiased (98) biased (99) consistent (100) unbiased (101) more efficient (102) unbiased (103) biased; inefficient (104) t_n; θ (105) UMVUE (106) uniqueness (107) Fisher and Neyman (108) Consistent (109) consistent (110) ΣX_i (111) more (112) $2/\pi$ (113) biased (114) unbiased (115) consistent (116) maximum likelihood (117) non-existent (118) best unbiased (119) most efficient (120) most efficient (121) unbiased; efficient (122) unbiased (123) consistent (124) $T_1 = T_2$ (125) better

SECTION-C

(1) b	(2) a	(3) b	(4) c	(5) a	(6) c
(7) d	(8) c	(9) c	(10) b	(11) c	(12) d
(13) d	(14) b	(15) c	(16) d	(17) b	(18) d
(19) b	(20) d	(21) d	(22) c	(23) a	(24) b
(25) b	(26) c	(27) b	(28) a	(29) c	(30) d
(31) b	(32) c	(33) d	(34) b	(35) c	(36) b
(37) d	(38) a	(39) c	(40) b	(41) c	(42) a
(43) d	(44) b	(45) a	(46) b	(47) c	(48) c
(49) d	(50) a	(51) d	(52) b	(53) c	(54) b
(55) a	(56) b	(57) d	(58) c	(59) a	(60) b
(61) c	(62) b	(63) a	(64) d	(65) c	(66) a
(67) b	(68) d	(69) c	(70) a	(71) b	(72) b
(73) a	(74) d	(75) c	(76) c	(77) a	(78) b
(79) d	(80) a	(81) c	(82) c	(83) a	(84) b
(85) a	(86) b	(87) d	(88) b	(89) b	(90) d
(91) c	(92) b	(93) b	(94) c	(95) d	(96) c
(97) c	(98) a	(99) a	(100) b	(101) c	(102) b
(103) a	(104) d	(105) c	(106) b	(107) d	(108) c
(109) a	(110) b	(111) b	(112) d	(113) d	(114) a
(115) a					

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Chapter **11**

Testing Parametric Hypotheses

SECTION-A

Short Essay Type Questions

Q. 1 Write a note on the exigency of testing of hypothesis.

Ans. There is always some contention about the value(s) of a parameter or the relationship between parameters. When parametric values are unknown, we estimate them through sample values. If the sample value is exactly the same as per our contention, there is no hitch in accepting it. And if it is far from our contention, there is no reason to accept it. But the problem arises when the sample provides a value which is neither exactly equal to the parametric value, nor too far. In that situation one has to develop some procedure(s) which enables one to decide whether to accept a contended (hypothetical) value or not on the basis of sample values. Such a procedure is known as testing of hypothesis.

There can be more than one procedure to test a hypothesis. If so, which one to choose out of many tests at hand. For this purpose, many properties of tests have been put forward. A test with maximum virtues is most preferred one. So we have to deal with hypothesis, test procedures and their properties. The theory of testing parametric statistical hypotheses was originally set forth by J. Neyman in 1928 and Karl Pearson in 1933.

Q. 2 What is meant by the test of a statistical hypothesis?

Ans. A statistical test of a hypothesis *H* is a rule or procedure which makes one to decide about the acceptance or rejection of the hypothesis *H*.

Q. 3 What are the principle steps involved in statistical test.

Ans. A statistical test mainly involves four steps:

(i) Evolving a test statistic

(ii) To know the sampling distribution of the test statistic

(iii) Setting of hypotheses testing conventions

(iv) Establishing a decision rule that lead to an inductive inference about the probable truth.

Q. 4 What is the difference between a research or scientific hypothesis and statistical hypothesis?

Ans. Research or scientific hypothesis is the hypothesis which a researcher postulates to meet his objectives. Whereas statistical hypothesis is a testable formulations of research or scientific hypotheses or the functional relation about the population parameter(s).

Q. 5 Define parametric statistical hypothesis.

Ans. A statistical hypothesis is an assertion about a parameter of a population or parameters of two or more populations. For instance, the life of the electric bulb in general, is 2,000 hours, *i.e.*, $\mu = 2000$ hours. Two bulb manufacturing processes produce bulbs of the same average life, etc., *i.e.*, $\mu_1 = \mu_2$.

In other words, a statistical hypothesis is an assertion about the distribution of one or more variables.

Q. 6 Define null and alternative hypothesis.

Ans. According to Fisher, any hypothesis tested for its possible rejection is called a null hypothesis and is denoted by H_0.

Alternative hypothesis is a statement about the population parameter or parameters which provides an alternative to the null hypothesis within the range of pertinent values of the parameters. It is denoted by H_1 or H_A. The idea of alternative hypothesis was propounded by J. Neyman, If H_0 is accepted, H_1 is rejected and *vice-versa*.

Q. 7 Define simple and composite hypotheses.

Ans. If a statistical hypothesis completely specifies a distribution, it is known as *simple hypothesis*, otherwise *composite hypothesis*. For a normal distribution $N(\mu, \sigma^2)$ with σ^2 known, the hypotheses $H_0 : \mu = 20$ vs. $H_1 : \mu = 30$ lead to simple hypotheses. Here H_0 and H_1, both are simple hypotheses. But generally H_1 is composite because H_1 is given as $\mu \neq \mu_0, \mu > \mu_0$ or $\mu < \mu_0$.

Q. 8 Define and elaborate two types of errors in testing of hypotheses.

Ans. There is a probability of committing an error in making a decision about a hypothesis. Hence, two types of errors are defined as follows:

First kind of error. Reject H_0 when it is true.

Second kind of error. Accept H_0 when H_1 is true. First kind of error is also named as Type I error or *rejection error*, and the second kind of error as *acceptance error*. Probability of Type I error is denoted by α and probability of Type II error by β. Notationally,

$$P\,(\text{reject } H_0/H_0 \text{ is true}) = \alpha$$

$$P\,(\text{accept } H_0/H_1 \text{ is true}) = \beta$$

From the definitions, it is apparent that if H_0 is accepted, then only type II error is committed and if H_0 is rejected, only type I error is committed.

Let us consider a practical problem. If a medicine is administered to a few patients of a particular disease to cure them and the medicine is curing the disease, but it is claimed that it has no effect or has an adverse effect, and hence it is discontinued. *This is type I error.* On the contrary, the medicine has adverse effect and is claimed to have good effect and the treatment is continued. *This is type II error.* If we delve into the consequences of both types of errors, we find type II error is more serious than type I error. Hence, α may be relatively large than β. In view of these facts, effort is made to minimise β even for certain risk of type I error usually 0.01, 0.05 or more.

Q. 9 Discuss power of a test and power function.

Ans. For testing a hypothesis H_0 against H_1, the test with probabilities α and β of type I and II errors respectively, the quantity $(1 - \beta)$ is called the power of the test.

The power of the test depends upon the difference between the parameter value specified by H_0 and the actual value of the parameter. $1 - \beta$ can be expressed as a function of the true parameter, say, θ. If T is a test of H_0, the power function of T is defined as the probability of rejection of H_0 when the distribution, from which the sample is drawn, is parameterised by θ and denoted by $P_T(\theta)$. In terms of second kind of error, the function $1 - \beta(\theta)$ is known as the *power function*. Whereas the function $\beta(\theta)$ is known as the *operating characteristic function* or OC function.

Q. 10 Define and discuss the level of significance.

Ans. The level of significance may be defined as the probability of type I error which we are ready to tolerate in making a decision about H_0. Briefly,

$$P\,(\text{reject } H_0/H_0 \text{ is true}) = \alpha.$$

It is our endeavour to carry out a test which minimises both types of errors. Unfortunately for a given set of observations, both the errors cannot be controlled simultaneously. Hence, it is a general practice to assign a bound to type I error and to minimise type II error. Thus, one chooses a value of α lying between 0 and 1 which is known as the level of significance.

While choosing a level of significance, one should also consider the power of a test against various alternatives. If the power of a test is too low, one should choose a higher value of α say 0.1, 0.2, 0.5, etc.

Q. 11 Define the size of a test.

Ans. The size of a non-randomised test is defined as the size of the critical region. Practically, it is numerically the same as the level of significance.

Q. 12 Clarify the concept of critical region.

Ans. Let Ω denote the sample space of observations or the potential set of data, *i.e.*, $\Omega = \{x_1, x_2, ..., x_n\}$ where $(x_1, x_2, \ldots, x_n)$ is a possible value of $X_1, X_2, ..., X_n$. Then sample space is specified by the test statistic under null hypothesis H_0. The space Ω is divided into two types of region, ω and $(\Omega - \omega)$. *The level of significance is the size of the critical region* ω. The critical region is also known as the *region of rejection*. The region of rejection may be situated on both the tails or on only one tail depending on the alternative hypothesis. Also the region $(\Omega - \omega)$ is called the *acceptance region*. If the value of the test statistic lies in the acceptance region, the null hypothesis H_0 is accepted and if it lies in the critical region, H_0 is rejected.

Also a critical region of size α which minimises β among all critical regions which do not exceed α is called *best critical region* (BCR).

Q. 13 How can a decision about one-tailed and two-tailed test be taken?

Ans. If an alternative hypothesis is such that it leads to two-sided alternatives to the null hypothesis, it is said to be a *two-tailed test*. For instance, testing $H_0 : \mu = 20$ vs. $H_1 : \mu \neq 20$ leads to two-sided test as μ can be greater than 20 or less than 20. In this situation half of the area of critical region lies on the left tail and half on the right tail. If α is the area of the critical region, $\alpha/2$ is the area on both the tails.

Again, if the alternative hypothesis provides one-sided alternative to H_0, e.g., $H_0 : \mu = 20$ vs. $H_1 : \mu > 20$ or $H_1 : \mu < 20$, the critical region of size α lies only on one tail, specifically an area equal to α lies on the right tail given by $\omega : z > z_\alpha$ when H_1 is $\mu > 20$ and on the left tail when H_1 is $\mu < 20$, an area equal to α given $z < -Z_\alpha$.

Q. 14 Distinguish between non-randomised and randomised test.

Ans. A test T of a hypothesis H is said to be non-randomised test if the decision about the rejection or acceptance of H is based on a test statistic. H is rejected if the test statistic lies in the critical region otherwise accepted.

A randomised test is one in which no test statistics is involved. The decision about H is taken on the basis of some predicted criteria. For instance, it is decided that H_0 will be accepted if on tossing the coin falls with head and rejected if the coin fall with tail. But randomised tests are seldom used.

Q. 15 Throw light on the role of degrees of freedom, and give its definition.

Ans. A test statistic always involves estimates(s) of the parameter(s) under test and estimated values do depend on the sample *vis-a-vis* the sample size. Hence, number of observations plays an important role in testing of hypothesis. Also in a large number of cases, the estimate approaches the true parameter as the sample size increases. In view of these facts it becomes necessary to take into account the sample size while testing a hypothesis.

The number of independent observations in a set is called *degrees of* freedom (d.f.).

In other words, degrees of freedom may be defined as the number of observations in a set minus the number of restrictions imposed on them.

In testing of hypothesis or interval estimation, the critical value of a statistic is obtained from the table of the distribution of the test statistic for the prefixed level of significance and corresponding d.f. of the test statistic.

Q. 16 What is a critical function?

Ans. A function $\psi_T(x_1, x_2, \ldots, x_n)$ of the sample values $x_1, x_2, \ldots, x_n$ where T is a test of a hypothesis

H_0 is said to be a critical function of a randomised test T if,

$$\psi_T(x_1, x_2, \ldots, x_n) = P\,[\mathrm{H}_0 \text{ is rejected} \mid (x_1, x_2, \ldots, x_n) \text{ is observed}]$$

On the other hand, the critical function in a non-randomised test divides the sample space into three regions, one the region of rejection, the other the region of acceptance and the third, the boundary between the region of acceptance and the region of rejection. In this situation, either the boundary is considered to be included in the region of rejection or a randomised test is applied.

Q. 17 What do you understand by optimum test?

Ans. A test is said to be optimum if it minimises both the errors α and β. Unfortunately, no such test is available as the reduction in one type of error causes the increase in the other. Hence, in normal practice a test which minimise β or maximises (1 – β) for a desired low level of α is considered to be an *optimum* or *best* test.

Q. 18 Name different properties of a test.

Ans. For comparing different tests of a hypothesis *H*, it becomes necessary to look into the properties of the tests. Main properties of the statistical non-randomised tests are as follows:

(i) Most powerful (MP)
(ii) Uniformly most powerful (UMP)
(iii) Unbiased
(iv) Uniformly most powerful unbiased (UMPU)
(v) Minimax
(vi) Admissible

Q. 19 Define most powerful test.

Ans. Let Ω be the sample space and ω the critical region and $\overline{\omega} = \Omega - \omega$, the acceptance region.

A test T of $H_0: \theta = \theta_0$ vs. $H_1: \theta = \theta_1$ (simple vs. simple) based on the sample values x: $(x_1, x_2, \ldots, x_n)$ is said to be a most powerful test if and only if,

$$P(x \,\varepsilon\, \omega \mid H_0) = \int_\omega L_0\, dx = \alpha$$

and for any other test T^* of the same size α,

$$P_T(x \,\varepsilon\, \omega \mid H_1) \geq P_{T^*}(x \,\varepsilon\, \omega_1 \mid H_1)$$

where ω_1 satisfies the condition,

$$\int_{\omega_1} L_0\, dx = \alpha$$

The critical region α satisfying the above conditions is known as the *most powerful* critical region of size α.

Q. 20 When do you call a test uniformly most powerful?

Ans. A test T of $H_0: \theta = \theta_0$ vs. $H_1: \theta \neq \theta_0$ is said to be a uniformly most powerful test of size α if,

$$P(x \,\varepsilon\, \omega \mid H_0) = \int_\omega L_0\, dx = \alpha$$

and $\quad P_T(x \,\varepsilon\, \omega \mid H_1) \geq P_T(x \,\varepsilon\, \omega_1 \mid H_1)$ for all $\theta \neq \theta_0$

where ω_1 is any other critical region such that,

$$\int_{\omega_1} L_0\, dx = \alpha$$

The critical region α satisfying the above conditions is called *uniformly most powerful* (UMP) *region*.

Q. 21 State Neyman-Pearson lemma and give its utility.

Ans. The lemma given by J. Neyman and E.S. Pearson provides the most powerful test for testing a simple null hypothesis against a simple alternative hypothesis.

Let $X_1, X_2, \ldots, X_n$ be a random sample from a density $f(x; \theta)$ where $\theta \,\varepsilon\, \Theta$, the parametric space. Θ consists of only two elements θ_0 and θ_1, *i.e.*, $\Theta = \{\theta_0, \theta_1\}$. Also the joint probability density function of $X_1, X_2, \ldots, X_n$,

$$L(x|\theta) = f(x_1, \theta)\, f(x_2, \theta), \ldots, f(x_n, \theta)$$

$L(x \mid \theta)$ is the likelihood function of sample observations for a parameter value of θ where $x = x_1, x_2, \ldots, x_n$.

If there exists a critical region ω of size α and a non-negative number K such that

$$\frac{L(x|\theta_1)}{L(x|\theta_0)} > K \text{ for every } x \in \omega$$

and $$\frac{L(x|\theta_1)}{L(x|\theta_0)} \le K \text{ for every } x \notin \omega$$

Then ω is said to be the best critical region (BCR) of size α for testing $H_0: \theta = \theta_0$ vs. $H_1: \theta = \theta_1$.

Any test T corresponding to the BCR ω is the most powerful test of size α of the hypothesis H_o against H_1.

Neyman-Pearson lemma helps to determine the size of type I and type II errors for the given range of x. Also it helps to find out the power and power function of a test in case of testing a simple null hypothesis against a simple alternative.

Q. 22 Using Neyman-Pearson lemma, find the best critical region for the test of hypothesis $H_0: \mu = \mu_0$ vs. $H_1: \mu = \mu_1$ for the normal population $f(x; \mu, \sigma^2)$ when σ^2 is known in the cases, (i) $\mu_0 < \mu_1$ (ii) $\mu_0 > \mu_1$. Also find the power of the test.

Ans. Let $X_1, X_2, \ldots, X_n$ be a random sample from the population under consideration. Adopting usual notations and using Neyman-Pearson lemma, BCR is obtained as,

$$\frac{L(x|\mu_1)}{L(x|\mu_0)} = \frac{e^{-\frac{1}{2\sigma^2}\Sigma(x_i-\mu_1)^2}}{e^{-\frac{1}{2\sigma^2}\Sigma(x_i-\mu_0)^2}} > k$$

or $$e^{-\frac{1}{2\sigma^2}\left\{\Sigma(x_i-\mu_1)^2-\Sigma(x_i-\mu_0)^2\right\}} > k$$

Taking logarithm and solving we obtain,

$$-\frac{n}{2\sigma^2}\left(\mu_1^2 - \mu_0^2\right) + \frac{1}{\sigma^2}(\mu_1 - \mu_0)\,\Sigma x_i > \log k$$

$$\bar{x}\,(\mu_1 - \mu_0) > \frac{\sigma^2}{n}\log k + \frac{1}{2}\left(\mu_1^2 - \mu_0^2\right)$$

$$\bar{x} > \frac{\sigma^2}{n\,(\mu_1 - \mu_0)}\log k + \frac{1}{2}(\mu_1 + \mu_0)$$

Case (i). When $\mu_0 < \mu_1$, BCR is determined as,

$$\bar{x} > \lambda_1$$

Case (ii). When $\mu_0 > \mu_1$, BCR is determined by,

$$\bar{x} < \lambda_2$$

where λ_1 and λ_2 are equal to the right hand of the inequality under the two situations. We know, under $H_0, \bar{x} \sim N\left(\mu_0, \frac{\sigma^2}{n}\right)$ and under $H_1, \bar{x} \sim N\left(\mu_1, \frac{\sigma^2}{n}\right)$. Using Neyman-Pearson lemma, the value of λ_1 can be obtained by the relation,

$$P(\bar{x} > \lambda_1 | H_0) = \frac{\sqrt{n}}{\sigma\sqrt{2\pi}}\int_{\lambda_1}^{\infty} e^{-\frac{n}{2\sigma^2}(\bar{x}-\mu_0)^2}\,d\bar{x} = \alpha \quad \text{and}$$

for the value of λ_2,

$$P(\bar{x} < \lambda_2 | H_0) = \frac{\sqrt{n}}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\lambda_2} e^{-\frac{n}{2\sigma^2}(\bar{x}-\mu_0)^2}\,d\bar{x} = \alpha$$

Substituting $\frac{\sqrt{n}(\bar{x} - \mu_0)}{\sigma} = Z$, the integral relation for λ_1 is

$$\frac{1}{\sqrt{2\pi}}\int_{\sqrt{n}(\lambda_1-\mu_0)/\sigma}^{\infty} e^{-\frac{1}{2}z^2}\,dz = \alpha$$

and for λ_2,

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\sqrt{n}(\lambda_2-\mu_0)/\sigma} e^{-\frac{1}{2}z^2}\,dz = \alpha$$

Suppose λ_α is a value which is obtained from the table of area under standard normal curve such that the integral,

$$\frac{1}{\sqrt{2\pi}}\int_{\lambda_\alpha}^{\infty} e^{-\frac{1}{2}z^2}\,dz = \alpha$$

Therefore, by equivalence,

$$\lambda_\alpha = \frac{\sqrt{n}\,(\lambda_1 - \mu_0)}{\sigma}$$

or $$\lambda_1 = \mu_0 + \lambda_\alpha \frac{\sigma}{\sqrt{n}}$$

Similarly, $\lambda_2 = \mu_0 - \lambda_\alpha \dfrac{\sigma}{\sqrt{n}}$

For the power of the test,

$$P(x \,\varepsilon\, \omega | H_1) = P(\bar{x} > \lambda_1 | H_1) = 1 - \beta$$

$$1 - \beta = \frac{\sqrt{n}}{\sigma\sqrt{2\pi}} \int_{\lambda_1}^{\infty} e^{-\frac{n}{2\sigma^2}(\bar{x}-\mu_1)^2} d\bar{x}$$

Now substituting $\dfrac{\sqrt{n}|\bar{x} - \mu_1|}{\sigma} = z,$

$$1 - \beta = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{n}(\lambda_1 - \mu_1)/\sigma}^{\infty} e^{-\frac{1}{2}z^2} dz$$

Putting the value of λ_1, obtained for BCR,

$$1 - \beta = \frac{1}{\sqrt{2\pi}} \int_{\sqrt{n}\left(\mu_0 + \frac{\sigma}{\sqrt{n}}\lambda_\alpha - \mu_1\right)/\sigma}^{\infty} e^{-\frac{1}{2}z^2} dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{\lambda_\alpha - \frac{\sqrt{n}}{\sigma}(\mu_1 - \mu_0)}^{\infty} e^{-\frac{1}{2}z^2} dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{z_\alpha}^{\infty} e^{-\frac{1}{2}z^2} dz$$

where $z_\alpha = \lambda_\alpha - \dfrac{\sqrt{n}}{\sigma}(\mu_1 - \mu_0)$

$$= 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z_\alpha} e^{-\frac{1}{2}z^2} dz$$

$$= 1 - \phi\ (z_\alpha)$$

where the value of $\phi\ (z_\alpha)$ can be obtained from the table for the area under the standard normal curve. In a practical problem either of the two situations will occur, *i.e.*, either $\mu_0 < \mu_1$ or $\mu_0 > \mu_1$.

Q. 23 Given the nine sample values 4.5, 6.5, 3.8, 4.2, 7.7, 8.5, 9.4, 5.3, 3.9 from a normal distribution with mean μ and variance 4. Find the best critical region for testing H_0: $\mu = 4$ vs. H_1: $\mu = 5$ of size 0.05. Also calculate the power of the test.

Ans. It is given that,

$$n = 9,\ \mu_0 = 4,\ \sigma = 2 \text{ and } \alpha = 0.05.$$

From the table we have to search a value of λ_α such that the area on the right tail of the standard normal curve is 0.05. Therefore, $\lambda_\alpha = 1.64$. Using the same notations as in Q. No. 22,

$$\lambda_1 = \mu_0 + \frac{\sigma}{\sqrt{n}}\lambda_\alpha$$

$$= 4 + \frac{2}{3} \times 1.64$$

$$= 5.09$$

Hence the best critical region is 5.09 to ∞.

For the power of the test,

$$z_\alpha = 1.645 - \frac{3}{2}(5 - 4)$$

$$= 1.645 - 1.5$$

$$= 0.145$$

The power of the test,

$$1 - \beta = 1 - \phi(0.145)$$

$$= 1 - 0.05765$$

$$= 0.94235$$

Q. 24 When a test is called a minimax test?

Ans. A test T of $H_0: \theta = \theta_0$ vs. $H_1: \theta = \theta_1$ is said to be a minimax test if for any other T^*, the following inequality holds.

Max $[R(T, \theta_0),\ R(T, \theta_1)] \leq$ Max $[R(T^*, \theta_0),\ R(T^*, \theta_1)]$ where $R(T, \theta)$ is the risk in using the test T when θ is the true value of the parameter. So is $R(T^*, \theta)$.

Q. 25 Define an unbiased test.

Ans. A test T of the null hypothesis $H_0: \theta \,\varepsilon\, \Theta_0$ vs. $H_1: \theta \,\varepsilon\, \Theta_1$ is said to be an unbiased test if the probability of rejecting H_0 when it is false is at least as much as the probability of rejecting H_0 when it is true, *i.e.*,

$$\sup_{\theta \varepsilon \Theta_0} P_T(\theta) \leq \inf_{\theta \varepsilon \Theta_1} P_T(\theta)$$

where $\Theta_0 \cup \Theta_1 = \Theta$ and $\Theta_0 \cap \Theta_1 = \phi$.

Here we would like to introduce a term, *unbiased critical region*. A critical region which is such that the power of a test based on it is never less than its size is called unbiased critical region and the test corresponding to it is called unbiased test. In terms of probability, the conditions for testing H_0 against H_1 for an unbiased test of size α are:

$$P\{x \,\varepsilon\, \omega | H_0\} \le \alpha$$

$$P\{x \,\varepsilon\, \omega | H_1\} \ge \alpha$$

A test is said to be *uniformly unbiased or completely unbiased* if the power of the test is always greater than its size α.

Q. 26 Define uniformly most powerful unbiased test.

Ans. If among the class of unbiased tests, there exists a test, which is uniformly most powerful, is known as *uniformly most powerful unbiased test* (UMPUT). Neyman-Pearson called a critical region corresponding to a UMPU test, the *critical region of type A_1*. The advantage of UMPU test is that even if UMP test do not exist, UMPU test may exist.

Q. 27 What is meant by an admissible test procedure?

Ans. A test procedure δ is said to be admissible if there exists no other test procedure δ' whose risk for testing a hypothesis about a parameter θ is less than the risk of δ for testing the same hypothesis about θ. Notationally test procedure δ is admissible if,

$$R(\theta, \delta) \le R(\theta, \delta') \text{ for all } \theta$$

$$R(\theta, \delta) < R(\theta, \delta') \text{ for some } \theta$$

Q. 28 Give the names of the methods of testing of hypotheses.

Ans. The names of different methods of testing of hypotheses are:

(i) Likelihood ratio test
(ii) Student's t-test
(iii) Normal deviate test or Z-test
(iv) Chi-square test
(v) F-test
(vi) Bayes test
(vii) Sequential probability ratio test (SPRT)

Q. 29 Discuss the general approach of likelihood ratio test.

Ans. Let $X_1, X_2, \ldots, X_n$ be a random sample from a density $f(x;\theta)$ where $\theta = (\theta_1, \theta_2, \ldots, \theta_m)$. Suppose one is interested in testing the hypothesis H_0: $\theta = \theta_0$ where $\theta_0 \,\varepsilon\, \theta$. In likelihood ratio test we consider the likelihood functions under H_0 and under the entire parametric space. The ratio,

$$\lambda(x) = \frac{L(x|\theta_0)}{L(x|\theta)}$$

is called the *likelihood ratio*. The value of λ (x) lies in the interval (0, 1).

The critical region for the test statistic is λ $(x) \le k$, where k is determined from the distribution g (λ) of λ to provide a test of size α, *i.e.*,

$$\int_0^k g_{H_0}(\lambda)\, d\lambda = \alpha$$

Q. 30 Given a sample $x_1, x_2, \ldots, x_n$ from a normal population having mean μ and variance σ^2, test H_0 : $\mu = \mu_0$ by the method of likelihood ratio test.

Ans. The likelihood functions under H_0 and under θ are to be written first.

We know that the maximum likelihood estimates of μ and σ^2 are,

$$\hat{\mu} = \bar{x}$$

and $$\hat{\sigma}^2 = \frac{1}{n}\sum(x_i - \bar{x})^2 = s^2$$

Likelihood function,

$$L(x|\hat{\mu}, \hat{\sigma}^2) = \left(\frac{1}{2\pi s^2}\right)^{n/2} e^{-\frac{1}{2}\Sigma\left(\frac{x_i - \bar{x}}{s}\right)^2}$$

for $i = 1, 2, \ldots, n$

$$= \left(\frac{1}{2\pi s^2}\right)^{n/2} e^{-n/2}$$

Under H_0, the estimate

$$\hat{\sigma}^2 = \frac{1}{n}\sum(x_i - \mu_0)^2$$

$$= \frac{1}{n}\sum(x_i - \bar{x} + \bar{x} - \mu_0)^2$$

$$= s^2 + (\bar{x} - \mu_0)^2$$

Hence under H_0, the likelihood function

$$L(x|\mu_0, \hat{\sigma}^2) = \left[2\pi\left\{s^2 + (\bar{x} - \mu_0)^2\right\}\right]^{-n/2} e^{-n/2}$$

The likelihood ratio,

$$\lambda = \frac{L(x|\mu_0, \hat{\sigma}^2)}{L(x|\hat{\mu}, \hat{\sigma}^2)}$$

$$= \frac{\left[2\pi\left\{s^2 + (\bar{x} - \mu_0)^2\right\}\right]^{-n/2} e^{-n/2}}{(2\pi s^2)^{-n/2} e^{-n/2}}$$

$$\lambda = \left[\frac{1}{1 + \frac{(\bar{x} - \mu_0)^2}{s^2}}\right]^{n/2}$$

or $$\lambda^{2/n} = \frac{1}{1 + \frac{t^2}{n-1}}$$

where t is student's-t with $(n - 1)$ d.f.

Therefore, λ is a monotone decreasing function of t^2. In this situation we can use distribution of t^2 instead of λ. Rejecting 100 α per cent largest values of t^2 amounts to rejecting 100 α per cent smallest values of λ. Hence, we can use equal tail t-test. It is a UMPU test for H_0.

Q. 31 Given a random sample $x_1, x_2, \ldots, x_n$ from a normal population with p.d.f $f(x; \mu, \sigma^2)$, test $H_0 : \sigma = \sigma_0$ vs $H_1 : \sigma \neq \sigma_0$ applying likelihood ratio test.

Ans. The likelihood ratio

$$\lambda = \frac{L(x|\hat{\mu}, \sigma_0^2)}{L(x|\hat{\mu}, \sigma^2)}$$

$$= \frac{\left(\sigma_0\sqrt{2\pi}\right)^{-n} e^{-\frac{1}{2\sigma_0^2}\Sigma(x_i - \bar{x})^2}}{\left(\sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n}} \cdot \sqrt{2\pi}\right)^{-n} e^{-n/2}}$$

$$= n^{-\frac{n}{2}}\left[\frac{\Sigma(x_i - \bar{x})^2}{\sigma_0^2}\right]^{n/2} e^{-\frac{1}{2}\left\{\frac{\Sigma(x_i - \bar{x})^2}{\sigma_0^2} - n\right\}}$$

Suppose $$u = \frac{\Sigma(x_i - \bar{x})^2}{\sigma_0^2}$$

$$\lambda = n^{-n/2} u^{n/2} e^{-u/2} e^{n/2}$$

$$= c u^{n/2} e^{-u/2}$$

where c is a constant

The critical region for the test statistic can be found by the relation,

$$u^{n/2} e^{-u/2} \leq k$$

Let the two roots of the above be C_1 and C_2. Then the critical regions are determined by the relations,

$$u < C_1 \text{ and } u \geq C_2$$

under H_0, the variable u is distributed as χ^2 with $(n - 1)$ d.f. and the distribution does not depend on the unspecified parameters. We can decide about H_0 by comparing the value of λ with the critical value of χ^2 for α level of significance and $(n - 1)$ d.f. If $\lambda \geq \chi^2_{\alpha, n-1}$, reject H_0, otherwise accept H_0.

Q. 32 How can you apply likelihood ratio test in case of large samples?

Ans. Suppose we want to test $H_0 : \theta = \theta_0$ where $\theta_0 \varepsilon \Theta_0$, some interval on the real line of the parametric space Θ.

Let $x_1, x_2, \ldots, x_n$ be a large sample of size n ($n > 30$) and $\hat{\theta}$ be the ML estimator of θ_0. Then,

$$\lambda = \frac{L(x|\theta_0)}{L(x|\hat{\theta})}$$

If the first and second derivatives w.r.t. θ in Θ of the distribution of x exist, under H_0, $-2 \log \lambda$ is approximately distributed as χ^2 with $(n-1)$ d.f. The test may be performed in the usual manner.

Q. 33 Describe Student's *t*-test.

Ans. Student's *t* distribution was given and first used by W.S. Gosset in 1908. The statistician W.S. Gosset is better known by his pseudonym, *student*.

Suppose we want to test $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$ on the basis of a small random sample $x_1, x_2, \ldots, x_n$ of size n $(n < 30)$ from a normal population. $N(\mu, \sigma^2)$ with σ^2 unknown for predecided level α.

Obviously it is a two tailed test.

For testing H_0, student's t statistic is,

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s}, -\infty \leq t \leq \infty$$

where $s = \sqrt{\frac{\Sigma_i (x_i - \bar{x})^2}{(n-1)}}$

for $i = 1, 2, \ldots, n$.

Definition. Student's t is the deviation of estimated mean from its population mean expressed in terms of standard error. The test criteria is; reject H_0 if $t \geq t_{\alpha/2, n-1}$ or $-t \leq -t_{\alpha/2, n-1}$, otherwise accept H_0.

Further, if one wants to test $H_0: \mu = \mu_0$ vs. H_1, $\mu > \mu_0$ or $\mu < \mu_0$, the test statistic remains the same. The only difference in the test procedure is that the critical value of t is only on one side.

In case of $H_1: \mu > \mu_0$, the test criteria is that reject H_0 if $t \geq t_{\alpha, n-1}$.

Q. 34 What is a normal deviate test?

Ans. A test of $H_0: \mu = \mu_0$ of a normal population $N(\mu, \sigma^2)$ against an alternative hypothesis H_1 can be tested by normal deviate test provided the population variance σ^2 is known or the sample drawn is large say, 30 or more. When the sample size is large enough, it is supposed that the sample variance is almost equal to population variance. The statistic Z for testing $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$ is

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}, \qquad -\infty \leq Z \leq \infty$$

where n is the sample size on which $\bar{x}$ is based. In case σ is not known and $n \geq 30$, σ is replaced by its estimated value s.

The variable $Z \sim N(0, 1)$. For the two tailed test, reject H_0 if $Z \geq Z_{\alpha/2}$ or $Z \leq -Z_{\alpha/2}$. But for a one-sided test when $H_1: \mu > \mu_0$, reject H_0 if $Z \geq Z_\alpha$ and when $H_1: \mu < \mu_0$, reject H_0 if $Z \leq -Z_\alpha$.

Q. 35 How do you test the equality of two means of normal populations $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ when σ_1^2 and σ_2^2 are known?

Ans. To test $H_0: \mu_1 = \mu_2$ vs. H_1: or $\mu_1 \neq \mu_2$ or $\mu_1 > \mu_2$ or $\mu_1 < \mu_2$, the test statistics is

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} - \infty \leq z \leq \infty$$

where n_1 and n_2 are the sample sizes on which $\bar{x}_1$ and $\bar{x}_2$ are biased.

The decision criteria remain same as in case of one mean.

Q. 36 What are the assumptions about *t*-test?

Ans. There are five assumptions about *t*-test as given below:

(i) The random variable X follows normal distribution or the sample is drawn from a normal population.

(ii) All observations in the sample are independent.

(iii) The sample size is small, *i.e.*, less than 30 as an usual practice. Also the sample should not contain less than five observations.

(iv) The hypothetical value μ_0 of μ is a correct value of population mean.

(v) The sample values are correctly measured and recorded.

Q. 37 What are the properties of t-test?

Ans.

(i) Student's t-test is a robust test. By a robust test we mean a test which is not vitiated much even if all the assumptions made about the test do not fully hold good.

(ii) Student's t-test for testing H_0: $\mu = \mu_0$ vs, H_1: $\mu = \mu_1$ for an arbitrary σ from a normal population provides an uniformly most powerful unbiased test.

Q. 38 A manufacturer of dry cells claimed that the life of their cells is 24.0 hours. A sample of 10 cells had mean life of 22.5 hours with a standard deviation of 3.0 hours. On the basis of available information, test whether the claim of the manufacturer is correct. [Given : $t_{0.05,\,9} = 2.2623$]

Ans. Assuming that the lifetime of cells is distributed normally. Here we test

$$H_0{:}\mu = 24 \text{ vs. } H_1{:}\mu \neq 24$$

by student's t-test.

$$t = \frac{(24.0 - 22.5)\times 3}{3.0}$$

$$= 1.5$$

The calculated value of t is less than the tabulated value of t since $t_{0.05,\,9} = 2.262$. Hence we accept H_0, *i.e.*, the claim of the manufacturer is correct.

Q. 39 Give the procedures for testing the equality of two normal populations mean when the independent samples are drawn from the populations $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ in cases, (i) $\sigma_1^2 = \sigma_2^2 = \sigma^2$ (ii) $\sigma_1^2 \neq \sigma_2^2$.

Ans. Let the two independent small samples of sizes n_1 and n_2 be $x_{11}, x_{12}, \ldots, x_{1n_1}$ and $x_{21}, x_{22}, \ldots, x_{2n_2}$ respectively. The test statistic for testing H_0 : $\mu_1 = \mu_2$ vs. H_1 : $\mu_1 \neq \mu_2$.

Case (i). $\sigma_1^2 = \sigma_2^2 = \sigma^2$ (unknown) is

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where $\bar{x}_1 = \frac{1}{n_1}\sum_{i=1}^{n_1} x_{1i}, \bar{x}_2 = \frac{1}{n_2}\sum_{j=1}^{n_2} x_{2j}$

and $$s_p^2 = \frac{\sum_i (x_{1i} - \bar{x}_1)^2 + \sum_j (x_{2j} - \bar{x}_2)^2}{(n_1 + n_2 - 2)}$$

$$= \frac{\left\{\sum_i x_{1i}^2 - \frac{(\Sigma_i x_{1i})^2}{n_1}\right\} + \left\{\sum_j x_{2j}^2 - \frac{(\Sigma_j x_{2j})^2}{n_2}\right\}}{(n_1 + n_2 - 2)}$$

$$= \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{(n_1 + n_2 - 2)}$$

where s_1^2 and s_2^2 are the variances of the first and second samples respectively.

Statistic t has $(n_1 + n_2 - 2)$ d.f.

The test criteria are, reject H_0 if $t \geq t_\alpha, (n_1 + n_2 - 2)$, otherwise accept H_0, where α is the predecided level of significance.

Case (ii). When $\sigma_1^2 \neq \sigma_2^2$. t-test for H_0 can be performed by two approaches. One is W.G. Cochran's approximate t-test and the other Berhans–Fisher test.

Following usual notations, statistics under Cochran's approximate test is,

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

For making a decision about H_0 *vis-a-vis* H_1, calculated value of t is compared with t^* where,

$$t^* = \frac{\frac{s_1^2}{n_1} \times t_{\alpha, n_1 - 1} + \frac{s_2^2}{n_2} \times t_{\alpha, n_2 - 1}}{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Reject H_0 if $t \geq t^*$, otherwise accept H_0.

[*Note:* If $n_1 = n_2 = n$, $t^* = t_\alpha$, $n - 1$]

Case (iii). When $\sigma_1^2 \neq \sigma_2^2$, it is not possible to get the standard error of $(\bar{x}_1 - \bar{x}_2)$ and hence student's t-test cannot be used directly. Also $\{(n_1-1)s_1^2 + (n_2-1)s_2^2\}$ is not distributed as $\chi^2\,\sigma^2$. In the want of any clear-cut test statistics, W.V. Behrens and later R.A. Fisher suggested an alternate procedure known as Behrens-Fisher test.

Let $\bar{x}_1, \bar{x}_2$, be the sample means and s_1^2, s_2^2, the sample variances based on samples of sizes n_1 and n_2 from two normal populations $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively where $\sigma_1^2 \neq \sigma_2^2$.

Let us define,

$$t_1 = \frac{\bar{x}_1 - \mu_1}{s_1/\sqrt{n_1}} \text{ and } t_2 = \frac{\bar{x}_2 - \mu_2}{s_2/\sqrt{n_2}}$$

which are distributed with $(n_1 - 1)$ and $(n_2 - 1)$ d.f., respectively. Suppose

$$d = \frac{(\bar{x}_1 - \mu_1) - (\bar{x}_2 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

Substitute $\dfrac{s_1}{\sqrt{n_1}} = r\sin\theta$ and $\dfrac{s_2}{\sqrt{n_2}} = r\cos\theta$.

Thus, $r^2 = \left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)$ and $\tan\theta = \dfrac{s_1}{s_2}\sqrt{\dfrac{n_2}{n_1}}$

using the above relations,

$$(\bar{x}_1 - \mu_1) = \frac{s_1}{\sqrt{n_1}} t_1 \text{ and } \bar{x}_2 - \mu_2 = \frac{s_2}{\sqrt{n_2}} t_2$$

$$d = \frac{\dfrac{s_1 t_1}{\sqrt{n_1}} - \dfrac{s_2 t_2}{\sqrt{n_2}}}{r}$$

$$= \frac{r t_1 \sin\theta - r t_2 \cos\theta}{r}$$

$$= t_1 \sin\theta - t_2 \cos\theta$$

Thus, it is evident that d depends on μ_1 and μ_2 and is independent of σ_1^2 and σ_2^2. If $f_n(t)$ be the p.d.f. of t, the probabilities that the two samples will have values t_1 and t_2, respectively, lying within the specified limits is,

$$\int\int f_{(n_1-1)}(t_1) f_{n_2-1}(t_2)\, dt_1\, dt_2$$

The integral being over (t_1, t_2) plane.

Let us assume that for some d_0, the region is given by

$$t_1 \sin\theta - t_2 \cos\theta > d_0$$

and determine d_0 such that the double integral is equal to α, the predecided level of significance. For a given α, d_0 will be a function of n_1, n_2 and θ only. Now substituting back, we obtain

$$\frac{\bar{x}_1 - \mu_1}{r} - \frac{\bar{x}_2 - \mu_2}{r} > d_0$$

$$(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2) > r d_0$$

$$-(\mu_1 - \mu_2) > -(\bar{x}_1 - \bar{x}_2) + r d_0$$

$$(\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) - r d_0$$

with the help of the above inequality, we can decide about the null hypothesis. If $(\bar{x}_1 - x_2) > r d_0$, then $\mu_1 \leq \mu_2$ will have a confidence coefficient less than α. Similarly if $(\bar{x}_1 - \bar{x}_2) < r d_0$, then $\mu_1 \geq \mu_2$ will have a confidence coefficient less that α. The value of d_0 are available in Fisher and Yates tables for different values of θ. For a sufficiently small value of α, we reject H_0 if $|\bar{x}_1 - \bar{x}_2| > r d_0$.

The main objection of Behrens-Fisher test is that $\dfrac{s_1}{s_2}$ has been taken as a constant which is not a correct supposition. But here it is worth pointing out that the power of Behrens-Fisher test is more than the power of Cochran's test.

Q. 40 Given the statistics of two samples drawn from two normal populations $N\left(\mu_1, \sigma_1^2\right)$ and $N\left(\mu_2, \sigma_2^2\right)$ as,

$$n_1 = 6, \bar{x}_1 = 25 \text{ and } s_1^2 = 36.0$$

$$n_2 = 8, \bar{x}_2 = 20 \text{ and } s_2^2 = 25.0$$

Test $H_0: \mu_1 = \mu_2$ vs. $H_1 = \mu_1 \neq \mu_2$ under two situations (i) $\sigma_1^2 = \sigma_2^2$, (ii) $\sigma_1^2 \neq \sigma_2^2$

[Given: $t_{0.05,12} = 2.18; t_{0.05,13} = 2.16$
$t_{0.05,5} = 2.571; t_{0.05,7} = 2.365$]

Situation (i): $\sigma_1^2 = \sigma_2^2$.

The statistic

$$t = \frac{25-20}{5.44\sqrt{\frac{1}{6}+\frac{1}{8}}}$$

$$= 1.70$$

where

$$s_p^2 = \frac{5 \times 36 + 7 \times 25}{6+8-2}$$

$$= 29.58$$

$$s_p = 5.44$$

Since $t < 2.18$, the tabulated value of t for 12 d.f. and $\alpha = 0.05$, we accept H_0, *i.e.*, there is no significant difference between the two population means.

Situation (ii): $\sigma_1^2 \neq \sigma_2^2$

First we apply Cochran's test. The test statistic,

$$t = \frac{25-20}{\sqrt{\frac{36}{6}+\frac{25}{8}}}$$

$$= 1.66$$

Value of

$$t^* = \frac{6 \times 2.571 + 3.125 \times 2.365}{6 + 3.125}$$

$$= 2.50$$

Since $t = 1.66 < 2.50$. H_0 is accepted.

Now we apply Behrens-Fisher test.

From the given statistics,

$$r^2 = \frac{36}{6} + \frac{25}{8} = 9.125$$

$$\therefore \quad r = 3.02$$

$$\tan\theta = \frac{6}{5} \times \sqrt{\frac{8}{6}} = 1.38$$

$$\theta = \tan^{-1} 1.38 = 60.08°$$

Also $\bar{x}_1 - \bar{x}_2 = 5$.

For $\alpha = 0.05$, d.f. $n_1 - 1 = 5$, $n_2 - 1 = 7$ and $\theta = 60°$, the value of d_0 is 2.50. Thus,

$$rd_0 = 3.02 \times 2.50$$

$$= 7.55$$

$$\bar{x}_1 - \bar{x}_2 = 5 < 7.55$$

Hence we conclude that there is no significant difference between μ_1 and μ_2.

Q. 41 When do you use paired t-test and how to apply it?

Ans. Let us consider two variables X_1 and X_2 which are normally distributed and there exists a correlation ρ between them. In practice, the observation are taken on the same item or the items which are paired before taking the observations. Let the difference between the paired values $X_1 - X_2 = d$. $(X_1 - X_2)$ is distributed with mean μ_d and variance $(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2)$.

In paired t-test, we test $H_0: \mu_D = 0$ vs. $H_1: \mu_D \neq 0$.

The test based on n paired values $\left(x_{11}, x_{21}\right)$, $\left(x_{12}, x_{22}\right) \ldots, \left(x_{1n}, x_{2n}\right)$ is

$$t = \frac{\bar{d}}{s_d/\sqrt{n}}$$

t has $(n-1)$ d.f.

where, $d_i = x_{1i} - x_{2i}$ for $i = 1, 2, \ldots, n$.

$$\bar{d} = \frac{1}{n}\sum_{i=1}^{n} d_i \text{ and } s_d^2 = \frac{1}{n-1}\sum_i \left(d_i - \bar{d}\right)^2$$

Test criteria is, reject H_0 if $|t| \ge t_{\alpha,n-1}$, otherwise accept H_0.

Q. 42 The measurements of diameter of 8 cylindrical rods by vernier calliper and micrometer were as follows:

Vernier reading (X_1):	2.265	2.267	2.264	2.267
	2.268	2.263	2.264	2.258
Micrometer reading:	2.270	2.268	2.269	2.273
(X_2):	2.270	2.270	2.268	2.268

Test whether the difference between measurements of diameter by vernier and micrometer is significant or not. [Given: $t_{7,05} = 2.365$]

Ans. The difference $(X_2 - X_1)$.

d: 0.005, 0.001, 0.005, 0.006, 0.002, 0.007, 0.004, 0.010

$$\bar{d} = \frac{0.040}{8} = 0.005,$$

$$s_d^2 = \frac{1}{7}\left\{0.000256 - \frac{(0.04)^2}{8}\right\}$$

$$= 0.000008$$

$$\therefore \qquad s_d = 0.0028$$

The statistic,

$$t = \frac{0.005 \times \sqrt{8}}{0.0028}$$

$$= 5.05$$

The calculated value of $t = 5.05$ is greater than the tabulated value 2.365 of t for 7 d.f. and $\alpha = 0.05$. Hence, H_0 is rejected. It confirms that there is a significant difference between the readings made by vernier and micrometer at $\alpha = 0.05$. .

Q. 43 How can one test the hypothesis for a hypothetical value of proportion in a class of binomial population on the basis of a sample.

Ans. The interest lies in testing the hypothesis that the proportion P of items in a class C_1 out of two classes in the population is p_0 or not on the basis of a sample of size n. Here we test,

$$H_0: P = p_0 \text{ vs. } H_1: P \ne p_0.$$

H_0 can be tested by Z-test. The condition is that the sample size should be large. For a binomial distribution, to test $H_0: p = \frac{1}{2}, n$ should be 10 or more. Let n_1 be the number of items in the class C_1 and n_2 in class C_2 out of n. Thus $n_1 + n_2 = n$. Also the proportion of items in C_1 is $\hat{p} = \frac{n_1}{n}$ and in C_2, $\hat{q} = \frac{n_2}{n}$.

The test statistic

$$Z = \frac{\hat{p} - p_0}{\sqrt{p_0\, q_0/n}}$$

where, $q_0 = 1 - p_0$ and $Z \sim N(0, 1)$.

The test criteria is reject H_0 if $Z \ge Z_{\alpha/2}$ or $-Z \le -Z_{\alpha/2}$ where α is the prefixed level of significance. In case, H_1: $P > p_0$, reject H_0 if $Z \ge Z_\alpha$. In case, H_1: $P < p_0$, reject H_0 if $Z < -Z_\alpha$.

Alternative approach: Instead of working with the proportions, it is possible to deal with the number of items (outcomes) belonging to a class associated with p. Taking the number of items x in class C_1, the test statistic for testing H_0 is,

$$Z = \frac{(x + 0.5) - np_0}{\sqrt{n\, p_0\, q_0}} \text{ if } x < np_0$$

use $(x - 0.5)$ if $x > np_0$.

The test criteria remain the same as above. Addition of 0.5 to x makes the approximation of binomial to normal distribution more exact.

Q. 44 In crossing of 30 white dams with brown sires, it is expected that half of the calves will be white and the rest will be brown. But the experiment showed that out of 30 calves, 20 were brown and 10 white. Can it be believed that there will be in general 50 per cent brown calves?

Ans. Here we have to test $H_0: p = \frac{1}{2}$ vs. $H_1: p \ne \frac{1}{2}$.

It means $p_0 = \frac{1}{2}, q_0 = \frac{1}{2}$. Also $\hat{p} = \frac{20}{30} = \frac{2}{3}$.

To test H_0, the statistic

$$Z = \frac{2/3 - 1/2}{\sqrt{\left(\frac{1}{2}\times\frac{1}{2}\right)\Big/30}}$$

$$= 1.82$$

The critical value of Z for 5 per cent level of significance is 1.96 which is greater than 1.82. Hence we accept H_0. This reveals that the expectation of 50 per cent brown calves is correct.

Alternatively H_0 is tested by taking the number of calves.

As per the question $x = 20$, $np_0 = 30\times\frac{1}{2} = 15$, $x > np_0$. The statistic

$$Z = \frac{(20-0.5)-15}{\sqrt{30\times\frac{1}{2}\times\frac{1}{2}}}$$

$$= 1.64$$

Again Z = 1.64 < 1.96. We accept H_0.

Q. 45 How will you test the equality of two proportion of items in the same class on the basis of two independent samples drawn from two populations?

Ans. Here we are to test $H_0 : P_1 = P_2$ vs. $H_1 : P_1 \neq P_2$ on the basis of two samples of sizes n_1 and n_2 from populations A and B respectively. Let there be two classes C_1 and C_2. The number of items belonging to different classes are displayed in the table below:

	Classes		
	C_1	C_2	
Sample from A	O_1	O_2	n_1
Sample from B	O'_1	O'_2	n_2
	$O_1 + O'_1$	$O_2 + O'_2$	$n_1 + n_2$

The observed proportions in class C_1 for items from populations A and B are, $p_1 = \frac{O_1}{n_1}, q_1 = \frac{O_2}{n_1}$ and

$p_2 = \frac{O_1'}{n_2}, q_2 = \frac{O_2'}{n_2}$ respectively.

H_0 against H_1 can be tested by the statistic,

$$Z = \frac{|p_1 - p_2|}{\sqrt{\hat{p}\,\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

Under $H_0, P_1 = P_2, \hat{p} = \frac{O_1 + O_1'}{n_1 + n_2}, \hat{q} = 1 - \hat{p}$. The term,

$\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$ is the standard error of $(p_1 - p_2)$.

In case H_0 is doubted to be true, the standard error of $(p_1 - p_2)$ should be restandardised using

$$s_{p_1 - p_2} = \sqrt{\frac{p_1\,q_1}{n_1} + \frac{p_2\,q_2}{n_2}}$$

The decision about H_0 can be taken in the usual manner.

Q. 46 A survey was conducted in two types of colonies, one consisting of normal residents and the other of slum dwellings to know whether the prevalence of Tuberculosis (TB) in slum area is more than the normal area.

A sample of 600 persons in normal area had 28 persons suffering from TB where in a sample of 400 persons in slum area, 96 persons had TB. Do the data support our assertion?

Ans. Here we have to test $H_0 : P_1 = P_2$ vs. H_1: $P_1 < P_2$. The given information can be tabulated as follows:

	Classes		
	TB	No TB	
Normal area	28	572	600
Slum area	96	304	400
	124	876	1000

From the table,

$$p_1 = \frac{28}{600} = 0.047, q_1 = 0.953$$

$$p_2 = \frac{96}{400} = 0.24, q_2 = 0.76$$

Under $H_0, \hat{p} = \frac{124}{1000} = 0.124, \hat{q} = 0.876$

The statistic

$$Z = \frac{|0.047 - 0.24|}{\sqrt{0.124 \times 0.876\left(\frac{1}{600} + \frac{1}{400}\right)}}$$

$$= \frac{0.193}{0.0213}$$

$$= 9.06$$

Here we have to apply the one-tailed test. If 1 per cent level of significance is chosen, the tabulated value of $Z_\alpha = 2.33$ which is less than 9.06. Hence we reject H_0. It means that the prevalence of TB in slum residential area is more than the normal area.

Q. 47 How can the hypothesis that the variance of a normal distribution has a specified value σ_0^2 be tested?

Ans. Here we test $H_0: \sigma^2 = \sigma_0^2$. vs. $H_1: \sigma^2 \neq \sigma_0^2$. Let there be a random sample $x_1, x_2, \ldots, x_n$. H_0 can be tested by χ^2-test subject to the condition that $n <$ 30.

The test statistic

$$\chi^2 = \frac{\Sigma_i (x_i - \bar{x})^2}{\sigma_0^2} \text{ for } i = 1, 2, \ldots, n.$$

$$= \frac{(n-1)s^2}{\sigma_0^2}$$

χ^2 has $(n - 1)$ d.f.

The decision criteria is, reject H_0 if $\chi^2 \geq \chi^2_{\alpha/2,(n-1)}$ or $\chi^2 < \chi_{(1-\alpha/2),(n-1)}$, otherwise accept H_0. Where α is the prefixed level of significance.

In case of one-tailed test, *i.e.*, testing $H_0: \sigma^2 = \sigma_0^2$ vs. $H_1: \sigma^2 < \sigma_0^2$, reject H_0 if $\chi^2 \geq \chi^2_{(1-\alpha),(n-1)}$, otherwise accept H_0.

Again for testing $H_0: \sigma^2 = \sigma_0^2$ vs. $H_1: \sigma^2 > \sigma_0^2$, reject H_0 if $\chi^2 \geq \chi^2_{\alpha,n-1}$, otherwise accept H_0.

Q. 48 A manufacturer claims that any of his lot of items cannot have a variance more than 1 cm^2. A sample of 25 items has a variance of 1.2 cm^2. Test whether the claim of the manufacturer is correct.

Ans. Here we test $H_0: \sigma^2 = 1$ vs. $H_1: \sigma^2 \neq 1$.

The test statistic

$$\chi^2 = \frac{24 \times 1.2}{1}$$

$$= 28.8$$

For $\alpha = 0.05$, $\chi^2_{0.05,24} = 36.41$ which is greater than $\chi^2 = 28.8$. Hence accept H_0. This confirms the claim of the manufacturer.

Q. 49 Give an appropriate test for testing the hypothesis $H_0: \sigma = \sigma_0$ vs. $H_1: \sigma \neq \sigma_0$ of a normal population with unknown variance σ^2.

Ans. The appropriate test for testing H_0 against H_1 is Z-test. The statistic

$$Z = \frac{s - \sigma}{s/\sqrt{2n}}$$

for large n.

$s/\sqrt{2n}$ is the standard error of the sample standard deviation s.

Decision about H_0 can be taken by the set procedure.

Q. 50 Give the test statistic for testing the equality of two standard deviations of the normal populations.

Ans. Let n_1 and n_2 be the sizes of two large samples from two normal populations $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$. Also σ_1^2 and σ_2^2 are unknown.

The statistic for testing $H_0: \sigma_1 - \sigma_2 = 0$ vs. H_i: $\sigma_1 - \sigma_2 \neq 0$ is,

$$Z = \frac{|s_1 - s_2|}{\sqrt{\frac{s_1^2}{2n_1} + \frac{s_2^2}{2n_2}}}$$

where s_1^2 and s_2^2 are the variance based on large samples. Also $Z \sim N(0, 1)$.

The decision about H_0 is taken in the usual manner.

Q. 51 What do you understand by the test of goodness of fit?

Ans. In test of hypotheses or estimation of parameters it is usually assumed that the random variable follows a particular distribution like Binomial, Poisson, normal distribution, *etc.* But often the need is felt to confirm whether our assumption is true or not. So on the basis of outcomes of a trial or observational data, Chi-square test is performed which measures the discrepancy between the observed frequencies and theoretically determined frequencies from the assumed distribution for the same event. If the discrepancy is not large, it is considered that our ass›mption about the distribution of the variable is correct, otherwise not.

Here we test H_0: the data have come from the assumed distribution.

vs. H_1: H_0 is not true.

If $O_1, O_2, \ldots, O_k$ are the observed frequencies and $E_1, E_2, \ldots, E_K$ are the corresponding expected frequencies, the test statistic

$$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i} \text{ for } i = 1, 2, \ldots, k$$

$$= \sum_i \frac{O_i^2}{E_i} - n$$

χ^2 has $(n - 1)$ d.f.

where $\sum_i O_i = n$.

The expected frequencies are obtained by finding the probability of an event to which an O_i belongs and then by multiplying the probability by n, we get the corresponding E_i. Thus $E_i = np_i$. Here it should be kept in mind that $\Sigma O_i = \Sigma E_i = n$. The decision criteria is, reject H_0 if $\chi^2 \geq \chi^2_{\alpha, k-1}$ otherwise accept H_0.

Q. 52 Is it necessary to consult χ^2-table to decide about the null hypothesis.

Ans. No, because if the calculated value of chi-square is less than or equal to the degrees of freedom for chi-square, there is no need to consult χ^2-table. In this situation, null hypothesis be accepted as such. The reason being that under the null hypothesis, the expected value of calculated chi-square is the degree of freedom of χ^2 and only much higher value of χ^2 will lead to rejection of the null hypothesis.

Q. 53 How to apply the Chi-square test in case of multinomial distribution?

Ans. A common problem is to test the hypothesis

$$H_0: p_i = p_{i0} \text{ for } i = 1, 2, \ldots, k$$

where k is the number of classes. Such a problem is often confronted in genetics and sociometery. A geneticist expects a definite ratio of offspring types out of a set of crossing. A sociologist expects certain ratio of occurrences of certain events. In all these problems, H_0 can be tested by Chi-square test.

Suppose O_i represents the observed frequency in the i^{th} class and E_i is the corresponding expected (theoretical or hypothetical) frequency where $E_i = np_{i0}$ under H_0. The value of p_{i0} can be calculated by the ratio of occurrence of frequencies in k-classes given as $r_1 : r_2 : \ldots : r_k$. If $\Sigma_i r_i = r$, $p_{i0} = \frac{r_i}{r}$ and $E_i = \frac{n}{r} r_i$.

The test statistic

$$\chi^2 = \sum_i \frac{(O_i - E_i)^2}{E_i} \qquad \text{for } i = 1, 2, \ldots, k$$

χ^2 is distributed with $(k - 1)$ d.f.

The decision about H_0 is taken as usual.

Q. 54 In a breeding experiment, the ratio of offsprings in four classes was expected to be 1 : 3 : 3 : 9. The experiment yielded the data as follows:

Classes:	AA	Aa	aA	aa
No. of offsprings:	8	29	37	102

Test whether the given data is in agreement with the hypothetical ratio.

Ans. Here we have $n = 176$, $r_1 = 1$, $r = 3$, $r_3 = 3$, $r_4 = 9$ and $r = 16$.

The expected frequencies can be worked out as,

$$E_1 = \frac{176}{16} \times 1 = 11, E_2 = \frac{176}{16} \times 3 = 33,$$

$$E_3 = 33, E_4 = \frac{176}{16} \times 9 = 99.$$

Thus

$$\chi^2 = \frac{(8-11)^2}{11} + \frac{(29-33)^2}{33} + \frac{(37-33)^2}{33} + \frac{(102-99)^2}{99}$$

$$= 0.818 + 0.485 + 0.485 + 0.091$$

$$= 1.879$$

For $\alpha = 0.05$, $\chi^2_{0.05,3} = 7.81$ which is greater than the calculated value of $\chi^2 = 1.879$. Hence, we accept H_0. It suggests that the data support the expected ratio in the four classes.

Q. 55 What is a contingency table?

Ans. A contingency table is a two-way table in which the columns are classified according to one criterion or attribute and rows are classified according to the other criterion or attribute. Each cell contains that number of items 0_{ij} possessing the qualities of the i^{th} row and j^{th} column, where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, p$. In such a case, the contingency table is said to be of order $(m \times p)$. Each row or column total is known as *marginal total*. Also the sum of row totals $\sum_i R_i$ is equal to the sum of column totals $\sum_j C_j$, *i.e.*, $\sum_i R_i = \sum_j C_j = n$, (the sample size). Contingency table helps to test the independence of two attributes.

Q. 56 What is an incomplete contingency table?

Ans. If in a contingency table, one or more cells are having zero count, then it is an incomplete contingency table.

Q. 57 What is the difference between structural and random zeros.

Ans. If there is a zero in *a* cell and this cell count has expected value zero *i.e.* the probability of having any observation in the cell is zero, then such a zero is a structural zero.

On the other hand, if the expected value *i.e.* the probability of having a count in the cell is greater than zero, then the zero present in a cell is a random zero.

Q. 58 Give a contingency table showing structural and random zeros.

Ans. Consider the following (3 × 4) contingency table.

	Factor B			
Factor A	B_1	B_2	B_3	B_4
A_1	0	0	0	0
A_2	5	10	0	11
A_3	9	12	8	10

The expected value of any cell count of first row is zero. Hence such zero counts in the first row are structural zeros. Whereas the zero count in the cell (2, 3) is a random zero.

Technically, the structural and random zeros are to be treated in the same way.

Q. 59 Is a contingency table only a two-dimensional table?

Ans. No. A contingency table may be a 3, 4 or more dimensional table. In general, it is a multidimensional table. As an example, we provide a 3 × 2 × 3 hypothetical contingency table below:

Social		*Employment type*		
group	*Sex*	*Technical*	*Skilled*	*unskilled*
High	Male	28	15	9
	Female	5	13	11
Middle	Male	34	23	45
	Female	25	18	51
Low	Male	7	12	106
	Female	1	2	61

Q. 60 What is an explanatory variable in a contingency table?

Ans. Many times the aim of analysis in a contingency table is to explain the variation in one of the variables through the variation of the other variable. Thus, those variables which are used in a contingency table to explain the variation in the response variable, are called explanatory variables. For example, the variation in alcohol consumption, the response variable, may be explained through the variation in explanatory variables like, social groups, marital status, age groups etc.

Q. 61 How will you test the independence of two attributes?

Ans. Suppose n randomly selected items are arranged in a contingency table given below:

	Columns						
Rows	B_1	B_2	...	B_j	...	B_p	Total
A_1	O_{11}	O_{12}	...	O_{1j}	...	O_{1p}	R_1
A_2	O_{21}	O_{22}	...	O_{2j}	...	O_{2p}	R_2
$\vdots$	$\vdots$			$\vdots$		$\vdots$	$\vdots$
A_i	O_{i1}	O_{i2}	...	O_{ij}	...	O_{ip}	R_i
$\vdots$	$\vdots$						$\vdots$
A_m	O_{m1}	O_{m2}	...	O_{mj}	...	O_{mp}	R_m
Total	C_1	C_2	...	C_j	...	C_p	n

Corresponding to each O_{ij}, find the expected frequency E_{ij} under H_0 by the formula,

$$E_{ij} = \frac{R_i \times C_j}{n}$$

Here we test H_0: Attributes A and B are independent.

vs. H_1: Attributes A and B are associated.

H_0 can be tested by the statistic

$$\chi^2 = \sum_{i=1}^{m} \sum_{j=1}^{p} \frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}}$$

χ^2 has $(m - 1)(p - 1)$ d.f.

The test criteria is, reject H_0 if $\chi^2 \geq \chi^2_{\alpha.(m-1)(p-1)}$ otherwise accept H_0.

Q. 62 Give the formula for calculating statistic χ^2 in case of contingency table of order (2 × 2).

Ans. Suppose the contingency table of order (2 × 2) is as displayed below:

	Columns		
Rows	B_1	B_2	Total
A_1	a	b	$a + b$
A_2	c	d	$c + d$
Total	$a + c$	$b + d$	$a + b + c + d = n$

The direct formula for calculating the value of statistic $-\chi^2$ is,

$$\chi^2 = \frac{n(ad - bc)^2}{(a+b)(c+d)(a+c)(b+d)}$$

χ^2 has 1 d.f.

Q. 63 If there are only two classes C_1 and C_2 having frequencies a and b which are hypothesised to occur in the ratio $r_1 : r_2$, how can you calculate the value of statistic-χ^2?

Ans. Suppose $\frac{r_1}{r_2} = r$. The direct formula for χ^2-statistic to test the validity of the hypothetical ratio $r_1 : r_2$ is,

$$\chi^2 = \frac{(a - rb)^2}{r(a+b)}$$

χ^2 has 1 d.f.

Q. 64 What is Yates' correction and how to apply it?

Ans. When the expected frequency in a cell of (2 × 2) contingency table is small say, less than 5, the continuity of χ^2 distribution is disturbed. To remove this deficiency, Yates in 1934 suggested that 0.5 be added to the small frequency for which the expected frequency is less than 5 and other cell frequencies be adjusted by adding and subtracting 0.5 in such a way that the marginal totals remain the same. After adjustment, the value of χ^2 is calculated in the usual manner.

An alternative formula is suggested which automatically takes care of the adjustments in cell frequencies. The formula is,

$$\chi^2 = \frac{n\left(|ad-bc|-\frac{n}{2}\right)^2}{(a+b)(c+d)(a+c)(b+d)}$$

Here it is worth pointing out that the value of χ^2 after adjustment in cell frequencies by a quantity 0.5 and the value of χ^2 obtained by the alternative formula are equal. Yates' correction is applicable only in case of contingency table of order (2 × 2).

Q. 65 Following table provides data with regard to stature of the fathers and their first sons at the age of 25 years.

		Stature of fathers		
		Tall	Short	Total
Stature	Tall	8	2	10
of sons	Short	7	6	13
	Total	15	8	23

Test that the stature of sons is independent of the stature of the fathers.

Ans. The expected frequency corresponding to the cell frequency 2 is less than 5. Hence, Yates' correction will have to be applied.

H_0: The stature of son is independent of the stature of father.

vs. H_1 : H_0 is not true.

The new configuration of the table under Yates' correction is,

7.5	2.5	10
7.5	5.5	13
15	8	23

The statistic,

$$\chi^2 = \frac{23(7.5\times 5.5-7.5\times 2.5)^2}{10\times 13\times 15\times 8}$$
$$= 0.746$$

The value of the statistic by the alternative formula is,

$$\chi^2 = \frac{23(|48-14|-11.5)^2}{10\times 13\times 15\times 8}$$
$$= 0.746$$

For $\alpha = 0.05$, $\chi^2_{0.05,1} = 3.841$. Since the calculated value of χ^2 is less than tabulated value, we accept H_0. It reveals that the stature of sons is independent of the stature of their fathers.

Q. 66 What is Dandekar's correction for continuity of χ^2 in a contingency table of order (2 × 2).

Ans. V.M. Dandekar's evolved a different correction method to maintain the continuity of χ^2 when the expected frequency in one or more cells are less than 5 in a contingency table of order (2 × 2). From a (2 × 2) table calculate χ_0^2, χ_{-1}^2, χ_{+1}^2 for the observed configuration and those obtained by decreasing and increasing the smallest frequency by unity respectively. It should be kept in mind that under any change in configuration, the marginal totals remain same. Using the three Chi-square values, calculate the value of corrected χ^2 by the formula,

$$\chi_c^2 = \chi_0^2 - \frac{\chi_0^2-\chi_{-1}^2}{\chi_{+1}^2-\chi_{-1}^2}\left(\chi_{+1}^2-\chi_0^2\right)$$

Dandekar's correction, in general is slightly better than Yates' correction, although the Yates' correction is easier to apply.

Q. 67 For the problem given in Q. No. 65, test H_0 by applying Dandekar's Correction.

Ans. Here we calculate directly χ_c^2.

$$\chi_0^2 = \frac{23(48-14)^2}{10\times 13\times 15\times 8}$$
$$= 1.704$$

$$\chi_{-1}^2 = \frac{23(63-6)^2}{10\times 13\times 15\times 8}$$
$$= 4.790$$

$$\chi_{+1}^2 = \frac{23(35-24)^2}{10\times 13\times 15\times 8}$$
$$= 0.178$$

$$\chi_c^2 = 1.704 - \frac{1.704 - 4.790}{0.178 - 4.790}(0.178 - 1.704)$$

$$= 2.725$$

Calculated value of $\chi^2 = 2.725$ is less than the tabulated value of $\chi^2_{0.05} = 3.841$. Hence H_0 is accepted. It means that the stature of sons is independent of the strature of their fathers.

Q. 68 Describe Fisher's exact test for testing the hypothesis of independence of attributes in a (2 × 2) contingency table.

Ans. Fisher's exact test consists of calculating the probability of the configurations of the contingency table of order (2 × 2) such as

		B		
		B_1	B_2	
A	A_1	*a*	*b*	$a + b$
	A_2	*c*	*d*	$c + d$
		$a + c$	$b + d$	$a + b + c + d = n$

from smallest observed frequency up to zero by reducing each time the smallest cell frequency by unity keeping the marginal totals fixed. The probability for a configuration say, with *d* being smallest can be obtained by the formula,

$$p_d = \frac{\binom{a+b}{a}\binom{c+d}{c}}{\binom{n}{a+c}}$$

$$= \frac{(a+b)!(c+d)!(a+c)!(b+d)!}{n!a!b!c!d!}$$

Calculate $P = p_d + p_{d-1} + \dots p_1 + p_0$, which is the probability corresponding to one tail of the distribution. Reject H_0 if $P \le \alpha$, otherwise accept H_0. Fisher's exact test can be used to test (i) independence of attributes (ii) the equality of proportions in two classes.

The chief defect of Fisher's exact test is that it involves too much of computation, particularly when the smallest frequency is more than 2.

Q. 69 Write the purpose of coefficient of contingency, and how is it measured?

Ans. When the hypothesis of independence of attributes in a contingency table is rejected by performing χ^2-test, it ensures the association between two attributes. One is not satisfied merely with association but is also interested in the strength of association. Hence, there is a desideratum for a measure of the strength of association. For this a measure known as *coefficient of contingency* was evolved by Karl Pearson in 1904. The coefficient of contingency

$$C = \sqrt{\frac{\chi^2}{n + \chi^2}}$$

where *n* is the sample size. The value of *C* lies between 0 and 1 and never attains 1. $C = 0$ indicates complete dissociation. A value near to 1 shows a high degree of association. Actually *C* is calculated only when H_0 is rejected.

Q. 70 If Chi-square test indicates dependence, how can one detect which cells in a ($p \times q$) contingency table contribute most to departure from independence?

Ans. For detecting whether the contribution in departure from independence of an individual cell (*i, j*) is significant or not, we commonly find the square root of the individual terms in the chi-square statistic, *i.e.*, calculate $\left(O_{ij} - E_{ij}\right)/\sqrt{E_{ij}}$ as an indicator. But such an indicator has no yardstick for claiming which of the indicator is large or small. As a matter of fact, it follows no standard distribution. Hence $\left(O_{ij} - E_{ij}\right)$ is standardised such that it follows a standard normal distribution - so we use the expression,

$$\frac{O_{ij} - E_{ij}}{\sqrt{E_{ij}\left(1 - p_{i.}\right)\left(1 - p_{.j}\right)}}$$

These standardised values are called residuals or more exactly standardised residuals.

If the value of an individual residual is greater than the 97.5 per cent value of standard normal distribution, *i.e.*, 1.96, the contribution of an individual cell is significant otherwise not.

[*Note:* Where p_{ij} is the proportion of frequency in (i, j) cell.]

Q. 71 What treatment is metted to Chi-square having large degrees of freedom?

Ans. If the d.f. for χ^2 are 100 or more, then Chi-square can be approximated to a standard normal deviate using the relation,

$$Z = \sqrt{2\chi^2} - \sqrt{2k-1}$$

where k is the d.f. for χ^2 and $Z \sim N(0, 1)$. Here H_0 is tested by one-sided normal deviate test.

Q. 72 Give a suitable test for testing the homogeneity (equality) of several population variances.

Ans. Suppose one is interested to test the homogeneity of k population variances, *i.e.*, to test

$$H_0: \sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2$$

vs. H_1: at least any two of them are not equal.

Out of various test procedures, the most accepted test procedure is one given by M.S. Bartlett in 1937.

Suppose $s_1^2, s_2^2, \ldots, s_k^2$ are the estimated variances of $\sigma_1^2, \sigma_2^2, \ldots, \sigma_k^2$ based on $(n_1 - 1), (n_2 - 1), \ldots, (n_k - 1)$ d.f. respectively. Let s_p^2 is the pooled estimated variance where,

$$s_p^2 = \frac{\sum_{i=1}^{k}(n_i - 1)s_i^2}{\sum_{i=1}^{k}(n_i - 1)}$$

To test H_0, we make use of χ^2-test where

$$\chi^2 = \log_e 10[\log_{10} s_p^2 \sum_{i=1}^{k}(n_i - 1) - \sum_{i=1}^{k}(n_i - 1) \times \log_{10} s_i^2]$$

$$= 2.3026\,[\log_{10} s_p^2 \sum_{i=1}^{k}(n_i - 1) - \sum_{i=1}^{k}(n_i - 1) \times \log_{10} s_i^2]$$

χ^2 has $(k - 1)$ d.f.

It has been proved that χ^2 has an upward bias. Hence a correction factor C is calculated by the formula,

$$C = 1 + \frac{1}{3(k-1)}\left[\sum_{i=1}^{k}\frac{1}{n_i - 1} - \frac{1}{\sum_{i=1}^{k}(n_i - 1)}\right]$$

Corrected value of Chi-square,

$$\chi_c^2 = \chi^2 / C$$

Using χ_c^2 distributed with $(k - 1)$ d.f., the decision about H_0 is taken in the usual manner.

Q. 73 How do you test the equality of variances of two normal populations?

Ans. Here we want to test

$$H_0: \sigma_1^2 = \sigma_2^2 \text{ vs. } H_1: \sigma_1^2 \neq \sigma_2^2$$

Let s_1^2 and s_2^2 be the estimates of σ_1^2 and σ_2^2 based on sample sizes n_1 and n_2 respectively. H_0 can be tested by the F-test where

$$F = \frac{s_1^2}{s_2^2}$$

F has d.f. $(n_1 - 1), (n_2 - 1)$ where s_1^2 is taken to be larger variance.

If $F \geq F_{(1-\alpha/2).\{(n_1-1).(n_2-1)\}}$, reject H_0, otherwise accept H_0 .

In case H_1 is one-sided, *i.e.*, $H_1: \sigma_1^2 > \sigma_2^2$, reject H_0 if

$$F \geq F_{(1-\alpha).\{(n_1-1).(n_2-1)\}}$$

Again if $H_1: \sigma_1^2 < \sigma_2^2$, reject H_0 if

$$F \leq F_{(\alpha).\{(n_1-1).(n_2-1)\}}$$

The value of statistic F is never negative.

Q. 74 How will you test the equality of several normal population means?

Ans. Let there be k normal populations $N\left(\mu_1, \sigma_1^2\right)$.

$N\left(\mu_2,\sigma_2^2\right),\ldots,N\left(\mu_k,\sigma_k^2\right)$. We have to test

$$H_0: \mu_1=\mu_2=\ldots=\mu_k$$

vs. H_1: at least two means are not equal.

Under the assumption of homogeneity of variances, *i.e.*, $\sigma_1^2=\sigma_2^2=\ldots=\sigma_k^2$. F-test under H_0 can be performed in the following manner. The statistic

$$F=\frac{\text{Between sample variances}}{\text{within sample variances}}$$

Suppose

$Y_{ij}=$ j^{th} observation in the i^{th} sample for $i = 1, 2, \ldots, k$ and $j = 1, 2, \ldots, n_i$.

$n_i=$ i^{th} sample size

$$\overline{Y}_i=\sum_{j=1}^{n_i} Y_{ij}/n_i = i^{\text{th}} \text{ sample mean.}$$

$$\overline{Y}=\sum_{i=1}^{k} n_i\overline{Y}_i/\Sigma n_i = \text{ over all mean.}$$

In notational form,

$$F=\frac{\sum_{i=1}^{k} n_i\left(\overline{Y}_i-\overline{Y}\right)^2/(k-1)}{\sum_{i=1}^{k}\sum_{j=1}^{n_i}\left(Y_{ij}-\overline{Y}_i\right)^2\Big/\sum_{i=1}^{k}(n_i-1)}$$

Reject H_0, if $F \ge F_{(1-\alpha),(\nu_1,\nu_2)}$ where $\nu_1 = k-1$ and $\nu_2=\sum_{i=1}^{k}(n_i-1)$.

Q. 75 What do you understand by analysis of variance?

Ans. Analysis of variance is a device to split the total variance of an experiment or trial into component variances responsible for contributing towards total variance. The gap between total variance and sum of component variances is attributed to experimental (random) error and so is true for degrees of freedom. Analysis of variance utilises F-test. Each component variance is tested against error variance and conclusion is drawn in the same way as we do in F-test for equality of two variances or several means. Analysis of variance is abbreviated as ANOVA.

Q. 76 Give skeleton analysis of variance table.

Ans. The skeleton ANOVA table is as given below:

Source of variation	*Degrees of freedom*	*Sum of squares*	*Mean sum of squares*	*Variance ratio*
Due to	*d.f.*	*S.S.*	*M.S.*	*F-value*
A				
B				
⋮				
C				
Error				
Total				

In practice column captions are used only in abbreviated form.

Q. 77 Define Bayes' test.

Ans. A test T_g of $H_0:\theta=\theta_0$ vs. $H_1:\theta=\theta_1$ is said to be a Bayes' test with regard to a prior distribution $g=P(\theta=\theta_1)$ if for any other test T, the following inequality holds.

$$(1-g)R\left(T_g,\theta_0\right)+gR\left(T_g,\theta_1\right)\le(1-g)R(T,\theta_0)+gR(T,\theta_1)$$

where $R\left(T_g,\theta_0\right)$ and $R\left(T_g,\theta_1\right)$ are the risks in choosing the test T_g under H_0 and H_1 respectively. In the same way $R(T,\theta_0)$ and $R(T,\theta_1)$ are the risks in choosing the test T under H_0 and H_1, respectively.

Q. 78 Give in brief the idea of sequential probability ratio test (SPRT).

Ans. In case of tests based on fixed sample size, it is generally not possible to determine the optimum sample size so that no extra observations are recorded except those necessitated to reach a decision. Moreover, sometimes the sample is too small to arrive at a right decision. To overcome this problem, Professor A. Wald innovated in 1947 sequential probability ratio test. In this test procedure, a decision about H_0 is taken after each successive observation. Hence in SPRT, sample size n is a random variable. There are three types of decisions with which an investigator comes across namely, reject H_0, accept H_0 or continue sampling. The process is terminated as soon as a decision either to reject or to accept H_0 is taken. The test procedure is as follows:

Suppose one wants to test $H_0:\theta=\theta_0$ vs. $H_1:\theta=\theta_1$.

Let $X_1, X_2, \ldots, X_m$ be the sample selected up to the m^{th} stage where x's are i.i.d. with p.d.f. $f(x\,;\theta)$. The test criteria is that for any two numbers A_0 and A_1 such that $A_0 < A_1$, the process of taking observations is continued if the ratio of joint p.d.f. under H_1 to the joint p.d.f. under H_0 satisfies the following inequality.

$$A_0 < \prod_{i=1}^{m} \frac{f(x_i,\theta_1)}{f(x_i,\theta_0)} < A_1$$

H_0 is accepted if $\prod_{i=1}^{m} f(x_i,\theta_1)/f(x_i,\theta_0) \le A_0$ or rejected if $\prod_{i=1}^{m} f(x_i,\theta_1)/f(x_i,\theta_0) \ge A_1$ and the process of taking observations is terminated. If α and β are the probabilities of rejecting H_0 when it is true and accepting H_0 when H_1 is true, the decision is taken in the following manner.

$$\text{Accept } H_0 \text{ if } \prod_{i=1}^{m} \frac{f(x_i;\theta_1)}{f(x_i;\theta_0)} \le \frac{\beta}{1-\alpha}$$

$$\text{Reject } H_0 \text{ if } \prod_{i=1}^{m} \frac{f(x_i;\theta_1)}{f(x_i;\theta_0)} \ge \frac{1-\beta}{\alpha}$$

and continue sampling if

$$\frac{\beta}{1-\alpha} < \prod_{i=1}^{m} \frac{f(x_i;\theta_1)}{f(x_i;\theta_0)} < \frac{1-\beta}{\alpha}.$$

It has been proved that the process of taking observations terminates with probability one and on an average there is saving in sample number.

Note: Testing of hypotheses with regard to correlation coefficient(s) and regression coefficient(s) are given in the respective chapters. Analysis of variance is given in the chapter on experimental designs and otherwise also.

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s) in the blanks:

1. There can be __________ procedure(s) to test the same hypothesis.
2. The theory of testing parametric hypothesis was first originated by __________ in __________.
3. Besides Neyman, the other pioneer worker in the field of testing of hypothesis was __________.
4. A hypothesis is an __________ about the parameter of a population.
5. The hypothesis which is under test for possible rejection is called ___________ hypothesis.
6. A hypothesis contrary to null hypothesis is known as __________ hypothesis.
7. The idea of alternative hypothesis was propounded by __________.
8. The hypothesis $H_0:\theta=\theta_0$ vs. $H_1:\theta=\theta_1$ is a __________ hypothesis against __________ alternative hypothesis.
9. The hypothesis $H_0:\theta>\theta_0$ is a __________ hypothesis.
10. There can be only __________ types of errors in taking a decision about H_0.
11. Type __________ error is more severe than type __________ error.
12. __________ kind of error is minimised for

a prefixed level of __________ kind of error.

13. Probability of first kind of error is called the __________ of the test.

14. Probability of type I error is called ________.

15. Rejecting H_0 when H_0 is true is __________ error.

16. Accepting H_0 when H_0 is false, is a ________ error.

17. Whether a test is one-sided or two-sided depends on __________ hypothesis.

18. The test statistic __________ in case of one-tailed and two-tailed test.

19. If β is the probability of type II error, the power of the test is __________.

20. With usual notation, the function $\beta(\theta)$ is known as __________ function.

21. Level of significance lies between __________ and __________.

22. The choice of level of significance is akin to the __________ of the test.

23. Critical region is also known as __________.

24. A statistical test is a __________ to decide about H_0.

25. A randomised test does not involve any __________.

26. A test in which the decision about a hypothesis H is based on a statistic is called __________ test.

27. A null hypothesis is rejected if the value of a test statistic lies in the __________.

28. There cannot be two __________ regions in any testing problem.

29. One-sided alternative hypothesis leads to one region of __________.

30. The size of a test is equal to the area of the __________.

31. The number of independent values in a set of values is known as __________.

32. Degrees of freedom in a test takes care of the __________.

33. A test which minimises the two types of error is termed as __________ test.

34. Does an__________ _ test exist in real life?

35. A test T is said to be __________ if there is no other test of which the power is more than that of T.

36. A test T of size α is said to be a __________ test if there is no other test of the same size whose power is more than that of T.

37. Neyman-Pearson lemma provides a ________ test of simple null hypothesis against a simple alternative.

38. Neyman-Pearson lemma helps to determine the size of __________ and __________ errors for the given range of the variable X.

39. With the help of Neyman-Pearson lemma, one can determine the __________ and __________ of a test of testing H_0 against H_1.

40. If the maximum risk of a test T under simple H_0 and H_1 is not more than the maximum risk of any other test T^* under simple H_0 and H_1, the test T is said to be __________ test.

41. If a test procedure T is such that the probability of rejecting H_0 when it is false is at least as much as the probability of rejecting H_0 when it is true, T is said to be an __________ test.

42. Among the class of unbiased tests, a test which is uniformly most powerful is called a __________ test.

43. A critical region corresponding to a UMPU test is the critical region of __________.

44. The name critical region of type A_1 was given by __________.

45. If a critical region is such that the power of the test based on it is never less than its size, the critical region is known as ________ region.

46. If a test procedure δ for testing a hypothesis about a parameter θ is such that its risk is never greater than any other test δ' for testing the same hypothesis about θ and is

less for some θ, then δ is said to be an __________ test.

47. The term critical function is related to __________ test.

48. A function $\psi_T(x_1, x_2, \ldots, x_n)$, where T is a test of H_0, is equal to the probability of rejecting H_0 when $x_1, x_2, \ldots, x_n$ is observed, $\psi_T(x_1, x_2, \ldots, x_n)$ is called a _________.

49. The critical value of a test statistic is a _________ point between the region of acceptance and the region of rejection.

50. A normal deviate test does not utilise _________.

51. A test based on the outcome of tossing of a coin is a _________ test.

52. If the likelihood ratio is λ, the variable -2 log λ is approximately distributed as _________.

53. Student's *t*-test is applicable in case of _________ samples.

54. Statistic-*t* is defined as deviation of sample mean from population mean expressed in terms of _________.

55. The formula for student's-*t* statistic is _________.

56. Student's *t* is valid in case the variable *x* follows _________ distribution.

57. *t* has $(n - 1)$ d.f. when all the *n* observations in the sample are _________.

58. The minimum number of observations required for *t*-test in a sample is _________.

59. Student's *t*-test is a _________ test.

60. *t*-test for a simple null hypothesis against a simple alternative for an arbitrary population standard deviation provides a _________ test.

61. Student's-*t* test based on two samples of sizes n_1 and n_2 for testing the equality of two normal population means when the populations have same variances has degrees of freedom equal to _________.

62. Sample variances for testing equality of two normal population means with unequal variances cannot be _________.

63 Approximate *t*-test for testing the equality of two normal populations means with unequal variances was given by _________.

64. Beside Cochran's approximate test for testing H_0: $\mu_1 = \mu_2$ of two populations $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ when $\sigma_1^2, \neq \sigma_2^2$, the other test is _________ test.

65. Behrens and Fisher gave *t*-test for unequal population variances _________.

66. Fisher gave the test __________ than Behrens for testing $H_0: \mu_1 = \mu_2$ when $\sigma_1^2, \neq \sigma_2^2$.

67. Behrens was _________ to give *t*-test for testing $H_0: \mu_1 = \mu_2$ when $\sigma_1^2, \neq \sigma_2^2$.

68. Behrens-Fisher test is also not free from _________.

69. The power of Behrens-Fisher test is _______ than Cochran's approximate *t*-test.

70. Paired *t*-test is applicable only when the observations are _________.

71. The two variables say, *X* and *Y* in paired *t*-test are _________.

72. In paired *t*-test, one test the significance of population _________.

73. When the population standard deviation is known, the hypothesis about population mean is tested by _________.

74. If the sample drawn from a population is large, then the hypothesis about μ can be tested by _________.

75. The validity of a hypothetical value of proportion in a class of dichotomous population can be tested by _________.

76. By adding ± 0.5 to the number of items belonging to a class in *Z*-test approximates the binomial distribution to __________ distribution.

77. To test whether the data follow an assumed distribution or not is known as a test of _________.

78. The value of χ^2-statistic depends on the difference between ________ and ________ frequencies.

79. The value of Chi-square varies from ________ to ________.

80. The value of coefficient of contingency lies between ________ and ________.

81. The value of coefficient of contingency never attains the value ________.

82. In a multinomial population with k classes, the degrees of freedom for χ^2 is ________.

83. In working with a contingency table of order 4×5, the d.f. for χ^2 is ________.

84. The d.f. for χ^2 while dealing with a contingency table of order (2×2) is ________.

85. Direct formula for χ^2 in case of contingency table of order (2×2) with usual notations is ________.

86. Direct formula for testing the validity of a hypothetical ratio r for frequencies in two classes is ________.

87. If in a (2×2) contingency table, the expected frequency in a cell is less than 5, ________ of χ^2 distribution is disturbed.

88. For small expected frequency in a 2×2 frequency table, ________ suggested a correction for continuity.

89. Dandekar's correction is applicable when the expected frequency in a cell of 2×2 contingency table is ________.

90. In general Dandekar's correction is ________ than Yates' correction.

91. Dandekar's correction is not so frequently used because its ________ is bit complicated.

92. The value of statistic χ^2 under Yates' correction can directly be obtained by the formula ________.

93. Fisher's test of independence of attributes is exact in the sense that it calculates the ________ of the configurations and no distribution is approximated.

94. Fisher exact test involves too much ________.

95. Coefficient of contingency measures the degree of ________.

96. When the value of χ^2 is zero, the value of coefficient of contingency is ________.

97. Coefficient of contingency is calculated only when H_0 is ________ by χ^2-test.

98. The value of coefficient of contingency near unity ensures ________ of association between attributes.

99. Chi-square can be approximated to standard normal distribution only if the d.f. for chi-square are ________ or more.

100. Homogeneity of several population variances can be tested by ________ test.

101. Bartlett's test utilises ________ test.

102. There is an________ bias in Bartlett's chi-square statistic.

103. Equality of two population variances can be tested by ________.

104. The ratio s_1^2/s_2^2 of two sample variances follows ________ under the hypothesis $\sigma_1^2 = \sigma_2^2$.

105. Equality of several normal populations mean can be tested by ________.

106. Analysis of variance utilises ________.

107. Abbreviated form of analysis of variance is ________.

108. Mean sum of square due to a component factor is nothing but its ________.

109. Error sum of square is obtained by ________.

110. In sequential probability ratio test, sample size is a ________.

111. In SPRT, decision is taken after each ________ observation.

112. Sequential probability ratio test was invented by ________ in ________.

113. Sequential probability ratio test is based on the functions of ________.

114. Bayes' test with regard to a given prior distribution utilises ________.

115. If the difference in sample means of two groups A and B of size 12 each is 5.42 units and the standard deviation of mean difference is 2 units, to test the significance of mean difference, you would prefer to apply ________.

116. Suppose there are two groups A and B of same size. A received special treatment and B is kept as control. Then, the null hypothesis for comparing two groups is ________.

117. On the basis of a sample of farm workers, the hypothesis that 50 per cent of workers are farm owners can be tested by ________.

118. Whether five brands of fertilizers have equal mean effect can be tested with the help of ________.

119. The average life of electric bulbs is 1600 hours with S.D. = 112 hours. It is desired that 95 per cent bulbs should not fall short of the average life by more than 1 per cent, the required sample size is ________.

[Given: $z_{0.05} = -1.64$]

120. A box contains 10 switches out of which θ are non-defective. Test H_0: $\theta = 5$ vs. H_1: $\theta = 4$. Also H_0 will be rejected if the two switches drawn at random with replacement are defective. The size of the test is ________.

121. For the problem given in Q. No. 120, the size of type II error is ________.

122. The daily consumption of diesel of a transporter is considered to be exponentially distributed. The hypothesis that the average consumption H_0: $\theta = 1000$ litres/day is to be tested against H_1: $\theta = 2000$. If the consumption on a randomly selected day is 1500 litres or more, H_0 is rejected. The size of the test is ________.

123. For the problem in Q. No. 122, the power of the test is ________.

124. If p is the probability of turning up a head in tossing of a coin. To test H_0: $p = 0.5$ vs. H_1: $p = 0.6$, H_0 is rejected if there are 7 or more heads in 10 trials. The size of the test is ________.

125. For the problem in Q. No. 124, the power of the test is ________.

126. For the problem is Q. No. 124, the size of type II error is ________.

127. A large population of heights of person is distributed with mean 66 inches and S.D. = 10 inches. A sample of 400 persons had mean height = 62 inches. The data ________ the hypothesis H_0: $\mu = 66$ inches.

128. Two cattle feeds A and B are compared on two groups of cows of the same breed of size 32 and 45. The average increases in milk yield are 10 litres and 15 litres per week with variances 4 and 5 litres^2 respectively. The hypothesis of the same average effect of feeds A and B is ________ at $\alpha = 0.01$.

129. If in a box of 20 transistors, θ are defective. The hypothesis $\theta = 10$ is rejected if any two transistors selected at random with replacement are defective. If H_1: $\theta = 20$, the type II error is ________.

130. The probability that the sample mean should not differ by population mean by more than $\sigma/2$ units is 0.95 where $\sigma/2$ is the population standard deviation. For the validity of the statement, the sample size should be ________.

131. Each particle attracts other particle is not a ________ hypothesis.

132. Fifty persons suffered from tuberculosis (TB) in a village. Out of which 20 died. The hypothesis that 50 per cent TB patients die is ________ at 5 per cent level of significance.

133. For the given configuration,

	B_1	B_2
A_1	20	10
A_2	15	15

the value of statistic χ^2 is ________.

134. Out of two groups consisting of 200 and 300 urban and rural males, 100 and 150 favoured family planning. From the data it is concluded at 95 per cent level that proportion of persons in urban and rural populations favouring family planning is ________.

135. Suppose the yields of 12 and 8 plots of two varieties of wheat are assumed to be normally distributed as $N\left(\mu_1, \sigma^2\right)$ and $N\left(\mu_2, \sigma^2\right)$. The average yield from 12 plots is 30 q/ha. with a S.D. = 3.0 q/ha and 8 plots 25 q/ha with a S.D. = 2.0 q/a. After statistical test it is concluded that both the fertilizers are __________ effective at $\alpha = 0.05$.

[Given, $t_{0.05,\,18} = 2.101$]

136. A sample of 30 items has a variance of 8 units. The hypothesis of the true variance of 10 units is __________ at $\alpha = 0.05$.

137. The variances of daily consumption of electricity in two cities in the month of June were 130 units and 170 units. The statistical test of H_0 that the variances in two cities are equal against H_1 that they are not equal reveals that H_0 is __________ at $\alpha = 0.05$.

138. A sample of 30 items was distributed in a contingency table of order (2 × 2). The value of χ^2 on computation was 7.84. The value of coefficient of contingency is __________.

139. The interest lies to test H_0: $p = 1/2$ vs. H_1: $p = 2/3$ in tossing of a coin. H_0 is rejected if there are more than 4 heads out of 6 tossings. The size of type I error is __________.

140. For the problem given in Q. No. 139, the size of type II error is __________.

141. For the problem given in Q. No. 139, the power of the test is __________.

142. It is desired that the population standard deviation should not differ from sample standard deviation by more than 4 per cent. To be 95.44 per cent confident, the minimum sample size should be __________.

143. A large population has mean 5 inches and S.D. = 2 inches. A large sample of 400 persons from this population has mean = 3.5 inches. The data __________ the value of population mean.

144. An investigator aspires that the sample standard deviation should not differ from population standard deviation by more than 5 per cent. To be 99.73 per cent confident, the sample size should be __________.

145. Critical value of t __________ as the sample size increases.

146. A test of hypothesis provides a __________ about the probable truth.

147. Research hypotheses are the __________ which a researcher postulates.

148. The critical value of chi-square for any level of significance is __________ its degrees of freedom.

149. A contingency table can be __________.

150. A contingency table having a few empty cells will be called __________ contingency table.

151. A zero in a cell of a contingency table having its expected value zero is called __________ zero.

152. If a cell of a contingency table having zero count has its expected frequency greater than zero, then such a zero count is termed as __________.

153. If the calculated value of chi-square statistic is less than its degrees of freedom, then we can straightaway _________ the null hypothesis.

154. Significant contribution of an individual cell towards dependency of attributes can be detected with the help of __________.

155. Paired t-test is applicable in case of ________ samples only.

156. A value of a standardised residual greater than 1.96 will indicate the __________ contribution of the cell.

157. An insurance company claims that it settles the claims on an average in six days. To test the validity of the claim, H_0: _________ and H_1: _________ are the only suitable hypothesis.

158. A Dean of a college wants to verify whether the grades awarded by a professor follow the bell-shaped curve. The appropriate test for verification is _________ test.

159. The rejection region in F-test lies on _________.

160. A government agency claims that only 20 per cent families are below poverty line. Whereas a citizen's body claims that this percentage is much higher. To ascertain about the claims, you will like to apply _________.

SECTION-C

Multiple Choice Questions

Select the correct alternative out of given ones:

Q. 1 The idea of testing of hypothesis was first set forth by:
(a) R.A. Fisher
(b) J. Neyman
(c) E.L. Lehman
(d) A. Wald

Q. 2 In 1933, the theory of testing of hypotheses was propounded by:
(a) R.A. Fisher
(b) J. Neyman
(c) E.L. Lehman
(d) Karl Pearson

Q. 3 A hypothesis may be classified as:
(a) simple
(b) composite
(c) null
(d) all the above

Q. 4 The hypothesis under test is:
(a) simple hypothesis
(b) alternative hypothesis
(c) null hypothesis
(d) none of the above

Q. 5 Whether a test is one-sided or two-sided depends on:
(a) alternative hypothesis
(b) composite hypothesis
(c) null hypothesis
(d) simple hypothesis

Q. 6 A wrong decision about H_0 leads to:
(a) one kind of error
(b) two kinds of error
(c) three kinds of error
(d) four kinds of error

Q. 7 Power of a test is related to:
(a) type I error
(b) type II error
(c) types I and II errors both
(d) none of the above

Q. 8 If θ is the true parameter and β the type II error, the function $\beta(\theta)$ is known as:
(a) power function
(b) power of the test
(c) operating characteristic function
(d) none of the above

Q. 9 Level of significance is the probability of:
(a) type I error
(b) type II error
(c) not committing error
(d) any of the above

Q. 10 In terms of type II error β and θ, the true parameter, the function $1 - \beta(\theta)$ is called:
(a) power of the test
(b) power function
(c) OC function
(d) none of the above

Q. 11 Out of the two types of error in testing, the more severe error is:
(a) type I error
(b) type II error
(c) both (a) and (b) are equally severe

(d) no error is severe

Q. 12 Area of the critical region depends on:

(a) size of type I error

(b) size of type II error

(c) value of the statistic

(d) number of observations

Q. 13 Critical region of size α which minimised β amongst all critical regions of size α is called:

(a) powerful critical region

(b) minimum critical region

(c) best critical region

(d) worst critical region

Q. 14 A test based on a test statistic is classified as:

(a) randomised test

(b) non-randomised test

(c) sequential test

(d) Bayes test

Q. 15 Size of critical region is known as:

(a) power of the test

(b) size of type II error

(c) critical value of the test statistics

(d) size of the test

Q. 16 Degrees of freedom is related to:

(a) no. of observations in a set

(b) hypothesis under test

(c) no. of independent observations in a set

(d) none of the above

Q. 17 A critical function provides the basis for:

(a) accepting H_0

(b) rejecting H_0

(c) no decision about H_0

(d) all the above

Q. 18 A test which maximises the power of the test for fixed α is known as:

(a) optimum test

(b) randomised test

(c) Bayes test

(d) likelihood ratio test

Q. 19 A test T which is at least as powerful as any other test of the same size, is called:

(a) best test

(b) most powerful test

(c) uniformly most powerful test

(d) none of the above

Q. 20 Neyman-Pearson lemma provides:

(a) an unbiased test

(b) a most powerful test

(c) an admissible test

(d) minimax test

Q. 21 A test T for which maximum risk under H_0 & H_1 is not more than the maximum risk of any other test T^* under H_0 and H_1 is called:

(a) an unbiased test

(b) uniformly most powerful test

(c) an admissible test

(d) minimax test

Q. 22 With usual notations the condition for unbiased test is:

(a) $\underset{\theta \varepsilon \Theta_0}{\text{Sup}}\, P_T(\theta) \le \underset{\theta \varepsilon \Theta_1}{\text{Inf}}\, P_T(\theta)$

(b) $\underset{\theta \varepsilon \Theta_0}{\text{Sup}}\, P_T(\theta) \ge \underset{\theta \varepsilon \Theta_1}{\text{Inf}}\, P_T(\theta)$

(c) $\underset{\theta \varepsilon \Theta_0}{\text{Sup}}\, P_T(\theta) = \underset{\theta \varepsilon \Theta_1}{\text{Inf}}\, P_T(\theta)$

(d) none of the above

Q. 23 A uniformly most powerful test among the class of unbiased test is termed as:

(a) minimax test

(b) minimax unbiased test

(c) uniformly most powerful unbiased test

(d) all the above

Q. 24 A test procedure δ for testing a hypothesis about a parameter θ whose risk is not more than the risk of any test procedures δ' for all θ and is definitely less for some θ is called:

(a) minimax test

(b) admissible test

(c) most powerful test

(d) optimum test

Q. 25 The ratio of the likelihood function under H_0 and under the entire parametric space is called:

(a) probability ratio

(b) sequential probability ratio

(c) likelihood ratio
(d) none of the above

Q. 26 Student's t-test was invented by:
(a) R.A. Fisher
(b) G.W. Snedecor
(c) W.S. Gosset
(d) W.G. Cochran

Q. 27 Student's t-test is applicable in case of:
(a) small samples
(b) for samples of size between 5 and 30
(c) large samples
(d) none of the above

Q. 28. A population is distributed as $N = (\mu, 10.24)$. A sample of 576 items has a mean 4.7. The value of the statistic Z to test H_0: $\mu = 5.2$ is:
(a) 3.75
(b) 28.125
(c) −3.75
(d) none of the above

Q. 29 It was claimed that the average life of dry battery cells is 60 hours. A large sample of 441 cells had mean life 42 hours with a variance of 81 hours^2. Do the data ascertain the claim:
(a) the claim is refuted
(b) the claim is accepted
(c) no decision is possible
(d) none of the above

Q. 30 A sample of 12 specimen taken from a normal population is expected to have a mean 50 mg/cc. The sample has a mean 64 mg/cc with a variance of 25. To test H_0: $\mu = 50$ vs. H_1: $\mu \neq 50$, you will use:
(a) Z-test
(b) χ^2-test
(c) F-test
(d) t-test

Q. 31 For the problem given in Q. No. 30, the test reveals that for $\alpha = 0.05$, H_0 should be:
[Given $t_{0.05, 11} = 2.201$]
(a) rejected
(b) accepted
(c) left undecided
(d) none of the above

Q. 32 Student's t-test is applicable only when:
(a) the variate values are independent
(b) the variable is distributed normally
(c) the sample is not large
(d) all the above

Q. 33 To test H_0: $\mu = \mu_0$ vs. H_1: $\mu > \mu_0$ when the population S.D. is known, the appropriate test is:
(a) t-test
(b) Z-test
(c) chi-test
(d) none of the above

Q. 34 For two populations $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$ with σ^2 unknown, the test statistics for testing H_0: $\mu_1 = \mu_2$ based on small samples with usual notations is:

(a) $$t = \frac{\overline{X} - \overline{X}_2}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

(b) $$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{\sigma^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

(c) $$t = \frac{\overline{X}_1 - \overline{X}_2}{s_p\sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

(d) any of the above

Q. 35 Test of hypothesis H_0: $\mu = 70$ vs. H_1: $\mu > 70$ leads to:
(a) one-sided left-tailed test
(b) one-sided right-tailed test
(c) two-tailed test
(d) none of the above

Q. 36 Testing H_0:$\mu = 1500$ against $\mu < 1500$ leads to:
(a) one-sided lower tailed test
(b) one-sided upper tailed test
(c) two-tailed test
(d) all the above

Q. 37 Testing H_0: $\mu = 100$ vs. H_1: $\mu \neq 100$ leads to:

(a) one-sided upper tailed test
(b) one-sided lower tailed test
(c) two-tailed test
(d) none of the above

Q. 38. If there are two populations $N\left(\mu_1, \sigma_1^2\right)$ and and $N\left(\mu_2, \sigma_2^2\right)$ the two samples from them have means $\overline{X}_1, \overline{X}_2$ and variances s_1^2 and s_2^2 based on n_1 and n_2 observations respectively, the hypothesis $H_0: \mu_1 = \mu_2$ vs. $H_1: \mu_1 \neq \mu_2$ when $\sigma_1^2 \neq \sigma_2^2$ can be tested by:
(a) Cochran's test
(b) Behrens-Fisher test
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 39 Cochran's test testing H_0: $\mu_1 = \mu_2$ for two normal populations with heterogeneous variances is:
(a) an exact test
(b) an approximate test
(c) a random tesι
(d) an unreliable test

Q. 40 Behrens-Fisher test for testing H_0: $\mu_1 = \mu_2$ vs. H_1: $\mu_1 \neq \mu_2$ for two normal populations with unequal variances is:
(a) better than Cochran's test
(b) inferior than Cochran's test
(c) as powerful as Cochran's test
(d) a hoax

Q. 41 Paired t-test is applicable when the observations in the two samples are:
(a) paired
(b) correlated
(c) equal in number
(d) all the above

Q. 42 The mean difference between 9 paired observations is 15.0 and the standard deviation of differences is 5.0. The value of statistic t is:
(a) 27
(b) 9
(c) 3
(d) zero

Q. 43 The degrees of freedom for statistic-t for paired t-test based on n pairs of observations is:
(a) $2(n-1)$
(b) $n-1$
(c) $2n-1$
(d) none of the above

Q. 44 To test an hypothesis about proportions of items in a class, the usual test is:
(a) t-test
(b) F-test
(c) Z-test
(d) none of the above

Q. 45 To test H_0: $P = 0.4$ vs. H_1 $P \neq 0.4$ in binomial population, there are eight persons out of fifteen who favoured a proposal. The value of statistic-Z is:
(a) 5.813
(b) 1.08
(c) 7.32
(d) none of the above

Q. 46 Standard error of the difference of proportions $(p_1 - p_2)$ in two classes under the hypothesis H_0: $P_1 = P_2$ with usual notations is:

(a) $\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

(b) $\sqrt{\hat{p}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

(c) $\hat{p}\hat{q}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$

(d) $\sqrt{\frac{p_1 q_1}{n_1}+\frac{p_2 q_2}{n_2}}$

Q. 47 Formula for the standard error of the difference between proportions $(p_1 - p_2)$ under the hypothesis $H : P_1 \neq P_2$ with usual notation is:

(a) $\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

(b) $\hat{p}\hat{q}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$

(c) $\hat{q}\hat{q}\left(\frac{1}{n_1}+\frac{1}{n_2}\right)$

(d) $\sqrt{\frac{p_1 q_1}{n}+\frac{p_2 q_2}{n_2}}$

Q. 48 The formula in general for testing the hypothesis for proportions H_0: $P_1 = P_2$ vs. H_1: $P_1 \neq P_2$ is:

(a) $Z = \frac{p_1 - p_2}{s_{(p_1 - p_2)}}$

(b) $Z = \frac{p_1 - p_2}{s^2_{(p_1 - p_2)}}$

(c) $Z = \frac{p_1 - p_2}{s_{p_1} - s_{p_2}}$

(d) none of the above

Q. 49 The hypothesis that the population variance has a specified value can be tested by:

(a) F-test
(b) Z-test
(c) χ^2-test
(d) None of the above

Q. 50 The test statistic to be used to test H_0: $\sigma^2 = C$ vs. H_1: $\sigma^2 \neq C$ with usual notations is:

(a) $\chi^2 = \frac{(n-1)s^2}{C^2}$

(b) $\chi^2 = \frac{(n-1)s^2}{C}$

(c) $\chi^2 = \frac{ns^2}{C^2}$

(d) all the above

Q. 51 Statistic-χ^2 to test H_0: $\sigma^2 = \sigma_0^2$ is based on a sample of size n has degrees of freedom equal to:

(a) $n - 1$
(b) n
(c) $(n + 1)$
(d) none of the above

Q. 52 Degrees of freedom for statistic-χ^2 in case of contingency table of order (2×2) is

(a) 3
(b) 4
(c) 2
(d) 1

Q. 53 In a multinomial distribution with 4 classes, the degrees of freedom for χ^2 is:

(a) 3
(b) 4
(c) 2
(d) 1

Q. 54 Test statistic for testing H_0: $\sigma = c$ vs. H_1: $\sigma \neq c$ is:

(a) $z = \frac{s - c}{s/\sqrt{n}}$

(b) $z = \frac{s - c}{s/\sqrt{2n}}$

(c) $z = \frac{s - c}{s\sqrt{2n}}$

(d) none of the above

Q. 55 The hypothesis H_0: $\sigma_1 = \sigma_2$ vs. H_1: $\sigma_1 > \sigma_2$ can be tested by the statistic:

(a) $Z = \frac{|s_1 - s_2|}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}}$

(b) $Z = \frac{|s_1 - s_2|}{\sqrt{\frac{s_1}{2n_1+\frac{s_2}{2n_2}}}}$

(c) $Z = \frac{|s_1 - s_2|}{\sqrt{\frac{s_1^2}{2n_1}+\frac{s_2^2}{2n_2}}}$

(d) none of the above

Q. 56 Formula for χ^2 for testing a null hypothesis in a multinomial distribution with usual notations is:

(a) $\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$

(b) $\chi^2 = \sum_{i=1}^{k} \frac{O_i^2}{E_i} - n$

(c) $\chi^2 = \sum_{i=1}^{k} \frac{O_i^2}{np_i} - n$

(d) all the above

Q. 57 The statistic-χ^2 with usual notations in case of contingency table of order ($m \times p$) is given by the formula:

(a) $\chi^2 = \sum_{i=1}^{m} \sum_{j=1}^{p} \frac{(O_{ij} - E_{ij})^2}{E_{ij}}$

(b) $\chi^2 = \sum_{i=1}^{m} \sum_{j=1}^{p} \frac{O_{ij}^2 - E_{ij}^2}{E_{ij}}$

(c) $\chi^2 = \sum_{i=1}^{m} \sum_{j=1}^{p} \left(\frac{O_{ij} - E_{ij}}{E_{ij}} \right)^2$

(d) all the above

Q. 58 Degrees of freedom for Chi-square in case of contingency table of order (4 × 3) are:

(a) 12
(b) 9
(c) 8
(d) 6

Q. 59 The degrees of freedom for χ^2 in case of dischotomised frequencies are:

(a) 4
(b) 2
(c) 1
(d) 0

Q. 60 Formula for statistic-χ^2 for the binomial frequencies a and b to occur in a specified ratio r : 1 is:

(a) $\frac{r(a-b)^2}{r(a+b)}$

(b) $\frac{(a-rb)^2}{r(a+b)}$

(c) $\frac{(a-rb)^2}{r(a+b)^2}$

(d) $\frac{(ar-b)^2}{r(a+b)}$

Q. 61 If the binomial frequencies 35 and 9 are expected to occur in the ratio 3 : 1, the value of statistic Chi-square is approximately equal to

(a) 0.48
(b) 5.10
(c) 1.45
(d) none of the above

Q. 62 An exact test for testing the independence of attributes in a contingency table of order (2 × 2) was given by:

(a) Karl Pearson
(b) Pascal
(c) Demoivre
(d) R.A. Fisher

Q. 63 An exact test for testing the independence of attributes in a contingency table of order (2 × 2) is based on the calculation of:

(a) the value of statistic-χ^2
(b) probabilities of configurations
(c) the value of statistic-Z
(d) none of the above

Q. 64 Fisher's exact test can be used to test:

(a) the independence of two attribute
(b) equality of proportions in two classes
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 65 Fisher's exact test is preferably used when:

(a) a cell frequency is small
(b) all cell frequencies are small
(c) both (a) and (b)
(d) none of the above

Q. 66 Coefficient of contingency is calculated when:
(a) the attributes are independent
(b) the attributes are associated
(c) both (a) and (b)
(d) neither (a) and (b)

Q. 67 The value of coefficient of contingency lies between:
(a) 0 and ∞
(b) 0 and 1
(c) 0 to 100
(d) – 1 and +1

Q. 68 When the value of coefficient of contingency $C = 0$, it shows:
(a) complete dissociation amongst attributes
(b) complete association amongst attributes
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 69 When coefficient of contingency $C = 1$, it indicates:
(a) high degree of association
(b) low degree of association
(c) low degree of dissociation
(d) nothing

Q. 70 For a sample of n individuals, formula for coefficient of contingency is

(a) $C = \sqrt{\dfrac{\chi^2}{\chi^2 + (n-1)}}$

(b) $C = \sqrt{\dfrac{\chi^2}{n\chi^2}}$

(c) $C = \dfrac{\chi^2}{\chi^2 + n}$

(d) $C = \sqrt{\dfrac{\chi^2}{\chi^2 + n}}$

Q. 71 When d.f. for χ^2 are 100 or more, Chi-square is approximated to:
(a) t-distribution
(b) F-distribution
(c) Z-distribution
(d) none of the above

Q. 72 Homogeneity of several variances can be tests by:
(a) Bartlett's test
(b) Fisher's exact test
(c) F-test
(d) t-test

Q. 73 Statistics-χ^2 under Barlett's test has:
(a) a downward bias
(b) an upward bias
(c) zero bias
(d) none of the above

Q. 74 Equality of two population variances can be tested by:
(a) Bartlett's test
(b) F-test
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 75 Equality of several normal population means can be tested by:
(a) Bartlett's test
(b) F-test
(c) χ^2-test
(d) t-test

Q. 76 The ratio of between sample variance and within sample variance fpllows:
(a) F-distribution
(b) χ^2-distribution
(c) Z-distribution
(d) t-distribution

Q. 77 Analysis of variance utilises:
(a) F-test
(b) χ^2-test
(c) Z-test
(d) t-test

Q. 78 Customarily the large variance in the variance ratio for F-statistic is taken:
(a) in the denominator
(b) in the numerator
(c) either way
(d) none of the above

Q. 79 In sequential probability ratio test (SPRT), the sample size is:

(a) fixed
(b) fixed but small
(c) fixed but large
(d) a random variable

Q. 80 In SPRT, decision about the hypothesis H is taken:
(a) after each successive observation
(b) after a fixed number of observations
(c) at least after five observations
(d) when the experiment is over

Q. 81 To decide about H_0, SPRT involves:
(a) one region only
(b) two regions only
(c) three regions
(d) four regions

Q. 82 SPRT was initiated by:
(a) R.A. Fisher
(b) A. Wald
(c) G.W. Snedecor
(d) Thomas Bayes

Q. 83 The decision criteria in SPRT depends on the functions of:
(a) type I error
(b) type II error
(c) type I and II errors
(d) none of the two types of errors

Q. 84 If in a contingency table of order (2 × 3), the frequencies are such that $x + y + z = N$ and $x' + y' + z' = N$, the value of statistic-χ^2 is:

(a) $(x-x')^2+(y-y')^2+(z-z')^2$

(b) $\dfrac{(x-x')^2}{2}+\dfrac{(y-y')^2}{2}+\dfrac{(z-z')^2}{2}$

(c) $\left(\dfrac{x-x'}{x+x'}\right)^2+\left(\dfrac{y-y'}{y+y'}\right)^2+\left(\dfrac{z-z'}{z+z'}\right)^2$

(d) $\dfrac{(x-x')^2}{x+x'}+\dfrac{(y-y')^2}{y+y'}+\dfrac{(z-z')^2}{z+z'}$

Q. 85 Formula for Chi-square statistic in a 2 × 2 contingency table under Yates' correction is

(a) $n\left(|ad-bc|-\dfrac{n}{2}\right)^2 \Big/ D$

(b) $n\left(ad-bc-\dfrac{n}{2}\right)^2 \Big/ D$

(c) $\left(ad-bc-\dfrac{n}{2}\right)^2 \Big/ D$

(d) $n\left(\dfrac{ad-bc-\dfrac{n}{2}}{D}\right)^2$

where, $D = (a + b)(b + d)(a + c)(c + d)$.

Q. 86 The value of statistic Chi-square for a contingency table

	B_1	B_2
A_1	2	28
A_2	13	7

after Yates' correction is
(a) 625/28
(b) 4225/252
(c) 845/2520
(d) none of the above

Q. 87 For testing H_0: $\mu_1 = \mu_2$, the value of the statistic $|\bar{x}-\bar{y}| \Big/ \left(\sigma\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}\right)$ lies between 1.96 and 2.58, then H_0 is:
(a) rejected at 5% level of significance
(b) accepted at 5% level of significance
(c) rejected at 1% level of significance
(d) any of the above

Q. 88 If two samples of size 9 and 11 have means 6.8 and 8.8, and variances 36 and 25 respectively from two populations $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$, the absolute value of statistic t for testing H_0: $\mu_1 = \mu_2$ is:
(a) 0.148
(b) 1.83
(c) 0.81
(d) none of the above

Q. 89 Assume that the daily sales of petrol follows exponential distribution. The hypothesis H that the sales of petrol is 1000 litres per day is tested against the hypothesis that it is 1500 litres per day. If the sales on a day is 1200 litres or more, H_0 is rejected, the size of type I error is

(a) $1 - e^{-1.2}$
(b) $e^{1.2}$
(c) $e^{-1.2}$
(d) none of the above

Q. 90 For the problem given in Q. No. 89, the size of type II error is:

(a) $1 - e^{-0.8}$
(b) $1 - e^{0.8}$
(c) $e^{-0.8} - 1$
(d) none of the above

Q. 91 For the problem given in Q. No. 89, the power of the test is:

(a) $1 - e^{-0.8}$
(b) $e^{-0.8}$
(c) $-e^{-0.8}$
(d) $2 - e^{-0.8}$

Q. 92 In tossing of a coin, let the probability of turning up a head be p. The hypotheses are H_0: $p = 0.4$ vs. H_1: $p = 0.6$. H_0 is rejected if there are 5 or more heads in six tosses. Then the size of type I error is:

(a) 0.041
(b) 0.037
(c) 0.029
(d) none of the above

Q. 93 For the test given in Q. No. 92, the size of type II error is:

(a) 0.767
(b) 0.762
(c) 0.233
(d) none of the above

Q. 94 For the problem given in Q. No. 92, the power of the test is:

(a) 0.767
(b) 0.762
(c) 0.233
(d) none of the above

Q. 95 It is desired that the sample standard deviation should not differ from population standard deviation by more than 3 per cent. To have 68.26 per cent confidence, the sample size should not be less than:

(a) 1111
(b) 556
(c) 8889
(d) none of the above

Q. 96 Two samples, one from urban and the other from rural adult males of sizes 400 and 600 had standard deviations 165 cm and 175 cm respectively. Test of hypothesis of equality of standard deviations in the two populations at 5 per cent level is:

(a) accepted
(b) rejected
(c) no decision about H_0
(d) none of the above

Q. 97 Two samples of size 10 and 8 had sample means 18 cm and 12 cm with variances 25 and 16. Supposing that the samples have been drawn from normal populations $N\left(\mu_1, \sigma_1^2\right)$ and $N\left(\mu_2, \sigma_2^2\right)$, the value of statistic-t for testing H_0: $\mu_1 = \mu_2$ under $\sigma_1^2 \neq \sigma_2^2$ is:

(a) 2.67
(b) 2.75
(c) 1.33
(d) 2.83

Q. 98 A manufacturer claims that his items could not have a large variance. 18 of his items has a variance = 0.033. The value of statistic χ^2 to test H_0: $\sigma^2 = 1$ is:

(a) 30.30
(b) 5.55
(c) 0.56
(d) none of the above

Q. 99 The variance due to two treatments in an experiment is 480 and the error variance is 60.0 with 14 d.f. Test for the equality of treatments effect reveals that:

[Given $F_{0.05, (1, 14)} = 4.60$]

(a) treatments are equally effective
(b) treatments differ significantly
(c) no conclusion
(d) none of the above

Q. 100 It is expected that 50 per cent people of a city are cinema goers. A survey of 1600 people revealed that 35 per cent people go to cinema. The value of statistic-Z is:
(a) 12.0
(b) 6.0
(c) 12.58
(d) −12.0

Q. 101 A normal population has a mean of 0.5 and S.D. = 6.0. The probability that the sample mean of 625 items of a sample will be negative is:
(a) 0.0188
(b) 0.365
(c) 0.4812
(d) 0.135

Q. 102 The claimed average life of electric bulbs is 2000 hours with a S.D. = 250 hours. To make 95 per cent sure that the bulbs should not fall below the claimed average life by more than 5 per cent, the sample size should be:
(a) 24
(b) 16
(c) 41
(d) none of the above

Q. 103 Given the sample statistics,

$$n_1 = 400, \bar{x}_1 = 24.50, s_1 = 2.5$$

$$n_2 = 500, \bar{x}_2 = 20.0, s_2 = 2.0$$

The value of test statistic to test H_0: $\mu_1 = \mu_2$, when $\sigma_1^2 = \sigma_2^2$ is:
(a) $Z = 44.47$
(b) $Z = 8.97$
(c) $Z = 30.0$
(d) none of the above

Q. 104 From the sample statistics given Q. No. 103, the hypothesis H_0: $\sigma_1 = \sigma_2$ is:
(a) accepted
(b) rejected
(c) not possible to be tested
(d) none of the above

Q. 105 From a population of 200 items with S.D. = 3.0, a sample of size n is drawn. If it is desired that the sample mean differs from population mean by 1 unit or more is controlled at $\alpha = 0.05$, the value of smallest n is:
(a) 35
(b) 6
(c) 12
(d) 312

Q. 106 An experiment consisted of two storage practices. In the first storage practice, out of 100 fruits 5 were putrefied and in the second, out of 150 fruits, 5 were putrefied. Can it be concluded that the second storage practice is better than first?
(a) No
(b) Yes
(c) not possible to decide
(d) none of the above

Q. 107 A die is thrown 60 times and number of times the following faces were obtained.

Faces:	1	2	3	4	5	6
No. of times:	14	7	5	8	10	16

Can the die be regarded as fair?

[Given: $\chi^2_{0.05,5} = 11.07$]

(a) The die is not fair
(b) The die is fair
(c) no conclusion
(d) none of the above

Q. 108 For the problem given in Q. No. 107, the value of statistic χ^2 is:
(a) 1.8
(b) 9.0
(c) 40.96
(d) none of the above

Q. 109 For the problem given in Q. No. 107, d.f. for statistic χ^2 are:
(a) 6
(b) 4
(c) 5
(d) 1

Q. 110 Given the following eight sample values $-4, -3, -3, 0, 3, 3, 4, 4$, the value of student's t-test H_0: $\mu = 0$ is:
(a) 2.73
(b) 0.97
(c) 3.30
(d) 0.41

Q. 111 If all frequencies of classes are same, the value of χ^2 is:
(a) 1
(b) ∞
(c) zero
(d) none of the above

Q. 112 The value of statistic χ^2 is zero if and only if:
(a) $\sum_i O_i = \sum_i E_i$
(b) $O_i = E_i$ for all i
(c) E_i is large
(d) all the above

Q. 113 The range of statistic-χ^2 is:
(a) -1 to $+1$
(b) $-\infty$ to ∞
(c) 0 to ∞
(d) 0 to 1

Q. 114 Range of statistic-t is
(a) -1 to 1
(b) $-\infty$ to ∞
(c) 0 to ∞
(d) 0 to 1

Q. 115 Range of the variance ratio F is:
(a) -1 to 1
(b) $-\infty$ to ∞
(c) 0 to ∞
(d) 0 to 1

Q. 116 The mean and S.D. of a set of values are 15 and 5 respectively. Then the value of student's-t is calculated to test H_0: $\mu = 10$. If each sample value is increased by 2, the value of t will be:
(a) decreased
(b) increased
(c) same
(d) all the above

Q. 117 A random sample of size 20 from a normal population gives a mean 42 and a variance 25. To test that the population standard deviation is 8, the value of statistic-χ^2 is:
(a) 7.42
(b) 15.62
(c) 51.20
(d) none of the above

Q. 118 Degrees of freedom for χ^2 in Q. No. 117 is:
(a) 20
(b) 24
(c) 7
(d) 19

Q. 119 A coin is tossed 400 times and it turns up head 216 times. The hypothesis that the coin is unbiased can be tested by:
(a) χ^2 test
(b) Z-test
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 120 The best critical region consists of:
(a) extreme positive values
(b) extreme negative values
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 121 Reduction in the size of a test results into:
(a) decrease in its power
(b) increase in its power
(c) no change in its power
(d) all the above

Q. 122 A sample of 36 measurements shows a standard deviation of 0.07. Test of hypothesis H_0 that the true standard deviation is 0.05 against that it is not at 5 per cent level of significance reveals that:
(a) H_0 is accepted
(b) H_0 is rejected
(c) not possible to test
(d) none of the above

Q. 123 25 persons were found suffering from cancer in a city and only 10 saved. Can it be concluded at 95 per cent confidence level that in general 50 per cent person suffering from cancer were saved against that it is less.
(a) No

(b) Yes
(c) no test is available
(d) none of the above

Q. 124 Statistic $Z\frac{\bar{x}-\bar{y}}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$ is used to test the null hypothesis:
(a) H_0: $\mu_1 + \mu_2 = 0$
(b) H_0: $\mu_1 - \mu_2 = 0$
(c) H_0: $\mu = \mu_0$ (a constant)
(d) none of the above

Q. 125 In a contingency table, the expected frequencies are computed under:
(a) null hypothesis H_0
(b) alternative hypothesis H_1
(c) H_0 and H_1 both
(d) no consideration of hypothesis

Q. 126 Which of the following is a research hypothesis?
(a) Two population means are equal
(b) Intelligent people have more reading habit
(c) Population correlation coefficient is zero
(d) Two populations follow the same distribution.

Q. 127 Which one of the following with usual notations is not a statistical hypothesis.
(a) $H{:}\sigma^2 = \sigma_0^2$
(b) $H{:}\sigma_1^2 > \sigma_1^2$
(c) $H{:}\rho_1 = \rho_2$
(d) H: People suffering from T.B. belong to the poor section of the society.

Q. 128 A contingency table having a zero count is called:
(a) a complete contingency table
(b) an incomplete contingency table
(c) abnormal contingency table
(d) none of the above

Q. 129 If the calculated value of chi-square is greater than its degrees of freedom, then
(a) null hypothesis be accepted directly
(b) null hypothesis be rejected straightaway
(c) χ^2-table be consulted to arrive at a decision about the null hypothesis
(d) all the above.

Q. 130 A zero count in a cell of a contingency table having its expected frequency zero is called:
(a) structural zero
(b) random zero
(c) false zero
(d) all the above

Q. 131 A zero in a cell of a contingency table having a finite expected value is termed as:
(a) structural zero
(b) random zero
(c) false zero
(d) none of the above

Q. 132 Calculated value of chi-square less than its degrees of freedom leads to:
(a) acceptance of H_0 directly
(b) rejection of H_0 straightaway
(c) no decision about H_0
(d) none of the above

Q. 133 For testing that a bivariate random sample has come from an uncorrelated population, the appropriate test is:
(a) normal deviate test
(b) χ^2-test
(c) F-test
(d) t-test

Q. 134 A variable which is used in a contingency table to explain the response variable is known as:
(a) random variable
(b) discrete variable
(c) explanatory variable
(d) dummy variable

Q. 135 In general a contingency table is a:
(a) one-dimensional table
(b) two-dimensional table
(c) three-dimensional table
(d) multi-dimensional table

Q. 136 The expression for knowing the significant contribution of a cell in a $(p \times q)$ contingency table with usual notations is:

(a) $\left(O_{ij} - E_{ij}\right)/\sqrt{E_{ij}}$

(b) $\sqrt{\left(O_{ij} - E_{ij}\right)/E_{ij}}$

(c) $\left(O_{ij} - E_{ij}\right)/\left[E_{ij}\left(1 - p_{i.}\right)\left(1 - p_{.j}\right)\right]^{1/2}$

(d) $\left(O_{ij} - E_{ij}\right)/E_{ij}\, p_i \cdot p \cdot j$

Q. 137 For testing $H_0 : \sigma = \sigma_0$ in a normal population $N(0, \sigma^2)$, a critical region based on sample $X_1, X_2, \ldots, X_n$ is $\Sigma X_i^2 < K$. For which alternative hypothesis does this provide uniformly most powerful test?

(a) $\sigma \neq \sigma_0$
(b) $\sigma^2 = \sigma_0^2$
(c) $\sigma < \sigma_0$
(d) $\sigma > \sigma_0$

Q. 138 Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(0, \sigma^2)$. Consider maximum likelihood ratio test for which the critical region is given as $\Sigma X_i^2 > K$. The alternative hypothesis against $H_0 : \sigma = \sigma_0$ which leads to an uniformly most powerful test is:

(a) $\sigma \neq \sigma_0$
(b) $\sigma^2 = \sigma_0$
(c) $\sigma < \sigma_0$
(d) $\sigma > \sigma_0$

Q. 139 In testing the equality of several population means by F-test, the assumption is:

(a) populations are continuous
(b) population variances are homogeneous
(c) populations are correlated
(d) all the above

Q. 140 The test of goodness of fit of an assumed population means:

(a) testing a hypothesis about its mean
(b) testing a hypothesis about its variance
(c) to make a comparison between observed and expected number of observations
(d) none of the above.

ANSWERS

SECTION-B

(1) more than one (2) J. Neyman; 1928 (3) Karl Pearson (4) assertion (5) null (6) alternative (7) J. Neyman (8) simple; simple (9) composite (10) two (11) second; first (12) second; first (13) size (14) level of significance (15) type I (16) type II (17) alternative (18) remains same (19) $1 - \beta$ (20) operating characteristic (21) 0; 1 (22) power (23) region of rejection (24) rule or procedure (25) test statistic (26) non-randomised (27) critical region (28) acceptance (29) rejection (30) critical region (31) degrees of freedom (32) sample size (33) optimum (34) optimum (35) most powerful (36) uniformly most powerful (37) most powerful (38) type I; type II (39) size; power (40) minimax (41) unbiased (42) UMPU (43) type A_1 (44) Neyman and Pearson (45) unbiased critical (46) admissible (47) randomised (48) critical function (49) border (50) degrees of freedom (51) randomised (52) chi-square (53) small (54) standard error (55) $\left(\bar{x} - \mu\right)\sqrt{n}/s$ (56) normal (57) independent (58) five (59) robust (60) UMPU (61) $n_1 + n_2 - 2$ (62) pooled (63) W.G. Cochran (64) Behrens-Fisher (65) separately (66) later (67) first (68) objection or lacuna (69) more (70) paired (71) correlated (72) mean difference (73) Z-test (74) Z-test (75) Z-test (76) normal (77) goodness of fit (78) observed; expected (79) 0; ∞ (80) 0; 1 (81) unity (82) $k - 1$ (83) 12 (84) 1 (85) $n(ad - bc)^2/(a + b)(c + d)(a + c)(b + d)$ (86) $(a - rb)^2/r\,(a+b)$ (87) continuity (88) Yates (89) small (90) better (91) calculation (92) $n\left(|ad - bc| - \frac{n}{2}\right)^2 \Big/ (a + b)(c + d)(a + c) \times (b + d)$ (93) probabilities (94) computation (95) association (96) zero (97) rejected (98) high degree (99) 100 (100) M.S. Bartlett (101) Chi-square (102) upward (103) F-test (104) F-distribution (105) F-test (106) F-test (107) ANOVA (108) variance (109) difference (110) random variable (111) successive (112) A. Wald; 1947 (113) two type of error (114) risk function (115) paired t-test (116) $\overline{X}_A = \overline{X}_B$ (117) Chi-square

or Z-test (118) analysis of variance

(119) 132 $\left[\text{Hint.} \dfrac{-16}{112/\sqrt{n}} = -1.64\right]$ (120) 1/4

$$\left[\text{Hint.}\,\alpha = p\,(\text{reject } H_0/H_0) = \frac{\binom{5}{1}\binom{5}{1}}{\binom{10}{1}\binom{10}{1}} = \frac{1}{4}\right]$$

(121) 16/25 $[\text{Hint.}\,\beta = p\,(\text{reject } H_0/H_1)$

$$= \frac{\binom{4}{1}\binom{4}{1}}{\binom{10}{1}\binom{10}{1}} + \frac{\binom{4}{1}\binom{6}{1}}{\binom{10}{1}\binom{10}{1}} + \frac{\binom{6}{1}\binom{4}{1}}{\binom{10}{1}\binom{10}{1}} = \frac{16}{25}\Bigg]$$

(122) $e^{-1.5}\left[\text{Hint. } f(x;\theta) = \theta e^{-\theta x} = \int_{1500}^{\infty} \dfrac{1}{1000} e^{-x/1000} dx\right]$

(123) $e^{-3/4}\left[\text{Hint. } 1-\beta = P(\text{reject } H_0/H_1) = \int_{1500}^{\infty} \dfrac{1}{2000} e^{-x/1000} dx\right]$

(124) 0.172 $\left[\text{Hint.}\,\alpha = P(x \geq 7/H_0) = \dfrac{1}{2^{10}} \times \sum_{x=7}^{10} \binom{10}{x}\right]$

(125) 0.382 $\left[\text{Hint. Power} = \sum_{x=7}^{10} \binom{10}{x}\left(\dfrac{3}{4}\right)^x \times \left(\dfrac{2}{5}\right)^{10-x}\right]$

(126) 0.618 (127) do not support (128) rejected (129) zero (130) 16 (131) statistical (132) accepted (133) 1.71 (134) same (135) not equally (136) accepted (137) accepted (138) 0.455 (139) 7/64 (140) 0.649 (141) 0.351 (142) 125 (143) do not support (144) 1800 (145) decreases (146) decision rule (147) objectives (148) greater than (149) multi-dimensional (150) incomplete (151) structural (152) random zero (153) accept (154) standardised residuals (155) dependent (156) significant (157) $\mu = 6$; $\mu > 6$ (158) chi-square test (159) right tail (160) Z-test

SECTION-C

(1) b (2) d (3) d (4) c (5) a (6) b
(7) b (8) c (9) a (10) b (11) b (12) a
(13) c (14) b (15) d (16) c (17) d (18) a
(19) c (20) b (21) d (22) a (23) c (24) b
(25) c (26) c (27) b (28) c (29) a (30) d
(31) a (32) d (33) b (34) c (35) b (36) a
(37) c (38) c (39) b (40) a (41) d (42) b
(43) b (44) c (45) d (46) a (47) d (48) a
(49) c (50) b (51) a (52) d (53) a (54) b
(55) c (56) d (57) a (58) d (59) c (60) b
(61) a (62) d (63) b (64) c (65) c (66) b
(67) b (68) a (69) a (70) d (71) c (72) a
(73) b (74) c (75) b (76) a (77) a (78) b
(79) d (80) a (81) c (82) b (83) c (84) d
(85) a (86) b (87) a (88) c (89) c (90) a
(91) b (92) a (93) a (94) c (95) b (96) b
(97) d (98) c (99) b (100) d (101) a (102) b
(103) c (104) b (105) a (106) b (107) b (108) b
(109) c (110) d (111) c (112) b (113) c (114) b
(115) c (116) b (117) a (118) d (119) c (120) c
(121) a (122) b (123) a (124) b (125) a (126) b
(127) d (128) b (129) c (130) a (131) b (132) a
(133) d (134) c (135) d (136) c (137) c (138) d
(139) b (140) d

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Chapter **12**

Nonparametric Statistical Methods

SECTION-A

Short Essay Type Questions

Q. 1 Give a brief account of the need of nonparametric statistical methods.

Ans. Parametric statistical methods are based on stringent assumptions about the population from which the sample has been drawn. Particularly the assumptions like form of the probability distribution, accuracy of observations, etc., are more common. Also, the parametric methods are applicable primarily to the data which are measured in interval or ratio scale. In practice, however, stringent assumptions are seldom fully valid. Moreover, the measurements are often made on nominal or ordinal scale.

If the assumption do not hold good or the data do not meet the requirement of parametric statistical methods, nonparametric methods come to the rescue of the worker. Nonparametric methods entail very mild assumptions like continuity and symmetry of the distribution. Also most of the nonparametric methods are applicable for ordered statistics.

Q. 2 Distinguish between nonparametric methods and distribution free methods.

Ans. Those statistical methods, which are not concerned with the estimation or testing of hypothesis for one or more parameters of probability distribution functions are termed as nonparametric methods. Those inferences whose validity do not rest on the form of specific probability distribution of the population from which the sample has been drawn are termed as distribution free methods. These two terms are not synonyms. But the statistical methods applied in the two cases are almost same and they are interchangeably used.

Q. 3 When should the nonparametric methods be preferably used?

Ans. Nonparametric methods be used when one or more of the following situations exist:

(i) The hypothesis does not involve a parameter of the probability function of the population.

(ii) The observations are not as accurate as required for a parametric inference. Also when the measurements are on the nominal or ordinal scale.

(iii) The assumptions necessary for a validity of a parametric method are suspected to hold good. For example, the assumption of normal population is doubtful.

(iv) One wants to avoid complicated analysis of data.

(v) One is interested in quick results.

Q. 4 What are the advantages of nonparametric methods?

Ans. There are many advantages of nonparametric methods over parametric ones. The advantages can precisely be delineated as under:

(i) Any inference based on the parametric analysis which does not uphold the underlying assumptions necessitated for it will be erroneous. In such a situation nonparametric methods can safely be applied.

(ii) If the measurement scale of data is nominal or ordinal, nonparametric methods çan be used.

(iii) In case the measurements are not so accurate as to apply parametric methods, nonparametric methods perform better.

(iv) With so-called *dirty data* (contaminated observations, outliers, etc.), many nonparametric methods are appropriate.

(v) There is no restriction for minimum size of sample for nonparametric methods for valid and reliable results.

(vi) Nonparametric methods require minimum assumption like continuity of the sampled population.

(vii) The analysis of data is simple and involves little computation work.

(viii) Nonparametric test may be quite powerful even if the sample size is small.

(ix) Nonparametric test are inherently robust against certain violation of assumptions.

Q. 5 What are the disadvantages of non-parametric procedures?

Ans. Nonparametric procedures are also not free from demerits. Some of the main disadvantages are as follows:

(i) Because of the simplicity of nonparametric procedures, they are often used even if appropriate parametric methods are available.

(ii) All nonparametric methods are not as simple as they are claimed to be.

(iii) It is not possible to determine the actual power of a nonparametric test due to the want of actual situation.

Q. 6 How can one judge the relative performance of two tests?

Ans. In many situations there can be more than one tests which appear appropriate for the test of a hypothesis. Then there is a need to fix some criteria to choose one out of many alternative tests. The asymptotic relative efficiency (ARE) is a single measure which provides satisfactory results for comparing the performance of two tests based on large samples. The concept of ARE was given by S.J.G. Pitman in 1961, and hence ARE is often named as Pitman efficiency. The calculation of ARE is based on the classical distribution and assumption of the nonparametric tests that have parametric analogues.

The *asymptotic relative efficiency* of test A relative to a test B can be defined as the limiting value of n_B/n_A where n_B and n_A are the sample sizes required for the tests A and B to have the same power.

Q. 7 What is power efficiency?

Ans. The *power efficiency* of test A relative to a test B is the ratio of sample sizes n_B/n_A where both the tests are for the same H_0 and H_1 and having same power of the tests A and B. Since it is difficult to calculate power efficiency, ARE is more frequently used.

Q. 8 What is meant by tied observations?

Ans. Under the assumption of continuous distribution, no two observations can be equal. But due to the rounding of figures, precision of measuring instruments, inaccuracy of measurements, etc., some observations seldom occur which are exactly equal in magnitude. Such observations are called *tied observations.* Due to tie among observations, one faces the problem in awarding ranks to them.

Q. 9 How to surmount the problem of tied observations?

Ans. There are different approaches to overcome the problem of tied observations. Five important approaches are discussed over here.

(i) Midranks approach. Under this approach, each group of tied observations is ranked as if they are all distinguishable and then take the average of the ranks of tied values and assign the same average rank to the tied observations of that group. This is the most frequently used methods of breaking the ties. When the mid rank method is used, a correction of ties is often applied in most of the tests and measures of associations.

(ii) Average statistics approach. If there are K groups of different observations in a set and i^{th} group is of size r_i, there are in all $\prod_{i=1}^{k} r_i!$ possible arrangements of values. Now calculate the test statistic for each arrangement and take their average. Use this average value for taking the decision. Under this approach, the test statistic will have the same mean but smaller variance. This method is generally not used because it usually involves too much computation.

(iii) Least favourable statistic approach. Instead of averaging the test statistics for all possible arrangements under tied observations, one might choose that statistics value out of all which minimises the probability of rejection. Under this approach, there is least chance of committing type I error.

(iv) Range of probability approach. In this method one chooses two values of test statistics, the least and most favourable values. But a decision is possible only if both the values fall either inside or outside the region of rejection. If not so, the method fails.

(v) Omitting the tied observations. This is the simplest but risky approach. In this method one discards all tied values and reduce the sample by that number. If the number of tied observations is small as compared to the sample size, the test is not affected much otherwise, there is a loss of information and the prodecure introduces bias towards rejection of the null hypothesis.

Q. 10 What assumptions are generally made for a nonparametric test?

Ans. Following assumptions are most commonly made about any nonparametric test:

(i) The sample at hand is a random sample drawn from a population whose median is unknown.

(ii) All the observations in the sample are independent.

(iii) The variable of interest is continuous.

(iv) The sampled population is symmetric.

(v) The observations are measured at least on ordinal scale.

Q. 11 What are the basic steps involved in any nonparametric test of hypothesis?

Ans. Various basic steps involved in a test are:

(i) First of all one should look for the assumptions necessary for the validity of a test procedure.

(ii) The sample data required should be collected.

(iii) The null and alternative hypotheses should be established.

(iv) The test statistic or procedure should be decided.

(v) The decision criteria should be fixed to decide about the rejection or acceptance of H_0 *vis-a-vis* H_1.

(vi) The interpretation to the conclusions drawn should be given.

Q. 12 What do you understand by ordered statistics?

Ans. Let $X_1, X_2, ..., X_n$ be a random sample from a continuous population with probability of any X's being equal to zero. If $x_{(1)}$ is the smallest X value in $X_1, X_2, ..., X_n$ $x_{(2)}$ the next larger value and $x_{(n)}$, the highest, then the set of values $x_{(1)}, x_{(2)}, ..., x_{(n)}$ is called the ordered statistics. This is an ascending order. If one wants he can put the observations in descending order as $x_{(n)}, x_{(n-1)}, ..., x_{(1)}$. If $x_{(r)}$ denotes the r^{th} ordered value for $r = 1, 2, ..., n$, the ordered statistics deals with the properties of $x_{(r)}$. Ordered statistics is extremely useful in nonparametric methods.

The ordered statistics do not possess the same probability distribution as the original variable X. Also the values in the ordered statistics are not independent even if original variate values are independent as is always true in case of random samples.

Q. 13 Give the names of various nonparametric tests and statistics.

Ans. The names of the nonparametric tests and statistics are as follows:

One sample nonparametric tests:

(i) Kolmogorov–Smirnov test
(ii) Ordinary sign test
(iii) Wilcoxon signed-rank test
(iv) Runs test

Two or more samples nonparametric tests and statistics:

(i) Kolmogorov–Smirnov two-sample test
(ii) Sign test for paired samples
(iii) Wilcoxon paired sample signed-rank test
(iv) Median test
(v) Wald-Wolfowitz runs test
(vi) Mann-Whitney U-test
(vii) Mcnemar's test
(viii) Cochran's Q-test
(ix) Mood's test for dispersion
(x) Moses test for dispersion
(xi) Kruskal-Wallis one way analysis
(xii) Friedman's method of two way analysis
(xiii) Jonckheere Terpstra test
(xiv) Page's test
(xv) Spearman's rank correlation
(xvi) Kendall's correlation coefficient τ
(xvii) Coefficient of concordance
(xviii) Brown and Mood's test
(xix) Mood's test
(xx) Theil's test
(xxi) Confidence interval
(xxii) Confidence band

Q. 14 Describe briefly Kolmogorov–Smirnov test of goodness of fit in case of one sample.

Ans. Let $X_1, X_2, ..., X_n$ be a random sample from an unknown continuous population having the cumulative distribution function $F(x)$. Also let the ordered statistics be $x_{(1)}, x_{(2)}, ..., x_{(n)}$. The *K-S* test is based on Glivenko-Cantelli theorem which states that *the step function $S_n(x)$ with jumps occurring at the values $x_{(1)}, x_{(2)}, ..., x_{(n)}$ of the ordered statistics for the sample approaches the true distribution for all X.* Kolmogorov-Smirnov used this theorem and compared the empirical distribution function $S_n(x)$ of the sample for any value of x with the population c.d.f. under H_0, i.e., $F_0(x)$.

The hypothesis under test is,

$$H_0 : F(x) = F_0(x) \text{ vs. } H_1 : F(x) \neq F_0(x)$$

where $F_0(x)$ is a completely specified continuous distribution.

To test H_0, the numerical difference $|s_n(x) - F_0(x)|$ is used in *K-S* test. Since the difference depends on x, the *K-S* statistic D_n is taken to be the supremum of such differences, *i.e.*,

$$D_n = \underset{\text{overall } x}{\text{Sup}} |s_n(x) - F_0(x)|$$

Under H_0, the statistic D_n has a distribution which is independent of the c.d.f. $F(x)$ that defines H_0. The statistic D_n is distribution free.

To take a decision about H_0, the test criteria are, reject H_0 if $D_n \geq D_{n,\alpha}$ (tabulated value), otherwise accept H_0.

Q. 15 Compare the Chi-square test of goodness of fit with Kolmogorov-Smirnov test.

Ans. The Chi-square test is also one of the very popular test of goodness of fit. If we compare the two, we find that:

(i) χ^2-test is specially meant for categorical data whereas *K-S* statistics are for random samples from continuous populations. However, when the data are categorical, the two tests can interchangeably be used.
(ii) Chi-square requires categorical data, whereas the *K-S* statistic utilises each of the n observations. Hence, the *K-S* makes better use of available information :han Chi-square statistic.
(iii) The Kolmogorov-Smirnov statistic is more flexible than Chi-square statistic as it can be used to determine minimum sample size and confidence band.
(iv) In *K-S* test, we can use one side test also which is not possible in Chi-square test.
(v) The *K-S* test is easier to apply.
(vi) The Chi-square test also comes in the categroy of parametric tests whereas *K-S* test is only a nonparametric test.

Q. 16 How can you perform ordinary sign test?

Ans. Let $x_{(1)}, x_{(2)}, ..., x_{(n)}$ be the ordered sample values from a population $F(x)$ and M be its median.

Also $P(X = M) = 0$. Here we test, H_0: $M = M_0$ vs. H_1: $M \neq M_0$ where M_0 is the given value of the median and hence $P(X > M_0) = P(X < M_0) = 0.5$.

So we can test $H_0 : P(X > M_0) = P(X < M_0)$

vs. $H_1 : P(X > M_0) \neq P(X < M_0)$

To perform the sign test, find the differences $(X_{(i)} - M_0)$ for $i = 1, 2, \ldots, n$ and consider their signs. Suppose the number of +ve signs is r and negative signs, $(n - r)$. For the purpose of test we consider only positive signs. So r follows binomial distribution. Also the null hypothesis H_0 changes to $p = 0.5$.

So the hypothesis under test amounts to testing,

$$H_0 : p = 0.5 \quad \text{vs.} \quad H_1 : p \neq 0.5$$

The test criterion is, reject H_0 if $r \geq r_{\alpha/2}$ where $r_{\alpha/2}$ is the critical value at significance level α. $r_{\alpha/2}$ is the smallest integer which satisfies the conditions,

$$\sum_{r=r_{\alpha/2}}^{n} \binom{n}{r} \left(\frac{1}{2}\right)^{r} \left(\frac{1}{2}\right)^{n-r} \leq \alpha/2$$

or $r \leq r'_{\alpha/2}$ is the smallest integer such that

$$\sum_{r=0}^{r'_{\alpha/2}} \binom{n}{r} \left(\frac{1}{2}\right)^{r} \left(\frac{1}{2}\right)^{n-r} \leq \alpha/2$$

For one-sided test use r_α instead of $r_{\alpha/2}$. Rest of the test procedure remains the same.

Large sample case: If $n \geq 25$, normal deviate test is applied to decide about H_0. Z is given by the statistic,

$$Z = \frac{(r + 0.5) - np_0}{\sqrt{np_0\, q_o}} \quad \text{where } r < np_0$$

$$= \frac{(r - 0.5) - np_0}{\sqrt{np_0\, q_o}} \quad \text{where } r > np_0$$

where $p_0 = 0.5$ and $q_0 = 1 - 0.5 = 0.5$

The decision about H_0 is taken in a usual manner as described with parametric tests.

Q. 17 How to resolve the problem of zero differences in sign test?

Ans. In practice zero differences seldom occur. To resolve this problem, the best way is to discard zero differences and reduce the sample size by that number. Another way is to count half zeros as positive and half zeros as negative.

Q. 18 Following are the yields of maize in q/ha recorded from an experiment and arranged in ascending order with median $M = 20$.

15.4, 16.4, 17.3, 18.2, 19.2, 20.9, 22.7, 23.6, 24.5.

Test H_0: $M = 20$ vs. H_1 : $M \neq 20$ at $\alpha = 0.05$

[Given: $P(x \leq 4) = 0.50$]

Ans. To test H_0, we find the difference $(X - 20)$ and write their signs.

$$- - - - - + + + +$$

Here $n = 9$, $r = 4$

$P(x \leq 4) = 0.5$ is greater than $\alpha = 0.05$. Hence H_0 is not rejected at 5 per cent level of significance.

Q. 19 How Wilcoxon's signed-rank test differ from sign test and how to perform it?

Ans. Ordinary sign test was based only on the direction of differences ignoring their magnitudes. But Wilcoxon's signed rank test takes into consideration, the both. This test is more sensitive and powerful than ordinary sign test.

To perform the test for $H_0 : M = M_0$ vs. H_1: $M \neq M_0$, find the differences $d_i = X_{(i)} - M_0$ for $i = 1, 2, \ldots, n$. d_i will be distributed symmetrically about the median zero so that +ve and –ve differences of equal absolute magnitude have equal probabilities of occurrences. The steps of the test are as follows:

Step-1. Arrange the differences in ascending order ignoring the sign and rank them from 1 to n.

Step-2. Now assign the signs to the ranks which the original differences possessed.

Step-3. Suppose the sum of ranks of +ve d_i's is T^+ and that of ranks of –ve d_i's is T^-. For a symmetric distribution, it is expected that T^+ and T^- will approxiamtely be equal.

Also $$T^+ + T^- = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Choose smaller of the T^+ and T^- for conducting the test. Suppose T^+ is smaller.

Step-4. Denote the rank of $|d_i|$ by $r(|d_i|)$ and a variable Z_i such that,

$$Z_i = \begin{bmatrix} 1 & \text{if } d_i > 0 \\ 0 & \text{if } d_i < 0 \end{bmatrix}$$

Therefore, $$T^+ = \sum_{i=1}^{n} Z_i r(|d_i|)$$

Step-5. If we take the subscript on the original sample such that all $|d_i|$ for $i = 1, 2, \ldots, n$ are ordered statistics replace $r(|d_i|)$ by i and Z_i by $Z_{(i)}$ where

$$Z_{(i)} = \begin{bmatrix} 1 \text{ if } d_i \text{ with rank } i \text{ is } +\text{ve} \\ 0 \text{ if } d_i \text{ with rank } i \text{ is } -\text{ve} \end{bmatrix}$$

Then, $$T^+ = \sum_{i=1}^{n} i Z_{(i)}$$

$Z_{(i)}$ are independent Bernoulli variables but are not identically distributed. $Z_{(i)}$ has mean p_i and variance $p_i q_i$ and Cov $(Z_{(i)}, Z_{(j)}) = 0$ for $i \neq j$. T^+ has mean

$\sum_{i=1}^{n} i\, p_i$ and variance $\sum_{i=1}^{n} i\, p_i (1 - p_i)$.

Under $H_0, p_i = \frac{1}{2}$ and hence

$$E(T^+) = \frac{1}{2}\sum_{i=1}^{n} i = \frac{n(n+1)}{4}$$

and

$$\text{var}(T^+) = \frac{1}{4}\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{24}$$

If T^- is smaller, same treatment can be given. Let $T = \min(T^+, T^-)$. If T_α is a number such that $P(T < T_\alpha) = \alpha$ (the level of significance). The test criteria for testing

$$H_0 : M = M_0 \quad \text{vs.} \quad H_1 : M \neq M_0$$

is, find the critical value of T from the table for the sample size n and prefixed level of significance α. If $T^+ < T_\alpha$, reject H_0, otherwise accept H_0. If the alternative hypothesis leads to one-tailed test, the critical value of T from the table be consulted for given α for one-tailed test.

Q. 20 For the problem given in Q. No. 18, test

$$H_0 : M = 20 \quad \text{vs.} \quad H_1 : M > 20.$$

[Given $T_{0.05,\,9} = 6$]

Ans. The differences $X_{(i)} - 20$ are:

−4.6, −3.6, −2.7, −1.8, −0.8, 0.9, 2.7, 3.6, 4.5.

The ordered sequence of numbers ignoring the sign and their ranks with original signs are as follows:

0.8, 0.9, 1.8, 2.7, 2.7, 3.6, 3.6, 4.5, 4.6

−1, 2, −3, 4.5, −4.5, 6.5, −6.5, 8., −9

Thus, $T^+ = 21$ and $T^- = 24$

$T^+ = 21 > 6$ (Table value), so we accpt H_0. It means the median yield of maize in general is 20 q/ha.

Q. 21 Define a run in a sequence of symbols.

Ans. A run is a sequence of symbols followed and preceded by other type of symbols or no symbols. For example, a sequence FMMFFMMMFF of symbols F & M has five runs.

Q. 22 What indication can one get from the number of runs?

Ans. The number of runs in a sequence are indicative of randomness. Any set pattern of symbols in a sequence shows lack of randomness. In other words too many or too less runs show lack of randomness.

Q. 23 Give runs test for randomness.

Ans. Let **a** and **b** be the symbols to show the kind of items, individuals or numbers forming a sequence. The hypothesis, H_0 : the symbols occur in random order vs. H_1 : the symbols occur in a set pattern, can be tested by runs test. The test procedure is as follows:

Suppose the sample of size n contains n_1 symbols of one kind say, a and n_2 symbols of the other kind say, b. Thus $n_1 + n_2 = n$. Also suppose, the number of runs of **a** symbols is r_1 and of **b** symbols r_2 and $r_1 + r_2 = r$.

To decide about H_0, the value of r is compared with the critical number of runs obtained from tables. These tables provide lower and upper critical values of the number of runs. If the observed number of runs in a sample lies in between these critical values, H_0 is not rejected and if outside these critical values, H_0 is rejected.

Q. 24 Following is a sequence of heads (H) and tails (T) in tossing of a coin 14 times.

HTTHHHTHTTHHTH

Test whether the heads, and tails occur in random order. [Given: For $\alpha = 0.05$, $r_L = 2$, $r_U = 12$]

Ans. For the given sequence,

The sample size,	$n = 14$
No. of heads,	$n_1 = 8$
No. of tails,	$n_2 = 6$
No. of runs of H,	$r_1 = 5$
No. of runs of T,	$r_2 = 4$

Thus, $r = 5 + 4 = 9$

Since the observed value of $r = 9$ lies between the critical values 3 and 12, we accept H_0. It means that the heads and tails occur in random order or it can be said that the coin is unbiased.

Q. 25 How can one use the Kolmogorov-Smirnov test for two sample problem?

Ans. When two samples are drawn from two continuous populations F_1 and F_2, it is desired to test whether two samples have come from identical populations, *i.e.*, one wants to test,

$$H_0: F_1(x) = F_2(x) \text{ for all } x$$

$$\text{vs. } H_1: F_1(x) \neq F_2(x) \text{ for some } x$$

Under the *K-S* test, the decision about H_0 is taken on the basis of the distance between the empirical distributions of the two sample. If $S_{n_1}(x)$ and $S_{n_2}(x)$ are the empirical distributions of two samples of sizes n_1 and n_2 from the populations F_1 and F_2 respectively, the *K-S* statistic is,

$$D_{n_1,n_2} = \max_{\text{over all } x} \left|S_{n_1}(x) - S_{n_2}(x)\right|$$

Decision about H_0 is taken by comparing D_{n_1,n_2} with critical value of D for n_1 and n_2 sample sizes and α level of significance from the corresponding table in the usual manner.

Q. 26 In what way the ordinary sign test can be used for paired samples?

Ans. Let $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ be the paired sample observations.

Suppose $\quad d_i = x_i - y_i$ for $i = 1, 2, \ldots, n$.

Here it is assumed that population of d_i's is continuous and obviously $P\,(d = M_d) = 0$.

The hypothesis to be tested is

$$H_0: P\left(d > M_d^0\right) = P\left(d < M_d^0\right)$$

$$\text{vs. } H_1: P\left(d > M_d^0\right) \neq P\left(d < M_d^0\right)$$

So in two sample case, instead of dealing with the sample values, one has to deal with the differences d and perform the test in the same way as we do for one sample case.

Q. 27 How can one apply Wilcoxon's signed-ranked test for matched-paired samples?

Ans. Under matched-paired samples, the differences d within n paired sample values (x_i, y_i) for $i = 1, 2, \ldots, n$ are assumed to have come from continuous and symmetric population differences. If M_d is the median of the population of differences and expected to possess a known value M_d^0, we test

$$H_0: M_d = M_d^0 \quad \text{vs.} \quad H_1: M_d \neq M_d^0$$

Now rest of the test procedure remains same as in case of one sample test. Here, our variable is d instead of X.

Q. 28 How to test the equality of location parameters of two populations by the median test?

Ans. The median test for testing the equality of location parameters of two populations does not require the restriction of paired observation as we have in matched pair sign test. It is a more general test. If we have random samples $X_1, X_2, \ldots, X_{n_1}$ and

$Y_1, Y_2, \ldots, Y_{n_2}$ from two populations F_X and F_Y respectively, then we have to test

$$H_0: F_X(x) = F_Y(x) \quad \text{for all } x$$

$$\text{vs.} \quad H_1: F_X(x) = F_Y(x - \delta) \quad \text{for all } x \text{ and } \delta \neq 0$$

where δ is the shift in the location parameter which is the median. The procedure for median test is as follows:

Step-1. Combine the two samples and arrange the pooled observations in order.

Step-2. Find the median of the combined samples, say it is θ.

Step-3. Count the number of X's and Y's on the left of θ. Suppose there are u X's and v Y's to the left of θ. Obviously there are $(n_1 - u)\,X$'s and $(n_2 - v)\,Y$'s not to the left of θ.

Step-4. Calculate the probability of the event that $u + v = t$ observations are on the left of θ. If p is the probability for any observation to be on the left of θ, then $P(u+v=t)$ is

$$f(t) = \binom{n_1 + n_2}{t} p^t (1-q)^{n_1+n_2-t}$$

for $t = 0, 1, 2, \ldots, (n_1 + n_2)$

Also the conditional distribution of u given t is,

$$f(u|t) = \frac{\binom{n_1}{u}\binom{n_2}{v}}{\binom{n_1+n_2}{t}}$$

for $u = 0, 1, 2, \ldots, n_1$

It is worth noting that $f(u|t)$ follows hypergeometric distribution. This expression gives the conditional probability of u where $t = \dfrac{n}{2}$ for even n and $t = \dfrac{n-1}{2}$ for odd n.

Step-5. The test based on u, the number of X-observations which are less than θ in the combined sample is called median test. Under H_0, the probability distribution of $U = u$ is,

$$f_U(u) = \binom{n_1}{u}\binom{n_2}{v} \Big/ \binom{n_1+n_2}{t}$$

for $u = 0, 1, 2, \ldots, n_1$ and $t = \dfrac{n}{2}$.

Step-6. The decision criteria for the test of size α are:

reject H_0 if $u \le c$ or $u \ge c'$ for $H_1: \delta \neq 0$

reject H_0 if $u \le c'_\alpha$ for $H_1: \delta > 0$

reject H_0 if $u \ge c_\alpha$ for $H_1: \delta < 0$

where $P(u \le c) + P(U \ge c') = \alpha$

Also c_α and c'_α are respectively the largest and smallest integers such that $P(U \le c_\alpha) \le \alpha$ and $P(U \ge c'_\alpha) \le \alpha$. The critical values of c, c', c_α and c'_α are also tabulated by Lieberman and Owen in1961. As an alternative, a simple approach to decide about H_0 is to calculate the probability $f_U(u)$ and compare it with α, the prefixed level of significance. For a two-tailed test $f_U(u) \le \alpha/2$, reject H_0, otherwise accept H_0.

Again for a one-tailed test, if $f_U(u) \le \alpha$, reject H_0, otherwise not.

If $n \ge 10$, H_0 can be tested by Z-test. The variable U is distributed with mean $= \dfrac{n_1 t}{n}$ and variance $= \dfrac{n_1 n_2 (n-t)}{n^2(n-1)}$. Hence for large n,

$$Z = \frac{U - \dfrac{n_1 t}{n}}{\sqrt{\dfrac{n_1 n_2 (n-t)}{n^2 (n-1)}}}$$

where $Z \sim N(0, 1)$

Decision about H_0 can be taken in the usual way.

Q. 29 Is it possible to apply the median test for testing the identicalness of more than two-sampled populations? If yes, how to perform this test?

Ans. The median test is applicable for testing the identicalness of more than two populations. The procedure for performing the median test is same as in the case of two populations. Let us consider in general k populations and random samples of sizes $n_1, n_2, \ldots, n_k$ from the populations $F_1(x), F_2(x), \ldots, F_k(x)$ respectively.

Here we test

$$H_0: \ F_1(x) = F_2(x) = \ldots = F_k(x) \text{ in resepct of median}$$

vs. H_1 : at least two of them are not same.

As a test procedure, pool all the samples and find the median θ. Then count how many observations for each sample are to the left of θ. Consider a random variable U_i denoting the number of observations which are to the left of θ in the i^{th} sample for $i = 1, 2, \ldots, k$.

Supposing

$$r = \sum_{i=1}^{k} u_i = \begin{bmatrix} \dfrac{N}{2} & \text{if } N \text{ is even} \\ \dfrac{N-1}{2} & \text{if } N \text{ is odd} \end{bmatrix}$$

where $N = \sum_i n_i$

Under H_0, all $\binom{N}{r}$ possible sets of observations are equally likely. Hence,

$$f(n_1, u_2, \ldots, u_k | r) = \frac{\binom{n_1}{u_1}\binom{n_2}{u_2}\cdots\binom{n_k}{u_k}}{\binom{N}{r}}$$

Under H_0, all u_i should be equal. Hence, if one or more u_i's differ largely from their expected u_i, H_0 should be rejected. But this approach is vague. So the best thing is to calculate the probabilites for observed and extreme values of $u_1, u_2, \ldots, u_k$ and cumulate these probabilities. In practice, if the sum of probabilities so calculated is less than the desired level α, reject H_0, otherwise H_0 is not rejected.

Calculation of probabilities is a tedious job. Hence, an alternative approach is better provided the number of observations N is not less than 25 and no individual sample has less than 5 observations. To test H_0, we apply the Chi-square test. The test statistic for $2K$ categories is

$$q = \sum_{i=1}^{K}\sum_{j=1}^{2} \frac{\left(f_{ij} - e_{ij}\right)^2}{e_{ij}}$$

where $f_{i1} = u_i =$ no. of observations in the i^{th} sample to the left θ.

and $f_{i2} = n_i - u_i =$ no. of observations in the i^{th} sample not to the left θ.

As a common procedure,

$$e_{i1} = \frac{r}{N} n_i$$

$$e_{i2} = \frac{N-r}{N} n_i$$

Substituting the value of f_{ij} and e_{ij}, the test statistic

$$q = \frac{N^2}{r(N-r)} \sum_{i=1}^{K} \frac{\left(u_i - \frac{r}{N} n_i\right)^2}{n_i}$$

q is approximately distributed as χ^2 with $(K - 1)$ d.f. The decision about H_0 can be taken in the usual manner.

It has been found that there is an upward bias in q as a χ^2-approximation. Hence a correction is incorporated in q, *i.e.*, to multiply q by $\left(\frac{N-1}{N}\right)$. This makes the approximation more verile.

Q. 30 The pulse rates of 6 persons without any medication and of 7 persons after 3 days of medication were as follows:

Pulse rate
without
medication (X): 120, 104, 72, 182, 88, 96
Pulse rate
after 3 days
medication (Y): 122, 108, 105, 130, 140, 136, 84.

Test whether the distribution of pulse rate of persons before and after medication is same at $\alpha = 0.05$.

Ans. To test $H_0: F_X(x) = F_Y(x)$ vs. $H_1: F_X(x) =$

$F_Y(x-\alpha)$, we combine the two samples and find the median θ of the combined samples.

72, 84, 88, 96, 104, 105, 108, 120, 122, 130, 136, 140, 182.

Here, $\theta = 108, n_1 = 6, n_2 = 7, u = 4, v = 2, t = 6$

The probability

$$f_U(u) = \frac{\binom{6}{4}\binom{7}{3}}{\binom{13}{6}}$$

$$= 0.184$$

Since $f_U(u) = 0.184 > \alpha/2$, we accept H_0.

This leads to the conclusion that the distribution of pulse rates before and after medication with regard to median are same.

Q. 31 Describe Wald-Wolfowitz runs test for identicalness of two populations.

Ans. Wald-Wolfowitz runs test uses the data of two random samples $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ of sizes m and n from two populations F_1 and F_2 respectively.

The hypothesis under test is

H_0 : the populations F_1 and F_2 are identical

vs. H_1 : the two populations F_1 and F_2 differ in any respect whatsoever, the measure of location or dispersion.

Notationally we test,

$H_0: \ F_X(x) = F_Y(x)$ for all x

vs. $H_1: \ F_X(x) \neq F_Y(x)$ for some x

Suppose $m = 5$ and $n = 6$. Let the combined ordered statistic is

$$XX|YY|X|Y|X|YYY|X.$$

So there are 7 runs in all.

Suppose the total number of runs in the pooled ordered arrangement of mX's and nY's are denoted by the random variable R. Now to take a decision about H_0, the critical region for Wald-Wolfowitz runs test of level α is given by

$$R \le r_\alpha$$

where r_α is chosen to the largest integer such that under $H_0, P(R \le r_\alpha) \le \alpha$.

The decision rule is, reject H_0 at $\alpha/2 = 0.025$ (say) if the computed value of R is less than or equal to the tabulated value of r for m and n sample sizes.

Large sample approximation. When either m or n is greater than 20, critical value of r is not available in the table. Hence in such a situation, distribution of R can be approximated to normal distribution with,

$$\text{mean}(R) = \frac{2mn}{m+n} + 1$$

and
$$\text{var}(R) = \frac{2mn(2mn - m - n)}{(m+n)^2(m+n-1)}$$

Thus, the normal deviate

$$Z = \frac{R - \text{mean}(R)}{\sqrt{\text{var}(R)}}$$

where $Z \sim N(0, 1)$.

The decision about H_0 is taken in the usual manner.

Q. 32 Explicate Mann-Whitney U-test for testing the identicalness of two populations.

Ans. Let the two random samples from two populations be $X_1, X_2, \ldots, X_{n_1}$ and $Y_1, Y_2, \ldots, Y_{n_2}$.

The hypothesis whether the two samples have come from two identical populations can be tested by Mann-Whitney U-test. It is a good substitute for t-test when the conditions imposed on parent populations are not met. This test is based on the criterion of the magnitude of Y's in relation to X's or vice versa.

Here we test

$H_0: \ F_X(x) = F_Y(x)$ for all x

vs. $H_1: \ F_X(x) \neq F_Y(x)$ for some x.

We may use one-sided test if there is some prior information whether $F_X(x) > F_Y(x)$ or $F_X(x) < F_Y(x)$. Also assume that $F_X(x)$ and $F_Y(x)$ both are continuous. It avoids the chance of tied observations.

Under the Mann-Whitney procedure, combine the data of two samples and arrange them in ascending order. Keep track which observation belongs to which sample. Statistic U is defined as the number of times

Y's precede the X's in the combined sequence of $(n_1 + n_2)$ variate values. Here we define an indicator variable D_{ij} to compute U

$$\text{where,}\quad D_{ij} = \begin{bmatrix} 1 \text{ if } Y_j < X_i \text{ for } i = 1, 2, \ldots, n_1 \\ 0 \text{ if } Y_j > X_i \text{ for } j = 1, 2, \ldots, n_2 \end{bmatrix}$$

$$\text{and}\qquad U = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} D_{ij}$$

Obviously D_{ij} are the Bernoulli variables with $P(D_{ij} = 1) = P(Y < X) = \pi$.

If H_0 is true for all x, $P(X < Y) = P(Y < X) = \dfrac{1}{2} = \pi$.

Hence, we have to test

$$H_0 : \pi = \frac{1}{2} \text{ for all } x, \quad \text{vs. } H_1 : \pi \neq \frac{1}{2} \text{ for some } x.$$

Decision about H_0 can be taken by comparing the calculated value of U with the tabulated value of U for sample sizes n_1 and n_2, and at α level of significance.

Alternative approach. The value of U can also be obtained by considering the sum of the ranks S_2 of Y's (the case when Y precedes X) in the ordered combined sequence. The formula for U is,

$$U = n_1 n_2 + \frac{n_2 (n_2 + 1)}{2} - S_2$$

Similarly, if the ranks of X's are counted (the case when X precedes Y), the value of U' can be obtained by the formula,

$$U' = n_1 n_2 + \frac{n_1 (n_1 + 1)}{2} - S_1$$

where S_1 is the sum of rank's of X's in the combined sequence of X's and Y's.

It is interesting to point out that the value of U obtained by the use of indicator variable D_{ij} or directly by the formula are same. Also there exists a relation between U and U' which is given as,

$$U' = n_1 n_2 - U$$

This relation helps to obtain U' when U is known and vice-versa without redoing the whole calculations and is often required for getting the probabilities under Mann-Whitney U-test or the critical value of U for different values of n_1 and n_2.

Large sample case. If n_1 and n_2 are so large that the tabulated probabilities or critical values of U are not available, then the distribution of U can be approximated to normal distribution with

$$\text{mean } (U) = \frac{n_1 n_2}{2} \text{ and var } (U) = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12}$$

The test statistic is,

$$Z = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}}$$

where $Z \sim N(0, 1)$

The decision about H_0 can be taken in the usual way.

Q. 33 Given the score of two groups of persons, the one under placebo and other under drug are as follows:

Scores under placebo (X)	Scores under drug (Y)
10	20
13	14
12	7
15	9
16	17
8	18
6	19
	25
	24

Test that the distributions of scores under placebo and under drug are identical.

Ans. We test

$$H_0 : F_X(x) = F_Y(x) \text{ vs. } H_1 : F_X(x) \neq F_Y(x)$$

The combined ordered sequence is,

	6,	7,	8,	9,	10,	12,	13,	14,
	X	Y	X	Y	X	X	X	Y
ranks	1	2	3	4	5	6	7	8
	15,	16,	17,	18,	19,	20,	24,	25
	X	X	Y	Y	Y	Y	Y	Y
ranks	9,	10,	11,	12,	13,	14,	15,	16,

Now find D_{ij} in case, Y precedes X.

$$D_{i1} = 6,\ D_{i2} = 5,\ D_{i3} = 2,\ D_{i4} = 0,\ D_{i5} = 0, \ldots$$
$$U = \Sigma D_{ij} = 6 + 5 + 2 + 0 + 0 = 13$$

For the given question, $n_1 = 7$, $n_2 = 9$.

The sum of ranks of $Y = S_2 = 95$

$$U = 7 \times 9 + \frac{9(9+1)}{2} - 95$$
$$= 13$$

Here it is shown that the value of U obtained in either way remains same.

The decision about H_0 can be taken by comparing the calculated value of U with the tabulated value of U from the table XIV.b, *Basic Statistics* by B.L. Agarwal.

Tabulated value of U for $n_1 = 7$, $n_2 = 9$ and $\alpha = 0.05$ is 12. Since the critical value of U is less than the calculated value, we reject H_0. It leads to the conclusion that distribution of scores under the drug and placebo are not identical.

Q. 34 How can one perform Mcnemar's test for the significance of change?

Ans. Mcnemar's test to see the effect of some treatment, training or advertisement which brings about the change in attitude of individuals. This test is particularly useful when the measurements are on nominal or ordinal scale.

Here we test

H_0: There is no significant change in individuals after the treatment.

vs. H_1: There is a significant change in individuals after the treatment.

As a test procedure, prepare a (2 × 2) contingency table denoting a positive response by +ve sign and a negative response by – sign. So the table will be of the kind

		After Treat.	
		–	+
Before Treat.	+	A	B
	–	C	D

$(1, 1)^{th}$ and $(2, 2)^{th}$ cells give the number of individuals which showed change, *i.e.*, $(A + D)$.

Under null hypothesis, the expected frequencies corresponding to A and D will be $(A + D)/2$ and $(A + D)/2$ respectively. In case of contingency table, χ^2 test is applicable. Here we confine our attention to two cells which indicate a change. Thus the Chi-square test statistic is,

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$= \frac{\left(A - \frac{A+D}{2}\right)^2}{(A+D)/2} + \frac{\left(D - \frac{A+D}{2}\right)^2}{(A+D)/2}$$

$$= \frac{(A-D)^2}{(A+D)}$$

χ^2 has 1 d.f.

When the expected frequency in either cell A or D or both is small, use Yates' correction. Under Yates' correction, the statistic

$$\chi^2 = \frac{(|A-D|-1)^2}{(A+D)}$$

The decision about H_0 is taken in the usual manner.

Q. 35 An opinion survey was conducted about the GATT treaty from a group of 25 persons, whether they favour (+) it or against (–) it. They were exposed to certain lectures and discussions for a few weeks time. Again their opinion were taken about the GATT treaty. The results were as follows:

		After Lect. & Discussion	
		–	+
Initially	+	4	5
	–	2	14

Test whether there is a significant change in the opinion of persons. [Given: $\chi^2_{0.05,1} = 3.841$]

Ans. Here we test

H_0 : There is no change in opinion after lectures and discussions.

From the table A.9 of Daniel *Applied Nonparametric Statistics*, $M'_{.025,(6,7)} = 38$ and $M''_{0.975,(6,7)} = 130$. The calculated value of $M > 38$ and < 130. Hence, we accept H_0. It means that the two populations of pulse rate with and without medication are identical with regard to measure of dispersion.

Q. 38 Explicate Moses' test for equality of two population dispersion measures.

Ans. Moses' test is more general than Mood's test in the sense that one has not to assume the equality of medians of the two population. *Secondly*, it is not necessary that $n_1 < n_2$. Rest of the assumptions remain the same.

Here we have to test,

$$H_0:\sigma_1 = \sigma_2 \text{ vs. } H_1:\sigma_1 \neq \sigma_2$$

or $$H_0:\sigma_1 \leq \sigma_2 \text{ vs. } H_1:\sigma_1 > \sigma_2$$

or $$H_0:\sigma_1 \geq \sigma_2 \text{ vs. } H_1:\sigma_1 < \sigma_2$$

Test procedure is totally different from all procedures. In this we have two independent samples say, sample $X: X_1, X_2, \ldots, X_{n_1}$ and sample $Y: Y_1, Y_2, \ldots, Y_{n_2}$.

Different steps for the test procedure are:

Step-1. Divide X's at random into sub-samples of arbitrary chosen size k. Discard left over observations. Let the number of these sub-samples is m_1.

Step-2. Repeat step-1 for Y's. Let the number of sub-samples of Y's is m_2.

Step-3. For each subsample of X's, calculate the sum of squares of the deviations from mean *i.e.* $\Sigma\left(X-\overline{X}\right)^2$.

Step-4. For each sub-sample of Y's, calculate $\Sigma\left(Y-\overline{Y}\right)^2$.

Now the quantities $\Sigma\left(X-\overline{X}\right)^2$ and $\Sigma\left(Y-\overline{Y}\right)^2$ are our variate values with m_1 and m_2 sample sizes randomly. Let them be denoted as $u_1, u_2, \ldots, u_{m_1}$ and $v_1, v_2, \ldots, v_{m_2}$ respectively.

Step-5. Now apply Mann-Whitney test taking u's and v's as two independent samples of sizes m_1 and m_2 respectively.

Step-6. Calculate the statistic,

$$U = S - \frac{m_1(m_1+1)}{2}$$

where, S is the sum of ranks of u's.

Step-7. To decide about H_0, compare the calculated value of U with W of Moses table A.8 in Daniel's *Applied Nonparametric Statistics*.

For a two-sided test reject H_0, if $U \leq W_{\alpha/2,(m_1,m_2)}$ or $U \geq W_{1-\alpha/2,(m_1,m_2)}$, otherwise accept H_0.

For a one-sided test $H_1 : \sigma_1 > \sigma_2$, reject H_0 in favour of H_1 if $U \geq W_{1-\alpha,(m_1,m_2)}$, otherwise accept H_0.

For a one-sided hypothesis $H_1 : \sigma_1 < \sigma_2$, reject H_0 in favour of H_1 if $U \leq W_{\alpha,(m_1,m_2)}$, otherwise accept H_0.

Note: As regards the sub-sample size k, it should be large enough but not more than 10. Also K should be chosen in such a way that m_1 and m_2 are large enough to yield meaningful results.

Q. 39 In a study on goats, the body weight of 3-month-old 26 goats and of 6-month-old 22 goats were as follows:

Body weight (in kg) of 3-month-old goats (X): 13.0, 7.5, 8.0, 11.5, 6.4, 9.0, 12.0, 10.5, 12.8, 10.2, 7.6, 10.1, 13.0, 14.0, 7.4, 7.8, 10.5, 9.7, 12.5, 9.2, 9.7, 11.6, 14.4, 12.8, 12.5, 14.5.

Body weight (in kg) of 6-month old goats (Y): 11.3, 11.7, 17.0, 10.0, 9.0, 16.4, 15.3, 15.5, 13.8, 12.7, 14.7, 18.0, 12.5, 13.4, 13.7, 17.7, 13.3, 9.6, 14.7, 17.0, 16.5, 15.3.

Test by Moses' method whether two populations of goats have equal dispersion in weights at 5 per cent level of significance.

Ans. Here we test H_0: $\sigma_1 = \sigma_2$ vs. H_1: $\sigma_1 \neq \sigma_2$. Let us take $k = 5$.

So for the given data $m_1 = 5$ and $m_2 = 4$.

Now we draw sub-sample randomly and calculate the sum of squares of the deviations from mean for both the samples.

Sample No.	*Sub-samples (X)*	$\Sigma(X-\bar{X})^2$
1.	10.2, 11.6, 11.8, 7.8, 7.6	21.04
2.	12.5, 9.7, 9.7, 12.0, 7.5	16.29
3.	12.8, 10.2, 13.0, 14.5, 13.0	9.68
4.	6.4, 14.0, 11.5, 8.0, 10.5	35.43
5.	7.5, 9.2, 9.0, 10.5, 12.5	14.05

Sample No.	*Sub-samples (Y)*	$\Sigma(Y-\bar{Y})^2$
1.	11.3, 17.0, 15.3, 16.0, 13.7	19.89
2.	14.7, 18.0, 9.0, 14.7, 13.8	42.01
3.	10.0, 15.5, 16.5, 15.3, 11.7	31.28
4.	13.3, 17.0, 12.5, 9.6, 13.4	27.93

Discarded X-value = 10.1
Discarded Y-values = 12.7, 17.7

Combine the quantities $\Sigma(X-\bar{X})^2$ and $\Sigma(Y-\bar{Y})^2$ and arrange them in ascending order. The quantities for $\Sigma(X-\bar{X})^2$ are underlined to keep track.

Combined order statistics:	$\underline{9.68}$,	$\underline{14.05}$,	$\underline{16.29}$,	19.89,	$\underline{21.04}$,
ranks:	(1)	(2)	(3)	4	(5)
Combined order statistics:	27.93,	31.28,	$\underline{35.43}$,	42.01,	
ranks:	6	7	(8)	9	

sum of rank's of u's, *i.e.*, $S = 19$

The test statistic

$$U = 19 - 5(5+1)/2$$
$$= 4$$

Tabulated value of $W_{0.025,(5,4)} = 2$

Since $U = 4 > W_{0.025,(5,4)}$, we reject H_0. It means that the two populations of goats have unequal measures of dispersion.

Q. 40 When do you use Cochran's Q-test and how to apply it?

Ans. Cochran's Q-test is used to test the equality of certain treatments particularly when the observations can be dichotomatised say success coded as 1 and failure coded as 0.

We test

H_0 : The treatments are equally effective

vs. H_1 : All are not equally effective.

In the experiment, the treatments are applied to as many equal size groups of units as the number of treatments. Suppose there are r groups and each group contains k units. The observations can be arranged in a $r \times k$ table as follows:

Groups	*Treatments*						*Total*
	1	*2*	...	*j*	...	*k*	
1	x_{11}	x_{12}	...	x_{1j}	...	x_{1k}	R_1
2	x_{21}	x_{22}	...	x_{2j}	...	x_{2k}	R_2
⋮	⋮						⋮
i	x_{i1}	x_{i2}	...	x_{ij}	...	x_{ik}	R_i
⋮	⋮						⋮
r	x_{r1}	x_{r2}	...	x_{rj}	...	x_{rk}	R_r
Total	C_1	C_2	...	C_j	...	C_k	n

Suppose, $x_{ij} = 1$ if the treatment effect is favourable and

$x_{ij} = 0$ if it is not.

To test H_0, the test statistic

$$Q = \frac{k(k-1)\sum_{j=1}^{k} C_j^2 - (k-1)n^2}{kn - \sum_{i=1}^{r} R_i^2}$$

The statistic Q is approximately distributed as χ^2 with $(k-1)$ d.f. Here a precaution is required while approximating the distribution of Q to χ^2. The number of treatments should be at least 4 and total number of observations be at least 24. If it is less than 24, construct the exact distribution of Q or use special tables prepared by Tata and Brown in 1964 available in a monograph at University of Pennsylvania. Also if there are all 0's or 1's in any group (block), delete that group for the purpose of analysis provided the remaining observations are not less than 24.

Q. 41 Eight groups of a fruit each consisting of 4 fruits were stored, one under four different storage methods. The fruits were examined after a week. If the fruit was putrefied, it was coded as 1 and if not putrefied, then 0. The results of the experiments are presented in the following table.

	Storage Methods				
Groups	*I*	*II*	*III*	*IV*	*Total*
1	0	1	0	1	2
2	1	0	0	0	1
3	1	1	0	0	2
4	1	1	1	0	3
5	0	0	0	1	1
6	0	1	0	0	1
7	1	0	1	1	3
8	0	1	1	0	2
Total	4	5	3	3	15

Test whether there is any difference in storage methods in respect of putrefication.

[Given: $\chi^2_{0.05,3} = 7.81$]

Ans. We test,

H_0: All the storage methods are equally good.

vs. H_1: At least two of them differ significantly.

The test statistic

$$Q = \frac{4(4-1)\left(4^2+5^2+3^2+3^2\right)-3\times 15^2}{4\times 15-\left(2^2+1^2+2^2+3^2+1^2+1^2+3^2+2^2\right)}$$

$$= \frac{708-675}{60-33}$$

$$= 1.22$$

Since the calculated value $\chi^2 = 1.22 < 7.81$, the tabulated value, H_0 is accepted. It leads to the conclusion that all the storage methods are equally effective.

Q. 42 Give Kruskal-Wallis method of analysis for one way classification of data.

Ans. Kruskal-Wali test is one of the most frequently used method in nonparametric statistics for analysing data in one way classification. It is equivalent to one way analysis of variance in parametric methods.

We test the identicalness of k populations (in respect of medians) from which the independent samples have been drawn. There is no restriction on sample sizes.

Assumptions. Kruskal-Wallis test is based on the following assumptions:

1. The observations are independent within and between samples.
2. The variable under study is continuous.
3. The populations are identical except possibly in respect of median.

We test

H_0: All the populations are identical.

vs. H_1: At least one pair of populations do not have the same median.

Let there be k independent samples from k populations of sizes $n_1, n_2, ..., n_k$. The observations in k samples can always be presented in the tabular form as given below:

		Sample Numbers			
1	2	...	i	...	k
x_{11}	x_{21}	...	x_{i1}	...	x_{k1}
x_{12}	x_{22}	...	x_{i2}	...	x_{k2}
$\vdots$	$\vdots$		$\vdots$		$\vdots$
x_{1n_1}	x_{2n_2}	...	x_{in_i}	...	x_{kn_k}

Assign rank to each observation from 1 to $N = \sum_{i=1}^{n} n_i$ by pooling all the sample observations and writing them in ascending order. The sum of ranks is obviously equal to $\frac{N(N+1)}{2}$. Under H_0, the sum of the ranks would be divided in proportion to sample size among k samples.

For the i^{th} sample of size n_i, the expected sum of ranks is

$$\frac{n_i}{N}\frac{N(N+1)}{2} = \frac{n_i(N+1)}{2}$$

Suppose R_i is the actual sum of ranks of observations in sample i.

To test H_0, Kruskal-Wallis test statistic is a weighted sum of squares of deviations of the sum of ranks of the treatments from the expected sum of ranks, using reciprocals of sample size as the weights. The Kruskal-Wallis statistic in notational form is,

$$H = \frac{12}{N(N+1)}\sum_{i=1}^{k}\frac{1}{n_i}\left[R_i - \frac{n_i(N+1)}{2}\right]^2$$

$$= \frac{12}{N(N+1)}\sum_{i=1}^{k}\frac{R_i^2}{n_i} - 3(N+1)$$

The statistic H is approximately distributed as χ^2 with $(k - 1)$ d.f. subject to the condition that n_i should be large, *i.e.*, each n_i should not be less than 5.

The decision about H_0 can be taken in the usual manner.

Note: When $k = 2$, H test reduces to Mann-Whitney U-test. Some people call it Wilcoxon's U-test.

Q. 43 What adjustment has to be made for ties in Kruskal-Wallis H?

Ans. If there are a substantial number of ties, one must adjust H for ties. The adjustment factor

$$C = \frac{\Sigma T}{n\left(n^2 - 1\right)}$$

where $T = (t^3 - t)$ for t, the number of tied observations in a group and Σ is over all such groups. The corrected test statistic,

$$H_C = H/C.$$

Q. 44 Wool yield of three groups of ewe lambs under different feeding regimen (in kg) was as given below.

Feed I *(pasture)*	*Feed II* *Pasture* *+ concentrate*	*Feed III* *Pasture* *+ concentrate* *+ minerals*
8.78	5.16	9.92
6.23	9.64	14.74
7.65	6.52	7.93
5.10	5.38	11.90
5.95	6.80	10.77
8.50		7.37
10.20		7.08
		5.67

From the experimental data, test whether the three types of feeds affect the wool yield.

[Given: $\chi^2_{0.05,2} = 5.99$]

Ans. In this problem we have to test

H_0: The three feeds are equally effective

vs. H_1: At least two feeds do not have the same effect on the production of wool.

To test H_0 we first assign ranks to each yield value by pooling them and tabulate them as given below:

	Ranks	*Sums*	*Sample size*
Feed I	14, 6, 11, 1, 5, 13, 17	67	7
Feed II	2, 15, 7, 3, 8	35	5
Feed III	16, 20, 12, 19, 18, 10, 9, 4	108	8
			$N = 20$

The test statistic

$$H = \frac{12}{20\times 21}\left(\frac{67^2}{7} + \frac{35^2}{5} + \frac{108^2}{8}\right) - 3\times 21$$

$$= 3.98$$

Calculated $\chi^2 = 3.98 < 5.99$, the tabulated value of χ^2 at 5 per cent level of significance and 2 d.f., we do not reject H_0. It means that the three feeds are equally effective.

Q. 45 Delineate Friedman's two way analysis of variance by ranks.

Ans. When the assumptions necessitated for two way analysis of variance in parametric test do not hold good, the data can be analysed by Friedman's nonparametric procedure. The method utilises the ranks within a block. Suppose there are k treatments and r blocks, each block of size k. Each treatment occurs once in each block.

Assumptions for Friedman's test are same as per Kruskal-Wallis test.

The data for r blocks and k treatments can be presented in the following two way table.

Blocks	*Samples (Treatments)*			
	1	2	...	k
1	x_{11}	x_{12}	...	x_{1k}
2	x_{21}	x_{22}	...	x_{2k}
⋮	⋮			
r	x_{r1}	x_{r2}	...	x_{rk}

We test

H_0: All treatments have same effect

vs. H_1: All treatments do not have the same effect.

To test H_0, assign ranks to all observations from 1 to k (lowest to highest) in each block independently. Under H_0, it is expected that the sum of ranks in all columns will almost be the same. It the difference in sum of ranks of the columns is remarkable, it cannot be left to random variation. Hence, we have to delve deep. For this, Friedman gave the following test.

Suppose R_{ij} is the rank of the observation in i^{th} row and j^{th} column for $i = 1, 2, ..., r$ and $j = 1, 2, ..., k$. So the two way table for ranks can be displayed as below:

Blocks	*Treatments*				*Total*
	1	2	...	k	
1	R_{11}	R_{12}	...	R_{1k}	$k(k + 1)/2$
2	R_{21}	R_{22}	...	R_{2k}	$k(k + 1)/2$
⋮	⋮			⋮	⋮
r	R_{r1}	R_{r2}	...	R_{rk}	$k(k + 1)/2$
Total	R_1	R_2	...	R_k	$rk(k + 1)/2$

Friedman's test statistic under H_0 is,

$$F = \frac{12}{rk(k+1)} \sum_{j=1}^{k} R_j^2 - 3r(k+1)$$

The statistic F is approximately distributed as Chi-square with $k - 1$ d.f. Reject H_0, if χ^2_{cal} is greater than or equal to the tabulated value of χ^2 for α level of significance and $(k - 1)$ d.f., otherwise accept H_0.

Q. 46 What adjustment factor should be used in Friedman's test statistic F when there are ample number of tied observations in blocks?

Ans. In case of ample number of tied observations within blocks, an adjustment in Friedman's F-statistic has to be made.

The adjustment factor is

$$C = 1 - \sum_{i=1}^{r} \frac{T_i}{rk\left(k^2 - 1\right)}$$

where $T_i = \Sigma t_i^3 - \Sigma t_i$ where t_i is the number of tied observations for a given rank in the i^{th} block. The adjusted Friedman's test statistic

$$F_C = F/C$$

Q. 47 How can you make multiple comparison in Friedman's test when H_0 is rejected?

Ans. We can make comparison of all possible pairs of treatment effects by the following relation. For any two treatment rank totals R_j and R_j' for $j \neq j'$, the difference is significant if and only if,

$$\left|R_j - R_j'\right| \ge Z\sqrt{\frac{rk(k+1)}{6}}$$

where for α size of the test, Z is the standard normal deviate corresponding to $\alpha/k\,(k-1)$.

Q. 48 Four technicians determined the percent moisture content of the samples of a powder manufactured by five different companies. Their determinations of moisture contents were as follows:

	Moisture Content (%)				
	Companies				
Technicians	*1*	*2*	*3*	*4*	*5*
A	9	10	8	11	12
B	10	11	9	7	12
C	9	10	8	12	11
D	8	11	7	14	12

Test whether the per cent moisture content in the powder of five companies is same.

[Given: $\chi^2_{0.05,4} = 9.49$]

Ans. We have to test

H_0: The moisture content in the powder of five companies is same.

vs. H_1: The moisture content in the powder of at least two companies differ significantly.

Since the data are not normal, it is preferable to apply Friedman's method of two way analysis.

Assigning ranks to each observation for each technician independently, we get the following data of ranks:

	Companies				
Technicians	1	2	3	4	5
A	2	3	1	4	5
B	3	4	2	1	5
C	2	3	1	5	4
D	2	3	1	5	4
Total	9	13	5	15	18

The value of the test statistic

$$F = \frac{12}{4\times5\times6}\left(9^2+13^2+5^2+15^2+18^2\right) - 3\times4\times6$$

$$= \frac{824}{10} - 72$$

$$= 10.4$$

Since the computed value of $F = 10.4$ is greater than the tabulated value of $\chi^2_{0.05,4} = 9.49$, we reject H_0. This shows that the moisture content in the powder of five companies is not same.

Q. 49 How can you perform the Jonckheere-Terpstra test for ordered alternatives?

Ans. The Jonckheere-Terpstra test is a test specially designed for testing the null hypothesis of equality of treatment means against ordered alternatives. Notationally we test

$$H_0: \mu_1 = \mu_2 = \ldots = \mu_k \text{ vs. } H_1: \mu_1 \le \mu_2 \le \ldots \le \mu_k$$

where $\mu_1, \mu_2, \ldots, \mu_k$ are k treatment (sample) means.

Assumptions for this test are same as for Kruskal-Wallis test.

The test statistic is

$$J = \sum_{i<j} U_{ij}$$

where U_{ij} is the number of pairs of observations in which an observation X_{ia} under the treatment a is less than of all values under the treatment b. Also we compare all pairs of different treatments in a sequence.

To draw a decision about H_0, compare the value of computed J with the critical value of J for α level of significance, K treatments (populations) and n_1, $n_2, \ldots, n_k$ sample sizes. Reject H_0, if $J \ge J_{\alpha, n_1, n_2, \ldots, n_k}$, otherwise accept H_0.

Note: If two values are equal in a paired comparison, it gets a value 1/2.

Q. 50 For the problem given in Q. No. 44, test the equality of feeds effect against the assertion that feed I is inferior than feed II and feed II than III.

Ans. Let the feeds I, II and III mean effects be denoted by τ_1, τ_2 and τ_3 respectively.

to be made in rank correlation. If the number of tied observations in X for a particular rank is t_X. There can be more than one t_X in X-sample units. In the same way we can define t_Y. Now define the following values:

$$T_X = \frac{t_X^3 - t_X}{12}, \quad T_Y = \frac{t_Y^3 - t_Y}{12}$$

and $$\sum x^2 = \frac{n^3 - n}{12} - \sum T_X;$$

$$\sum y^2 = \frac{n^3 - n}{12} - \sum T_Y.$$

Σ is over all groups of tied values.

After incorporating the correction for ties, the amended formula for rank correlation is,

$$r_S' = \frac{\Sigma x^2 + \Sigma y^2 - \Sigma d_i^2}{2\sqrt{\Sigma x^2 \, \Sigma y^2}}$$

Q. 55 How can the significance of Spearman's rank correlation be tested?

Ans. Kendall in 1962 derived the frequency function of r_S and gave exact critical value r_S. But the approximate test of r_S which is the same as t-test for Pearsonian correlation coefficient is good enough for all practical purposes. Here we test $H_0{:}\rho_s = 0$ vs. $H_1{:}\rho_s \neq 0$.

The test statistic

$$t = \left[\frac{(n-2) r_s^2}{1 - r_S^2}\right]^{1/2}$$

$$= \frac{r_S\sqrt{n-2}}{\sqrt{1 - r_S^2}}$$

t has $(n-2)$ d.f.

The decision about H_0 is taken in the usual way.

Q. 56 Following are the ranks awarded to seven debators in a competition by two judges.

Debators	A	B	C	D	E	F	G
Ranks by judge I (X)	3	2	1	6	7	4	5
Ranks by judge II (Y)	5	6	3	7	4	2	1

Calculate the rank correlation and test its significance. [Given: $t_{0.05,5} = 2.571$]

Ans. First we find d's which are:

d: −2 −4 −2 −1 3 2 4

Also $$\sum_{i=1}^{7} d_i^2 = 54$$

Thus, $$r_S = 1 - \frac{6 \times 54}{7 \times 48}$$

$$= 0.036$$

To test $H_0{:}\rho_S = 0$ vs. $H_1{:}\rho_S \neq 0$, the statistic

$$t = \frac{0.036 \times \sqrt{7-2}}{\sqrt{1-(0.036)^2}}$$

$$= \frac{0.080}{0.9993}$$

$$= 0.080$$

Calculated value of $t = 0.080 < 2.571$, hence we accept H_0. It means that there is a dissociation between the ranks awarded by two judges.

Q. 57 What is Kendall's rank correlation coefficient and how to measure it?

Ans. Kendall's rank correlation coefficient 'τ' is suitable for the paired ranks as in case of Spearman's rank correlation. The condition is that both the variables X and Y be measured at least on ordinal scale.

One advantage of τ over r_S is that τ can be generalised to a partial correlation coefficient which is not possible in case of r_S.

The procedure for calculating τ consists of the following steps:

Step-1. Arrange the rank of the first set (X) in ascending order and rearrange the ranks of the second set (Y) in such a way that n pairs of rank remain the same.

Step-2. After operating step-1, the ranks of X are in natural order. Now we are left to determine how many pairs of ranks in the set Y are in their natural order and how many are not. A number is said to be in natural order if it is smaller than the succeeding number and is coded as +1, and also if it is greater than its succeeding number then it will not be taken in natural order and will be coded as −1. In this way all $\binom{n}{2}$ pairs of the set (Y) will be considered and assigned the values +1 and −1.

Step-3. Find the sum 'S' of all the coded values.

Step-4. The formula for Kendall's rank correlation coefficient τ is,

$$\tau = \frac{S}{\binom{n}{2}} = \frac{\text{Actual value}}{\text{Max. possible value}}$$

$$= \frac{2S}{n(n-1)}$$

Q. 58 Give the properties of Kendall's correlation τ.

Ans. Properties of Kendall's τ are:

(i) The range of τ is −1 to 1.

(ii) For large n, there exists an approximate relation between τ and r_S. The relation is,

$$\tau = \frac{2}{3} r_S$$

(iii) If $\tau = 1$, it means that there is perfect correspondence between the rankings awarded by two judges or the rankings based on the scores earned by the individuals.

(iv) If $\tau = -1$, it reveals that the ranking of X are just in reverse order of the rankings of Y.

(v) The distribution of τ approaches to normality more rapidly than r_S.

(vi) Calculation of τ is considered more cumbersome than r_S.

(vii) An important difference between τ and r_S is that τ provides an unbiased estimate of population rank correlation whereas r_S does not.

(viii) The distribution of $\hat{\tau}$ tends to normal more rapidly than r_S. So for moderate size samples, $\hat{\tau}$ may provide a more reliable z-test statistic than r_S.

(ix) For the same set of data, τ and r_S provide different numerical values but on testing hypothesis about them usually lead to the same conclusion.

Q. 59 How can the problem of tied observations be resolved while calculating Kendall's rank correlation τ?

Ans. Many times it is possible to grade two or more individuals differently and hence they receive the same rank. The ranks to the tied observations are assigned by using the midrank method. Also there can be more than one group of tied observations in X or Y or both. Hence, an adjustment is made in the formula for τ.

The formula after adjustment is,

$$\tau = \frac{S}{\sqrt{\frac{1}{2}n(n-1) - \Sigma T_X}\sqrt{\frac{1}{2}n(n-1) - \Sigma T_Y}}$$

where, $T_X = \frac{t_X(t_X - 1)}{2}$; t_X being the number of tied observations in one group of the variable X. Also there may be more than one such group.

Similarly $T_Y = \frac{t_Y(t_Y - 1)}{2}$ for the variable Y.

Q. 60 How can the significance of τ be tested?

Ans. Test of significance of τ means testing H_0: $\tau = 0$, *i.e.*, no correspondence between the ranks of X and Y.

If no correspondence exists, then for the natural order of ranks of X, all possible corresponding arrangements of Y are equally likely. Thus, H_0 can be tested under three alternative hypotheses namely,

H_{A+}: Direct association (consider right tailed probability)

H_{A-}: Inverse association (consider left tailed probability)

H_A: No association (consider two-tailed probability)

For the test of H_0 vs. H_A's of size α, direct tables A.20 of Daniel's *Applied Nonparametric Statistics* have been provided for n and α. This table gives the value of τ^*, the critical value of τ. If computed $\tau \geq \tau^*$ reject H_0, otherwise not.

Because of the symmetry of the distribution of τ, the same table is being used for either right or left tailed probability. As an usual practice, for two-sided test, we consult the table for n and $\alpha/2$.

Also for one-tailed test, consult the table for n and α and get the value of τ^*.

If computed τ is positive and greater than τ^*, reject H_0, otherwise not.

If computed τ is negative and smaller than $-\tau^*$, reject H_0, otherwise not.

Again for large n, the distribution of τ can be approximated to normal distribution. For $n > 10$, τ is taken to be distributed normally with mean,

$$\mu_\tau = 0$$

and variance, $$\sigma_\tau^2 = \frac{2(2n+5)}{9n(n-1)}$$

Hence, $$Z = \frac{\tau - 0}{\sqrt{\dfrac{2(2n+5)}{9n(n-1)}}}$$

or in terms of S,

$$Z = \frac{S}{\sqrt{n(n-1)(2n+5)/18}}$$

The decision about H_0 is taken in the usual way.

Q. 61 For the problem given in Q. No. 56, calculate Kendall's rank correlation τ and test its significance.

Ans. We write below the ranks of X in natural order and ranks of Y correspondingly.

X:	1	2	3	4	5	6	7
Y:	3	6	5	2	1	7	4

For this problem, $n = 7$

For S, take the rank 3 and give +1 or −1 value for all pairs with subsequent ranks of Y. $3 < 6$, give a number +1 ; $3 < 5$, again +1 ; $3 > 2$, give a number −1 and so on. Then choose 6 and take the pairs (6, 5), (6, 2), (6, 1), (6, 7) and (6, 4) and continue the process till we reach the last pair (7, 4). Proceedings in this manner,

$$\begin{aligned} S &= (+1 + 1 - 1 - 1 + 1 + 1) + (-1 - 1 - 1 + 1 - 1) \\ &\quad + (-1 - 1 + 1 - 1) + (-1 + 1 + 1) \\ &\quad + (+1 + 1) + (-1) \\ &= 2 - 3 - 2 + 1 + 2 - 1 \\ &= -1 \end{aligned}$$

Thus, $$\tau = \frac{-2 \times 1}{7 \times 6} = -0.048$$

To test the significance of τ, we test

$$H_0 : \tau = 0 \text{ vs. } H_1 : \tau \neq 0$$

From the table A.2 of Daniel's *Applied Nonparametric Statistics* for $n = 7$ and $\frac{\alpha}{2} = 0.025$, $\tau^* = 0.714$.

Since $-0.048 \not< -0.714$, we accept H_0. It reveals that there is no association amongst the ranks awarded by two judges.

Q. 62 Give the formula for calculating the Kendall's partial rank correlation coefficient between two variables X and Y eliminating the effect of the third variable Z.

Ans. The formula for calculating Kendall's partial rank correlation coefficient $\tau_{XY.Z}$ is analogous to formula for partial correlation coefficient $r_{12.3}$.

Thus,

$$\tau_{XY\cdot Z} = \frac{\tau_{XY} - \tau_{YZ}\,\tau_{XZ}}{\sqrt{\left(1-\tau_{YZ}^2\right)\left(1-\tau_{XZ}^2\right)}}$$

where $\tau_{XY}, \tau_{XZ}, \tau_{YZ}$ are Kendall's rank correlations. The value of $\tau_{XY\cdot Z}$ lies between −1 and +1. Similar formulae can be given for $\tau_{XZ\cdot Y}$ and $\tau_{YZ\cdot X}$.

Q. 63 What is coefficient of concordance and how is it calculated?

Ans. The measure for knowing the agreement or concordance between k ($k \geq 3$) sets of rankings of the same n individuals awarded by k judges or the same n individuals being measured for k variates and then ranked was given by M.G. Kendall.

As a general discussion, one can easily conceive that if all the k judges are in perfect agreement, the sum of the ranks of n individuals would be $1k$, $2k$, ..., nk and if they totally disagree, then the sum of ranks of individuals would be almost equal. From this, it is evident that the degree of agreement between the ranks awarded by k judges is reflected by the degree of variation between the n sums of ranks. So we find the sum of squares of the deviations of the actual n rank totals from the mean.

Under perfect agreement, the sum of squares of the deviations of the actual n rank totals from the average column total will be constant equal to,

$$KS_t = \sum_{j=1}^{n}\left[jk - \frac{k(n+1)}{2}\right]^2$$

$$= \frac{k^2 n(n^2-1)}{12}$$

The actual sum of squares of the deviations of observed columns totals from the average column total is equal to

$$S = \sum_{j=1}^{n}\left[R_j - \frac{k(n+1)}{2}\right]^2$$

where R_j is the sum of actual ranks of the j^{th} individual for $j = 1, 2, ..., n$.

The coefficient of concordance

$$W = \frac{S}{kS_t}$$

$$= \frac{12S}{k^2 n(n^2-1)}$$

The measure 'W' is called the *Kendall's coefficient of concordance.*

Q. 64 Mention the properties of coefficient of concordance 'W'.

Ans. Properties of W are:

(i) The value of W lies between 0 and 1. It is apparent that W is the ratio of the two sum of squares, and hence, it cannot be negative and greater than 1 because numerator can not exceed the denominator.

(ii) If $W = 0$, it means that there is no agreement between the rankings of the k judges.

(iii) If $W = 1$, it means that there is perfect concordance between the rankings of k sets.

(iv) A value between 0 and 1 can be interpreted accordingly.

(v) There exists a linear relation between average Spearman's rank correlation $r_{S_{av}}$ and coefficient of concordance W. By average of rank correlation $r_{S_{av}}$, we mean the average of r_S of all $\binom{K}{2}$ pairs of rankings. The relation is,

$$r_{S_{av}} = \frac{kW-1}{(k-1)}$$

(vi) W cannot take a value -1 as there is no such thing like perfect discordance when more than two sets of rankings are involved, *i.e.*, the rankings cannot all disagree perfectly.

Q. 65 What adjustments have to be made in W if the ties occur within samples and across samples?

Ans. If the ties occur across samples, no adjustment has to be made. But tied ranks do affect the value of W if they occur within a sample. First break the ties by midrank method and then think of the adjustment in the formula for W. If the proportion of ties is small, the effect on W is negligible and no adjustment is required. If the proportion of ties is large, an adjustment in the formula for W has to be made.

Suppose there are t tied ranks in a group and there are more than one group.

Then we find the quantity

$$T = \frac{\Sigma(t^3-t)}{12}$$

where Σ is taken over all groups of tied observations.

Various steps to determine 'a' and 'b' are as follows:

(1) Plot the paired values on a graph.

(2) Draw a vertical line through the median of X values. If some points fall on this line, shift this line to the right or left in such a way that as nearly as possible, the number of points on either side is equal.

(3) Compute the median of X and Y for both the groups separately. So we have two points, one in each group whose co-ordinates are the median values of X and Y.

(4) Plot both the points obtained in step (3) and join them. This line is our required line.

(5) If the median of the vertical deviations of the points from the line drawn in step (4) is not zero in both the groups, then shift the line to either side maintaining the slope in such a way that the median deviations on both the groups is zero or near zero.

(6) The value of a is the intercept of the final line and $b=(Y_1-Y_2)/(X_1-X_2)$ for any two points (X_1, Y_1) and (X_2, Y_2) on the final line.

Q. 69 Are there any tests available for testing the significance of regression coefficients?

Ans. Earlier it was believed that nonparametric tests are not available for testing the hypothesis about regression coefficients. But in 1950 and onwards, nonparametric tests were evolved by C.W. Brown, A.M. Mood and H. Theil to test the significance of regression coefficients.

Q. 70 Discuss Brown and Mood's method of testing hypothesis about the parameters of a simple regression equation.

Ans. Suppose the regression equation is,

$$Y = \alpha + \beta X + \varepsilon$$

Let $(X_1, Y_1),(X_2, Y_2),\ldots,(X_n, Y_n)$ be the n paired sample observations and on the basis of these observations we have to test the hypothesis about α and β.

The null hypothesis

H_0: $\alpha = \alpha_0$ and $\beta = \beta_0$ is to be tested against

H_1: $\alpha \neq \alpha_0$ and $\beta \neq \beta_0$

where α_0 and β_0 are the known values of regression constants.

Various steps of Brown and Mood's test procedure are:

Step-1. Plot the points (X_i, Y_i), for $i = 1, 2, \ldots, n$; as a scatter diagram.

Step-2. Draw the line $Y = \alpha_0 + \beta_0 X$ on the scatter diagram.

Step-3. Draw a vertical line on the X-axis at the median of X values.

Step-4. Count the number of points n_1 which lie above the hypothesised regression line $Y = \alpha_0 + \beta_0 X$ and to the left of the vertical line. Also count the number of points n_2 which lie to the right of vertical line and above the line $Y = \alpha_0 + \beta_0 X$.

Step-5. Once we know n_1, n_2 and n, the test statistic is,

$$\chi^2 = \frac{8}{n}\left\{\left(n_1 - \frac{n}{4}\right)^2 + \left(n_2 - \frac{n}{4}\right)^2\right\}$$

Here the test statistic χ^2 is distributed with 2 d.f. under H_0 and also if n is not too small. The decision about H_0 is taken in the usual manner.

The main feature of this test is that the decision about α and β is taken simultaneously, *i.e.*, both the values α_0 and β_0 are accepted or both are rejected.

Another interesting point of the test is that it does not require the estimates of α and β.

Q. 71 How can you test the hypothesis about regression coefficient alone by Mood's test?

Ans. Mostly the interest lies in testing the hypothesis about β alone of the regression line $Y = \alpha + \beta X + \varepsilon$. Hence, Mood evolved the nonparametric test for testing $H_0:\beta=\beta_0$ vs. $H_1:\beta\neq\beta_0$.

The test procedure may be described through the following steps.

Suppose (X_i, Y_i) are the n paired sample observations for $i = 1, 2, \ldots, n$.

Step-1. Plot the points (X_i, Y_i) as a scatter diagram.

Step-2. Plot the line $Y = a + \beta_0 X$ where a is median of the deviations $(Y_i - \beta_0 X_i)$. The line $Y = a + \beta_0 X$ can easily be fitted by plotting the line $Y = \beta_0 X$ and then drawing a line parallel to $Y = \beta_0 X$ at the intercept point $Y = a$.

Step-3. Draw a vertical line on X-axis through the median of the X-values.

Step-4. Count the number of points n_1 which lie above the line $Y = a + \beta_0 X$ and to the left of the vertical line.

Step-5. Once we know n_1 and n, the test statistic

$$\chi'^2 = \frac{16}{n}\left(n_1 - \frac{n}{4}\right)^2$$

The statistic χ'^2 follows Chi-square distribution with 1 d.f when n is fairly large say, $n \geq 20$.

Decision about H_0 is taken in the usual way.

Q. 72 What is Theils method for testing the significance of the regression coefficient in a regression line?

Ans. Suppose the simple linear regression model is $Y = \alpha + \beta X + \varepsilon$.

Theils proposed the method of testing

$$H_0{:}\beta = \beta_0 \text{ vs. } H_1{:}\beta \neq \beta_0$$

or $$H_0{:}\beta \leq \beta_0 \text{ vs. } H_1{:}\beta > \beta_0$$

or $$H_0{:}\beta \geq \beta_0 \text{ vs. } H_1{:}\beta < \beta_0$$

where β_0 is a known value of regression coefficient β.

Let $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$ be n pairs of observations on which the test is based.

Theils test is based on Kendall's τ statistic. Kendall's τ is calculated by comparing all possible pairs of the form $(X_i, Y_i - \beta_0 X_i)$ for $i = 1, 2, \ldots, n$.

Various steps of Theils procedure are:

Step-1. Arrange all pairs of modified observations $(X_i, Y_i - \beta_0 X_i)$ in ascending order in respect of X_i alone.

Step-2. Search those pairs of modified observations in which $(Y_i - \beta_0 X_i) < (Y_j - \beta_0 X_j)$ for $i < j$ are in natural order. Let this number of pairs be P also let Q be the number of pairs in which $(Y_i - \beta_0 X_i) > (Y_j - \beta_0 X_j)$ for $i < j$.

Step-3. Find the quantity

$$S = P - Q$$

Step-4. Calculate Kendall τ by the formula,

$$\hat{\tau} = \frac{2S}{n(n-1)}$$

Step-5. To take a decision about H_0, compare $\hat{\tau}$ with the critical value τ^* of τ for n d.f. and $\alpha/2$ level of significance in case of two-sided test available in table A.20 of Daniel's *Applied Nonparametric Statistics.*

If $|\hat{\tau}| \geq \tau^*$ reject H_0, otherwise accept H_0. Again for one-sided test, obtain the value of τ^* for n and α and take the decision in the same way.

Q. 73 How can the confidence interval for the population median using sign test be found out?

Ans. The confidence interval for the population median M with confidence coefficient $(1 - \alpha)$ 100 per cent can be found out in the following manner:

Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the ordered sample statistics. We have to determine two extreme values of X say, X_L and X_U such that the probability of M lying between X_L and X_U is $(1 - \alpha)$, *i.e.*, $P(X_L < M < X_U) = 1 - \alpha$.

If there are k positive differences among n differences $(X_i - M)$ for $i = 1, 2, \ldots, n$, then for all k which satisfy the relation,

$$P\left(k'_{\alpha/2} + 1 \leq k \leq k_{\alpha/2} - 1\right) = 1 - \alpha$$

iff, $$P\left[X_{(k'_{\alpha/2}+1)} < M < X_{(k_{\alpha/2})}\right] = 1 - \alpha$$

Using the normal approximation with continuity correction, we get the values of $k'_{\alpha/2}$ and $k_{\alpha/2}$ by the relations,

16. Most frequently used method of breaking the ties is the ________ method.

17. Average statistics approach is useful in ________ the problem of tied observations.

18. Under least favourable approach for coping with the problem of ties, there is least chance of ________.

19. Range of probability approach in resolving the problem of tied observations is ________ feasible.

20. If the number of observations is large and number of tied values is small, one can ________ the tied values.

21. Before we apply a test it is necessary to establish the ________.

22. Any test always requires ________.

23. The decision about any hypothesis is to be taken according to the ________.

24. When the observations are arranged in ascending or descending order, they are said ________.

25. The probability function of ordered statistics is ________ as that of original variables.

26. All the observations in ordered statistics are ________.

27. Kolmogorov-Smirnov test is based on ________ theorem.

28. Kolmogorov-Smirnov statistics can be used to determine ________ or ________.

29. Ordinary sign test is based on the sign of the deviations from ________.

30. Sign test utilises ________ distribution.

31. Wilcoxon signed-rank test is based on the ________ as well as ________ of the differences.

32. We use large sample Wilcoxon'x signed-rank test when the sample size is greater than ________.

33. A set of like symbols followed and preceded by different kind of symbols or no symbols is called a ________.

34. Runs test is a test of ________.

35. Too many runs are indicative of ________.

36. Too few runs are indicative of ________.

37. Any set pattern of symbols in a sequence shows ________.

38. Kolmogorov-Smirnov test for two samples is based on the distance between two ________ distributions.

39. Kolmogorov-Smirnov test for two samples is based on the ________ between two empirical distributions.

40. Sign test for paired samples is founded on the ________ between paired values.

41. By median test, one can test the equality of ________.

42. In median test the variable u, the number of X's to the left of median in the pooled sample for given t, the total number of observations to the left of the median follows ________ distribution.

43. For large samples, the variable U in median test has mean ________ and variance ________.

44. The critical values for Mann-Whitney U were tabulated by ________ in ________.

45. Procedures for testing identicalness of two populations by Wald-Wolfowitz runs test and for randomness in one sample population are ________.

46. By Mann-Whitney U-test, one tests the identicalness of ________.

47. With usual notations, the relation between U and U' is ________.

48. In case of large samples, Mann-Whitney U is distributed with mean ________ and variance ________.

49. When m or n in Wald-Wolfowitz runs test are greater than 20, the variable R representing the number of runs follows normal distribution with mean ________ and variance ________.

50. Mood's test is meant for testing the equality of two population ________.

51. In Mood's test, two population medians are taken to be ________.

52. Regarding sample size, the condition for the validity of Mood's test is ________.

53. Mood's test statistic M is the ________ of the deviations of ranks of variates of smaller sample from the overall mean rank.

54. Moses' test is meant for testing the equality of two populations ________.

55. In Moses' test, samples on two variates are divided into ________ of equal size.

56. The corrected sum of squares for sub-samples are taken as ________ in Moses' test.

57. Moses' test utilises ___________ test statistic.

58. Mcnemar test for change utilises only ________ frequencies of a (2 × 2) contingency table.

59. The test statistic used in Mcnemar's test follows ________ distribution.

60. When the cell's expected frequency under consideration are small, ________ correction is applied.

61. Cochran's Q-test is used in ________ analysis of data.

62. For Cochran's Q-test, the variate values should be ________.

63. Cochran's Q-statistic is approximately distributed as ________.

64. Kruskal-Wallis test is used for analysis of ________ classification of data.

65. Kruskal-Wallis test statistic is a ________ sum of squares.

66. If there are K sampled populations, Kruskal-Wallis statistic H is distributed as Chi-square with ________ d.f.

67. Friedman's nonparametric procedure is meant for ________ analysis of variance.

68. In Friedman's test, on tests the ________ of treatment effects.

69. Jonckheere-Terpstra test is applicable only for ________.

70. With usual notations, the Jonckheere test statistic is ________.

71. Page's test is applicable only when the alternative hypothesis is ________.

72. Following common notations, Page's test statistics is ________.

73. Rank correlation between sets of ranks was given by ________.

74. Formula for Spearman's rank correlation is ________.

75. The range of rank correlation is ________.

76. Spearman's rank correlation formula has to be ________ for tied values.

77. The significance of rank correlation can be tested by ________.

78. Kendall's rank correlation τ is based on the ________ of ranks.

79. Kendall's τ can be calculated by the formula ________.

80. For large sample size n, the relation between τ and r_S is ________.

81. Kendall's τ lies between ________.

82. $\tau = 1$ shows __________ correspondence between two sets of rankings.

83. Kendall's τ is ________ by tied ranks.

84. For large n, Kendall's τ follows normal distribution with mean ________ and variance ________.

85. Coefficient of concordance is a measure of ________ for three or more sets of rankings.

86. Coefficient of concordance was given by ________.

87. The limits of coefficient of concordance W are from ________.

88. If the coefficient of concordance $W = 1$, it indicates ________ concordance between k sets of rankings.

89. If $W = 0$, it shows ________ between k sets of rankings.

90. The linear relation between average rank

correlation $r_{S_{av}}$ and coefficient of concordance is ________.

91. The value of $W = -1$ ________ exist.

92. $W = -1$ cannot exist because there is noting like ________ between k sets of rankings.

93. Formula for W has ________ for tied ranks.

94. The hypothesis H_0: $W = 0$ can be tested by ________ test.

95. The test statistic for testing H_0: $W = 0$ is ________.

96. Statistics-χ^2 with n individuals for testing H_0: $W = 0$ has ________ d.f.

97. ________ test is meant for testing the significance of regression coefficient of a regression line.

98. Both the parameters of a linear regression model can simultaneously be tested by ________ test.

99. Brown and Mood's test applied for testing the significance of regression parameters utilises ________ statistic.

100. Theils test procedure for testing the significance of regression coefficient is based on ________.

101. The number of runs in the sequence *ABBAAABAABBBAAAB* is ________.

102. The residuals in a regression analysis may be positive or negative. The hypothesis whether these positive and negative residuals occur in random order can be tested by ________.

103. With usual notations, the correction factor in Kruskal-Wallis test statistic H is ________.

104. With usual notations, the correction factor for tied observations in Friedman's F-statistic is ________.

105. Friedman's test statistic adjusted for ties with usual notations is ________.

106. For large number of treatments $k > 8$ and number of blocks r large, Page statistics L is distributed normally with mean ________ and variance ________.

107. The region between $L_n(x)$ and $U_n(x)$, where $L_n(x) = S_n(x) - D_{n,\alpha}$, and $U_n(x) = S_n(x) + D_{n,\alpha}$ are the terms of Kolmogorov-Smirnov test, is called ________.

108. The Kolmogorov-Smirnov test is more ________ than Chi-square test.

109. There is no one-sided test in________ test.

110. Formula for Kendall's partial rank correlation $\tau_{XZ \cdot Y}$ is ________.

111. The range ot Kendall's partial rank correlation coefficient is ________.

112. When the number of treatments $k = 2$ in Kruskal-Wallis test, then H reduces to ________ or ________ test.

113. In Moses' test, the size of the sub-sample should not be more than ________.

114. Following are the ranks awarded by judges I and II to seven contestants.

Contestants:	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Judge I:	1	2	3	4	5	6	7
Judge II:	2	5	4	3	7	1	6

The rank correlation between the rank is ________.

115. For the problem given in Q. No. 114, Kendall's rank correlation τ is equal to ________.

116. A tranquilliser was given to 12 patients and their hours of sleep at night were as follows:

Patient No:	1	2	3	4	5	6
Hours of sleep:	3.9	7.6	10.0	6.1	8.2	6.2
Patient No:	7	8	9	10	11	12
Hours of sleep:	10.1	6.5	6.6	8.5	9.2	7.6

It is claimed that the medicine will induce median sleep of 7 hours. The value of Wilcoxon's signed-rank statistic T^+ is ________.

117. If Kendall's τ's between pairs of variables X,

Y and Z are $\tau_{XY} = 0.58$, $\tau_{XZ} = 0.45$ and $\tau_{YZ} = 0.24$, Kendall's partial rank correlation $\tau_{XY.Z}$ is equal to __________.

118. Ranks of five students in three subjects graded by three teachers grading 5 as best and 1 as worst were as follows:

	Students				
	A	*B*	*C*	*D*	*E*
Teacher *X*	2	1	3	4	5
Teacher *Y*	5	3	4	1	1
Teacher Z	4	2	5	3	1

For the three sets of ranks, the coefficient of concordance W is equal to __________.

119. Given $W = 0.5$, $K = 3$, $n = 6$, the computed value of χ^2 for testing the significance of W is equal to __________.

120. If the rank correlation $r_S = 0.40$ based on two sets of eight rankings, the value of statistic t for testing the significance of rank correlation is __________.

121. An alternative to t-test in nonparametric statistics is __________.

122. A unbiased estimate of population rank correlation coefficient is __________.

123. Spearman's rank correlation r_S is __________ estimate of population rank correlation coefficient.

124. Kendall's τ based on moderate sample size tends to __________.

125. Spearman's r_S and Kendall's τ on testing lead to __________ conclusion.

126. The significance of Kendall's τ is tested by __________.

127. In fitting a regression line through nonparametric method, the __________ of the deviation about the line is zero.

128. The significance of Spearman's r_S can be tested by __________.

129. The method of fitting a regression line through nonparametric approach was given by __________ and __________.

130. Nonparametric approach of fitting a regression line is a __________ method.

SECTION-C

Multiple Choice Questions

Select the correct alternative out of given ones:

Q. 1 Nonparametric methods are based on:
(a) mild assumptions
(b) stringent assumptions
(c) no assumptions
(d) none of the above

Q. 2 Most of the nonparametric methods utilise measurements on:
(a) interval scale
(b) ratio scale
(c) ordinal scale
(d) nominal scale

Q. 3 Ordered statistics is a sequence of:
(a) observations
(b) ranks
(c) natural numbers
(d) integers

Q. 4 Most commonly used assumption about the distribution of a variable is:
(a) continuity of the distribution
(b) symmetry of the distribution
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 5 The term dirty data implies:
(a) contaminated data
(b) inaccurate data
(c) data with outliers
(d) all the above

Q. 6 The concept of asymptotic relative efficiency was given by:

It is expected that the median production of lignite in India is 5 Mn. Tonnes/year. To test H_0: $M = 5.0$, the value of T^- in Wilcoxon's signed rank test is:

(a) 28
(b) 27
(c) 25
(d) 26

Q. 20 For the data given in Q. No. 19, the number of positive deviations for sign test will be:

(a) 5
(b) 6
(c) 4
(d) 7

Q. 21 If there are 10 symbols of two types, equal in number, the maximum possible number of runs is:

(a) 8
(b) 9
(c) 10
(d) none of the above

Q. 22 If there are 10 symbols of two types, equal in number, the minimum possible number of runs is:

(a) 5
(b) 3
(c) 1
(d) none of the above

Q. 23 A sequence of symbols shows lacks of randomness if there are:

(a) Too many runs
(b) Too few runs
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 24 Randomness of a sequence through runs test is adjudged by comparing the observed number of runs with:

(a) two critical values
(b) one upper critical value
(c) one lower critical value
(d) none of the above

Q. 25 If the number of symbols of either type is large, then one can apply:

(a) t-test
(b) normal deviate test
(c) Chi-square test
(d) none of the above

Q. 26 If the sample size in Wald-Wolfowitz runs test is large, the variate R is distributed with mean:

(a) $\dfrac{2m}{m+n}+1$

(b) $\dfrac{2n}{m+n}+1$

(c) $\dfrac{2mn}{n+n}$

(d) $\dfrac{2mn}{m+n}+1$

Q. 27 If the sample size in Wald-Wolfowitz runs test is large, the variate R is distributed with variance:

(a) $\dfrac{2mn(2mn-m-n)}{(m+n)(m+n-1)}$

(b) $\dfrac{2mn(2mn-m-n)}{(m+n)^2(m+n-1)}$

(c) $\dfrac{mn(2mn-m-n)}{(m+n)^2(m+n-1)}$

(d) $\dfrac{2mn(2mn-m-n)}{(m+n)(m+n-1)}$

Q. 28 Mann-Whitney test statistic U depends on the fact that:

(a) how many times Y's precede X's
(b) how many times X's precede Y's
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 29 The relation between U when Y precedes X and U' when X precedes Y in Mann-Whitney test is:

(a) $U + U' = n_1n_2$
(b) $U = n_1n_2 - U'$
(c) $U' = n_1n_2 - U$
(d) all the above

Q. 30 If n_1 and n_2 in Mann-Whitney test are large, the variable U is distributed with mean:

(a) $(n_1+n_2)/2$

(b) $(n_1-n_2)/2$

(c) $n_1 n_2/2$

(d) $n_1 n_2$

Q. 31 IF n_1 and n_2 are large in Mann-Whitney test, the variable U is distributed with variance equal to:

(a) $n_1 n_2(n_1+n_2+1)/12$

(b) $n_1 n_2(n_1+n_2-1)/12$

(c) $n_1 n_2(n_1+n_2)/12$

(d) $n_1 n_2(n_1 n_2+1)/12$

Q. 32 The hypothesis tested by Mood's test, is:

(a) equality of two standard deviations

(b) equality of two mean deviations about median

(c) equality of any two measure of dispersion

(d) all the above

Q. 33 Mood's test statistic is:

(a) sum of squares of the deviations of the ranks of pooled ordered statistic from their mean rank

(b) sum of the absolute deviations of the ranks of pooled ordered statistics from their mean

(c) sum of square of the deviations of the ranks of pooled sample ranks from their median

(d) none of the above

Q. 34 Mood's table provides at α level of significance and n_1, n_2 sample sizes:

(a) a critical value of M

(b) two critical values of M

(c) probabilities for Mood's statistic M.

(d) none of the above

Q. 35 In Mood's test, one can test the hypotheses:

(a) $H_0:\sigma_1=\sigma_2$ vs. $H_1:\sigma_1\neq\sigma_2$

(b) $H_0:\sigma_2\leq\sigma_2$ vs. $H_1:\sigma_1>\sigma_2$

(c) $H_0:\sigma_1\geq\sigma_2$ vs. $H_1:\sigma_1<\sigma_2$

(d) all the above

Q. 36 In Moses' test for testing the equality of two populations dispersion measure, the size of sub-samples should not be more than:

(a) 5

(b) 10

(c) 15

(d) 20

Q. 37 In Moses' test for equality of two populations dispersion, the variable for the test statistic is

(a) sum of square corrected for mean

(b) mean deviation from median

(c) mean deviation from mean

(d) any of the above

Q. 38 In Moses' test, the statistic used for making decision is

(a) Mood's M-statistic

(b) Wilcoxon's T statistic

(c) Mann-Whitney U statistic

(d) none of the above

Q. 39 Mcnemar test is a test of:

(a) change

(b) independence

(c) association of attributes

(d) none of the above

Q. 40 With usual notations, Mcnemar χ^2-statistics is:

(a) $\left(\frac{A-D}{A+D}\right)^2$

(b) $\frac{(A-D)^2}{A+D}$

(c) $\frac{|A-D|}{(A+D)^2}$

(d) $\frac{|A+D|}{(A-D)^2}$

Q. 41 Cochran's Q-test for equality of a number of treatments is applicable for a variable which is:

(a) continuous
(b) discrete
(c) dichotomous
(d) all the above

Q. 42 Cochran's Q-test is applicable only if the number of treatments under test are:

(a) not more than four
(b) not less than four
(c) at least two
(d) none of the above

Q. 43 Cochran's Q-test restricts to the minimum number of observations in the experiment, that number is:

(a) 20
(b) 24
(c) 25
(d) 4

Q. 44 Kruskal-Wallis analysis of data is meant for:

(a) one way classification
(b) two way classification
(c) non-classified data
(d) none of the above

Q. 45 While performing Kruskal-Wallis test, the ranks are assigned:

(a) independently to the observations for each treatment
(b) for observations in each block independently
(c) by pooling all the observations
(d) none of the above

Q. 46 The test statistics in the Kruskal-Wallis test is:

(a) weighted sum of squares of the deviations of the sum of treatments rank from the expected sum of ranks
(b) sum of squares of the deviations of the sum of treatments rank from the expected sum of ranks
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 47 The statistic H under the Kruskal-Wallis test is approximately distributed as:

(a) Student's t
(b) Snedecor's F
(c) Chi-square
(d) normal deviate-Z

Q. 48 When the number of treatments in Kruskal-Wallis test is two, the statistic H reduces to:

(a) Mann-Whitney U statistic
(b) Wilcoxon's U statistic
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 49 Kruskal-Wallis H with K treatments and n blocks which is approximated to Chi-square has d.f:

(a) $(n-1)$
(b) $(k-1)(n-1)$
(c) $(k-1)$
(d) $k(n-1)$

Q. 50 If the ties occur in the Kruskal-Wallis test, with usual notations, the correction C for ties is:

(a) $\Sigma T/n\left(n^2-1\right)$
(b) $\Sigma T/k\left(n^2-1\right)$
(c) $\Sigma T/kn(n-1)$
(d) none of the above

Q. 51 If C is the correction factor for ties in Kruskal-Wallis test statistic H, the corrected test statistic is:

(a) $H-C$
(b) H/C
(c) $H+C$
(d) $H \times C$

Q. 52 In Friedman's two way analysis of variance, the observations are ranked in:

(a) blocks
(b) treatments
(c) pooled observations
(d) none of the above

Q. 53 Friedmen's test statistic F based on an experiment with K treatments and r blocks and R_j $(j = 1, 2, ..., k)$ rank totals for treatments is:

(a) $F = \dfrac{12}{rk\,(k+1)} \sum_{j=1}^{k} R_j^2 - 3k(k+1)$

(b) $F = \dfrac{12}{kr\,(r+1)} \sum_{j=1}^{k} R_j^2 - 3r(k+1)$

(c) $F = \dfrac{1}{rk\,(k+1)} \sum_{j=1}^{k} R_j^2 - 3r(k+1)$

(d) $F = \dfrac{12}{rk\,(k+1)} \sum_{j=1}^{k} R_j^2 - 3r(k+1)$

Q. 54 Friedman's-F is distributed as:

(a) Snedecor's F
(b) student's t
(c) Chi-square
(d) none of the above

Q. 55 Degrees of freedom for Friedman's-F distributed as χ^2 based on k treatments and r blocks are:

(a) $(r-1)$
(b) $(k-1)$
(c) $r\,(k-1)$
(d) $k\,(r-1)$

Q. 56 Correction factor C for tied observations in Friedman's-F with usual notations is:

(a) $C = 1 - \sum_{i=1}^{r} \dfrac{T_i^2}{rk(k-1)}$

(b) $C = 1 - \sum_{i=1}^{r} \dfrac{T_i}{rk\left(k^2-1\right)}$

(c) $C = 1 - \sum_{i=1}^{r} \dfrac{T_i}{rk(k-1)}$

(d) none of the above

Q. 57 The quantity T_i in the formula for correction for ties 'C' with usual notations is:

(a) $\Sigma t_i^3 - \Sigma t_i$

(b) $\Sigma\left(t_i^3 - t_i\right)$

(c) both (a) and (b)
(d) neither (a) and nor (b)

Q. 58 The critical difference for multiple comparisons in Friedman's test with usual notations is:

(a) $Z\sqrt{\dfrac{rk^2(k+1)}{6}}$

(b) $Zr\sqrt{\dfrac{k(k+1)}{6}}$

(c) $Zk\sqrt{\dfrac{r(k+1)}{4}}$

(d) $Z\sqrt{\dfrac{rk(k+1)}{6}}$

Q. 59 In an experiment with 4 treatments and 5 blocks, the sum of the ranks for treatments were 12, 15, 6, 17.

Friedman's-F based on the given data is equal to:

(a) 23.28
(b) –5.6
(c) –68.06
(d) 8.28

Q. 60 The Jonckheere-Terpstra test for equality of k treatments mean is applicable when the alternative hypothesis is:

(a) at least two of them are not equal
(b) all of them differ from each other
(c) ordered in respect of treatment means
(d) none of the above

Q. 61 Page's test for equality of k treatments mean is applicable when the alternative hypothesis is:

(a) all treatment means differ from one another
(b) ordered
(c) at least two treatment means are not equal
(d) none of the above

Q. 62 With k treatments and R_j, the rank total for j^{th} treatment in a two way classification, the Page's test statistic L is:

(a) $\sum_{j=1}^{k} jR_j$

(b) $\sum_{j=1}^{k} R_j/j$

(c) $\sum_{j=1}^{k} j^2 R_j$

(d) none of the above

Q. 63 When number of treatments k is large ($k >$ 8), the Page's statistic L is distributed as normal variates with mean:

(a) $rk(k+1)/2$

(b) $rk(k+1)^2/2$

(c) $rk(r+1)^2/4$

(d) $rk(k+1)^2/4$

Q. 64 In case of large number of treatments k, Page's statistic L is distributed normally with variance:

(a) $r\left(k^3-k\right)/144(k-1)$

(b) $r\left(k^3-k\right)^2/144(k-1)$

(c) $r\left(k^2-k\right)^2/144(r-1)$

(d) none of the above

Q. 65 Rank correlation was given by:

(a) A.M. Mood

(b) M.G. Kendall

(c) L.E. Moses

(d) C.E. Spearman

Q. 66 Rank correlation was invented in the year:

(a) 1916

(b) 1925

(c) 1928

(d) none of the above

Q. 67 Formula for rank correlation between two sets of ranks with usual notations is:

(a) $r_S = 1 - \frac{6\Sigma_i d_i^2}{n\left(n^2-1\right)}$

(b) $r_S = 1 - \frac{6\Sigma_i d_i}{n\left(n^2-1\right)}$

(c) $r_S = 1 - \frac{6\Sigma_i d_i^2}{n(n-1)}$

(d) all the above

Q. 68 The range of r_S is:

(a) -1 to 1

(b) 0 to 1

(c) $-\infty$ to ∞

(d) 0 to ∞

Q. 69 If $r_S = 1$, means that:

(a) the ranks awarded by two judges are same

(b) there is perfect association between the ranks awarded by two judges

(c) all difference (d_i's) are zero

(d) all the above

Q. 70 If $r_S = 0$, it shows:

(a) no rank correlation between the ranks of two sets

(b) the ranks of two sets are independent

(c) $6\Sigma d_i^2 = n\left(n^2-1\right)$

(d) all the above

Q. 71 When there are only two individuals ranked by two judges, then the possible values of rank correlation r_S are:

(a) zero

(b) -1 or $+1$

(c) 1 or zero

(d) 0 and -1

Q. 72 When there are three items ranked by two investigators, the only possible values of rank correlation r_S are

(a) $-1, -\frac{1}{2}, \frac{1}{2}, 1$

(b) $-1, -\frac{1}{2}, 0, \frac{1}{2}, 1$

(c) $\tau = \frac{2}{3} r_S$

(d) $\tau = \frac{1}{3} r_S$

Q. 89 For large n, τ is distributed as:

(a) $N\left(0, \frac{2(2n+5)}{9n(n-1)}\right)$

(b) $N\left(\mu_\tau, \frac{2(2n+1)}{9n(n-1)}\right)$

(c) $N\left(0, \frac{2(2n+3)}{9n(n+1)}\right)$

(d) none of the above

Q. 90 Five students were ranked at their schooling period and college period as follows:

Students:	*A*	*B*	*C*	*D*	*E*
At schooling	2	3	1	5	4
At college	3	4	1	2	5

The Kendall's rank correlation τ between the ranks at schooling and at college is:

(a) $\tau = 7/10$

(b) $\tau = 2/5$

(c) $\tau = -1/5$

(d) none of the above

Q. 91 If $\tau = 2/5$, then r_S is approximately equal to:
(a) 2/5
(b) 1
(c) 3/5
(d) none of the above

Q. 92 Coefficient of concordance was invented by:
(a) A.M. Mood
(b) C. Spearman
(c) Sidney Siegel
(d) M.G. Kendall

Q. 93 Coefficient of concordance is calculated when there are:
(a) three or more sets of rankings
(b) three sets of rankings
(c) at least five sets of rankings
(d) exactly five sets of rankings

Q. 94 Formula for coefficient of concordance W with usual notations is:

(a) $W = \frac{12S}{k^2 n(n^2-1)}$

(b) $W = \frac{12S}{n(n^2-1)}$

(c) $W = \frac{6S}{k^2 n(n^2-1)}$

(d) $W = \frac{6S}{kn^2(n-1)}$

Q. 95 The value of coefficient of concordance W lies in the range:
(a) -1 to 1
(b) 0 to 1
(c) 0 to ∞
(d) $-\infty$ to 0

Q. 96 If $W = 1$, one can conclude that:
(a) there is no concordance between the rankings of various sets
(b) there is perfect concordance between the rankings of various sets
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 97 The value of $W = 0$ leads to the conclusion that:
(a) there is no concordance between the rankings of various sets
(b) there is perfect concordance between the rankings of various sets
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 98 The relation between the average rank correlation and coefficient of concordance is:
(a) linear
(b) quadratic
(c) cubic
(d) non-existent

Q. 99 If there are t tied ranks in one group of a set and there are more than one such groups in

(27) Glivenko Cantelli (28) sample size; confidence band (29) median (30) binomial (31) signs; magnitude (32) 25 (33) run (34) randomness (35) nonrandomness (36) nonrandomness (37) lack of randomness (38) empirical (39) distance (40) differences (41) location parameters (42) hypergeometric (43) $n_1 t/n; n_1 n_2 (n-t)/n^2 (n-1)$ (44) Lieberman and Owen; 1961 (45) identical (46) two populations (47) $U' = n_1 n_2 - U$

(48) $n_1 n_2/2; n_1 n_2 (n_1 + n_2 + 1)/12$

(49) $\dfrac{2mn}{m+n} + 1; \dfrac{2mn(2mn - m - n)}{(m+n)^2 (m+n-1)}$

(50) dispersions (51) equal (52) $n_1 \le n_2$ (53) sum of squares (54) dispersion (55) sub-samples (56) variate values (57) Mann-Whitney (58) two cell (59) Chi-square (60) Yates (61) one way (62) dichotomous (63) Chi-square (64) one way (65) weighted (66) $k - 1$ (67) two way (68) equality (69) ordered alternatives (70) $J = \sum\limits_{i<j} U_{ij}$ (71) ordered (72) $L = \sum\limits_{j} j R_j$ (73) C.E. Spearman (74) $1 - \dfrac{6 \Sigma d_i^2}{n(n^2-1)}$ (75) −1 to +1 (76) adjusted (77) *t*-test (78) natural order (79) $S \Big/ \dbinom{n}{2}$ (80) $\tau = \dfrac{2r_s}{3}$ (81) −1 to +1 (82) perfect (83) affected (84) 0, 2 $(2n + 5)/9n\,(n - 1)$ (85) association (86) M.G. Kendall (87) 0 to 1 (88) perfect (89) no agreement (90) $r_{Sav} = \dfrac{kW-1}{k-1}$ (91) does not (92) perfect discordance (93) to be adjusted (94) Chi-square (95) $\chi^2 = k\,(n-1)\,W$ (96) $n - 2$ (97) Mood's or Theil's (98) Brown's and Mood's (99) Chi-square (100) Kendall's τ (101) eight (102) runs test (103) $\dfrac{\Sigma T}{n(n^2-1)}$ (104) $1 - \dfrac{\Sigma T_i}{rk(k^2-1)}$

(105) $1 - \dfrac{F}{1 - \dfrac{\Sigma T_i}{rk(k^2-1)}}$

(106) $rk(k+1)^2/4; \dfrac{r(k^3-k)^2}{144(k-1)}$

(107) confidence band (108) flexible (109) Chi-square (110) $(\tau_{XZ} - \tau_{XY}\,\tau_{YZ}) \Big/ \sqrt{(1-\tau_{XY}^2)(1-\tau_{YZ}^2)}$ (111) −1 to 1 (112) Mann-Whitney; Wilcoxon's (113) 10 (114) 0.25 (115) 1/7 (116) 52.5 (117) 0.54 (118) 0.289 (119) 7.5 (120) 1.07 (121) Mann-Whitney *U*-test (122) Kendall's τ (123) a biased (124) normal distribution (125) same (126) *Z*-test (127) median (128) *t*-test (129) G.W. Brown; A.M. Mood (130) graphic

SECTION-C

(1) a (2) c (3) a (4) a (5) d (6) b
(7) c (8) a (9) d (10) d (11) a (12) d
(13) b (14) a (15) c (16) d (17) a (18) b
(19) d (20) b (21) c (22) d (23) c (24) a
(25) b (26) d (27) b (28) c (29) d (30) c
(31) a (32) c (33) a (34) b (35) d (36) b
(37) a (38) c (39) a (40) b (41) c (42) b
(43) b (44) a (45) c (46) a (47) c (48) c
(49) c (50) a (51) b (52) a (53) d (54) c
(55) b (56) b (57) c (58) d (59) d (60) c
(61) b (62) a (63) d (64) b (65) d (66) d
(67) a (68) a (69) d (70) d (71) b (72) a
(73) b (74) d (75) c (76) b (77) a (78) b
(79) d (80) c (81) a (82) b (83) c (84) b
(85) b (86) b (87) d (88) c (89) a (90) b
(91) c (92) d (93) a (94) a (95) b (96) b
(97) a (98) a (99) c (100) c (101) b (102) a
(103) c (104) b (105) a (106) b (107) c (108) c
(109) d (110) b (111) d (112) b (113) a (114) d
(115) c (116) b (117) a (118) b (119) d (120) b
(121) b (122) a (123) c (124) b (125) d (126) d
(127) b (128) d (129) c (130) a.

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Chapter **13**

Regression and Correlation Methods

SECTION-A

Short Essay Type Questions

Q. 1 Give the idea of regression methods.

Ans. Regression methods are meant to determine the best functional relationship between a dependent variable Y with one or more concomitant or related variable (s) X. The functional relationship of a dependent variable with one or more independent variables is called a *regression equation*. If the relationship between a dependent variable Y and an independent variable X is linear, it is known as *simple regression of* Y on X. The graph of the regression equation in general is known as the *curve of regression*. If the curve is a straight line, it is called the line of *regression*. The functional relationship of a variable Y with other two or more independent variables is termed as *multiple regression*. Often the independent variables are called *regressors* or *explanatory* variables. Also the dependent variable is frequently named as *regressed* or *response* variable.

The aim of regression model is to estimate as best as possible, the dependent variable Y from the independent variable (s) X. Hence, a regression equation is also called a *estimating* or *prediction equation. The line of average relationship* is another name given to the regression line.

In many situations, it is possible to take X as dependent variable and Y as independent variable. If so, the line of regression is of X on Y. For example, the height and weight of persons are the variables in which either can be taken as independent variable and the other as dependent variable. Hence, one can obtain two lines of regression namely the regression line of Y on X and of X on Y. But if the regression of Y on X is linear, it is not necessary that the regression of X on Y is also linear and vice-versa.

Q. 2 What is a regression model?

Ans. A functional relationship of a dependent variable Y with the independent variable(s) $X_1, X_2, \ldots, X_k$ involving the parameters $\beta_0 \beta_1, \beta_2, \ldots, \beta_k$ of the type.

$$Y = \psi(X_1, X_2, \ldots, X_k \mid \beta_0, \beta_1, \beta_2, \ldots, \beta_k) + \varepsilon$$

is called a regression model.

where ψ indicates the form of the equation and ε is a random variable distributed with mean 0 and variance σ_ε^2 and is known as the residual or error term. Such a regression equation is called mathematical, or statistical or probabilistic model. The main advantage of probabilistic model is that it enables one to draw inferences about the parameter $\beta_0\,\beta_1,\ldots,\beta_k$. As a special case, a function of the type

$$Y=\psi\left(X_1,X_2,\ldots,X_k\middle|\beta_0,\beta_1,\beta_2,\ldots,\beta_k\right)$$

is known as deterministic model.

More specifically, the statistical model for simple linear regression of Y on X is

$$Y=\beta_0+\beta_1X+\varepsilon$$

In this equation β_0 is the intercept which the line cuts on the axis of Y and β_1 is the slope of the line and is called the *regression coefficient*. ε is error which is distributed as $N\left(0,\sigma_e^2\right)$.

Q. 3 What are the assumptions made in a linear regression?

Ans. There are four assumptions made in a linear regression model:

(i) For each selected X, Y's are distributed normally and independently about the mean $\mu_{Y/X}=\eta=\beta_0+\beta_1X$ and variance $\sigma^2_{Y/X}$ = $\sigma^2_{Y\cdot X}$.

(ii) The population of values of Y corresponding to each selected X has mean μ which lies on the line

$$\mu=\beta_0+\beta_1\left(X-\overline{X}\right)$$
$$=\beta_0+\beta_1x.$$

(iii) The variable X is measured without error and is known exactly.

(iv) The homoscedasticity of the variance $\sigma^2_{Y/X}$ is assumed in a linear regression model and is same as σ_e^2.

Q. 4 Define regression coefficient.

Ans. The regression coefficient 'β' is a measure of change in dependent (response) variable Y corresponding to an unit change in independent variable X.

β is often written as β_{YX} to indicate that it is the regression coefficient of Y on X. Similarly β_{XY} denotes the regression coefficient of X on y. Often the suffix are omitted and are understood by themselves.

Q. 5 Who coined the term 'regression'?

Ans. The linear relation between two variables was named as 'regression' for the first time by the English scientist, Sir Francis Galton.

Q. 6 What is a scatter diagram?

Ans. When the pairs of values (X_i, Y_i) for i = 1, 2, ..., n are plotted on a graph paper, the points show the pattern in which they lie. Such a diagram is known as scatter diagram. If these points lie on a straight line, it is expected that there is a linear relationship between X and Y, otherwise not.

Q. 7 What do you understand by fitting of regression equation?

Ans. By fitting of a regression equation we mean to estimate the regression parameters $\beta_0,\beta_1,\ldots,\beta_k$ with the help of sample observational data in such a way that error ε is minimised.

Q. 8 How can the parameters β_0 and β_1 be estimated?

Ans. Most commonly adopted method is legendre's least squares approach. Under this approach, the residual sum of squares

$$\sum_i\varepsilon_i^2=\sum_i\left(y_i-\beta_0-\beta_1x_i\right)^2$$

where (x_i, y_i) are the n pairs of sample observations for i = 1, 2, ..., n.

is minimised by partially differentiating it w.r.t β_0 and β_1 respectively and equating them to zero. Also replace β_0 and β_1 by their estimates b_0 and b_1 respectively. In this way we get two equations,

$$\sum_i y_i=nb_0+b_1\sum_i x_i$$

$$\sum_i x_i\,y_i=b_0\sum_i x_i+b_1\sum_i x_i^2$$

These equations are known as *normal equations*. Solving these equations, one gets the values of b_0 and b_1 which are,

$$b_0=\overline{y}-b_1\,\overline{x}$$

and $$b_1 = \frac{\sum_i x_i y_i - \frac{(\Sigma_i x_i)(\Sigma_i y_i)}{n}}{\sum_i x_i^2 - \frac{(\Sigma_i x_i)^2}{n}}$$

$$= \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$$

$$= \frac{\text{cov}(X, Y)}{\text{var}(X)}$$

$$= \frac{s_{XY}}{s_X^2}$$

$$= r\frac{s_Y}{s_X}$$

If we study whole population, the value of $\beta_{YX} = \frac{\sigma_{XY}}{\sigma_X^2} = \rho\frac{\sigma_Y}{\sigma_X}$.

The estimated regression equation of Y on X is

$$\hat{Y} = (\bar{y} - b_1 \bar{x}) + b_1 X$$

or $$(\hat{Y} - \bar{y}) = b_1 (X - \bar{x})$$

Note. To obtain the regression equation of X on Y, replace Y by X and X by Y in the whole process.

Q. 9 Comment on the unbiasedness of regression estimates.

Ans. Taking the deviations from mean of x values for n pairs of observations, the regression equation can be given as,

$$y_i = \beta_0 + \beta_1 (x_i - \bar{x}) + \varepsilon_i$$

for $i = 1, 2, ..., n$

where $\varepsilon_i \sim N(0, \sigma_e^2)$ and $\bar{x} = \Sigma x_i / n$.

The estimate of β_0, *i.e.*, $b_0 = \bar{y} = \beta_0 + \bar{\varepsilon}$

$$E(b_0) = E(\bar{y}) = E(\beta_0 + \bar{\varepsilon}) = \beta_0$$

since $E(\bar{\varepsilon}) = \frac{1}{n} E\left(\Sigma_i \varepsilon_i\right) = 0$

The estimate of β_1 is,

$$b_1 = \frac{\Sigma_i y_i (x_i - \bar{x})}{\Sigma_i (x_i - \bar{x})^2}$$

$$= \frac{\Sigma_i [\beta_0 + \beta_1 (x_i - \bar{x}) + \varepsilon_i](x_i - \bar{x})}{\Sigma_i (x_i - \bar{x})^2}$$

$$= \beta_1 + \frac{\Sigma_i (x_i - \bar{x})\varepsilon_i}{\Sigma_i (x_i - \bar{x})^2}$$

$$E(b_1) = \beta_1$$

Hence, it has been proved that both the regression estimates are unbiased.

Q. 10 Give expressions for the variance of estimates of the constants of the regression line $Y = \beta_0 + \beta_1 X + \varepsilon$ and also of estimates of Y on X and X on Y.

Ans. Let there be n pairs of values (x_i, y_i) for $i = 1, 2, ..., n$ and $\varepsilon_i \sim N(0, \sigma_e^2)$. Then

$$V(b_1) = \frac{\sigma_e^2}{\Sigma_i (x_i - \bar{x})^2}$$

$$V(b_0) = \frac{\sigma_e^2}{n}$$

Also, $S_{YX}^2 = \frac{1}{n}\Sigma_i (Y_i - \overline{Y}_i)^2 = \sigma_y^2 (1 - r^2)$, similarly

$S_{XY}^2 = \frac{1}{n}\Sigma_i (X_i - \overline{X}_i)^2 = \sigma_x^2 (1 - r^2)$.

Q. 11 Delineate the properties of regression coefficients.

Ans. Different properties of regression coefficients are delineated below:

(i) The range of regression coefficient is $-\infty$ to ∞.

(ii) The correlation coefficient between two variable X and Y is the geometric mean of the two regression coefficients b_{YX} and b_{XY}. This is known as the *fundamental property* of regression coefficients.
Notationally,

$$\beta_{YX} = \rho\frac{\sigma_Y}{\sigma_X} \text{ and } \beta_{XY} = \rho\frac{\sigma_X}{\sigma_Y}$$

$$\beta_{YX}\cdot\beta_{XY} = \rho\frac{\sigma_Y}{\sigma_X}\cdot\rho\frac{\sigma_X}{\sigma_Y}$$

or $$\beta_{YX}\cdot\beta_{XY} = \rho^2$$

or $$\rho = \sqrt{\beta_{YX}\cdot\beta_{XY}}$$

For sample, $r = \sqrt{b_{YX}\cdot b_{XY}}$

(iii) The signs of regression coefficients and correlation coefficient are always the same. This is known as *signature property of regression coefficient.*
Notationally,

$$\beta_{YX} > 0 \Leftrightarrow \rho > 0 \text{ or } \beta_{XY} > 0 \Leftrightarrow \rho > 0$$

$$\beta_{YX} < 0 \Leftrightarrow \rho < 0 \text{ or } \beta_{XY} < 0 \Leftrightarrow \rho < 0$$

(iv) The arithmetic mean of the two regression coefficients β_{YX} and β_{XY} is greater than or equal to the correlation coefficient between the variables X and Y. This is known as the *mean property* of regression coefficients.
Notationally,

$$\frac{1}{2}(\beta_{YX} + \beta_{XY}) \geq \rho$$

(v) If one of the regression coefficient in a simple linear regression is greater than unity, the other is less than unity. This is known as the *magnitude property* of regression coefficients.
Notationally,

if $\beta_{YX} > 1 \Leftrightarrow \beta_{XY} < 1$

or if $\beta_{XY} > 1 \Leftrightarrow \beta_{YX} < 1$

(vi) If the variables X and Y are independent, the regression coefficients are zero. This is known as the *independence property* of regression coefficients. Notationally, if X and Y are independent, then

$$\rho = 0 \Leftrightarrow \sigma_{XY} = \sigma_{YX} = 0 \Leftrightarrow \beta_{YX} = \beta_{XY} = 0$$

(vii) Variance of the regression coefficient of Y on X is,

$$V(b_{YX}) = \frac{1}{n}\frac{\sigma_Y^2}{\sigma_X^2}(1-\rho^2).$$

Similarly, $$V(b_{XY}) = \frac{1}{n}\frac{\sigma_X^2}{\sigma_Y^2}(1-\rho^2)$$

Q. 12 At what point, the two lines of regression intersect?

Ans. The lines of regression of Y on X and X on Y intersect at the mean value of the variables, *i.e.*, their point of intersection is $(\overline{X}, \overline{Y})$. Obviously, the point $(\overline{X}, \overline{Y})$ satisfy both the equations, $(\hat{Y}-\overline{Y}) = b_{YX}(X-\overline{X})$ and $(\hat{X}-\overline{X}) = b_{XY}(Y-\overline{Y})$.

Q. 13 If we convert the two variables of a simple regression line into standard deviates, through which point, the two regression lines will pass.

Ans. The regression lines of Y on X and X on Y will pass through the origin.

Q. 14 What is the tangent of the angle between the two lines of regression?

Ans. If θ is the angle between the regression lines $(Y-\overline{Y}) = \beta_{YX}(X-\overline{X})$ and $(X-\overline{X}) = \beta_{XY}(Y-\overline{Y})$, then

$$\tan(\theta) = \frac{1-\rho^2}{\rho}\cdot\frac{\sigma_X\sigma_Y}{\sigma_X^2+\sigma_Y^2}$$

(i) If $\rho = \pm 1, \tan\theta = 0 \Rightarrow \theta = 0$. This shows, when there is perfect correlation between X and Y, the two lines of regression are coincident.

(ii) If $\rho = 0, \tan\theta = \infty \Rightarrow \theta = \frac{\pi}{2} = 90°$. This infers, when the variables X and Y are uncorrelated, the two lines of regression are perpendicular to each other. In terms of regression coefficients,

$$\tan(\theta) = \frac{1 - b_{YX} \cdot b_{XY}}{b_{YX} + b_{XY}}$$

(i) The two lines of regression are coincident if $b_{YX}.b_{XY} = 1$.

(ii) The two lines of regression are perpendicular to each other if the two regression coefficients are equal in magnitude but opposite in sign, *i.e.*,

$$b_{YX} = -b_{XY}$$

Q. 15 What is the effect of coding on regression coefficient?

Ans. If we subtract constants c_1 and c_2 from the variables X and Y and then divide them by suitable constants d_1 and d_2 respectively, the variables are said to be coded or linearly transformed. Thus, the coded variables are $X' = \frac{X - c_1}{d_1}$ and $Y' = \frac{Y - c_2}{d_2}$. It is trivial to prove that the regression coefficient is independent of origin but not of scale. The relation between regression coefficient without and with coding is,

$$\beta_{YX} = \frac{d_2}{d_1}\beta_{Y'X'} \text{ and } \beta_{XY} = \frac{d_1}{d_2}\beta_{X'Y'}$$

Q. 16 Where does one require the assumption of normality of ε *vis-a-vis* the variable Y.

Ans. The assumption of normality becomes essential while testing the significance of regression parameters or finding their confidence limits.

Also this assumption becomes necessary while testing the significance of regression coefficient by F-test. Under the assumption of normality, the regression mean sum of squares $b\sum_{i=1}^{n} u_i v_i$ is distributed as $\chi_1^2\,\sigma_e^2$ with 1 d.f. and residual mean sum of square 'S_e^2' is distributed as $\chi_2^2\,\sigma_e^2$ with $(n - 2)$ d.f. where $u_i = x_i - \bar{x}$ and $v_i = y_i - \bar{y}$.

Q. 17 How will you test the significance of regression coefficient?

Ans. The significance of the regression coefficient can be tested by student's t-test. Here we test,

$$H_0{:}\beta_{YX} = 0 \text{ vs. } H_1{:}\beta_{YX} \neq 0$$

To test H_0 against H_1, the test statistic is,

$$t = \frac{b_{YX}}{s_b}$$

t has $(n - 2)$ d.f.

where b_{YX} is the estimated value of β_{YX} and s_b is the standard error of $\beta_{YX} \cdot s_b$ can be computed by the formula,

$$s_b = \sqrt{\frac{s_e^2}{\sum_i u_i^2}}$$

where $u_i = x_i - \bar{x}$ and $v_i = y_i - \bar{y}$ for $i = 1, 2, \ldots, n$

Also, $s_e^2 = \frac{1}{(n-2)}\left\{\sum_i v_i^2 - b_{YX}\sum_i u_i\, v_i\right\}$

Substituting the value of b_{YX} and s_b, the value of t is available. On comparing the computed value of t with the tabulated value of t at α level of significance and for $(n - 2)$ d.f., the decision about H_0 is taken in the usual way.

Q. 18 Give a test for testing the significance of the intercept β_0.

Ans. The test of the hypothesis

$$H_0{:}\beta_0 = 0 \text{ vs. } H_1{:}\beta_0 \neq 0$$

can be tested by the statistic

$$t = \frac{b_0}{s_{b_0}}$$

where s_{b_0} is the standard error of an estimator b_0.

With usual notations,

$$s_{b_0}^2 = s_e^2\left\{\frac{1}{n} + \frac{\bar{x}^2}{\sum_i u_i^2}\right\}$$

The decision about the acceptance or rejection of H_0 is taken in the usual manner.

Q. 19 Give F-test for testing the significance of the regression coefficient.

Ans. The hypothesis

$$H_0 : \beta_{YX} = 0 \text{ vs. } H_1 : \beta_{YX} \neq 0$$

can be tested by the test statistics

$$F = \frac{b \sum_i u_i v_i}{s_e^2}$$

F has $(1, n-2)$ d.f.

To decide as to whether the regression coefficient is significant or not, the tabulated value of F for $(1, n-2)$ d.f. and α level of significance and computed value of F are compared. If $F \geq F_{\alpha(1,n-2)}$, reject H_0. It means that the value of regression coefficient is significant. Again if $F \leq F_{\alpha(1,n-2)}$, accept H_0. It reveals that the value of regression coefficient is non-significant.

Q. 20 What do you understand by regression function?

Ans. From minimisation problem it is known that $E\{Y - g(X)\}^2$ is minimum when $g(x) = E\{Y|X = x\}$. The graph of this minimum function $Y = g(x)$ is the *regression curve* of Y on X. The random function $g(x)$ is called the *regression function* of Y on X.

The regression curve of the mean of Y on X can be defined as the functional relation $y = E(Y|X = x)$. Also $E(Y|x) = \alpha + \beta X$ is called the line of regression of Y on X. In the same way, the *regression curve* of the mean of X on Y is defined as the functional relation $x = E\{X|Y = y\}$. Also $E(X|y) = \alpha' + \beta' Y$ is known as the line of regression of X on Y.

Q. 21 If $(X, Y) \sim BN(0, 0, \sigma_X^2, \sigma_Y^2, \rho)$, find the correlation between X^2 and Y^2.

Ans. For the given bivariate normal distribution, the two regression equations, of Y on X and of X on Y are,

$$Y = \rho \frac{\sigma_Y}{\sigma_X} X \text{ and } X = \rho \frac{\sigma_X}{\sigma_Y} Y$$

or $$Y^2 = \frac{\rho^2 \sigma_Y^2}{\sigma_X^2} X^2 \text{ and } X^2 = \rho^2 \frac{\sigma_X^2}{\sigma_Y^2} Y^2$$

These equations represent the linear regression equations of Y^2 on X^2 and of X^2 on Y^2 respectively. Thus, the correlation between X^2 and Y^2 is the geometric mean of the regression coefficients, *i.e.*,

$$\text{Corr.Coeff.} = \sqrt{\rho^2 \frac{\sigma_Y^2}{\sigma_X^2} \cdot \rho^2 \frac{\sigma_X^2}{\sigma_Y^2}}$$

$$= \rho^2$$

Hence, the correlation coefficient between X^2 and Y^2 is ρ^2.

Q. 22 Give the expression for $(1 - \alpha)$ 100 per cent confidence limits for the regression coefficient β_{YX}.

Ans. $(1 - \alpha)$ 100 per cent confidence limits for the regression coefficient β_{YX} with usual notations can be given as,

$$b_{YX} \mp s_b \cdot t_{\alpha,(n-2)}$$

where $t_{\alpha,\, n-2}$ is student's t-value obtained from t-table for a two-tailed test for α level of significance and $(n - 2)$ d.f.

Q. 23 What formula is used to find the confidence limits for the intercept β_0?

Ans. Following the usual principle of confidence interval estimation, the formula for $(1 - \alpha)$ 100 per cent confidence limits for the intercept β_0 is,

$$b_0 \mp s_{b_0} \cdot t_{\alpha,(n-2)}$$

where $t_{\alpha,(n-2)}$ is the tabulated t-value, from a t-table tabulated for two-tailed test, for α level of significance and $(n - 2)$ d.f.

Q. 24 For the regression equation $\mu_{Y/X} = \beta_0 + \beta_1 X$ estimated as $\hat{Y} = b_0 + b_1 X$, find $(1 - \alpha)$ 100 per cent confidence limits for $\mu_{Y/X} = \eta$ (say).

Test of linearity of regression can easily be performed through analysis of variance for one way classification. Thus the ANOVA table is as follows:

Source (*i*)	*d.f.* (*ii*)	*S.S.* (*iii*)	*M.S.* (*iv*) (*iii*) ÷ (*ii*)	*F-value* (*v*)
Due to regression	1	$b_1^2 \sum_i n_i (x_i - \bar{x})^2$		
Due to deviation from regression	$(p-2)$	$\sum_i n_i (y_i - \bar{y})^2$ $-b_1 \sum_i n_i (x_i - \bar{x})^2$	D	D/E
Between groups	$(p-1)$	$\sum_i n_i (y_i - \bar{y})^2$		
Error	$n-p$	$\sum_i \sum_j (y_{ij} - \bar{y}_i)^2$	E	
Total	$n-1$	$\sum_i \sum_j (y_{ij} - \bar{y})^2$		

If the *F*-value equal to *D/E* is greater than or equal to the tabulated value of *F* at α level of significance and for $\{(p-2), (n-p)\}$ d.f., we reject the hypothesis that regression is linear. Otherwise, the hypothesis of linearity is accepted.

Note. It is further emphasised that the significance of regression coefficient has nothing to reveal about the linearity of regression equation.

Q. 28 What do you understand by weighted regression?

Ans. If the assumption of homoscedasticity holds no longer, one cannot assume that $\sigma^2_{Y|X}$ is same for all *X*. Hence, the variance $\sigma^2_{Y|X}$ will be dependent of X_i. Let it be denoted by σ_i^2. The variance,

$$\sigma_i^2 = \sigma^2 / w_i$$

where w_i is a known constant and is called the weight of variance for x_i. Often one finds that σ_i^2 is proportional to X_i and hence it looks germane to take $w_i = \frac{1}{X_i}$ or $\sigma_i^2 = \sigma^2 X_i$.

If we consider the regression equation as,

$$Y_{ij} = \beta_0 + \beta_1 X_i + \varepsilon_{ij}$$

for $\quad i = 1, 2, \ldots, k$

$\qquad j = 1, 2, \ldots, n_i$

Following least square method for estimation of β_0 and β_1, the normal equations are

$$\left(\sum_{i=1}^{k} \frac{n_i}{X_i}\right) b_0 + n b_1 = \sum_{i=1}^{k} \sum_{j=1}^{n_i} \frac{y_{ij}}{X_i}$$

$$n b_0 + \left(\sum_{i=1}^{k} n_i X_i\right) b_1 = \sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij}$$

where $\sum_i n_i = n$.

Solving the normal equations it is trivial to get the values of b_0 and b_1.

Q. 29 The following measurements show the respective heights in inches of ten fathers and their eldest sons.

Father (X):	67,	63,	66,	71,	69,	65,	62,	70,	61,	72
Son (Y):	68,	66,	65,	70,	69,	67,	64,	71,	60,	63

(i) Find the regression line of son's height on father's height.

(ii) Estimate the height of son for the given height of father as 70 inches.

(iii) Test the significance of the regression coefficient β_{YX}.

(iv) Calculate 95 per cent confidence limits for β_{YX}.

(v) Test the significance of the intercept of the line *Y* on *X*.

(vi) Find the regression line of *X* on *Y*.

(vii) At what point the two lines of regression intersect?

Ans. To fit in the regression lines and test the significance of regression coefficients, etc., we first do the following computations.

$n = 10$, $\Sigma X = 666$, $\Sigma Y = 663$, $\Sigma X^2 = 44490$,

$\Sigma Y^2 = 44061$, $\Sigma XY = 44224$.

$$\bar{X} = 66.6, \bar{Y} = 66.3, \Sigma u_i^2 = 44490 - \frac{(666)^2}{10} = 134.4$$

$$\Sigma v_i^2 = 44061 - \frac{(663)^2}{10} = 104.1,$$

$$\Sigma u_i v_i = 44224 - \frac{666 \times 663}{10} = 68.2$$

(i) The regression coefficient

$$b_{YX} = \frac{68.2}{134.4}$$
$$= 0.507$$

Regression line of son's height on Father's height is,

$$(\hat{Y} - 66.3) = 0.507(X - 66.6)$$

$$\hat{Y} = 0.507X + 32.53$$

(ii) The estimate of son's height for $X = 70$ is,

$$\hat{Y} = 0.507X\,70 + 32.53$$

$$\hat{Y} = 68.02 \text{ inches}$$

(iii) To test

$$H_0 : \beta_{YX} = 0 \text{ vs. } H_1 : \beta_{YX} \neq 0$$

$$s_e^2 = \frac{1}{8}\left\{104.1 - \frac{(68.2)^2}{134.4}\right\}$$
$$= 8.69$$

$$s_b^2 = \frac{8.69}{134.4}$$
$$= 0.0646$$

$$\therefore \quad s_b = 0.2543$$

Thus, the statistic

$$t = \frac{0.507}{0.2543}$$
$$= 1.99$$

Tabulated $t_{0.05,\,8} = 2.306$, which is greater than 1.99. Hence, the regression coefficient is non-significant.

(iv) The confidence limits for β_{YX} are,

$$\left.\begin{matrix} b_L \\ b_U \end{matrix}\right] = 0.507 \mp 0.2543 \times 2.306$$

Therefore, $b_L = -0.079$ and $b_U = 1.093$

(v) To test

$$H_0 : \beta_0 = 0 \text{ vs. } H_1 : \beta_0 \neq 0$$

we compute,

$$s_{b_0}^2 = 8.69\left\{\frac{1}{10} + \frac{(66.6)^2}{134.4}\right\}$$
$$= 287.66$$

$$\therefore \quad s_{b_0} = 16.96$$

The statistic,

$$t = \frac{32.53}{16.96}$$
$$= 1.918$$

Tabulated $t_{0.05,\,8} = 2.306$, which is greater than 1.918. Hence, the intercept β_0 is nonsignificant.

(vi) For the regression line of X on Y,

$$b_{XY} = \frac{68.2}{104.1}$$
$$= 0.655$$

The regression line of X on Y is,

$$(\hat{X} - 66.6) = 0.655(Y - 66.3)$$

$$\hat{X} = 0.655Y + 23.174$$

(vii) The two lines of regression intersect at the point (66.6, 66.3).

Q. 30 What is meant by curvilinear regression?

Ans. If the test for linearity of regression model reveals that linear fit is inadequate, one is bound to desiderate for some non-linear model. Under this, a scientist often tries one of the following or other function which is non-linear in nature:

(i) Second degree curve of parabola,

$$Y = \alpha + \beta X + \gamma X^2$$

(ii) Exponential growth curve or compound interest function,

$$Y = a\beta^X$$

(iii) Exponential decay model,

$$Y = \alpha\beta^{-X} \text{ for } \beta < 1$$

curves (ii) and (iii) becomes linear functions by taking logarithm.

(iv) Logistic growth curve,

$$\frac{1}{Y} = \alpha\beta^{X} + \gamma \text{ for } \alpha, \beta, \gamma > 0$$

(v) Compertz function,

$$\log_e Y = \log_e \gamma + \beta^{X} \log_e \alpha \text{ for } \alpha, \beta, \gamma > 0$$

(vi) Mitscherlich function,

$$Y = \gamma\left(1 - e^{\alpha - \beta X}\right) \text{ for } \alpha, \beta, \gamma > 0$$

(vii) Cubic Parabola

$$Y = \alpha + \beta X + \gamma X^2 + \delta X^3$$

(viii) Equilateral hyperbola, asymptotic to a line parallel to X-axis,

$$Y = \frac{1}{\alpha + \beta X}$$

(ix) Equilateral hyperbola, asymptotic to a line parallel to Y-axis.

$$Y = a + b\left(\frac{1}{X}\right)$$

(x) Equilateral hyperbola, asymptotic to lines parallel to both the axes,

$$\frac{1}{Y} = a + b\left(\frac{1}{X}\right)$$

(xi) The curve with convex or concave bend,

log Y = a + b logX

Convex from above if b is positive,

Concave from above if b is negative.

Q. 31 What is an orthogonal polynomial and how to fit it?

Ans. A polynomial ϕ_k in X of degree k is said to be an orthogonal polynomial if corresponding to any other polynomial ϕ_j for $j = 1, 2, ...,$ the conditions, $\Sigma \phi_j \phi_k = 0$ holds good. The summation (Σ) runs over all admissible values of X. Here we shall confine our discussion to the orthogonal polynomials in which the X values are equally spaced and the common difference is unity. If it is not unity, it can be made so by dividing each value of X by the common difference. The method of fitting the orthogonal polynomial is as follows.

Let the orthogonal polynomial to be fitted is,

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + ... + \beta_k X^k$$

The above equation is also called the *parabola of degree k*. Here it is not known what should actually be the value of k or in other words, what should be the degree of the polynomial. One way is to fit in the first degree equation and test the significance of the coefficient of X. If it is significant, add second degree term and test the significance of its coefficient. If it is significant, add third degree term and test the significance of the coefficient of X^3 and continue this process till two consecutive coefficients come out to be non-significant. But this process is very tedious in the sense that every time a fresh calculations have to be made. The tedium of this method can be overcome by using the method of fitting of orthogonal polynomial with X's at equal intervals,

$$\hat{Y} = a_0 + a_1 X + a_2 X^2 + ... + a_k X^k$$

can be expressed as,

$$\hat{Y} = a_0 + a_1 \phi_1' + a_2 \phi_2' + ... + a_k \phi_k'$$

where ϕ_i's ($i = 1, 2, ..., k$) are orthogonal polynomials and a_i are constants such that,

$$a_0 = \frac{1}{n}\sum_j Y_j = \overline{Y} \text{ and } a_i = \frac{\Sigma Y_i \phi_i'}{\Sigma \phi_i'^2}$$

for $i = 1, 2, ..., k$

Elaborately, the orthogonal polynomials are,

$$\phi_1 = \left(X - \overline{X}\right)$$

$$\phi_2 = \left(X - \overline{X}\right)^2 - \frac{1}{12}\left(n^2 - 1\right)$$

$$\phi_3 = \left(X - \overline{X}\right)^3 - \frac{1}{20}\left(3n^2 - 7\right)\left(X - \overline{X}\right)$$

A recurrence relation between ϕ_r and ϕ_{r+1} can be established as,

$$\phi_{r+1} = \phi_1\phi_r - \frac{r^2\left(n^2-1\right)}{4\left(4r^2-1\right)}\phi_{r-1}$$

The relation between ϕ_r and ϕ'_r is,

$$\phi'_r = \lambda_r\phi_r$$

where λ is the smallest number for which ϕ_r assumes only the integral values. The advantage of the method of orthogonal polynomials is that we can add terms one by one sparingly without redoing the whole calculations. Substituting the values ϕ'_i 's, a_0 and a_i's, the orthogonal polynomial is fitted.

The values of ϕ_i's, λ and $\Sigma\phi_i^2$ can be obtained from Fisher and Yates tables. A part of the table is given below for three values of n:

	$n=3$		$n=4$			$n=5$			
	ϕ_1	ϕ_2	ϕ_1	ϕ_2	ϕ_3	ϕ_1	ϕ_2	ϕ_3	ϕ_4
	–1	1	–3	1	–1	–2	+2	–1	1
	0	–2	–1	–1	3	–1	–1	2	–4
	1	1	1	–1	–3	0	–2	0	6
			3	1	1	1	–1	–2	–4
						2	2	1	1
λ's	1	3	2	1	10/3	1	1	5/6	35/12
$\Sigma\phi_{ji}^2$	2	6	20	4	20	10	14	10	70

The fitted orthogonal polynomial is of the type,

$$\hat{y} = a_0 + a_1\lambda_1\left(X-\overline{X}\right) + a_2\lambda_2\left\{\left(X-\overline{X}\right)^2 - \frac{n^2-1}{12}\right\}$$

$$+a_3\lambda_3\left\{\left(X-\overline{X}\right)^3 - \left(X-\overline{X}\right)\left(\frac{3n^2-7}{20}\right)\right\}+\ldots$$

Reduction in sum of squares due to first, second, third, ... degree terms can be calculated by the formula for testing the significance of the first, second, third exponents, etc.

Reduction in sum of square due to j^{th} power of X is equal to

$$a_j\left(\sum_i Y_i\phi_{ji}\right)$$

Total sum of squares = $\sum_i\left(Y_i-\overline{Y}\right)^2$

Sum of squares due to deviation from regression = Total S.S – S.S. due to different exponent terms. Significance of each exponent term can be tested by F-test in an analysis of variance table and the decision about the highest power of the orthogonal polynomial can be taken accordingly.

Q. 32 How can you fit in a second degree equation (parabola)?

Ans. A second degree parabola is,

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \varepsilon$$

Suppose (X_i, Y_i) for i = 1, 2, ..., n are n paired observations. The regression parameters β_0, β_1 and β_2 can be estimated as b_0, b_1 and b_2 using Legendre's method of least squares. The normal equations under least squares method are:

$$\sum_i Y_i = nb_0 + b_1\Sigma X_i + b_2\sum_i X_i^2$$

$$\sum_i X_i Y_i = b_0\sum_i X_i + b_1\sum_i X_i^2 + b_2\sum_i X_i^3$$

$$\sum_i X_i^2 Y_i = b_0\sum_i X_i^2 + b_1\sum_i X_i^3 + b_2\sum_i X_i^4$$

Solving these equations, the values of b_0, b_1 and b_2 come out to be,

$$b_1 = \frac{s_{1y}s_{22}-s_{2y}s_{12}}{s_{11}s_{22}-s_{12}^2}, b_2 = \frac{s_{2y}s_{11}-s_{1y}s_{12}}{s_{11}s_{22}-s_{12}^2}$$

and $\quad b_0 = \overline{Y} - b_1\overline{X} - b_2\overline{X}^2$

where $s_{1y} = \sum_i X_i Y_i - n\overline{X}\overline{Y}, s_{2y} = \sum_i X_i^2 Y_i - n\overline{X}^2\overline{Y}$

$$s_{11} = \sum_i X_i^2 - n\overline{X}^2, s_{12} = \sum_i X_i^3 - n\overline{X}^3$$

and $\quad s_{22} = \sum_i X_i^4 - n\left(\overline{X}^2\right)^2$

Substituting the values of b_0, b_1 and b_2, we obtain the equation of the parabola or second degree polynomial as is called the *coefficient of determination* and $(1 - \rho^2)$, the *coefficient of nondetermination*. Also the quantity $\sqrt{1-\rho^2}$ is called the *coefficient of alienation*.

Q. 37 Find the limits of ρ.

Ans. We know that error mean sum of squares,

$$\sigma_e^2 = \sigma_Y^2\left(1-\rho^2\right)$$

Since any sum of squares cannot be negative,

$$1-\rho^2 \geq 0$$

$$-\rho^2 \geq -1$$

or $$\rho^2 \leq 1$$

or $$\rho \leq \pm 1$$

Thus, the limits of ρ are from −1 to 1 and so is true for *r*.

Alternative proof. By Schwarz inequality, we know.

$$\{E(x,y)\}^2 \leq E\left(x^2\right)E\left(y^2\right)$$

or $$\left\{\frac{E(x,y)}{\sqrt{E\left(x^2\right)E\left(y^2\right)}}\right\}^2 \leq 1$$

$$\rho^2 \leq 1$$

or $$-1 \leq \rho \leq 1$$

Q. 38 Interpret the values of ρ as −1, 1 and 0.

Ans. If ρ = 1, it means that *Y* is proportional to *X* which ensures perfect positive linear association. In this case all points in a scatter diagram lie on a straight line extending from left bottom to the right top. It means as *X* increases, *Y* also increases. If ρ = −1, it means as *X* increases, *Y* decreases. This ensures perfect negative association. In this situation *Y* is proportional to $\frac{1}{X}$. All the points lie on a straight line extending from left top to the right bottom.

The value ρ = 0, confirms the lack of linear association between two variables. In this case, all the points are scattered on a graph and hardly any three points lie in a straight line.

Q. 39 Discuss the properties of correlation coefficient.

Ans. The properties of correlation coefficient are:

(i) Correlation coefficient is a pure number, *i.e.*, it has no unit.

(ii) The correlation coefficient ρ (or *r*) ranges from −1 to 1.

(iii) The correlation between two variables is known as simple correlation or correlation of zero order.

(iv) It is not affected by coding (linear transformation) of variables or variate values.

(v) The relation between the correlation coefficient '*r*' and the regression coefficients b_{YX} and b_{XY} is,

$$r = \sqrt{b_{YX} \cdot b_{XY}}$$

(vi) The sign of *r* will be the same as that of b_{YX} or b_{XY}.

(vii) If the two variables are independent, the correlation coefficient between them is zero but the converse is not true.

(viii) If ρ (or *r*) = 0, it shows that the relationship between the variables *X* and *Y* is not linear.

(ix) The relationship between the correlation coefficient with a regression coefficient is,

$$\beta_{YX} = \rho\frac{\sigma_Y}{\sigma_X};$$

$$\beta_{XY} = \rho\frac{\sigma_X}{\sigma_Y}$$

(x) Arithmetic mean of two regression coefficients is always greater than the positive correlation coefficient between the variables. Symbolically,

$$\frac{1}{2}\left(\rho\frac{\sigma_Y}{\sigma_X} + \rho\frac{\sigma_X}{\sigma_Y}\right) \geq \rho$$

$$\sigma_Y^2 + \sigma_X^2 \geq 2\sigma_X\,\sigma_Y$$

Ans. Probable error is useful in roughly having an idea about the significance of coefficient of correlation. If $r < 3$ P.E. (r), the correlation coefficient is definitely not significant. If $r > 6$ P.E., r is definitely significant. This approach is seldom used as t-test is an exact test for testing the signi-ficance of correlation coefficient.

Q. 48 How will you test the significance of the correlation coefficient ρ?

Ans. The hypothesis,

$$H_0 : \rho = 0 \quad \text{vs.} \quad H_1 : \rho \neq 0$$

can be tested by t-test where the statistic,

$$t = \frac{r}{s_r}$$

$$= \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

t has $(n - 2)$ d.f. The decision about the hypothesis can be taken in the usual manner.

Q. 49 How can one perform the test for a specified value of population correlation coefficient?

Ans. The test of the hypothesis

$$H_0 : \rho = \rho_0 \quad \text{vs.} \quad H_1 : \rho \neq \rho_0$$

can be performed by using Fisher's transformation.

Suppose the estimated value of ρ is r and is based on n pairs of observed values.

Fisher's Z-transformation for the values ρ_0 and r are:

$$Z_r = \frac{1}{2}\log_e\left(\frac{1+r}{1-r}\right) = \tanh^{-1} r$$

$$= 1.1513\left\{\log_{10}(1+r) - \log_{10}(1-r)\right\}$$

Similarly,

$$Z\rho_0 = 1.1513\left\{\log_{10}(1+\rho_0) - \log_{10}(1-\rho_0)\right\}$$

The transformed values of Z corresponding to different values of correlation coefficient can also be obtained from the tables. Fisher told that Z_r is distributed normally with mean Z_{ρ_0} and variance $1/(n - 3)$. Also he pointed out that for small n, Z_{ρ_0}, the mean of Z_r is somewhat biased and hence can be improved by using the transformation

$$Z_{\rho_0} = \frac{1}{2}\log_e\left(\frac{1+\rho_0}{1-\rho_0}\right) + \frac{\rho_0}{2(n-1)}$$

The test statistic

$$Z = \frac{Z_r - Z_{\rho_0}}{1/\sqrt{n-3}}$$

$$= \left(Z_r - Z_{\rho_0}\right)\sqrt{n-3}$$

The decision about H_0 can be taken in the usual way.

Q. 50 Give the formula for the confidence limits for the population correlation coefficient ρ.

Ans. Following usual notation, $(1 - \alpha)$ 100 per cent confidence limits can be obtained by the formula,

$$\left.\begin{matrix} Z_L \\ Z_U \end{matrix}\right] = Z_r \mp Z_{1-\alpha/2}\frac{1}{\sqrt{n-3}}$$

The limits for ρ can be obtained by retransformation.

Q. 51 How can the equality of the correlation coefficients of two bivariate populations be tested?

Ans. The hypothesis

$$H_0 : \rho_1 = \rho_2 \text{ vs. } H_1 : \rho_1 \neq \rho_2$$

can be tested by making use of Fisher's Z-transformation.

Suppose r_1 and r_2 are the estimates of ρ_1 and ρ_2 obtained from the samples of sizes n_1 and n_2, respectively.

Under Fisher's Z-transformation,

$$Z_{r_1} = \frac{1}{2}\log_e\left(\frac{1+r_1}{1-r_1}\right) = \tanh^{-1} r_1$$

and $$Z_{r_2} = \frac{1}{2}\log_e\left(\frac{1+r_2}{1-r_2}\right) = \tanh^{-1} r_2$$

Fisher showed that $\left(Z_{r_1} - Z_{r_2}\right)$ is distributed with mean

$$\frac{\rho}{2(n_1-1)} - \frac{\rho}{2(n_2-1)}$$

$$r_c = \pm\sqrt{\pm\frac{2c-n}{n}}$$

where c is the number of positive signs of the product of deviations for X and Y, n the number of deviations. Also the same sign within and outside the square root has to be used. If $(2c - n)$ is negative, use negative sign and vice-versa.

Q. 55 Following are the data of Gross National Product (GNP) and Net National Product (NNP) at current prices from 1971-72 to 1979-80.

Year:	1971-72	1972-73	1973-74	1974-75	1975-76	1976-77	1977-78	1978-79	1979-80
GNP (X): Rs. (crore)	38983	42993	53501	63051	66375	71432	69760	65842	87058
NNP (X): Rs. (crore)	36999	36629	38486	38958	42799	41566	47233	54711	53468

Find the coefficient of concurrent deviation.

Ans. We prepare the following table to calculate r_c.

Year	*GNP (X)*	*NNP (Y)*	*Signs of deviations from previous year for X*	*Signs of deviations from previous yaer for Y*	*Product of signs of deviations*
1971-72	38983	36999			
1972-73	42993	36629	+	–	–
1973-74	53501	38486	+	+	+
1974-75	63051	38958	+	+	+
1975-76	66375	42799	+	+	+
1976-77	71432	41566	+	–	–
1977-78	69760	47233	–	+	–
1978-79	65842	54711	–	+	–
1979-80	87058	53468	+	–	–
$n = 8$					$c = 3$

Having calculated n and c, the coefficient of concurrent deviation,

$$r_c = \pm\sqrt{\pm\frac{2\times 3-8}{8}}$$

$$= -\sqrt{\frac{1}{4}}$$

$$= -0.5$$

Q. 56 Give the formula for coefficient of correlation where we can make use of the variance of difference $(X - Y)$.

Ans. The formula for the coefficient of correlation using the variance of the difference $(X - Y)$ is,

$$\rho = \frac{\sigma_X^2 + \sigma_Y^2 - \sigma_{X-Y}^2}{2\sigma_X \sigma_Y}$$

and its estimate

$$r = \frac{s_X^2 + s_Y^2 - s_{X-Y}^2}{2s_X s_Y}$$

The main feature of this formula is that it does not involve covariance term and the value of r obtained by this formula is same as that obtained by Karl Pearson's formula.

Q. 57 Yield of foodgrains in kg/ha in Kharif and Rabi seasons from 1975-76 to 1984-85 were as follows:

Year	*Kharif (X)*	*Rabi (Y)*
1975-76	89	104
1976-77	94	109
1977-78	94	116
1978-79	78	105
1979-80	93	120
1980-81	95	119
1981-82	92	130
1982-83	106	131
1983-84	104	134
1984-85	105	142

Year	X	$X-\bar{X}$	$(X-\bar{X})^2$	Y	$Y-\bar{Y}$	$(Y-\bar{Y})^2$	$Y-X=Z$	$Z-\bar{Z}$	$(Z-\bar{Z})^2$	$(X-\bar{X})$ $\times(Y-\bar{Y})$
1975-76	89	−6	36	104	−17	289	15	−11	121	102
1976-77	94	−1	1	109	−12	144	15	−11	121	12
1977-78	94	−1	1	116	−5	25	22	−4	16	5
1978-79	78	−17	289	105	−16	256	27	1	1	272
1979-80	93	−2	4	120	−1	1	27	1	1	2
1980-81	95	0	00	119	−2	4	24	−2	4	0
1981-82	92	−3	9	130	9	81	38	12	144	−27
1982-83	106	11	121	131	10	100	25	−1	1	110
1983-84	104	9	81	134	13	169	30	4	16	117
1984-85	105	10	100	142	21	441	37	11	121	210
Total	950	00	642	1210	00	1510	260	00	546	803

(i) Find the coefficient of correlation by using variance of differences and also verify that this value is same as that of Pearsonian correlation coefficient.

(ii) Judge the significance of correlation coefficient by probable error.

(iii) Test the significance of the correlation coefficient by t-test. [Given: $t_{0.05,\,8} = 2.306$]

(iv) Find 95% confidence limits for ρ.

Ans. To calculate the value of r, we prepare the above table:

$$n = 10, \bar{X} = \frac{950}{10} = 95.0, \bar{Y} = \frac{1210}{10} = 121.0,$$

$$\bar{Z} = \frac{260}{10} = 26.0$$

$$s_X^2 = \frac{642}{9} = 71.33, s_Y^2 = \frac{1510}{9} = 167.78,$$

$$s_{Y-X}^2 = s_Z^2 = \frac{546}{9} = 60.67.$$

(i) The coefficient of correlation by the formula,

$$r = \frac{s_X^2 + s_Y^2 - s_{Y-X}^2}{2 s_X\, s_Y}$$

$$r = \frac{71.33 + 167.78 - 60.67}{2\sqrt{71.33 \times 167.78}}$$

$$= \frac{178.44}{218.80}$$

$$= 0.8155$$

Pearsonian correlation coefficient,

$$r = \frac{\Sigma\left(X-\bar{X}\right)\left(Y-\bar{Y}\right)}{\sqrt{\Sigma\left(X-\bar{X}\right)^2 \Sigma\left(Y-\bar{Y}\right)^2}}$$

$$= \frac{803}{\sqrt{642 \times 1510}}$$

$$= \frac{803}{984.59}$$

$$= 0.8155$$

The above results show that the coefficient of correlation obtained by the formula using the variance of the differences and by Pearsonian formula are exactly the same.

(ii) Probable error,

$$\text{P.E} = 0.6745\,\frac{1-(0.8155)^2}{\sqrt{10}}$$

$$= \frac{0.2260}{3.1623}$$

$$= 0.07145$$

Again 6 × P.E. = 6 × 0.07145

$$= 0.4287$$

Since 0.8155 > 0.4287, the coefficient of correlation is definitely significant.

(iii) To test the hypothesis,

$$H_0:\rho=0 \text{ vs. } H_1:\rho\neq 0$$

The test statistic,

$$t=\frac{0.8155\sqrt{10-2}}{\sqrt{1-(0.8155)^2}}$$

$$=\frac{2.3066}{0.5788}$$

$$=3.9851$$

Since the calculated value of t = 3.9851 is greater than the tabulated value of t = 2.306 for $\alpha = 0.05$ and 8 d.f., we reject H_0. It means that the correlation between the per hectare productions of Rabi and Kharif crops is significant.

(iv) To find 95 per cent confidence limits for ρ, we first find Z_L and Z_U and then retransform to r.

$$\left.\begin{matrix}Z_L\\ Z_U\end{matrix}\right]=1.1586\mp\frac{1.96}{2.646}$$

$$=1.1586\mp 0.7407$$

$$Z_L=0.4179, Z_U=1.8993$$

∴ Lower limit of ρ = 0.4 (approx.)

Upper limit of ρ = 0.955 (approx.)

Q. 58 How can the correlation coefficient between the two variables X and Y be calculated by the method of least squares?

Ans. By the method of least squares, we find the estimated value $\hat{Y}$ of Y by simple regression equation. Then find the deviations $d=Y-\hat{Y}$ and Σd^2. Calculate,

$$S_Y^2=\frac{\Sigma d^2}{N}$$

$$\sigma_Y^2=\frac{1}{N}\left(Y_i-\overline{Y}\right)^2$$

The formula for coefficient of correlation by the method of least squares is,

$$\rho=\sqrt{1-\frac{s_Y^2}{\sigma_Y^2}}$$

If we estimate X by the regression of X on Y as $\hat{X}$, the formula is,

$$\rho=\sqrt{1-\frac{s_X^2}{\sigma_X^2}}$$

where, $s_X^2=\dfrac{\Sigma d^2}{N}$ and $\sigma_X^2=\dfrac{1}{N}\left(X_i-\overline{X}\right)^2$

Q. 59 Calculate coefficient of correlation by the method of least squares for the following paired values of X and Y variables. Also verify that this value of r is same as that obtained by Pearson's formula.

X :	10	12	13	17	18
Y :	5	6	7	9	13

Ans. To calculate the coefficient of correlation by the method of least squares and otherwise, we prepare the following table.

X	$X-\overline{X}$	$(X-\overline{X})^2$	Y	$Y-\overline{Y}$	$(Y-\overline{Y})^2$	$(X-\overline{X})$ $\times(Y-\overline{Y})$	$\hat{X}$	$X-\hat{X}=d$	d^2
10	−4	16	5	−3	9	12	11	−1	1
12	−2	4	6	−2	4	4	12	0	0
13	−1	1	7	−1	1	1	13	0	0
17	3	9	9	1	1	3	15	2	4
18	4	16	13	5	25	20	19	−1	1
70	00	46	40	00	40	40	70	0	6

Ans. Tetrachoric correlation is suitable when both the variables X and Y are dichotomous. This measure of linear association between X and Y is based on the assumption that the underlying distribution is bivariate normal. Here the actual observations on X and Y are not available but their distribution of frequencies in a (2×2) contingency table are known. A measure of correlation in this situation is called tetrachoric correlation.

Q. 65 Explicate autocorrelation and its measure.

Ans. Often in time series data, it has been observed that the figures of consecutive periods are correlated. For example, the area under irrigation in a year is correlated to its previous year irrigated area, the area under high yielding varieties (HYV) in a year is correlated to the previous year area under HYV, etc. In this situation, the assumption of independent errors of linear probabilistic model is refuted, *i.e.*, if the linear model is,

$$Y = \alpha + \beta X + \varepsilon$$

$$E(\varepsilon_i \varepsilon'_i) \neq 0 \text{ for } i \neq i'$$

The simplest form of autocorrelation is the linear dependence of error on its previous year value. Symbolically,

$$e_t = be_{t-1} + \varepsilon_t$$

This is known as the first order autoregressive scheme. Thus, a measure of linear association between the errors of consecutive periods is given by autocorrelation. Autocorrelation is also known as *serial correlation*. The formula for autocorrelation is

$$r_A = \frac{\Sigma e_t e_{t-1}}{\sqrt{\Sigma e_t^2 \cdot \Sigma e_{t-1}^2}}$$

Q. 66 Why are multiple linear regression equations needed?

Ans. In real life problems, a variate value does not depend only on one independent variable but on many variables. Hence to estimate a dependent variable, it is not enough to include only one independent variable only but many independent variables. For example, the cost of a produced unit depends on the cost of raw material, labour cost, cost of energy, transportation, etc. In this situation, to estimate the cost of production, it is necessary to include all those variables in the regression model which add to the cost of the unit produced.

Q. 67 Give mathematical model for multiple linear regression and the method for its fitting.

Ans. A mathematical model with dependent variable Y and k independent variables $X_1, X_2, \ldots, X_k$ can be given as,

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + e$$

This equation is called *multiple linear regression equation* or *prediction equation* with Y as predictant and $X_1, X_2, \ldots, X_k$ as predictors. In this model e is the error term and for all types, e_i are i.i.d normal variates with mean 0 and variance σ_e^2, *i.e.*, $e \sim N(0, \sigma_e^2)$.

To fit in the regression model, suppose there are n composite sample observations which can be presented in the following format:

Composite sample No.	*Variables*						
	Y	X_1	X_2	...	X_j	...	X_k
1	y_1	x_{11}	x_{21}	...	x_{j1}	...	x_{k1}
2	y_2	x_{12}	x_{22}	...	x_{j2}	...	x_{k2}
⋮	⋮	⋮	⋮		⋮		⋮
i	y_i	x_{1i}	x_{2i}	...	x_{ji}	...	x_{ki}
⋮	⋮	⋮	⋮		⋮		⋮
n	y_n	x_{1n}	x_{2n}	...	x_{jn}	...	x_{kn}
Total	$\sum_i y_i$	$\sum_i x_{1i}$	$\sum_i x_{2i}$	...	$\sum_i x_{ji}$ ...		$\sum_i x_{ki}$

where $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, k$. To fit in the regression equation by the method of least squares and supposing that $b_0, b_1, b_2, \ldots, b_k$ are the estimates of $\beta_0, \beta_1, \beta_2, \ldots, \beta_k$, respectively, the set of normal equations is,

$$\sum_i y_i = nb_0 + b_1 \sum_i x_{1i} + b_2 \sum_i x_{2i} + \ldots + b_k \sum_i x_{ki}$$

$$\sum_i x_{1i} y_i = b_0 \sum_i x_{1i} + b_1 \sum_i x_{1i}^2 + b_2 \sum_i x_{1i} x_{2i} + \ldots + b_k \sum_i x_{1i} x_{ki}$$

$$\sum_i x_{2i}\, y_i = b_0 \sum_i x_{2i} + b_1 \sum_i x_{1i} x_{2i} + b_2 \sum_i x_{2i}^2 + \ldots + b_k \sum_i x_{2i}\, x_{ki}$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$\sum_i x_{ki}\, y_i = b_0 \sum_i x_{ki} + b_1 \sum_i x_{1i} x_{ki} + b_2 \sum_i x_{2i} x_{ki} + \ldots + b_k \sum_i x_{ki}^2$$

In matrix form, the above equations after transposal are,

$$\begin{bmatrix} n & \Sigma x_{1i} & \Sigma x_{2i} & \cdots & \Sigma x_{ki} \\ \Sigma x_{1i} & \Sigma x_{1i}^2 & \Sigma x_{1i} x_{2i} & \cdots & \Sigma x_{1i} x_{ki} \\ \Sigma x_{2i} & \Sigma x_{1i} x_{2i} & \Sigma x_{2i}^2 & \cdots & \Sigma x_{2i} x_{ki} \\ \vdots & & & & \vdots \\ \Sigma x_{ki} & \sum_i x_{1i} x_{ki} & \Sigma x_{2i} x_{ki} & \cdots & \Sigma x_{ki}^2 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix} = \begin{bmatrix} \Sigma y_i \\ \Sigma x_{1i} y_i \\ \Sigma x_{2i} y_i \\ \vdots \\ \Sigma x_{ki} y_i \end{bmatrix}$$

$$(X'X)_{(k+1)\times(k+1)}\, B_{(k+1)\times 1} = (X'Y)_{(k+1)\times 1}$$

If we denote the regression equation as

$$Y = X\beta + e$$

and its estimated equation as

$$Y = XB,$$

then the set of normal equations can be written as,

$$X'XB = X'Y$$

or

$$B = (X'X)^{-1} X'Y$$

where the matrices $X'X, B$ and $X'Y$ are as indicated above. The matrix $X'X$ is known as the *coefficient matrix.*

From the first normal equation we get,

$$b_0 = \bar{y} - b_1 \bar{x}_1 - b_2 \bar{x}_2 \ldots b_k \bar{x}_k$$

On substituting the value of b_0 and representing the deviation from mean of the respective variables as,

$$x_{ji} - \bar{x}_j = u_{ji} \text{ and } y_i - \bar{y} = v_i$$

The set of normal equations reduces to,

$$\underset{A_{k\times k}}{\begin{bmatrix} \Sigma u_{1i}^2 & \Sigma u_{1i} u_{2i} & \cdots & \Sigma u_{1i} u_{ki} \\ \Sigma u_{1i} u_{2i} & \Sigma u_{2i}^2 & \cdots & \Sigma u_{2i} u_{ki} \\ \vdots & & & \\ \Sigma u_{1i} u_{ki} & \Sigma u_{2i} u_{ki} & \cdots & \Sigma u_{ki}^2 \end{bmatrix}} \underset{B_{k\times 1}}{\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix}} = \underset{Y_{k\times 1}}{\begin{bmatrix} \Sigma u_{1i} v_i \\ \Sigma u_{2i} v_i \\ \vdots \\ \Sigma u_{ki} v_i \end{bmatrix}}$$

In matrix notation, the normal equations are,

$$AB = Y$$

or

$$B = A^{-1} Y$$

Suppose A is non-singular and its inverse is

$$A^{-1} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1k} \\ c_{21} & c_{22} & \cdots & c_{2k} \\ \vdots & & & \\ c_{k1} & c_{k2} & \cdots & c_{kk} \end{bmatrix}$$

Matrix A^{-1} is symmetric which means $c_{ij} = c_{ij}$. Hence, the matrix equations leading to the solution of b's are,

$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1k} \\ c_{21} & c_{22} & \cdots & c_{2k} \\ \vdots & & & \\ c_{k1} & c_{k2} & \cdots & c_{kk} \end{bmatrix} \begin{bmatrix} \Sigma u_{1i} v_i \\ \Sigma u_{2i} v_i \\ \vdots \\ \Sigma u_{ki} v_i \end{bmatrix}$$

Thus,

$$b_1 = c_{11}\, \Sigma u_{1i} v_i + c_{12}\, \Sigma u_{2i} v_i + \ldots + c_{1k}\, \Sigma u_{ki} v_i$$

$$b_2 = c_{21}\, \Sigma u_{1i} v_i + c_{22}\, \Sigma u_{2i} v_i + \ldots + c_{2k}\, \Sigma u_{ki} v_i$$

$$\vdots$$

$$b_k = c_{k1}\, \Sigma u_{1i} v_i + c_{k2}\, \Sigma u_{2i} v_i + \ldots + c_{kk}\, \Sigma u_{ki} v_i$$

Substituting the values of $b_0, b_1, b_2, \ldots, b_k$, the estimated linear regression equation is,

$$\hat{Y} = \bar{y} + b_1 (X_1 - \bar{x}_1) + b_2 (X_2 - \bar{x}_2) + \ldots + b_k (X_k - \bar{x}_k)$$

From this equation Y can be predicted for given values of $X_1, X_2, \ldots, X_k$. In multiple linear regression equation, each of β_j (or b_j) is known as *partial* *regression coefficient.* Its range is from $-\infty$ to ∞.

Definition. Partial regression coefficient β_j is the measure of change in dependent variable Y corresponding to a unit increase in the independent variable X_j whereas the other independent variables are kept fixed.

Note: When $k = 2$, the equation of multiple linear regression reduces to plane of regression.

Q. 68 How will you test the significance of a partial regression coefficient?

Ans. To test the hypothesis

$$H_0:\beta_j = 0 \text{ vs. } H_1:\beta_j \neq 0$$

for $j = 1, 2, ..., k$

The test statistic is,

$$t = \frac{b_j}{s_{b_j}}$$

Statistic t has (n−k−1) d.f.

where, $s_{b_j}^2 = s_e^2 \, c_{jj}$

c_{jj} is the $(j, j)^{\text{th}}$ cell element of A^{-1} and

$$s_e^2 = \frac{\sum_i \left(Y_i - \hat{Y}_i\right)^2}{(n-k-1)}$$

$$= \frac{1}{n-k-1}\left(\Sigma v_i^2 - R^2\,\Sigma v_i^2\right)$$

whereas,

$$R^2\,\Sigma v_i^2 = b_1\,\Sigma u_{1i}v_i + b_2\,\Sigma u_{2i}v_i + \ldots + b_k\,\Sigma u_{ki}v_i$$

The quantity $R^2 \sum_i v_i^2$ is known as the *regression sum of squares,*

$$s_{b_j} = \sqrt{s_{b_j}^2}$$

Decision about H_0 is taken in the usual way. If H_0 is rejected, it means that the contribution of X_j in estimating Y is significant. If H_0 is accepted, it means that the inclusion of X_j in the equation is redundant.

Q. 69 How can you test the equality of two partial regression coefficients.

Ans. Sometime one wants to test whether the contribution of two independent variables X_j and X_l in estimating Y is same or not. Thus, the hypothesis

$$H_0: \beta_j = \beta_l \quad \text{vs.} \quad H_1: \beta_j \neq \beta_l$$

for $j \neq l = 1, 2, ..., k$

can be tested by the statistic

$$t = \frac{b_j - b_l}{s_{b_j - b_l}}$$

t has $(n - k - l)$ d.f.

where $s_{b_j-b_l}^2 = s_e^2\left(c_{jj} + c_{ll} - 2c_{jl}\right)$.

The decision about H_0 can be taken in the usual manner.

Q. 70 How can the significance of partial regression coefficient be tested simultaneously?

Ans. The hypothesis

$$H_0: \quad \beta_1 = \beta_2 = \ldots = \beta_k = 0$$

vs. H_1: at least one of β_j is not zero

for $j = 1, 2, ..., k$

can be tested through analysis of variance. The ANOVA table is,

Source	*d.f.*	*S.S.*	*M.S.*	*F-value*
Regression	k	$R^2 \sum_i v_i^2$	$\dfrac{R^2 \sum_i v_i^2}{k}$	$\dfrac{(n-k-1)R^2 \sum_i v_i^2}{k(\sum_i v_i^2 - R^2 \sum_i v_i^2)}$
Residual (Deviation from regression)	$(n-k-1)$	$\sum_i v_i^2 - R^2 \sum_i v_i^2$	$\dfrac{\sum_i v_i^2 - R^2 \sum_i v_i^2}{(n-k-1)}$	
Total	$n-1$	$\sum_i v_i^2$		

The statistic F in the last column of ANOVA has $(k, n-k-1)$ d.f. If $F_{Cal} \geq F_{\alpha,(k,n-k-1)}$, reject H_0. It means, all β's are not zero. Again if $F_{Cal} < F_{\alpha(k,n-k-1)}$, accept H_0. This indicates that the independent variables $X_1, X_2, \ldots, X_k$ are not suitable for predicting Y

Q. 71 Delineate the term correlation index.

Ans. The ratio,

$$R^2 = \frac{\text{S.S due to regression}}{\text{Total corrected S.S}}$$

$$= \frac{R^2 \sum_i v_i^2}{\sum_i v_i^2}$$

is referred to as *correlation index* or *coefficient of determination as* R^2 is analogous to r^2 in simple linear regression.

Q. 72 In what ways each potential parameter in a model can be specified?

Ans. Each potential parameter in a model can be specified in either of the three categories, namely, a free parameter, a fixed parameter or a constrained parameter.

Q. 73 What is a free parameter?

Ans. A free parameter is that parameter which is unknown and is thereby to be estimated.

Q. 74 Define a fixed parameter.

Ans. A fixed parameter is that parameter which is not free but has a fixed value either 0 or 1.

Q. 75 Explicate a constrained parameter.

Ans. A parameter which is unknown but is constrained to be equal to one or more parameters is called a constrained parameter.

Q. 76 Define multiple correlation coefficient and explain its utility.

Ans. Multiple correlation coefficient

$$R = +\sqrt{R^2}$$

Definition-1. It may be defined as a measure of linear association of a variable Y (say) with all other X-variables.

Definition-2. Multiple correlation coefficient is the simple correlation between Y and $\hat{Y}$, where $\hat{Y} = \overline{Y} + b_1X_1 + b_2X_2 + \ldots + b_kX_k$.

The range of R is from 0 to 1.

The value of R is indicative as to whether the linear regression equation is a good fit or not. If R is near to 1, the regression equation is a good fit otherwise not.

Q. 77 Express multiple correlation coefficient in terms of simple correlation coefficients.

Ans. If $X_1, X_2, \ldots, X_k$ are k variables and the sample correlation coefficient between any two variables X_i and X_j for $i, j = 1, 2, \ldots, k$ is r_{ij}, correlation matrix P consists of the elements which are the simple correlation coefficients among all possible pairs of variables X_i and X_j. Also $r_{ij} = r_{ji}$. Thus, the correlation matrix,

$$P = \begin{bmatrix} 1 & r_{12} & r_{13} & \cdots & r_{1k} \\ r_{21} & 1 & r_{23} & \cdots & r_{2k} \\ r_{31} & r_{32} & 1 & \cdots & r_{3k} \\ \vdots & & & & \\ r_{k1} & r_{k2} & r_{k3} & \cdots & 1 \end{bmatrix}$$

The multiple correlation coefficient of a variable X_j with rest of the variables can be obtained by the formula,

$$R_{j\cdot 12\ldots(j-1)(j+1)\ldots k} = \left(1 - \frac{|P|}{P_{jj}}\right)^{1/2}$$

where $|P|$ is the determinant of the correlation matrix P and P_{jj} is the cofactor of r_{jj}.

Here it is revealed that the sign of $|P|$ and P_{jj} will always be the same. Also, $|P| \leq P_{jj}$. If both these conditions do not hold good, the value of R (suffix omitted) will become greater than 1 which is an impossibility.

The range of R is 0 to 1. If $R = 0$, it means X_j has no linear relationship with rest of the variables and

$R = 1$, shows that X_j has perfect linear relationship with the remaining variables. Also $R_{1.23}$ is never less than any of simple correlation coefficients r_{12}, r_{13} and r_{23}, *i.e.*, $R_{1.23} \geq r_{12}, r_{13}, r_{23}$.

Q. 78 If there are three variables X_1, X_2 and X_3, express the multiple correlation coefficient of a variable with the other two.

Ans. Following usual notations, the formulae for multiple correlation coefficients $R_{1.23}, R_{2.13}$ and $R_{3.12}$ in terms of r_{12}, r_{13} and r_{23} are:

$$R_{1.23} = \sqrt{\frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{13}r_{23}}{1 - r_{23}^2}}$$

$$R_{2.13} = \sqrt{\frac{r_{12}^2 + r_{23}^2 - 2r_{12}r_{23}r_{13}}{1 - r_{13}^2}}$$

$$R_{3.12} = \sqrt{\frac{r_{13}^2 + r_{23}^2 - 2r_{13}r_{23}r_{12}}{1 - r_{12}^2}}$$

Q. 79 Fit a plane of regression in a trivariate population with the help of correlation coefficients.

Ans. Let the equation of the plane involving three random variables X_1, X_2 and X_3 be,

$$X_1 = a + b_{12.3}X_2 + b_{13.2}X_3 + e_i$$

Let the means and standard deviations of X_1, X_2 and X_3 be $\overline{X}_1, \overline{X}_2, \overline{X}_3$ and $\sigma_1, \sigma_2, \sigma_3$ respectively. Also r_{12}, r_{13} and r_{23} be the simple correlation coefficients between three pairs of variables. In the above equation $b_{12.3}$ and $b_{13.2}$ are known as the partial regression coefficients. Without loss of generality we can assume that X_1, X_2 and X_3 are measured from their respective means. Thus, the estimates of $b_{12.3}$ and $b_{13.2}$ by the principle of least squares are:

$$b_{12.3} = -\frac{\sigma_1}{\sigma_2}\frac{P_{12}}{P_{11}} \text{ and } b_{13.2} = -\frac{\sigma_1}{\sigma_3}\frac{P_{13}}{P_{11}}$$

where P_{ij} is the cofactor of r_{ij} in the determinant,

$$\begin{vmatrix} 1 & r_{12} & r_{13} \\ r_{12} & 1 & r_{23} \\ r_{13} & r_{23} & 1 \end{vmatrix} \qquad \because r_{ij} = r_{ji}$$

Thus,

$$b_{12.3} = \frac{\sigma_1}{\sigma_2} \cdot \frac{r_{12} - r_{13}r_{23}}{1 - r_{23}^2} = -\frac{P_{12}}{P_{11}} \cdot \frac{\sigma_1}{\sigma_2}$$

$$b_{13.2} = -\frac{\sigma_1}{\sigma_3} \cdot \frac{r_{13} - r_{12}r_{23}}{1 - r_{23}^2} = -\frac{P_{13}}{P_{11}} \cdot \frac{\sigma_1}{\sigma_3}$$

Hence, the equation of plane of regression is,

$$\hat{X}_1 = b_{12.3}X_2 + b_{13.2}X_3$$

Shifting back to the origin, the equation becomes,

$$\left(\hat{X}_1 - \overline{X}_1\right) = b_{12.3}\left(X_2 - \overline{X}_2\right) + b_{13.2}\left(X_3 - \overline{X}_3\right)$$

which is,

$$\hat{X}_1 = a + b_{12.3}X_2 + b_{13.2}X_3$$

where, $a = \overline{X}_1 - b_{12.3}\overline{X}_2 - b_{13.2}\overline{X}_3$

The same equation of the plane can simply be obtained by the determinable equation,

$$\begin{vmatrix} \dfrac{\hat{X}_1 - \overline{X}_1}{\sigma_1} & \dfrac{X_2 - \overline{X}_2}{\sigma_2} & \dfrac{X_3 - \overline{X}_3}{\sigma_3} \\ r_{12} & 1 & r_{23} \\ r_{13} & r_{23} & 1 \end{vmatrix} = 0$$

Expanding the determinant, we get

$$\frac{\hat{X}_1 - \overline{X}_1}{\sigma_1}\left(1 - r_{23}^2\right) + \frac{X_2 - \overline{X}_2}{\sigma_2}\left[-\left(r_{12} - r_{13}r_{23}\right)\right] + \frac{\left(X_3 - \overline{X}_3\right)}{\sigma_3}\left(r_{12}r_{23} - r_{13}\right) = 0$$

$$\left(\hat{X}_1 - \overline{X}\right) + \left(X_2 - \overline{X}_2\right)\frac{P_{12}}{P_{11}} \cdot \frac{\sigma_1}{\sigma_2} + \left(X_3 - \overline{X}_3\right)\frac{P_{13}}{P_{11}} \cdot \frac{\sigma_1}{\sigma_3} = 0$$

Definition. Partial correlation coefficient may be defined as a measure of degree of association between any two variables out of a set of variables eliminating the common association of remaining variables from both of them.

Q. 83 Give the formula for partial correlation coefficient in terms of simple correlation coefficients.

Ans. If the correlation matrix for k variables $X_1, X_2, \ldots, X_k$ is,

$$P = \begin{bmatrix} 1 & r_{12} & r_{13} & \cdots & r_{1k} \\ r_{21} & 1 & r_{23} & \cdots & r_{2k} \\ r_{31} & r_{32} & 1 & \cdots & Y_{3k} \\ \vdots & & & & \\ r_{k1} & r_{k2} & r_{k3} & \cdots & 1 \end{bmatrix}$$

The partial correlation coefficient between any two variables X_i and X_j for $i \neq j = 1, 2, \ldots, k$ can be computed by the formula,

$$r_{ij \cdot 12 \ldots (i-1)(i+1) \ldots (j-1)(j+1) \ldots k} = \frac{P_{ij}}{\sqrt{P_{ii} P_{jj}}}$$

where, P_{ij}, P_{ii} and P_{jj} are the cofactors of r_{ij}, r_{ii} and r_{jj} respectively in the determinant of the correlation matrix P. The range of partial correlation coefficient is from -1 to 1.

Q. 84 Express partial correlation coefficient between any two variables eliminating the influence of the third variable out of the variables X_1, X_2 and X_3 in terms of simple correlation coefficients.

Ans. If r_{12}, r_{13} and r_{23} are the simple correlation coefficients between the variables X_1 and X_2; X_1 and X_3; X_2 and X_3 respectively, the formulae for the partial correlation coefficients are:

$$r_{12.3} = \frac{r_{12} - r_{13} r_{23}}{\sqrt{\left(1 - r_{13}^2\right)\left(1 - r_{23}^2\right)}}$$

$$r_{13.2} = \frac{r_{13} - r_{12} r_{23}}{\sqrt{\left(1 - r_{12}^2\right)\left(1 - r_{23}^2\right)}}$$

$$r_{23.1} = \frac{r_{23} - r_{12} r_{13}}{\sqrt{\left(1 - r_{12}^2\right)\left(1 - r_{13}^2\right)}}$$

Recall $r_{12} = r_{21}$, $r_{13} = r_{31}$ and $r_{23} = r_{32}$. Also the partial correlation coefficients $r_{12.3}, r_{13.2}$ and $r_{23.1}$ are called the ***first order partial correlation coefficients*** as there is only one figure in the suffix to the right of the dot.

Q. 85 **Write the formula for the partial correlation coefficient $r_{12.34}$ in case of multivariate study of the four variables X_1, X_2, X_3 and X_4 in terms of simple correlation coefficients.**

Ans. If $r_{12}, r_{13}, r_{14}, r_{23}, r_{24}$ and r_{34} are the simple correlation coefficients between two variables marked by the respective suffixes and $r_{12.4}$, $r_{13.4}$, $r_{23.4}$, $r_{12.3}$, $r_{14.3}$ and $r_{24.3}$ are the first order partial correlation coefficients, the partial correlation coefficient,

$$r_{12.34} = \frac{r_{12.4} - r_{13.4} r_{23.4}}{\sqrt{\left(1 - r_{13.4}^2\right)\left(1 - r_{23.4}^2\right)}}$$

or alternatively,

$$r_{12.34} = \frac{r_{12.3} - r_{14.3} r_{24.3}}{\sqrt{\left(1 - r_{14.3}^2\right)\left(1 - r_{24.3}^2\right)}}$$

$r_{12.34}$ or any other likewise partial correlation coefficient is called ***second order partial correlation coefficient*** as there are two figures in the suffix after the dot. So to calculate second order partial correlation coefficient, one has to compute first order partial correlation coefficients from simple correlation coefficients. Formulae for $r_{13.24}$, $r_{14.23}$, etc., can be written by simply changing the suffixes.

Q. 86 Expatiate part correlation.

Ans. Part correlation is the correlation between two variables with variable(s) controlled for in one of the two variables in which the correlation is worked out.

More simply, in partial correlation $r_{12.3}$, the effect of variable 3 is eliminated from both the variables 1 estimated by the use of _________ eliminating the effect of concomitant variables.

51. Least square estimate of β_{YX} is _________.

52. Least square estimate of β_0 is _________.

53. If ε of the regression model $Y = \beta_0 + \beta_1 X + \varepsilon$ is distributed as $N\left(0, \sigma_\varepsilon^2\right)$, the variance of the estimate b_1 of β_1 is _________.

54. If the error $\varepsilon \sim N\left(0, \sigma_\varepsilon^2\right)$ in $Y = \beta_0 + \beta_1 X + \varepsilon$, the variance of b_0 is _________ where b_0 is an estimate of β_0.

55. The equation $Y = \gamma\left(1 - e^{\alpha - \beta X}\right)$ for $\alpha, \beta, \gamma >$ 0 represents _________.

56. $Y = \alpha + \beta X + \gamma X^2$ is the equation of a _________.

57. The function $\log_e Y = \log_e \gamma + \beta^X \log_e \alpha$ represents _________ curve.

58. The equation $Y = 1/(a + bX)$ represents _________.

59. The equation for exponential growth curve is _________.

60. The equation for logistic growth curve is _________.

61. In orthogonal polynomial, the _________ terms can be added one by one.

62. The method of fitting orthogonal polynomials was given by _________.

63. The idea of correlation was given by _________.

64. Karl Pearson defined correlation in the year _________.

65. Pearson's formula for correlation coefficient ρ_{XY} is _________.

66. If (X_i, Y_i) are n pairs of observations on the variables X and Y, the formula for correlation coefficient is _________.

67. The quantity ρ^2 (or r^2) is known as _________.

68. The quantity $(1 - \rho^2)$, where ρ is the correlation coefficient, is called _________.

69. If ρ is the correlation coefficient, the $\sqrt{1 - \rho^2}$ is termed as _________.

70. The range of Pearson's coefficient of correlation is _________.

71. If $\rho_{XY} = 1$, the line on the graph will extend from _________ to _________.

72. If $\rho_{XY} = -1$, it means that there is a _________ correlation between X and Y.

73. If $\rho = 0$, it depicts _________ association.

74. Correlation coefficient is a _________ number.

75. If each value of data is reduced by 10, the correlation coefficient between coded values is _________ as that of original values.

76. If the variate values of X are divided by 100 and Y values by 1000, the correlation between X and Y from coded values is _________ as that of original values.

77. If each value of X and Y is reduced by 5 and then divided by 10, the correlation between coded values is _________ as that of original observations.

78. The relation between the regression coefficient β_{YX} and correlation coefficient ρ is _________.

79. The regression coefficient β_{XY} can be expressed in terms of correlation coefficient ρ as _________.

80. The origin of correlation coefficient lies in _________ distribution.

81. The standard error of sample correlation coefficient r, based on n paired values, is _________.

82. The formula for probable error with usual notations is _________.

83. Probable error helps to know the _________ of correlation coefficient.

144. If the line of regression of Y on X is $Y = 2X + 1$ and of X on Y is $6X = Y - 3$, then corr $(X, Y) =$ ________.

145. If the lines of regression of Y on X is $4X - 5Y + 33 = 0$ and of X on Y is $20X - 9Y - 107 = 0$, the mean value $\overline{X}$ is ________ and $\overline{Y}$ ________.

146. If the regression line of Y on X is $2Y = 3X - 6$, the estimated value of Y for given value of $X = 10$ is ________.

147. The given values, $b_{YX} = \frac{1}{5}$ and $b_{XY} = 10$ are ________.

148. If the correlation coefficient is zero, the value of regression coefficient is ________.

149. Both the regression coefficients cannot exceed ________.

150. If $R_{1.23} = 1$, then $R_{2.13}$ is equal to ________.

151. The correlation between the number of blinds per year and the production of liquor per year is ________ correlation.

152. If the sum of the product of the deviation of X and Y from their means is zero, the correlation between X and Y is ________.

153. If the probable error is more than r, the correlation coefficient is ________.

154. If one regression coefficient is negative, the other would be ________.

155. The values $r_{12} = 0.6$, $r_{13} = -0.5$ and $r_{23} = 0.8$ obtained in a certain investigation are ________.

156. The multiple correlation coefficient in multiple regression analysis is equivalent to simple correlation between a variable Y and ________.

157. Two regression lines of standard normal deviates pass through the ________.

158. If $\rho = 0$, the regression line of Y on X will be parallel to ________ at a distance of ________.

159. The regression line of Y on X, the two uncorrelated standard normal deviates, is ________.

160. The two regression lines of two uncorrelated standard normal variates are ________ to each other.

161. The slope of regression lines of two standard normal deviates having zero correlation is ________.

162. If simple correlation coefficient is zero, then regression coefficient is equal to ________.

163. If $\rho = 1$ between X and Y, then the two regression lines may intersect at an angle of ________.

164. An unknown parameter in a model likely to be estimated is called a ________ parameter.

165. A parameter of a model, which can take values either zero or unity, is specified as ________ parameter.

166. A parameter in a regression model which is forced to be equal to some other parameter(s) is termed as ________ parameter.

167. A regression equation with a dependent variable and two independent variables is called a ________.

168. Out of three variables X_1, X_2 and X_3 if X_3 influences either X_1 or X_2, then to find the correlation between X_1 and X_2 eliminating the effect of X_3, one should preferably calculate ________.

169. The formula for part correlation $r_{1(3.2)}$ is ________.

170. Part correlation coefficient is always ________ than partial correlation coefficient.

SECTION-C

Multiple Choice Questions

Select the correct alternative out of given ones:

Q. 1 The term regression was introduced by:
(a) R.A. Fisher
(b) Sir Francis Galton
(c) Karl Pearson
(d) none of the above

Q. 2 If X and Y are two variates, there can be at most:
(a) one regression line
(b) two regression lines
(c) three regression lines
(d) an infinite number of regression lines

Q. 3 In a regression line of Y on X, the variable X is known as:
(a) independent variable
(b) regressor
(c) explanatory variable
(d) all the above

Q. 4 Regression equation is also named as:
(a) prediction equation
(b) estimating equation
(c) line of average relationship
(d) all the above

Q. 5 Scatter diagram of the variate values (X, Y) gives the idea about:
(a) functional relationship
(b) regression model
(c) distribution of errors
(d) none of the above

Q. 6 The estimate of β in the regression equation $Y = \alpha + \beta X + e$ by the method of least squares is:
(a) biased
(b) unbiased
(c) consistent
(d) efficient

Q. 7 The formula for the estimate of β in the regression equation $Y = \alpha + \beta X + \varepsilon$ is:
(a) $\text{cov}(X,Y)/V(X)$
(b) $r\sigma_Y/\sigma_X$
(c) $\Sigma(X_i-\overline{X})(Y_i-\overline{Y})/\Sigma(X_i-\overline{X})^2$
(d) all the above

Q. 8 In the regression line $Y = \alpha + \beta X$, β is called the:
(a) slope of the line
(b) intercept of the line
(c) neither (a) nor (b)
(d) both (a) and (b)

Q. 9 In the regression line $Y = \beta_0 + \beta_1 X$, β_0 is the:
(a) slope of the line
(b) intercept of the line
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 10. If β_{YX} and β_{XY} are two regression coefficients, they have:
(a) same sign
(b) opposite sign
(c) either same or opposite signs
(d) nothing can be said

Q. 11 The property that β_{YX} and β_{XY} and ρ have same signs, it called:
(a) fundamental property
(b) signature property
(c) magnitude property
(d) none of the above

Q. 12 The average of two regression coefficients is always greater than or equal to the correlation coefficient is called:
(a) fundamental property
(b) signature property
(c) magnitude property
(d) mean property

Q. 13 If $\beta_{YX} > 1$, then β_{XY} is:
(a) less than 1
(b) greater than 1
(c) equal to 1
(d) equal to 0

(a) $\sigma_e^2/(n-2)$

(b) $\sigma_e^2/\Sigma u_i^2$

(c) σ_e^2/n

(d) none of the above

Q. 51 If in a regression equation $Y = \alpha + \beta X + e$, $e \sim N\left(0, \sigma_e^2\right)$, the standard deviation of a, the estimate of α, is:

(a) σ_e^2/n

(b) $\sigma_e^2/\Sigma u_i^2$

(c) $\sigma_e/\sqrt{n}$

(d) none of the above

Q. 52 The function,

$$\frac{1}{Y} = \alpha\beta^X + \gamma \quad \text{for} \quad \alpha, \beta, \gamma > 0$$

represent:

(a) logistic growth curve
(b) compertz curve
(c) equilateral hyperbola
(d) exponential growth curve

Q. 53 The equation $Y = \alpha\beta^{-X}$ for $\beta < 1$ represents:

(a) exponential growth curve
(b) exponential decay curve
(c) a parabola
(d) none of the above

Q. 54 The function $Y = a + bX + cX^2 + dX^3$ represents:

(a) a hyperbola
(b) a exponential curve
(c) a parabola
(d) all the above

Q. 55 The mathematical function,

$$\frac{1}{Y} = a + b\left(\frac{1}{X}\right)$$

represents:

(a) an equilateral hyperbola
(b) exponential curve
(c) compertz curve
(d) none of the above

Q. 56 The model

$$Y = \gamma\left(1 - e^{\alpha - \beta X}\right) \text{ for } \alpha, \beta, \gamma > 0$$

is known as,

(a) equilateral hyperbola
(b) compertz curve
(c) Mistcherlich function
(d) none of the above

Q. 57 The mathematical function

$$Y = \gamma\alpha^{\beta^X} \quad \text{for } \alpha, \beta, \gamma > 0$$

represents:

(a) an equilateral hyperbola
(b) compertz curve
(c) Mistscherlich function
(d) none of the above

Q. 58 The advantage of orthogonal polynomial is:

(a) one can fit a polynomial of appropriate degree
(b) it saves time of computation
(c) easy to fit in the equation
(d) all the above

Q. 59 If the correlation between the two variables X and Y is negative, the regression coefficient of Y on X is:

(a) positive
(b) negative
(c) not certain
(d) none of the above

Q. 60 Given the two lines of regression as, $3X - 4Y + 8 = 0$ and $4X - 3Y = 1$, the means of X and Y are:

(a) $\overline{X} = 4, \overline{Y} = 5$

(b) $\overline{X} = 3, \overline{Y} = 4$

• (c) $\overline{X} = \frac{4}{3}, \overline{Y} = \frac{5}{4}$

(d) none of the above

Q. 61 The idea of product moment correlation was given by:

(a) R.A. Fisher
(b) Sir Francis Galton

(c) Karl Pearson
(d) Spearman

Q. 62 Correlation coefficient was invented in the year:
(a) 1910
(b) 1890
(c) 1908
(d) none of the above

Q. 63 The formula for simple correlation coefficient between the variables X and Y with usual notations is:
(a) $\text{cov}(X, Y)/\sqrt{V(X)V(Y)}$
(b) $\mu_{XY}/\sqrt{\mu_{XX}\mu_{YY}}$
(c) $\sigma_{XY}/\sigma_X\sigma_Y$
(d) all the above

Q. 64 The formula for calculating r from n paired sample values (X_i, Y_i) is:

(a) $$r = \frac{\Sigma(X_i - \overline{X})\Sigma(Y_i - \overline{Y})}{\sqrt{\Sigma(X_i - \overline{X})^2\,\Sigma(Y_i - \overline{Y})^2}}$$

(b) $$r = \frac{\Sigma X_i Y_i}{\Sigma X_i^2\,\Sigma Y_i^2}$$

(c) $$r = \frac{\Sigma(X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\Sigma(X_i - \overline{X})^2\,\Sigma(Y_i - \overline{Y})^2}}$$

(d) all the above

Q. 65 If ρ is the simple correlation coefficient, the quantity ρ^2 is known as:
(a) coefficient of determination
(b) coefficient of non-determination
(c) coefficient of alienation
(d) none of the above

Q. 66 If ρ is the simple correlation, the quantity $(1 - \rho^2)$ is called:
(a) coefficient of determination
(b) coefficient of non-determination
(c) coefficient of alienation
(d) none of the above

Q. 67 If ρ is the correlation coefficient, the quantity $\sqrt{1-\rho^2}$ is termed as:
(a) coefficient of determination
(b) coefficient of non-determination
(c) coefficient of alienation
(d) all the above

Q. 68 The unit of correlation coefficients is:
(a) kg/cc
(b) per cent
(c) non-existing
(d) none of the above

Q. 69 The range of simple correlation coefficient is:
(a) 0 to ∞
(b) $-\infty$ to ∞
(c) 0 to 1
(d) -1 to 1

Q. 70 If $\rho = 1$, the relation between the two variables X and Y is:
(a) Y is proportional to X
(b) Y is inversely proportional to X
(c) Y is equal to X
(d) none of the above

Q. 71 If $\rho_{XY} = 0$, the variables X and Y are:
(a) linearly related
(b) independent
(c) not linearly related
(d) none of the above

Q. 72 If $\rho_{XY} = -1$, the relation between X and Y is of the type:
(a) when Y increases, X also increases
(b) when Y decreases, X also decreases
(c) X is equal to $-Y$
(d) when Y increases, X proportionately decreases

Q. 73 The correlation between two variables is of order:
(a) 2
(b) 1
(c) 0
(d) none of the above

Q. 74 The geometric mean of the two regression coefficient b_{YX} and b_{XY} is equal to:

Q. 84 Z_r, the Fisher's transform of the correlation coefficient r based on a sample of size n in the test of H_0: $\rho = \rho_0$ is distributed as:

(a) $N\left(0, Z_{\rho_0}\right)$

(b) $N\left(Z_{\rho_0}, 1\right)$

(c) $N\left(Z_{\rho_0}, n-3\right)$

(d) $N\left(Z_{\rho_0}, \frac{1}{n-3}\right)$

Q. 85 The test statistic for testing H_0: $\rho = \rho_0$ with usual notations is:

(a) $Z = \frac{Z_r - Z_{\rho_0}}{1/(n-3)}$

(b) $Z = \frac{Z_r - Z_0}{1/(n-3)}$

(c) $Z = \frac{Z_r - Z_{\rho_0}}{1/\sqrt{n-3}}$

(d) none of the above

Q. 86 Test statistic for testing $H_0 : \rho_1 = \rho_2$ with usual notations is:

(a) $Z = \left(Z_{r_1} - Z_{r_2}\right) \Big/ \sqrt{\left(\frac{1}{n_1-3} + \frac{1}{n_2-3}\right)}$

(b) $Z = \left(Z_{r_1} - Z_{r_2}\right) \Big/ \left(\frac{1}{n_1-3} + \frac{1}{n_2-3}\right)$

(c) $Z = \frac{Z_{r_1}}{n_1-3} + \frac{Z_{r_2}}{n_2-3}$

(d) all the above

Q. 87 Homogeneity of three or more population correlation coefficients can be tested by:

(a) t-test
(b) Z-test
(c) χ^2-test
(d) F-test

Q. 88 Coefficient of concurrent deviation depends on:

(a) the signs of the deviations
(b) the magnitude of deviation
(c) both (a) and (b)
(d) none of (a) and (b)

Q. 89 The formula for coefficient of concurrent deviation with usual notations is:

(a) $\sqrt{\frac{2c-n}{n}}$

(b) $\pm\sqrt{\pm\frac{2c-n}{n}}$

(c) $\pm\sqrt{\frac{2c-n}{n}}$

(d) $\pm\sqrt{-\frac{2c-n}{n}}$

Q. 90 Formula for coefficient of correlation by making use of the variance of the difference $(X - Y)$ of the variables X and Y is:

(a) $\rho = \frac{\sigma_X^2 + \sigma_Y^2 + \sigma_{X-Y}^2}{2\sigma_X \sigma_Y}$

(b) $\rho = \frac{\sigma_X^2 + \sigma_Y^2 - \sigma_{X-Y}^2}{\sigma_X \sigma_Y}$

(c) $\rho = \frac{\sigma_X^2 + \sigma_Y^2 - \sigma_{X-Y}^2}{2\sigma_X^2 \sigma_Y^2}$

(d) $\rho = \frac{\sigma_X^2 + \sigma_Y^2 - \sigma_{X-Y}^2}{2\sigma_X \sigma_Y}$

Q. 91 The formula for calculating the correlation coefficient by the method of least squares with usual notations is:

(a) $\rho = \sqrt{1 - \frac{S_Y^2}{\sigma_Y^2}}$

(b) $\rho = \sqrt{1 - \frac{S_X^2}{\sigma_X^2}}$

(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 92 Correlation ratio is an appropriate measure of relationship between the two variables X and Y only if the functional relationship between them is:

(a) linear
(b) non-linear
(c) parabolic
(d) none of the above

Q. 93 The value of correlation ratio varies from:

(a) -1 to 1
(b) -1 to 0
(c) 0 to 1
(d) 0 to ∞

Q. 94 If all the observations lie in one group, the value of correlation ratio E^2 is:

(a) 0
(b) 1
(c) between 0 and 1
(d) between -1 and 1

Q. 95 If each group consists of one observation only, the value of correlation ratio is:

(a) 0
(b) 1
(c) between 0 to 1
(d) between -1 and 1

Q. 96 The relation between Pearson's correlation coefficient ρ and correlation ratio η^2 is:

(a) $\rho^2 \le \eta^2$
(b) $\rho^2 > \eta^2$
(c) $\rho^2 < \eta^2$
(d) none of the above

Q. 97 If there are k groups and each group consists on n observations, the limits of intraclass correlation are:

(a) 0 to 1
(b) $\dfrac{1}{n-1}$ to 1
(c) $-\dfrac{1}{n-1}$ to 1
(d) -1 to 1

Q. 98 From a given $(2 \times c)$ contingency table, the appropriate measure of association is:

(a) correlation ratio
(b) biserial correlation
(c) intraclass correlation
(d) tetrachoric correlation

Q. 99 When both the variables X and Y are dichotomous, a suitable measure of association between X and Y is:

(a) correlation ratio
(b) biserial correlation
(c) intraclass correlation
(d) tetrachoric correlation

Q. 100 Another name of autocorrelation is:

(a) biserial correlation
(b) serial correlation
(c) Spearman's correlation
(d) none of the above

Q. 101 A measure of linear association between the errors of consecutive periods is called:

(a) autocorrelation
(b) serial correlation
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 102 The idea of multiple regression equation strikes the mind only when:

(a) a variable Y depends on two independent variables only
(b) a variable Y depend on a number of independent variables
(c) the relation between a dependent variable and independent variables is not known
(d) all the above

Q. 103 A coefficient of any independent variable in a multiple linear regression equation is known as:

(a) partial regression coefficient
(b) multiple regression coefficient
(c) simple regression coefficient
(d) none of the above

Q. 104 The ratio of the regression sum of squares to the total sum of square is called:

(a) correlation index
(b) coefficient of determination
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 105 Significance of the partial regression coefficients can simultaneously be tested by:

(a) t-test
(b) Z-test
(c) χ^2-test
(d) F-test

Q. 106 A measure of linear association of a variable say, X_1 with a number of other variables $X_2, X_3, \ldots, X_k$ is known as:

(a) partial correlation
(b) multiple correlation
(c) simple correlation
(d) autocorrelation

Q. 107 The range of multiple correlation coefficient R is:

(a) 0 to 1
(b) 0 to ∞
(c) -1 to 1
(d) $-\infty$ to ∞

Q. 108 The range of a partial correlation coefficient is:

(a) 0 to 1
(b) 0 to ∞
(c) -1 to 1
(d) $-\infty$ to ∞

Q. 109 If the value of multiple correlation coefficient R is near to 1, it leads to the conclusion that:

(a) there is a lack of linear relationship
(b) Linear relation is a good fit
(c) there is a curvilinear relation
(d) all the above

Q. 110 The multiple correlation coefficient $R_{1.23}$ of X_1 with X_2 and X_3 variable is always:

(a) greater than the correlation coefficient of zero order
(b) less than the correlation coefficient of zero order
(c) equal to each of the correlation coefficient of zero
(d) all the above

Q. 111 If P is the correlation matrix, the formula for multiple correlation coefficient $R_{i.12\ldots k}$ in terms of determinants is:

(a) $\left(1-\frac{|P|}{P_{ii}}\right)$

(b) $\left(1-\frac{|P|}{P_{ii}}\right)^2$

(c) $\left(1-\frac{|P|}{P_{ii}}\right)^{1/2}$

(d) $\left(1-\frac{P_{ii}}{|P|}\right)^{1/2}$

Q. 112 The formula for multiple correlation coefficient $R_{2.13}$ in terms of simple correlation coefficients r_{12}, r_{13} and r_{23} is:

(a) $\frac{r_{12}^2+r_{23}^2-2r_{12}\,r_{23}\,r_{13}}{1-r_{13}^2}$

(b) $\sqrt{\frac{r_{12}^2+r_{23}^2-2r_{12}^2\,r_{13}^2\,r_{23}^2}{1-r_{13}^2}}$

(c) $\sqrt{\frac{r_{12}^2+r_{23}^2-2r_{12}\,r_{13}\,r_{23}}{1-r_{23}^2}}$

(d) $\sqrt{\frac{r_{12}^2+r_{23}^2-2r_{12}\,r_{23}\,r_{13}}{1-r_{13}^2}}$

Q. 113 The necessary and sufficient condition for the three planes, X_1 on X_2, X_3; X_2 on X_1, X_3 and X_3 on X_1, X_2 is:

(a) $r_{12}+r_{23}+r_{13}-2r_{12}\,r_{13}\,r_{23}=1$

(b) $r_{12}^2+r_{13}^2+r_{23}^2-2r_{12}\,r_{13}\,r_{23}=1$

(c) $r_{12}^2+r_{13}^2+r_{23}^2-2r_{12}\,r_{13}\,r_{23}=0$

(d) none of the above

Q. 114 In a multivariate study, the correlation between any two variables eliminating the effect of all other variables is called:

(a) simple correlation
(b) multiple correlation

(c) partial correlation
(d) partial regression

Q. 115 The range of partial regression coefficient is:
(a) 0 to 1
(b) −1 to 1
(c) 0 to ∞
(d) −∞ to ∞

Q. 116 If P is the correlation matrix and $X_1, X_2, \ldots, X_k$ are k variables, the correlation between X_1 and X_2 eliminating the influence of $X_3, X_4, \ldots, X_k$ can be calculated by the formula:

(a) $\left(1-\dfrac{P_{12}}{P_{11}\,P_{22}}\right)$

(b) $\dfrac{P_{12}}{P_{11}\,p_{22}}$

(c) $\sqrt{\dfrac{P_{12}}{P_{11}\,p_{22}}}$

(d) $\dfrac{P_{12}}{\sqrt{P_{11}\,p_{22}}}$

Q. 117 If X_1, X_2 and X_3 are three variables, the partial correlation between X_2 and X_3 eliminating the effect X_1 in terms of simple correlation coefficients is given by the formula:

(a) $r_{23.1}=\dfrac{r_{23}-r_{12}\,r_{13}}{\sqrt{\left(1-r_{12}^2\right)\left(1-r_{13}^2\right)}}$

(b) $r_{23.1}=\dfrac{r_{32}-r_{21}\,r_{31}}{\sqrt{\left(1-r_{21}^2\right)\left(1-r_{31}^2\right)}}$

(c) $r_{23.1}=\dfrac{r_{32}-r_{12}\,r_{13}}{\sqrt{\left(1-r_{12}^2\right)\left(1-r_{13}^2\right)}}$

(d) all the above

Q. 118 The partial correlation coefficient $r_{13.2}$ is called:
(a) first order partial correlation
(b) zero order partial correlation
(c) second order partial correlation
(d) none of the above

Q. 119 The partial correlation coefficient $r_{12.34}$ is called:
(a) zero order partial correlation
(b) first order partial correlation
(c) second order partial correlation
(d) third order partial correlation

Q. 120 The formula for the partial correlation coefficient $r_{13.24}$ is:

(a) $r_{13.24}=\dfrac{r_{13.4}-r_{12.4}\,r_{23.4}}{\sqrt{\left(1-r_{12.4}^2\right)\left(1-r_{23.4}^2\right)}}$

(b) $r_{13.24}=\dfrac{r_{13.2}-r_{14.2}\,r_{34.2}}{\sqrt{\left(1-r_{14.2}^2\right)\left(1-r_{34.2}^2\right)}}$

(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 121 A positive significant correlation between the number of shoes produced and the steel produced per year is:
(a) a nonsense correlation
(b) a spurious correlation
(c) a meaningless correlation
(d) all the above

Q. 122 Given the expected values for variables X *and* Y as:
$E(X) = 2$, $E(X^2) = 10$, $E(Y) = 3$, $E(Y^2) = 20$ and $E(XY) = 16$
we conclude that:
(a) correlation coefficient will be positive
(b) correlation coefficient will be negative
(c) expected values are incompatible
(d) none of the above

Q. 123 If X and Y are two independent variates with variances σ_X^2 and σ_Y^2 respectively, the correlation coefficient between X and $(X - Y)$ is equal to:

(a) $\sigma_{XY}/\sqrt{\sigma_X^2\,\sigma_Y^2}$

(b) $\sigma_X/\sqrt{\sigma_X^2+\sigma_Y^2}$

Q. 133 If $\sigma_X = 0.5$, $\sigma_Y = 1.5$ and $\sigma^2_{X-Y} = 1.25$, the coefficient of correlation between X and Y is:
(a) 1
(b) 1/4
(c) 1/2
(d) 5/6

Q. 134 If $\sigma_X = 0.5$, $\sigma_Y = 1.5$, $\sigma^2_{X-Y} = 1.25$, $\overline{X} = 8$ and $\overline{Y} = 6$, the regression line of Y on X is:
(a) $2Y = 5X - 28$
(b) $3Y = 2X - 1$
(c) $6X = 4 + 42$
(d) none of the above

Q. 135 If the correlation coefficient between two variables X and Y is very high, the two lines of regression are:
(a) far apart
(b) coincident
(c) near to each other
(d) none of the above

Q. 136 Regression coefficient is independent of the change of:
(a) scale
(b) origin
(c) both origin and scale
(d) neither origin nor scale

Q. 137 If the covariance between the two variables is positive, it means that:
(a) the variables would change in the same direction
(b) the variables would change in the opposite direction
(c) the variables would not change
(d) none of the above

Q. 138 If the correlation between two variables is zero, it implies that:
(a) two variable are independent
(b) two variables do not have negative correlation
(c) the two variables are not linearly related
(d) all the above

Q. 139 Correlation between the direction of deviations is calculated by the method of:
(a) product moments
(b) rank correlation
(c) coefficient of concurrent deviation
(d) Kendall's τ

Q. 140 The correlation between the two variables is unity, there is:
(a) perfect correlation
(b) perfect positive correlation
(c) perfect negative correlation
(d) no correlation

Q. 141 The quantity R^2 in reference to multivariate study is known as:
(a) correlation index
(b) coefficient of determination
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 142 Equality of k partial regression coefficient can simultaneously be tested with the help of:
(a) t-test
(b) χ^2-test
(c) Z-test
(d) analysis of variance

Q. 143 The partial correlation coefficient $r_{12.34}$ can be calculated with the help of:
(a) zero order correlation coefficient
(b) first order correlation coefficient
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 144 Which of the following relation is correct:
(a) $r_{12.34} = r_{13.24}$
(b) $r_{12.3} = r_{21.3}$
(c) $r_{13} = r_{23}$
(d) $r_{12.3} = r_{13.2}$

Q. 145 If we transform the variables of a simple regression line into standard normal deviates, then the two regression lines pass through the point:
(a) (0, 0)
(b) (1, 1)
(c) $(\overline{X}, \overline{Y})$
(d) $(\hat{X}, \hat{Y})$

Q. 146 The correlation between the five paired measurements (3, 6), $\left(\frac{1}{2}, 1\right)$, (2, 4), (1, 2), (4, 8) for the variables X and Y is equal to:

(a) 0
(b) 1
(c) ½
(d) –1

Q. 147 On the basis of the following paired values of the variables X and Y,

$$X: -1, -\frac{1}{2}, -\frac{1}{4}, \quad 0, \quad 1, \quad \frac{1}{2}, \quad \frac{1}{4}$$

$$Y: 1, \quad \frac{1}{4}, \frac{1}{16}, \quad 0, \quad 1, \quad \frac{1}{4}, \quad \frac{1}{16}$$

we can conclude that:

(a) the correlation between X and Y is 1.
(b) the variable are independent
(c) the variables are related
(d) the data are incompatible

Q. 148 A potential parameter of a model which is unknown and estimated is termed as:

(a) fixed parameter
(b) free parameter
(c) constrained parameter
(d) noise parameter

Q. 149 A parameter of model which can take the value either 1 or 2 is called:

(a) a free parameter
(b) non-centrality parameter
(c) constrained parameter
(d) fixed parameter

Q. 150 If an unknown potential parameter of a model is equalised to one or more parameters of the model, then the parameter is classified as:

(a) nuisance parameter
(b) constrained parameter
(c) free parameter
(d) non-centrality parameter

Q. 151 There are three continuous variables X_1, X_2 and X_3. It is known that the variable X_3 effects the variable X_1 but not X_2. Now to find the correlation between X_1 and X_2 eliminating the effect of X_3, you would work out:

(a) partial correlation
(b) multiple correlation
(c) part correlation
(d) simple correlation

Q. 152 For the same set of data for three variables X_1, X_2, and X_3, the relation between partial correlation $r_{12.3}$ and part correlation $r_{1(2.3)}$ is:

(a) $r_{12.3} > r_{1(2.3)}$
(b) $r_{12.3} = r_{1(2.3)}$
(c) $r_{12.3} < r_{1(2.3)}$
(d) no definite relation

Q. 153 Let the correlation coefficient between two variables X and Y be unity. Then the relation between the regression coefficients β_{YX} and β_{XY} that always holds is:

(a) $\beta_{YX} > \beta_{XY}$
(b) $\beta_{YX} < \beta_{XY}$
(c) $\beta_{YX} = \beta_{XY}$
(d) $\beta_{YX} \cdot \beta_{XY} = 1$

Q. 154 If the correlation between the variable X and Y is 0.5, then the correlation between the variables $2x - 4$ and $3 - 2y$ is:

(a) 1
(b) 0.5
(c) –0.5
(d) 0

Q. 155 Let the equations of the regression lines be expressed as $2X - 3Y = 0$ and $4Y - 5X = 8$. Then the correlation between X and Y is:

(a) $\sqrt{15/8}$
(b) $\sqrt{8/15}$
(c) $\sqrt{6/15}$
(d) $\sqrt{1/15}$

Q. 156 An investigator reports that the arithmetic mean of two regression coefficients of a regression line is 0.7 and correlation coefficient is 0.75. Are the results:

(a) valid

(b) invalid
(c) inconclusive
(d) none of the above

Q. 157 Given the two regression lines $X + 2Y = 5$ and $2X + 3Y = 8$ and $\sigma_Y^2 = 4$, the value of σ_X^2 is:
(a) 12
(b) $^{27}\!/_4$
(c) 6
(d) none of the above

Q. 158 Given $r_{12} = 0.6$, $r_{13} = 0.5$ and $r_{23} = 0.8$, the value of $r_{12.3}$ is,
(a) 0.4
(b) 0.72
(c) 0.38
(d) 0.47

Q. 159 If the sum of squares of the differences between ten ranks of two series is 33, then the rank correlation coefficient is:
(a) 0.967
(b) 0.725
(c) 0.80
(d) 0.67

Q. 160 If Var $(X + Y)$ = Var (X) + Var (Y), then the value of correlation coefficient r_{XY} is:
(a) 0
(b) 1
(c) –1
(d) 0.5

Q. 161 Let the coefficient of correlation be 0.7, then the coefficient of alienation is:
(a) 0.51
(b) 0.71
(c) 0.49
(d) none of the above

Q. 162 If Var $(X + Y)$ = Var $(X - Y)$, then the correlation between X and Y is equal to:
(a) 1
(b) 1/2
(c) 1/4
(d) 0

Q. 163 If $u + 3x = 5$, $2y - v = 7$, $r_{xy} = 0.12$, then the correlation r_{uv} is equal to:
(a) –0.12
(b) 0.12
(c) 0.08
(d) 1.0

Q. 164 Let the regression lines of Y on X and X on Y are respectively $Y = aX + b$ and $X = cY + d$, then the correlation coefficient between X and Y is:
(a) $\sqrt{c/a}$
(b) $\sqrt{a/c}$
(c) $\sqrt{ac}$
(d) $\sqrt{bd}$

Q. 165 Let the regression lines of Y on X and X on Y are respectively $Y = \alpha X + \beta$ and $X = \theta Y + \delta$. Then the ratio of the variances of X and Y is:
(a) θ/α
(b) $\sqrt{\theta/\alpha}$
(c) $\sqrt{\theta\alpha}$
(d) α/θ

Q. 166 If one regression coefficient of the two regression lines is greater than unity, the other will be:
(a) > 1
(b) 1
(c) < 1
(d) 1/2

Q. 167 If $R_{1.32} = 0$, then all total and partial correlation coefficients involving X_1 are:
(a) 0
(b) 1
(c) –1
(d) 1/2

Q. 168 The multiple correlation coefficient $R_{1.23}$ as compared to any simple correlation coefficients between the variables X_1, X_2 and X_3 is:
(a) less than any r_{12}, r_{13}, r_{23}
(b) not less than any r_{12}, r_{13} and r_{23}

(123) $\left(1-\frac{|P|}{P_{ii}}\right)^{1/2}$

(124) $\sqrt{\frac{r_{12}^2+r_{13}^2-2r_{12}\,r_{13}\,r_{23}}{\left(1-r_{23}^2\right)}}$

(125) $r_{12}^2+r_{13}^2+r_{23}^2-2r_{12}\,r_{13}\,r_{23}=1$

(126) eliminating (127) $P_{ij}\,/\,\sqrt{P_{ii}P_{jj}}$ (128) residual

(129) $\left(r_{12}-r_{13}\,r_{23}\right)\Big/\sqrt{\left(1-r_{13}^2\right)\left(1-r_{23}^2\right)}$ (130) first

(131) $\left(r_{12.3}-r_{14.3}\,r_{24.3}\right)\Big/\sqrt{\left(1-r_{14.3}^2\right)\left(1-r_{24.3}^2\right)}$ (132) second (133) -1 to 1 (134) 0 (135) 0 (136) $\left(\sigma_X^2-\sigma_Y^2\right)\big/\left(\sigma_X^2+\sigma_Y^2\right)$ (137) 0.25 (138) $1/\sqrt{n}$ (139) 0 (140) 0 (141) $\sigma^2(1+2r)/3$ (142) uncorrelated (143) origin; scale (144) $1/\sqrt{3}$ (145) 13; 17 (146) 12 (147) impossible (148) zero (149) 1 (150) 1 (151) nonsense (152) zero (153) nonsignificant (154) negative (155) not consistent (156) its linear estimate $\hat{Y}$ (157) origin (158) abscissa; intercept (159) x-axis (160) perpendicular (161) zero (162) zero (163) 45° (164) free (165) fixed (166) constrained (167) plane of regression (168) part correlation (169) $\left(r_{13}-r_{12}r_{23}\right)/\sqrt{1-r_{23}^2}$ (170) less.

SECTION-C

(1) b (2) b (3) d (4) d (5) a (6) b
(7) d (8) a (9) b (10) a (11) b (12) d
(13) a (14) b (15) c (16) a (17) c (18) b
(19) c (20) a (21) a (22) c (23) b (24) d
(25) b (26) a (27) d (28) d (29) b (30) c
(31) a (32) b (33) b (34) c (35) d (36) b
(37) b (38) b (39) a (40) a (41) c (42) b
(43) d (44) d (45) c (46) d (47) b (48) c
(49) c (50) b (51) c (52) a (53) b (54) c
(55) a (56) c (57) b (58) d (59) b (60) a
(61) c (62) b (63) d (64) c (65) a (66) b
(67) c (68) c (69) d (70) a (71) c (72) d
(73) c (74) a (75) b (76) c (77) d (78) b
(79) a (80) b (81) c (82) b (83) c (84) d
(85) c (86) a (87) c (88) a (89) b (90) d
(91) c (92) b (93) c (94) a (95) b (96) a
(97) c (98) b (99) d (100) b (101) c (102) b
(103) a (104) c (105) d (106) b (107) a (108) c
(109) b (110) a (111) c (112) d (113) b (114) c
(115) d (116) d (117) d (118) a (119) c (120) c
(121) d (122) c (123) b (124) d (125) a (126) a
(127) c (128) b (129) d (130) c (131) b (132) a
(133) d (134) a (135) c (136) b (137) a (138) c
(139) c (140) b (141) c (142) d (143) c (144) b
(145) a (146) b (147) c (148) b (149) d (150) b
(151) c (152) a (153) d (154) c (155) b (156) b
(157) a (158) c (159) c (160) a (161) b (162) d
(163) a (164) c (165) a (166) c (167) a (168) b
(169) c (170) c

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Chapter **14**

Measures of Association of Attributes

SECTION-A

Short Essay Type Questions

Q. 1 When do you apply association of attributes?

Ans. Correlation is a suitable measure of association when both the variables belong to a bivariate normal population. But in behavioural sciences one often deals with the variables which are not quantitatively measurable but can be cross-classified with respect to two or more classifications or polytomies such as eye colour, hair style, skin colour, liking of programmes, etc. If an attribute has only two classes, it is said to be *dichotomous*, and if it has many classes, it is called *manifold* classification. Sometimes polytomies may be arising from an underlying continuum. For instance, age may be divided into categories or classes but may be treated as qualitative classes. Hence, the need is often felt as to whether there is some association between different attributes or not. For this, some measures are developed specifically known as measures of association of attributes.

Q. 2 Give a brief idea of notations and terminology used in classification of attributes.

Ans. The system adopted to show the presence or absence of a attribute in an unit singly or in combination with other attributes is as follows: Generally, the presence of an attribute in an individual is indicated by the Latin letters A, B, C, etc., and its absence by the small Latin letter a, b, c, etc., or by Greek letters α, β, γ, etc. Two or more attributes present in an individual or individuals are indicated by the combination of Latin letters AB, AC, BC, ABC, etc. Also the presence of one attribute and the absence of the other will be represented by Ab or $A\,\beta$, aB or αB, ABc or $AB\,\gamma$, abc or $\alpha\,\beta\,\gamma$ etc. Ab represents the presence of A and absence of B in an individual. Similarly, the combination ABc or $AB\,\gamma$ represents the unit which shows the absence of the attribute C and presence of A and B. Likewise any other combination of letters can be interpreted.

Q. 3 What is meant by the order of classes?

Ans. The total of all frequencies denoted as N is known as the class of order zero. Whereas the classes A, a, B, b, etc., are called the classes of first order. At the same time, combination of any two letters showing the presence or absence of attributes are called the classes of second order, *e.g.*, AB, Ab, aB, ab (AB, $A\beta$, $\alpha\,B$, $\alpha\,\beta$), etc. Similarly, any combination like ABC, ABc, Abc, aBC, abc (ABC, $AB\,\gamma$, $A\beta\,\gamma$, $\alpha\,BC$, $\alpha\,\beta\,\gamma$), etc., is known as the class of third order and so on. The highest order classes are termed as *ultimate order classes*.

The total number of classes depends upon the number of attributes. For n attributes, the number of classes are 3^n. For $n = 1$, the three classes are A, a, N. For $n = 2$, the nine classes are N, A, a, B, b, AB, Ab, aB, ab and so on.

Q. 4 How are frequencies represented for various combination of attributes?

Ans. The number of individual or units belonging to a class is known as its *frequency*. For convenience and clear understanding, the frequency of a class is denoted by the letters in parentheses representing that cell. For example, the frequency of the cell representing the attributes AB is denoted by (AB). Similarly, for the class Ab or $A\beta$, frequency is denoted by (Ab) or $(A\beta)$, for the class ABC, frequency is (ABC), etc.

The frequency of a lower order class can always be expressed in terms of the higher order class frequencies. For three factors, A, B and C, all possible twenty-seven combinations of attributes belonging to different classes in the form of a pedigree can be displayed in the following manner:

Similarly other relations can be given.

Also it is obvious from the above relations that no higher order class frequency can exceed the lower order class frequency. For example,

$$(AB) \le (A); (AB) \le (B)$$

$$(ABC) \le (AB); (ABC) \le (AC); (ABC) \le (BC)$$

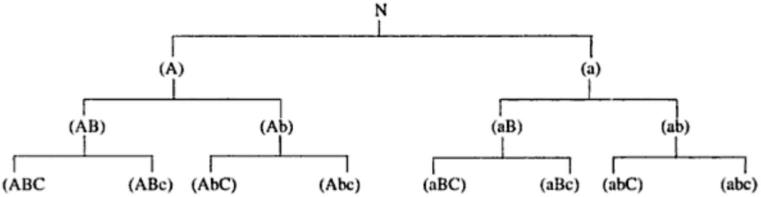
$$(abc) \le (ab); (abc) \le (ac), \text{ etc.}$$

Q. 5 How can the frequencies for various attributes be displayed in a contingency table?

Ans. If there are n attributes with dichotomy, there will be 2^n ultimate frequencies each represented by the combination of n letters, capital or small. Thus, the contingency table of order (2 × 2) for two attributes A and B can be displayed as given below:

	A	a	
B	(AB)	(aB)	(B)
b	(Ab)	(ab)	(b)
	(A)	(a)	(N)

Some of the missing frequencies can be determined with the help of the relations among frequencies.



Similarly relations can be given by taking N, B, b and N, C, c.

From the above relations, it is apparent that:

$(A) = (AB) + (Ab)$; $(a) = (aB) + (ab)$

$(AB) = (ABC) + (ABc)$; $(Ab) = (AbC) + (Abc)$

$(aB) = (aBc) + (aBc)$; $(ab) = (abC) + (abc)$

$N = (A) + (a) = (B) + (b) = (C) + (c)$

$N = (AB) + (Ab) + (aB) + (ab)$

$N = (ABC) + (ABc) + (AbC) + (Abc) + (aBC) + (aBc) + (abC) + (abc)$

Q. 6 You are given,

$(A) = 90$, $(AB) = 40$, $N = 150$ and $(b) = 80$.

Complete (2 × 2) contingency table.

Ans. We know,

$(A) = (AB) + (Ab)$

$90 = 40 + (Ab)$ or $(Ab) = 50$

$(b) = (Ab) + (ab)$

$80 = 50 + (ab)$ or $(ab) = 30$

$N = (B) + (b)$

$$
\begin{aligned}
150 &= (B) + 80 \text{ or } (B) = 70\\
(B) &= (AB) + (aB)\\
70 &= 40 + (aB) \text{ or } (aB) = 30\\
(a) &= (aB) + (ab)\\
&= 30 + 30 = 60
\end{aligned}
$$

Thus, the complete contingency table is,

	A	*a*	*Total*
B	40	30	70
b	50	30	80
Total	90	60	150

Q. 7 Explicate the method of expressing any class frequency in terms of other class frequency.

Ans. If we define the frequency of an attribute as the attribute multiplied by the total frequency N, *e.g.*, $A.N = (A)$, $AB.N = (AB)$, $aB.N = (aB)$, $Abc.N = (Abc)$, etc.

The frequency of any class in terms of the frequencies of other classes can be obtained by writing the attribute as such and its negation as $(1 - X)$, where X is an attribute, multiplied by N. Here we use the letters for attributes as *operators*. For example,

$$
\begin{aligned}
AB\,\gamma &= AB\,(1 - C)\cdot N = AB\cdot N - ABC\cdot N\\
&= (AB) - (ABC)\\
A\,\beta\,\gamma &= A\,(1 - B)\,(1 - C)\cdot N\\
&= A\cdot N - AB\cdot N - AC\cdot N + ABC\cdot N\\
&= (A) -(AB) - (AC) + (ABC)\\
(\alpha\,\beta\,C) &= (1 - A)\,(1 - B)\,C\cdot N\\
&= C\cdot N - AC\cdot N - BC\cdot N + ABC\cdot N\\
&= (C) - (AC) - (BC) + (ABC)
\end{aligned}
$$

Similarly,

$$
(\alpha\,\beta\,\gamma) = N - (A) - (B) - (C) + (AB) + (BC) + (AC) - (ABC)
$$

and so on.

Q. 8 What do you understand by inconsistency of data?

Ans. It is a bare fact that no frequency can be negative. Hence, the data are said to be inconsistent if they lead to any frequency obtained by the relations among frequencies as negative. Also no higher order class can have a greater frequency than the lower order class frequency.

If any frequency of an attribute or combination of attributes is greater than total frequency N, the data are inconsistent. Some of the examples of inconsistency of data are as follows:

If given that$(AB) > (A)$, then (Ab) is negative. Hence the data are inconsistent.

If given that $(AB) < (A) + (B) - N$, then (ab) will be negative. So the data are inconsistent.

If $(ABC) > (AB) + (AC) + (BC) - (A) - (B) - (C) + N$ then (abc) will be negative. Obviously, the data are inconsistent.

Some other relations for inconsistency of data are,

$$
\begin{gathered}
(ABC) < (AB) + (AC) - A\\
(ABC) < (AB) + (BC) - B\\
(ABC) < (AC) + (BC) - C\\
(AB) + (AC) + (BC) < (A) + (B) + (C) - N\\
(AB) + (AC) - (BC) > (A)\\
(AB) + (BC) - (AC) > (B)\\
(AC) + (BC) - (AB) > (C)
\end{gathered}
$$

etc.

Q. 9 What kind of associations among attributes are likely to occur?

Ans. There are three kinds of associations which possibly occur between attributes namely, (i) positive association; (ii) negative association, and (iii) no association.

(i) In positive association, the presence of one attribute is accompanied by the presence of other attribute(s). For example, health and hygiene, education and intelligence are positively associated.

In case of positive association, the observed frequency of the combined positive attributes is greater than the expected frequencies, *i.e.*, $(AB) > \dfrac{(A)(B)}{N}$. Similar relations hold for other attributes.

(ii) When the presence of an attribute say, A ensures the absence of another attribute say, B or vice-versa, the attributes A and B are said to be negatively associated. For instance, the vaccination and occurrence of disease for which the vaccine is meant for, are negatively associated. In this situation, the actual frequency of the cell for combination of attributes is smaller than the expected frequency, *i.e.*, $(AB) < \frac{(A)(B)}{N}$.

(iii) If the two attributes are such that the presence or absence of one attribute has nothing to do with the absence or presence of the other, they are said to be independent. For instance, skin colour and intelligence of persons are independent attributes. If the two attributes A and B are independent, then

$$(AB) = \frac{(A)(B)}{N}$$

or $$(ab) = \frac{(a)(b)}{N}$$

etc.

Q. 10 Name different methods of measures of association.

Ans. There are mainly five methods of measures of association:

(i) Proportion method

(ii) Method of probability

(iii) Yule's coefficient of association

(iv) Coefficient of colligation

(v) Coefficient of contingency

Q. 11 Give proportion method for finding the association between two attributes.

Ans. Let us consider two attributes A and B. To find out the association between A and B by proportion method, one has to find the proportion of B's in A's and B's in non-A's, *i.e.*, to calculate $\frac{(AB)}{(A)}$ and $\frac{(aB)}{(a)}$. If $\frac{(AB)}{(A)} = \frac{(aB)}{(a)}$, A and B are independent. Again if $\frac{(AB)}{(A)} > \frac{(aB)}{(a)}$, there is a positive association between A and B. On the contrary, if $\frac{(AB)}{(A)} < \frac{(aB)}{(a)}$, then attributes A and B are negatively associated. It only indicates the kind of association but fail to provide the extent of association.

Q. 12 How to know the nature of association by the method of probability?

Ans. The method utilises the probability of occurrence of an attribute or combination of attributes to determine the expected frequencies. The expected frequency is the product of the probability of an event and total number of possible outcomes. For instance, if A and B are two attributes, and N, the total number of cases (outcomes), the expected frequency of the event AB through probability is $\frac{(A)\times(B)}{N}$ or for aB is $\frac{(a)\times(B)}{N}$, etc. To decide about the nature of association, the criteria are as given below:

Observed frequency	*Expected frequency*	*Relation*	*Type of association*
(AB)	$\frac{(A)\times(B)}{N}$	$(AB) = \frac{(A)\times(B)}{N}$	A and B are independent
(AB)	$\frac{(A)\times(B)}{N}$	$(AB) > \frac{(A)\times(B)}{N}$	+ve association
(AB)	$\frac{(A)\times(B)}{N}$	$(AB) < \frac{(A)\times(B)}{N}$	–ve association
(aB)	$\frac{(a)\times(B)}{N}$	$(aB) > \frac{(a)(B)}{N}$	+ve association
(ab)	$\frac{(a)\times(b)}{N}$	$(ab) > \frac{(a)\times(b)}{N}$	+ve association

and so on.

The negative association is also termed as *dissociation.*

This method does not give the idea of degree of association between attributes.

Q. 13 Give Yule's coefficient of association.

Ans. Yule's coefficient of association is named after its inventor G. Undy Yule. It is a relative measure of association between two attributes say, A and B. If (AB), (aB), (Ab) and (ab) represent the frequencies of all the four distinct combination of A, B, a and b, Yule's coefficient of association is given by the formula,

$$Q_{AB} = \frac{(AB)(ab)-(Ab)(aB)}{(AB)(ab)+(Ab)(aB)}$$

Q. 14 Delineate the properties of Yule's coefficient of association.

Ans.

(i) The value of Q_{AB} lies between -1 and 1.

(ii) If $Q = 1$, A and B has perfect positive association. This leads to the following relations,

$$(AB) = (A) \Rightarrow (Ab) = 0$$
$$(AB) = (B) \Rightarrow (aB) = 0$$

(iii) If $Q = -1$, A and B possess perfect negative association. This leads to the relation, $(AB) = 0$ or $(ab) = 0$.

(iv) If $Q = 0$, A and B are independent. In this situation,

$$(AB)\ (ab) = (Ab)\ (aB)$$

(v) Any other value between -1 and 1 gives the idea of degree of association between two attributes.

Q. 15 Discuss coefficient of colligation and give its relation with Q.

Ans. Coefficient of colligation as a measure of association was given by Professor Yule. If A and B are two attributes and (AB), (aB), (Ab) and (ab) are the class frequencies, the coefficient of colligation,

$$Y = \frac{1-\sqrt{\dfrac{(Ab)(aB)}{(AB)(ab)}}}{1+\sqrt{\dfrac{(Ab)(aB)}{(AB)(ab)}}}$$

The relation between Y and Q_{AB} is,

$$Q_{AB} = \frac{2Y}{1+Y^2}$$

The range of Y is from -1 to 1 and can be interpreted in the same way as Q.

Q. 16 Compare Yule's coefficients Y and Q.

Ans. Both the coefficients of association Y and Q hold the same properties. But the formula for Q is simple and easily calculable, and hence it is mostly used. Cell frequencies for Q or Y are easily ascertained by preparing a (2 × 2) contingency table.

Q. 17 What do you understand by partial association between attributes?

Ans. The need for partial association arises due to the fact that the association between two attributes may sometimes occurs due to the third attribute rather than the actual association between the two. For instance, the association amongst the prevalence of Tuberculosis (TB) and vaccination may be different in posh colonies and slum area.

The association between any two attributes A and B in the sub-populations say C and c of an universe are called *partial association*. It is usually denoted by $Q_{AB.C}$ and $Q_{AB.c}$.

The formula for partial associations between the attributes A and B in the sub-populations C and c in terms of class frequencies are:

$$Q_{AB.C} = \frac{(ABC)(abC)-(AbC)(aBC)}{(ABC)(abC)+(AbC)(aBC)}$$

$$Q_{AB.c} = \frac{(ABc)(abc)-(Abc)(aBc)}{(ABc)(abc)+(Abc)(aBc)}$$

Similarly the formulae for $Q_{AC.B}$, $Q_{AC.b}$, $Q_{BC.A}$ and $Q_{BC.a}$ can be given simply by interchanging the letters.

Q. 18 What is illusory association?

Ans. Sometimes the association between two attributes is not actually the association amongst them but due to some non-ascribable factors. Such an association is known as *illusory association*. This is due to the fact that there is no direct causal relationship between the attributes. Though partial association provides a check to the illusory association but not totally. An illusory association creates lot of confusion, and hence one should be wary of it. For instance, there is a high positive association between white colour people in Europe and sport of skiing. This is an illusory association because it is not the colour associated with skiing but the cold whether which is responsible for white colour people and snow covered hills.

Q. 19 Compare measure of association and correlation.

Ans. Association and correlation both have similarities and dissimilarities which are as follows:

(i) Both are relative measures.

(ii) Both the measures range from −1 to 1.

(iii) Measure of association is based on qualitative factors which are not quantitatively measurable. For instance, level of education and crime. In association no variables are involved.

(iv) Correlation measures the relationship (association) between the factors which we can measure quantitatively. In this measure, actually the variables are involved.

(v) Measure of association is based merely on frequencies of various polytomies whereas in correlation actual paired measurements are used.

(vi) In association, the frequencies of joint attributes should be more than the expected frequency. It has nothing to do with the large or small frequency of the joint attribute.

(vii) Correlation is a measure of proportional increase or decrease in the two variables simultaneously.

(viii) Association of attributes is measured in terms of total or partial association.

(ix) Correlation is measured in terms of simple, multiple and partial correlation coefficient.

Q. 20 Comment on coefficient of contingency.

Ans. Coefficient of contingency is connected with the Chi-square statistic. The coefficient of contingency is worked out when chi-square test confirms the association between attributes. If so, the formula for coefficient of contingency is,

$$C = \sqrt{\frac{\chi^2}{\chi^2 + n}}$$

where, n is the total sample size (sum of all ultimate frequencies) and χ^2, the calculated value of the statistic. C lies between 0 to 1 but never attains the value unity. A value of C near to 1 shows a great degree of association between the two attributes and a value near to zero shows no association.

If we put, $\phi = \chi^2/n$, then

$$C = \sqrt{\frac{\phi}{\phi + 1}}$$

Q. 21 What is Tschuprow's coefficient?

Ans. Coefficient of contingency can never attain the value unity. Hence, Tschuprow gave another coefficient which is named after him as Tschuprow's coefficient which is given by the formula,

$$T = \sqrt{\frac{C^2}{(p-1)(q-1)\left(1-C^2\right)}}$$

where,

p = number of rows in the contingency table.

q = number of columns in the contingency table.

and $C^2 = \chi^2/\text{n}$.

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s) in the blanks:

1. If an attribute has two classes, it is said to be _______.

2. If an attribute has more than two classes, it is called to possess _________ classification.

3. The number of letters representing a class determines the _________ of the class.

4. The class represented by AC is of ________ order.

5. The total of all frequencies N is of order ________.

6. The order of the class represented by $\alpha\beta\gamma$ is of order ________.

7. If there are two attributes A and B and their negations a and b respectively, the frequency (A) in terms of the second order frequencies is equal to ________.

8. In case of two attributes A and B, the zero order frequency in terms of ultimate frequencies with usual notations is ________.

9. If we are given the frequencies for two attributes A and B as $(Ab) = 65$, $(A) = 55$ and $N = 100$, the given data are ________.

10. In case of consistent data, no class frequency can be ________.

11. If the frequency of the combined attributes is less than the expected frequency, they are ________.

12. If the attributes A and B are independent, the frequency (AB) is equal to ________.

13. If the proportion of B's in A's and of B's in non-A's is same, the attributes A and B are ________.

14. If all A's are B's and all B's are C's, then all A's are ________.

15. If $\frac{(A)}{N} = x, \frac{(B)}{N} = 2x$ and $\frac{(C)}{N} = 3x$ and $\frac{(AB)}{N} = \frac{(BC)}{N} = \frac{(AC)}{N} = y$, then neither x nor y can exceed ________.

16. If $(A) = 50$, $(B) = 60$, $(C) = 50$, $(AC) = 20$ and $N = 100$, the upper and lower limit of (BC) are ________ so that the data are consistent.

17. If the attributes A and B are independent, the relation between the frequencies (AB), (Ab), (aB) and (ab) of attributes is ________.

18. If the attributes A and B are independent, the value of (AB) in terms of (A) and (B) is ________.

19. If A and B are independent, Yule's coefficient Q is equal to ________.

20. If A and B are completely associated, then $(AB) =$ ________ and ________.

21. If A and B are two completely dissociated attributes, then $(AB) =$ ________ and/or ab = ________.

22. If two attributes A and B are completely dissociated, the value of Yule's coefficient Q is equal to ________.

23. If two attributes A and B are completely associated, then $(Ab) =$ ________ and/or $(aB) =$ ________.

24. If A and B are two completely associated attributes, Yule's coefficient of association between A and B is equal to ________.

25. If Yule's coefficient $Q = 0$, the coefficient of colligation $Y =$ ________.

26. If Yule's coefficient $Q = -1$, then coefficient of colligation Y is equal to ________.

27. The relation between Yule's coefficient Q and coefficient of colligation Y is ________.

28. If Yule's coefficient $Q = 1$, the coefficient of colligation Y is equal to ________.

29. If the frequencies for combination of two attributes are $(AB) = 40$, $(Ab) = 15$, $(aB) = 70$ and $(ab) = 30$, the value of Yule's coefficient of association Q is ________.

30. If $Q = 0.6$, the coefficient of colligation Y ________.

31. Given the data for two attributes A and B as, $N = 100$, $(ab) = 30$, $(B) = 60$ and $(a) = 70$, the other frequencies of a 2×2 contingency table are ________.

32. From the given data for two attributes as, $N = 60$, $(A) = 15$, $(B) = 40$ and $(AB) = 10$, one concludes that the attributes A and B are ________.

33. Data $(AB) = 20$, $(aB) = 50$, $(Ab) = 10$, $(ab) = 15$ leads to infer that the attributes A and B are ________.

34. For the data given in Q. No. 33, the attributes a and B are ________.

35. In the proportion method, the attributes A and B are independent if ________.

36. In the proportion method, the attributes A and B have frequencies such that $(AB)/(A) < (aB)/(a)$, then A and B are ________.

37. Method of proportion for association of attributes ________ the idea about the degree of association.

38. The signs of coefficient of association Q and coefficient of colligation Y are always ________.

39. The value of coefficient of colligation ________ the value of coefficient of association Q.

40. Formula for Tschuprow's coefficient with usual notations is ________.

41. The coefficient of contingency never ________ unity.

42. Coefficient of contingency never attains the value ________.

43. The association between two attributes in a population is called ________ association.

44. The association between two attributes in a sub-population is known as ________ association.

45. The range of partial association between attributes is ________.

46. The formula for partial association $Q_{BC.A}$ is ________.

47. Partial association eliminates the influence of the ________ attribute from the association between two attributes.

48. If two attributes are such that there exists an association between them even though there is casual relation, such an association is known as ________ association.

49. An association between skin colour and intelligence is an ________ association.

50. Association is meant for ________ whereas correlation is meant for ________.

51. Just like partial association there is also a parallel correlation measure named as ________ correlation.

52. Just like multiple correlation, there is ________ measure in association of attributes.

53. Correlation and association both are ________ measures.

54. Two attributes A and B are said to be positively associated in the population of C's iff ________.

55. If $N = 100$, $(A) = 60$, $(B) = 40$ and $(AB) = 24$, the attributes A and B are ________.

56. If $(A) = 25$, $(AB) = 15$, $(\alpha\beta) = 4$ and $(B) = 24$, the coefficient of association between A and B is ________.

57. Association and correlation are ________.

58. Limits of Yule's coefficient square are ________.

59. If A and B are positively associated, then the attributes α, β showing their absence are ________ positively associated.

60. If every individual that possesses an attribute A also possesses the attribute B, then coefficient of colligation between A and B is equal to ________.

61. If there is a perfect association between two attributes, Yule's coefficient Q is equal to ________.

62. If $\delta = (AB) - \dfrac{(A)(B)}{N}$, then δ in terms of ultimate class frequencies with usual notations is equal to ________.

63. If two attributes A and B are independent then $\dfrac{(A\alpha)}{(A)} = \dfrac{(\alpha B)}{(B)} =$ ________.

64. If A and B are two independent attributes, the

(a) $(AB) + (ab)$
(b) $(AB) + (Ab)$
(c) $(ab) + (AB)$
(d) none of the above

Q. 11 With three attributes A, B and C, the number of second order class frequencies is:
(a) six
(b) nine
(c) twelve
(d) fifteen

Q. 12 With three attributes A, B and C, the number of first order classes is:
(a) six
(b) nine
(c) twelve
(d) none of the above

Q. 13 In case of two attributes A and B, the class frequency (αB) in terms of other class frequencies can be expressed as:
(a) $(B) + (AB)$
(b) $(B) - (AB)$
(c) $(Ab) - (B)$
(d) $N - (AB)$

Q. 14 In case of three attributes A, B and C, the class frequency $(\alpha\beta\gamma)$ in terms of other class frequencies is:
(a) $(AB) + (AC) - (B) - (ABC)$
(b) $(ABC) - (B) + (AB) - (BC)$
(c) $(ABC) - (A) - (C) + (B)$
(d) $(B) - (AB) - (BC) + (ABC)$

Q. 15 With three factors A, B and C, the class frequency $(A\beta\gamma)$ in terms of other class frequencies can be expressed as:
(a) $(A) + (BC) - (AB) - (ABC)$
(b) $(A) + (AB) + (AC) - (ABC)$
(c) $(A) - (AB) - (AC) + (ABC)$
(d) $(A) + (AB) + (BC) + (ABC)$

Q. 16 With three factors A, B and C, the frequency $(\alpha\gamma)$ can be expressed in terms of frequencies with positive attributes as:
(a) $1 - (A) - (C) + (AC)$
(b) $N - (A) - (C) + (AC)$
(c) $N + (A) - (C) - (AC)$
(d) none of the above

Q. 17 With three attributes A, B and C, the frequency (β) in terms of positive attribute frequencies is:
(a) $N - (A) - (B) - (C)$
(b) $N - (A) - (C)$
(c) $N - (AC)$
(d) $N - (B)$

Q. 18 The class frequency $(\alpha\ \beta)$ in terms of frequencies of positive attribute is:
(a) $N - (A) - (B) + (AB)$
(b) $N - (A) - (B) + (C)$
(c) $N - (AB)$
(d) $N - (A) - (B)$

Q. 19 The class frequency $(\alpha\beta\gamma)$ in terms of positive attributes is:
(a) $N + (A) + (B) + (C) - (AB) - (AC) - (BC) + (ABC)$
(b) $N - (A) - (B) - (C) + (AB) + (AC) + (BC) + (ABC)$
(c) $N - (A) - (B) - (C) - (AB) - (AC) - (BC) + (ABC)$
(d) none of the above

Q. 20 If for two attributes A and B, $\frac{(AB)}{(A)} = \frac{(\alpha\beta)}{(\alpha)}$, then A and B are:
(a) independent
(b) positively associated
(c) negatively associated
(d) no conclusion

Q. 21 If for two attributes A and B, $(AB) > \frac{(A)(B)}{N}$, the attribute are:
(a) independent
(b) positively associated
(c) negatively associated
(d) none of the above

Q. 22 If in case of two attributes A and B, $(\alpha\beta) < \frac{(\alpha)(\beta)}{N}$, then the attributes are:
(a) independent
(b) positively associated

(b) $(AB) = 30$, $(Ab) = 30$, $(aB) = 110$, $(ab) = 290$

(c) $(AB) = 20$, $(Ab) = 30$, $(aB) = 30$, $(ab) = 20$

(d) $(AB) = 20$, $(Ab) = 80$, $(aB) = 120$, $(ab) = 280$

Q. 33 For three attributes A, B and C given that,

$$(A) = (B) = (C) = \frac{N}{2} \text{ and } (ABC) = (\alpha\beta\gamma),$$

the relation between (ABC), (AB), (AC), (BC) and N is:

(a) $(ABC) = (AB) + (AC) + (BC) - N$

(b) $(ABC) = (AB) + (AC) + (BC) - \frac{N}{2}$

(c) $2\,(ABC) = (AB) + (AC) + (BC) - N$

(d) $2\,(ABC) = (AB) + (AC) + (BC) - \frac{N}{2}$

Q. 34 For two attributes A and B and their negations a and b, if $\frac{(aB)}{a} < \frac{(AB)}{(A)}$, then A and B are:

(a) positively associated

(b) negatively associated

(c) independent

(d) non-conclusive

Q. 35 For two attributes A and B with their negations a and b, if $\frac{(aB)}{(a)} > \frac{(AB)}{(A)}$, then A and B are:

(a) positively associated

(b) negatively associated

(c) independent

(d) non-conclusive

Q. 36 Given the following contingency table for two attributes A and B is as:

	A	α
A	a	b
β	c	d

the formula for Yule's Q is:

(a) $Q = (ab - cd)/(ab + cd)$

(b) $Q = (ac - bd)/(ac + bd)$

(c) $Q = (ad - bc)/(ad + bc)$

(d) none of the above

Q. 37 If in case of two attributes A and B, the class frequency $(AB) = 0$, the value of Q is:

(a) 1

(b) −1

(c) 0

(d) any value between 0 and −1

Q. 38 If for two attributes A and B, the class frequency $(ab) = 0$, then Q is equal to:

(a) 1

(b) −1

(c) 0

(d) any value between 0 and −1

Q. 39 If for two attributes the class frequency $(Ab) = (aB) = 0$, the value of the coefficient of colligation is:

(a) 1

(b) −1

(c) 0

(d) none of the above

Q. 40 If for two attributes A and B, the class frequencies hold the relation $(AB)\,(ab) = (Ab)\,(aB)$, then the value of Q is:

(a) 1

(b) −1

(c) 0

(d) none of the above

Q. 41 If class frequencies between two attributes A and B hold the inequality, $(AB)\,(ab) > (aB)\,(Ab)$, then the value of Q is:

(a) 1

(b) −1

(c) 0

(d) any value between 0 and 1

Q. 42 If with usual notations for two attributes, the inequality $(AB)\,(ab) < (aB)\,(Ab)$ holds, then:

(a) $-1 \le Q \le 1$

(b) $-1 \le Q < 0$

(c) $0 \le Q \le 1$

(d) none of the above

Q. 43 If for two factors A and B, the class frequencies are such that $(AB)(ab) = (Ab)(aB)$, then:

(a) A and B are positively associated

(b) A and B are negatively associated

(c) A and B are independent

(d) none of the above

Q. 44 If A and B are positively associated, then:

(a) $(AB) > \frac{(A)(B)}{N}$

(b) $(AB) < (A)(B)/N$

(c) $(AB) = (A)(B)/N$

(d) none of the above

Q. 45 If the class frequencies for two attributes hold the relation $(AB)(ab) < (Ab)(aB)$, then A and B are:

(a) positively associated

(b) negatively associated

(c) independent

(d) none of the above

Q. 46 If two attributes A and B are such that the class frequencies hold the relation. $(AB)(ab) > (Ab)(aB)$, then A and B are:

(a) positively associated

(b) negatively associated

(c) independent

(d) none of the above

Q. 47 If all A's are B's, then the coefficient of association Q is equal to:

(a) 1

(b) −1

(c) 0

(d) ∞

Q. 48 If for two attributes A and B, all B's are a's, then the coefficient of association Q is equal to:

(a) 1

(b) −1

(c) 0

(d) ∞

Q. 49 Formula for coefficient of colligation between two attributes A and B with usual notations is:

(a) $Y = \dfrac{1+\sqrt{\dfrac{(aB)(Ab)}{(AB)(ab)}}}{1-\sqrt{\dfrac{(aB)(Ab)}{(AB)(ab)}}}$

(b) $Y = \dfrac{1-\sqrt{\dfrac{(aB)(Ab)}{(AB)(ab)}}}{1+\sqrt{\dfrac{(aB)(Ab)}{(AB)(ab)}}}$

(c) $Y = \dfrac{1-\dfrac{(aB)(Ab)}{(AB)(ab)}}{1+\dfrac{(aB)(Ab)}{(AB)(ab)}}$

(d) $Y = \dfrac{1-\sqrt{\dfrac{(AB)(ab)}{(Ab)(aB)}}}{1+\sqrt{\dfrac{(AB)(ab)}{(Ab)(aB)}}}$

Q. 50 The relation between Yule's Q and coefficient of colligation Y is:

(a) $Q = Y/(1 + Y^2)$

(b) $Q = 2Y/(1 + Y^2)$

(c) $Q = Y/(1 + 2Y^2)$

(d) $Q = 2Y/(1 + 2Y)$

Q. 51 If all A's are B's, the coefficient of colligation is equal to:

(a) 0

(b) −1

(c) 1

(d) ∞

Q. 52 If the class frequencies in a contingency table are such that the cross products are equal, the coefficient of colligation is equal to:

(a) 0

(b) −1

(c) 1

(d) ∞

Q. 53 If in a (2 × 2) frequency table for two attributes A and B, the frequency of the cell ab is zero, the coefficient of colligation is equal to:
(a) 0
(b) −1
(c) 1
(d) ∞

Q. 54 If $Y = 1$, then it implies that
(a) Either (aB) or (Ab) is zero
(b) $(aB) = (Ab)$
(c) $(aB)\,(Ab) = 1$
(d) all the above

Q. 55 Given the cell frequencies for two attributes as, $(AB) = 90$, $(aB) = 60$, $(Ab) = 180$ and $(ab) = 30$, the coefficient of colligation is:
(a) −1
(b) 2/3
(c) −1/3
(d) 0

Q. 56 Out of 200 persons of a locality, 100 were vaccinated to prevent TB. Out of 50 patients, 10 were vaccinated. Coefficient of association Q between vaccination and prevention from TB is:
(a) 5/7
(b) 5/11
(c) −5/11
(d) none of the above

Q. 57 Partial Association between two attributes refers to the association:
(a) with a third attribute
(b) in a third population
(c) in a sub-population
(d) none of the above

Q. 58 Partial association eliminates the influence of:
(a) third attribute
(b) wrong frequencies
(c) wrong calculations
(d) none of the above

Q. 59 If two attributes have no causative relationship, the association between them is known as:
(a) positive association
(b) negative association
(c) illusory association
(d) none of the above

Q. 60 Coefficient of contingency is a measure of:
(a) independence of attributes
(b) dependence of attributes
(c) correlation
(d) all the above

Q. 61 The numerical value of coefficient of contingency:
(a) lies between 0 and 1
(b) never attains the value 1
(c) can never be negative
(d) all the above

Q. 62 Another measure related to coefficient of contingency is:
(a) Yule's coefficient
(b) coefficient of correlation
(c) Tschuprow's coefficient
(d) all the above

Q. 63 In an investigation on immunisation of cattle from renderpest disease, the following results were obtained:

	Affected	*Not Affected*
Inoculated	12	26
Not-inoculated	16	6

The value of coefficient of contingency is:
(a) $C = 0.33$
(b) $C = 0.37$
(c) $C = 0.95$
(d) none of the above

Q. 64 For the problem given in Q. No. 63, Tschuprow's coefficient is:
(a) $T = 0.43$
(b) $T = 0.10$
(c) $T = 0.466$
(d) none of the above

Q. 65 If for two attributes A and B, $N = 140$, $(A) = 100$, $(b) = 105$ and $(AB) = 25$, the attributes A and B are:
(a) dependent
(b) positively associated
(c) negatively associated
(d) independent

ANSWERS

SECTION-B

(1) dichotomous (2) manifold (3) order (4) second (5) zero (6) three (7) $(AB) + (Ab)$ (8) $(AB) + (Ab) + (aB) + (ab)$ (9) inconsistent (10) negative (11) dissociated (12) $(A)\ (B)/N$ (13) independent (14) C's (15) 1/4 (16) $15 \le (BC) \le 35$ (17) $(AB)\ (ab) = (Ab)\ (aB)$ (18) $(A)\ (B)/N$ (19) zero (20) (A); (B) (21) zero; zero (22) −1 (23) zero; zero (24) 1 (25) zero (26) −1 (27) $Q = 2Y/\left(1+Y^2\right)$ (28) 1 (29) 0.066 (30) 1/3 (31) $(AB) = 20$, $(aB) = 40$, $(Ab) = 10$, $(A) = 30$ and $(b) = 40$ (32) independent (33) positively associated (34) dissociated (35) $(AB)/(A) = (aB)/(a)$ (36) negatively associated (37) does not give (38) same (39) never exceeds (40) $T = \sqrt{\dfrac{C^2}{\left(1-C^2\right)(p-1)(q-1)}}$ (41) exceeds (42) unity (43) total (44) partial (45) −1 to 1 (46) $\dfrac{(BCA)(bcA)-(BcA)(bCA)}{(BCA)(bcA)+(BcA)(bCA)}$ (47) third (48) illusory (49) illusory (50) attributes; variables (51) partial (52) no (53) relative (54) $(ABC) > (AC)\ (BC)/(C)$ (55) independent (56) –0.2 (57) not same (58) from 0 to 1 (59) not necessarily (60) 1 (61) −1 or 1 (62) $[(AB)(\alpha\beta)-(\alpha B)(A\beta)]/N$ (63) $\dfrac{(\alpha\beta)}{(\beta)}$ (64) $(AB)(\alpha\beta) = (\alpha\ B)\ (A\ \beta)$ (65) zero (66) greater than zero or positive (67) negative or less than zero (68) $\alpha\beta$; αB (69) inconsistent.

SECTION-C

(1) a	(2) b	(3) a	(4) c	(5) b	(6) d
(7) b	(8) c	(9) a	(10) b	(11) c	(12) a
(13) b	(14) d	(15) c	(16) b	(17) d	(18) a
(19) d	(20) a	(21) b	(22) c	(23) a	(24) b
(25) d	(26) a	(27) b	(28) c	(29) a	(30) c
(31) b	(32) d	(33) d	(34) a	(35) b	(36) c
(37) b	(38) b	(39) a	(40) c	(41) d	(42) b
(43) c	(44) a	(45) b	(46) a	(47) a	(48) b
(49) b	(50) b	(51) c	(52) a	(53) b	(54) a
(55) c	(56) a	(57) c	(58) a	(59) c	(60) b
(61) d	(62) c	(63) b	(64) a	(65) d	

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Chapter **15**

Interpolation and Extrapolation

SECTION-A

Short Essay Type Questions

Q. 1 Give the general idea about interpolation and extrapolation.

Ans. The method of estimating a value of the dependent variable Y corresponding to a given value of X has already been explained in regression analysis. The interpolation and extrapolation come under mathematical approach known as numerical analysis. This approach is nonprobabilistic. If a value of Y is to estimated for a given value of X and it lies within the range of X-values, it is called *interpolation*. If it lies outside the range of its given values, it is known as *extrapolation*. According to W.M. **Harper**, "Interpolation consists in reading a value which lies between two extreme points. Extrapolation means reading a value that lies outside the two extreme points."

Q. 2 What are the assumptions on which the interpolation and extrapolation are based?

Ans. Interpolation and extrapolation are based on the following assumptions:

(i) There is no violent fluctuation within the given series of data. This makes the interpolation more accurate.

(ii) There should be a regularity in variation of variate values.

(iii) The dependent and independent variables should have definite mathematical relationship of the type $y = f(x)$.

Q. 3 What are the uses of interpolation and extrapolation?

Ans. There are various uses of interpolation and extrapolation, which are delineated below:

(i) Mostly the periodic observations or data are collected. But often the need arises for the intermediary periods. Interpolation provides the estimates for such values.

(ii) The data are available only for a span of time. But there is often a need of values of the past or future periods. In such a situation extrapolation is helpful in estimating the values outside the series of data.

(iii) Often the data for some period or the value of independent variable get lost. In such a situation inverse interpolation comes to our rescue to estimating the missing value.

(iv) For planning, one is always curious to know the position of demand in advance. Extrapolation to a great extent fulfils this requirement. For example, we can regulate our production keeping in view the population forecast ten years ahead.

(v) For comparison of data, it is necessary that the data should belong to the same reference period. If it is not so, it is necessary that the data be transformed to the same period. Interpolation and extrapolation serves this purpose very well.

Q. 4 Out of interpolation and extrapolation which one is more frequently used, and why?

Ans. Interpolation is more frequently used than extrapolation. Also the estimates obtained by interpolation are more accurate than extrapolation because they are free from the unseen and unknown vagaries of the past and the future.

Q. 5 Name different kinds of approaches for interpolation.

Ans. There are two kinds of interpolation approaches namely:

(i) Graphical method
(ii) Algebraic methods.

Q. 6 Name different algebraic methods.

Ans. The algebraic methods of interpolation are as follows:

(i) Binomial expansion method
(ii) Parabolic curve method
(iii) Newton's formula for advancing differences
(iv) Newton's backward formula
(v) Newton's-Gauss forward formula
(vi) Newton's-Gauss backward formula
(vii) Newton's-method of divided differences
(viii) Lagrange's interpolating formula
(ix) Stirling's formula
(x) Bessels formula
(xi) Inverse interpolation.

Q. 7 On what factors, the accuracy of interpolation and extrapolation depends?

Ans. Interpolation and extrapolation are simply estimation procedures. To expect absolutely accurate results from them is to deceive ourselves. Anyhow, there are many factors on which the accuracy of interpolation and extrapolation depends. Besides, interpolation is usually more accurate then extrapolation. The factors responsible for accuracy can be summarised as follows:

(i) One should have an idea about the fluctuations occurring in a series of data.

(ii) The knowledge about the unusual events related to the data be gathered by the analyst.

(iii) A proper choice of an interpolation or extrapolation formula in largely responsible for the accuracy of the estimates.

Q. 8 Describe, in brief, the graphical method of interpolation.

Ans. In graphical method, the paired variate values of the variables X and $Y (= f(x))$ are plotted on the graph paper after choosing appropriate scales along abscissa and ordinate. The plotted points are joined in sequence through a smooth line or curve. The value of Y for any value of X is estimated by drawing a line at the given point $X = x_0$ parallel to Y-axis so long as it touches the line or curve. From the point of intersection, draw a line parallel to X-axis. The point at which this touches the Y-axis, the corresponding reading on Y-axis is the estimated value of Y for the given value of X.

Graphical method provides a good estimate if the relation between X and Y is linear. But the estimate becomes poorer as the plotted points distort from an exact line or curve.

Graphical method is not frequently used.

Q. 9 What special terms are used for independent and dependent variate values with reference to interpolation and extrapolation?

Ans. As for interpolation and extrapolation, the independent variate values x's are called *arguments* and dependent variate values y's which are a function of x, *i.e.*, $y = f(x)$ are known as *entries*.

Q. 10 How to make use of binomial expansion method for interpolation?

Ans. If we want to interpolate for y for a given value of x, the method is applicable only when the values of x advance with equal jumps. Let the given paired observations for the two variables be:

x	x_0	x_1	x_2	...	x_i	...	x_n
y	y_0	y_1	y_2	...	y_i	...	y_n

We define the n^{th} finite difference $\Delta^n_{y_0}$ through an operator E such that

$$\Delta^n y_0 = (E-1)^n y_0$$

If one entry say y_r corresponding to $(r + 1)^{\text{th}}$ argument is missing, then

$$\Delta^n y_0 = (E-1)^n y_0 = 0$$

$$\Delta^n y_0 = \left[E^n - \binom{n}{1}E^{n-1} + \binom{n}{2}E^{n-2} - \ldots + (-1)^n\right] \times y_0 = 0$$

Now taking $E^i_{y_0} = y_i$, we obtain

$$y_n - \binom{n}{1}y_{n-1} + \binom{n}{2}y_{n-2} \ldots + (-1)^n y_0 = 0$$

To estimate an unknown value, we choose n as the number of known values of y.

Also we shall have as many equations as the number of unknowns by taking $\Delta^n y_0 = \Delta^n y_1 = \Delta^n y_2 = \ldots = 0$. Solving these equations we obtain the missing values.

Q. 11 For the following data,

x:	1	2	3	4
y:	7	?	13	24

find the missing value by binomial expansion method.

Ans. Since 3 values are known, we would take third order finite difference zero. Thus,

$$y_3 - 3y_2 + 3y_1 - y_0 = 0$$

$$24 - 3 \times 13 + 3y_1 - 7 = 0$$

$$y_1 = \frac{22}{3}$$

Q. 12 Given the values of a variable x and $f(x)$, find the missing values.

x:	0	1	2	3	4	5
y:	230	220	?	330	?	500

Ans. Since four values are known, we have to take $\Delta^4_{y_0} = 0$ and $\Delta^4_{y_1}$ where,

$$y_0 = f(x_0) \text{ and } y_1 = f(x_1)$$

$$(E-1)^4 y_0 = y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$

$$(E-1)^4 y_1 = y_5 - 4y_4 + 6y_3 - 4y_2 + y_1 = 0$$

According to the given data,

$$y_0 = 230,\ y_1 = 220,\ y_2 = y_2,\ y_3 = 330,\ y_4 = y_4,\ y_5 = 500$$

Substituting the values in the two equations, we get,

$$y_4 + 6y_2 = 1970$$

$$4y_4 + 4y_2 = 2700$$

Solving the two equation we obtain the missing values as,

$$y_2 = 259 \text{ and } y_4 = 416$$

Q. 13 Describe in brief the parabolic method of interpolation and extrapolation.

Ans. In this method, a polynomial of degree n is always considered when $(n + 1)$ is the number of known entries y or $f(x)$, *i.e.*, we consider the polynomial.

$$y = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$$

This equation is called the parabola of degree n. Putting the given values of x's and corresponding y's in sequence, we get as many equations as the number of unknown constants $a_0, a_1, \ldots, a_n$. Solving these equations, we obtain the numerical values of a's. Substituting the values of $a_0, a_1, \ldots, a_n$, in the equation of the polynomial, we get the estimating polynomial. Now for any given value of x, y can be estimated. This method is also known as the *method of simultaneous equations*. This method is applicable almost in all situations for interpolation and extrapolation. Extrapolation is good if the situation outside the series, prevails similar to that of within the series, otherwise not.

Q. 14 Give merits and demerits of the parabolic method of interpolation and extrapolation.

Ans. Merits of parabolic method are:

(i) It is applicable in almost all situations.

(ii) The value of entry y can be estimated for any argument x.

(iii) The curve is a good fit if it passes through all the points.

Demerit of parabolic method is:

(i) The calculation becomes very lengthy if the interpolation has to be carried out for more than one value.

Q. 15 Following are the prices per kg of an item during the four years:

Year:	1981	1984	1986	1989
Prices:	5	8	11	13

Interpolate the price for the year 1987 by parabolic method.

Solution. Since four years (X) data are given, we have to use a parabolic equation of degree 3. The equation is,

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 \qquad \text{(A)}$$

Ans. Let us code years taking 1984 as origin and fit in the parabolic equation.

Years	*Coded value (x)*	*y*
1981	−3	5
1984	0	8
1986	2	11
1989	5	13

Substituting the values of x and y in the third degree equation.

$$5 = a_0 - 3a_1 + 9a_2 - 27a_3 \qquad \text{(i)}$$
$$8 = a_0 + a_1 \times 0 + a_2 \times 0 + a_3 \times 0 \qquad \text{(ii)}$$
$$11 = a_0 + 2a_1 + 4a_2 + 8a_3 \qquad \text{(iii)}$$
$$13 = a_0 + 5a_1 + 25a_2 + 125a_3 \qquad \text{(iv)}$$

Solving the 4 equations for a_0, a_1, a_2 and a_3, we obtain

$$a_0 = 8,\ a_1 = 1.5,\ a_3 = -1/30,\ a_2 = 1/15$$

Substituting the values of a_0, a_1, a_2 and a_3, the parabolic equation is,

$$y = 8 + 1.5x + \frac{1}{15}x^2 - \frac{1}{30}x^3 \qquad \text{(B)}$$

To estimate y for 1987, $x = 3$

Putting $x = 3$ in equation B,

$$y = 8 + 1.5 \times 3 + \frac{1}{15} \times 9 - \frac{1}{30} \times 27$$
$$= 12.2$$

The estimated price for the year 1987 is 12.2 which of course lies between 11 and 13.

Q. 16 What do you understand by finite differences for equal intervals.

Ans. We consider equal intervals for the independent variable x as it makes the differences comparable and workable. The difference between two consecutive values of $y = f(x)$ starting from the origin x_0 are called finite differences. x_0 may be in the beginning of a series or somewhere in its midway or even the last argument. The values preceding to x_0 are suffixed with negative figures and following it by positive figures like −1, −2, −3, ... and 1, 2, 3, ... The finite differences are denoted as,

$$y_1 - y_0 = \Delta y_0,\ y_2 - y_1 = \Delta y_1, y_3 - y_2 = \Delta y_2, \ldots$$

and $\quad y_0 - y_{-1} = \Delta y_{-1},\ \Delta y_{-1} - \Delta y_{-2} = \Delta y_{-2}, \ldots$

These are known as first order differences and are placed in between the position of two entries in the next column. Similarly, the differences,

$$\Delta y_{-1} - \Delta y_{-2} = \Delta^2 y_{-2}, \Delta y_{-2} - \Delta y_{-3} = \Delta^2 y_{-3}, \ldots$$

or $\Delta y_1 - \Delta y_0 = \Delta^2 y_0,\ \Delta y_2 - \Delta y_1 = \Delta^2 y_1 \ldots$ are called the differences of second order and so on.

Finite differences are frequently used in a large number of interpolation formulae.

Q. 17 What are divided differences?

Ans. The difference between two consecutive y-values divided by the interval length of the x-values is called divided difference.

Following usual, notations, the difference

$$\Delta y_i = \frac{y_j - y_i}{x_j - x_i}$$

The higher order divided differences are obtained by taking the differences of the preceding order difference and dividing them by the difference(s) of the corresponding x-values (arguments). Divided differences are extremely useful for data having unequally spaced x-values.

Argument x	*Entry* y	*I difference* Δy	*II difference* $\Delta^2 y$	*III difference* $\Delta^3 y$	*IV difference* $\Delta^4 y$	*V difference* $\Delta^5 y$
x_0	y_0					
		$y_1 - y_0 = \Delta y_0$				
x_1	y_1		$\Delta y_1 - \Delta y_0 = \Delta^2 y_0$			
		$y_2 - y_1 = \Delta y_1$		$\Delta^2 y_1 - \Delta^2 y_0 = \Delta^3 y_0$		
x_2	y_2		$\Delta y_2 - \Delta y_1 = \Delta^2 y_1$		$\Delta^3 y_1 - \Delta^3 y_0 = \Delta^4 y_0$	
		$y_3 - y_2 = \Delta y_2$		$\Delta^2 y_2 - \Delta^2 y_1 = \Delta^3 y_1$		$\Delta^4 y_1 - \Delta^4 y_0 = \Delta^5 y_0$
x_3	y_3		$\Delta y_3 - \Delta y_2 = \Delta^2 y_2$		$\Delta^3 y_2 - \Delta^3 y_1 = \Delta^4 y_1$	
		$y_4 - y_3 = \Delta y_3$		$\Delta^2 y_3 - \Delta^2 y_2 = \Delta^3 y_2$		
x_4	y_4		$\Delta y_4 - \Delta y_3 = \Delta^2 y_3$			
		$y_5 - y_4 = \Delta y_4$				
x_5	y_5					

In nutshell, divided differences are symmetric functions of their arguments. Also the divided differences are independent of the order of the arguments.

Q. 18 What kind of difference tables are usually prepared for interpolation?

Ans. Various kinds of difference tables frequently used for interpolation are:

(i) Diagonal difference table
(ii) Central difference table
(iii) Divided difference table.

Q. 19 Give the method of preparing a diagonal difference table.

Ans. In diagonal difference table, the first argument (first x-value) is taken as origin x_0 and corresponding entry (y-value) as y_0. The diagonal difference table with six values is displayed above:

The first differences $\Delta^n y_0 (n = 1, 2, ...)$ in columns are called diagonal differences.

Q. 20 Explain a central difference table.

Ans. In central difference table, the origin x_0 of the argument lies in the midst of the series and corresponding entry (y-value) is taken as y_0. The central difference table with five values has been prepared and presented below.

Argument x	*Entry* y	*I difference* Δy	*II difference* $\Delta^2 y$	*III difference* $\Delta^3 y$	*IV difference* $\Delta^4 y$
$x - 1$	y_{-2}				
		$y_{-1} - y_{-2} = \Delta y_{-2}$			
$x - 2$	y_{-1}		$\Delta y_{-1} - \Delta y_{-2} = \Delta^2 y_{-2}$		
		$y_0 - y_{-1} = \Delta y_{-1}$		$\Delta^2 y_{-1} - \Delta^2 y_{-2} = \Delta^3 y_{-2}$	
x_0	y_0		$\Delta y_0 - \Delta y_{-1} = \Delta^2 y_{-1}$		$\Delta^3 y_{-1} - \Delta^3 y_{-2} = \Delta^4 y_{-2}$
		$y_1 - y_0 = \Delta y_0$		$\Delta^2 y_0 - \Delta^2 y_{-1} = \Delta^3 y_{-1}$	
x_1	y_1		$\Delta y_1 - \Delta y_0 = \Delta^2 y_0$		
		$y_2 - y_1 = \Delta y_1$			
x_2	y_2				

Q. 21 Discuss a divided difference table.

Ans. When the argument x does not advance with equal interval, divided difference table has to be prepared for interpolation. The divided difference table is presented below by taking five values only for the purpose of illustration:

Argument	*Entry*	*I difference*	*II difference*	*III difference*	*IV difference*
x	y	$\underline{\Delta} y$	$\underline{\Delta}^2 y$	$\underline{\Delta}^3 y$	$\underline{\Delta}^4 y$
x_0	y_0				
		$\frac{y_1 - y_0}{x_1 - x_0} = \underline{\Delta} y_0$			
x_1	y_1		$\frac{\underline{\Delta} y_1 - \underline{\Delta} y_0}{x_2 - x_0} = \underline{\Delta}^2 y_0$		
		$\frac{y_2 - y_1}{x_2 - x_1} = \underline{\Delta} y_1$		$\frac{\underline{\Delta}^2 y_1 - \underline{\Delta}^2 y_0}{x_3 - x_0} = \underline{\Delta}^3 y_0$	
x_2	y_2		$\frac{\underline{\Delta} y_2 - \underline{\Delta} y_1}{x_3 - x_1} = \underline{\Delta}^2 y_1$		$\frac{\underline{\Delta}^3 y_1 - \underline{\Delta}^3 y_0}{x_4 - x_0} = \underline{\Delta}^4 y_0$
		$\frac{y_3 - y_2}{x_3 - x_2} = \underline{\Delta} y_2$		$\frac{\underline{\Delta}^2 y_2 - \underline{\Delta}^2 y_1}{x_4 - x_1} = \underline{\Delta}^3 y_1$	
x_3	y_3		$\frac{\underline{\Delta} y_3 - \underline{\Delta} y_2}{x_4 - x_2} = \underline{\Delta}^2 y_2$		
		$\frac{y_4 - y_3}{x_4 - x_3} = \underline{\Delta} y_3$			
x_4	y_4				

where $\underline{\Delta}$ ≡ is divided difference.

Q. 22 Discuss in brief Newton's formula of advancing differences for interpolation.

Ans. Newton's formula of advancing differences which is also known as Newton-Gregory forward formula for interpolation is explicated below:

Applicability

(1) The formula is applicable when the arguments advance with equal intervals such as x, $x + h$, $x + 2h$, ...

(2) The formula is more appropriate if the interpolating item, *i.e.*, the value of x lies in the beginning of the series.

(3) Newton's formula of advancing differences can also be used for extrapolation if the interpolation value lies slightly before the first value of x.

Difference table

Prepare diagonal difference table.

Formula

Suppose there are n arguments and n corresponding entries such as,

X : x_0, $x_0 + h$, $x_0 + 2h$, ... $x_0 + (n - 1)h$
Y : y_0, y_1, y_2, ... y_{n-1}

Let x be the argument for which the entry say, y_x is to be estimated. Newton's formula of advancing differences is,

$$y_x = y_0 + \binom{u}{1}\Delta y_0 + \binom{u}{2}\Delta^2 y_0 + \dots + \binom{u}{n-1}\Delta^{n-1} y_0$$

$$= y_0 + \sum_{r=1}^{n-1} \binom{u}{r} \Delta^r y_0$$

where, $u = \dfrac{x - x_0}{h}$

and $\dbinom{u}{r} = \dfrac{u(u-1)(u-2)\ldots(u-r+1)}{r(r-1)(r-2)\ldots3.2.1}$

Substituting the values of different terms in the formula, we get the estimated value of y_x.

Q. 23 Give Newton's backward formula.

Ans. This is also known as Newton-Gregory backward formula and is described below:

Applicability

Newton's backward formula is applicable:

(1) When the argument x advances with equal jumps.

(2) When the x-value to be interpolated lies near the end of the series.

(3) For extrapolation also if the extrapolating value x lies slightly beyond the last x-value of the series.

Difference table

Diagonal difference table is used in Newton's backward formula. But the differences used are in the reverse order.

Formula

If there are n arguments and n corresponding entries, Newton's backward formula for the entry y_x to be interpolated for the argument x is,

$$y_x = y_n + \binom{u}{1}\Delta_{y_{n-1}} + \binom{u+1}{2}\Delta^2_{y_{n-2}} + \binom{u+2}{3}\Delta^2_{y_{n-3}} + \ldots$$

$$= y_n + \sum_{r=1}^{n} \binom{u+r-1}{r} \Delta^r y_{n-r}$$

where, $u = \dfrac{x - x_n}{h}$

x_n – last x-value of the series

h – common difference between two consecutive x-values.

Q. 24 Describe the Newton-Gauss forward formula.

Ans. The Newton-Gauss forward formula is also known as *Gauss forward polynomial formula.* This is not much different from Newton's formula for advancing differences except that in this formula the origin x_0 is the nearest lower value of x in the series to the given value of x instead of the first value of the series.

Applicability

(1) This formula is preferably used when the interpolating value x lies in the middle of the series or in the upper half of the series.

(2) It is necessary that the arguments x-advance with equal increment.

Difference table

For this formula central difference table has to be used.

Formula

The formula with usual notations is,

$$y_x = y_0 + \binom{u}{1}\Delta y_0 + \binom{u}{2}\Delta^2 y_{-1} + \binom{u+1}{3}\Delta^3 y_{-1}$$

$$+ \binom{u+2}{4}\Delta^4 y_{-2} + \binom{u+2}{4}\Delta^5 y_{-2} + \ldots$$

where, $u = \dfrac{x - x_0}{h}$

and h is the constant interval between x-values. It is trivial to calculate y_x for a given value of x.

Q. 25 Discuss the Newton-Gauss backward method of interpolation.

Ans. This method is also known as the *Gauss backward polynomial formula.*

Applicability

(1) This method is applicable when the arguments have equal common difference between arguments.

method could help in this situation. But Lagrange's interpolating polynomial is another very good formula for interpolation as well as extrapolation.

Applicability

(1) This method has no restriction on the x-variable (argument) whether it should be equally spaced or not.

(2) Lagrange's method can be used for any value of x either for interpolation or extrapolation.

(3) Lagrange's interpolation formula can also be used to estimate the argument x for a given value of y. It means Lagrange's formula can be used for inverse interpolation also.

Difference table

It requires no difference table.

Formula

If $(n + 1)$ known values of y, which are a function of x, *i.e.*, $y = f(x)$ corresponding to $(n + 1)$ arguments, are as follows,

$$\begin{array}{llllll} x: & x_0, & x_1, & x_2, & \ldots, & x_n \\ y: & y_0, & y_1, & y_2, & \ldots, & y_n \end{array}$$

then for estimating y corresponding to a given value of x, the formula is,

$$y_x = \frac{(x-x_1)(x-x_2)(x-x_3)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x-x_3)\ldots(x_0-x_n)} y_0$$

$$+\frac{(x-x_0)(x-x_2)(x-x_3)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\ldots(x_1-x_n)} y_1$$

$$+\frac{(x-x_0)(x-x_1)(x-x_3)\ldots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\ldots(x_2-x_n)} y_2$$

$$\vdots \qquad\qquad \vdots$$

$$+\frac{(x-x_0)(x-x_1)(x-x_2)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\ldots(x_n-x_{n-1})} y_n$$

Substituting the value of given x and other known values of the arguments and entries, a good estimate of y is obtained. Lagrange's formula is most general formula.

The only demerit of this formula is that it entails heavy computational work.

Q. 29 Why do we require formulae for central interpolation?

Ans. Formulae propounded by Newton, Gauss, Lagrange and others are applicable for interpolation in general. But the question remains, how exact is the estimate in a particular situation. Therefore, there is a desideration for better and better formulae. In quest of the same, stirling and Bessel gave interpolation formulae which are more appropriate for values to be estimated lying in the mid or central part of the given series.

Q. 30 Enunciate stirlings formula and its implications.

Ans. Stirling's formula is applicable only when the values of the arguments x are equidistant. It is most suitable for interpolation near the middle of the tabulated values of the set. Let there be a set of $(2n + 1)$ values of the function $y = f(x)$ as,

$$x_{-n}, x_{-n+1}, \ldots, x_{-2}, x_{-1}, x_0, x_1, x_2, \ldots, x_n$$

$$y_{-n}, y_{-n+1}, \ldots, y_{-2}, y_{-1}, y_0, y_1, y_2, \ldots, y_n$$

The Stirling's formula is,

$$y_x = y_0 + \frac{u}{1!}\frac{\Delta y_0 + \Delta y_{-1}}{2} + \frac{u^2}{2!}\Delta^2_{y_{-1}}$$

$$+\frac{u(u^2-1^2)}{3!}\frac{\Delta^3_{y_{-1}}+\Delta^3_{y_{-2}}}{2} + \frac{u^2(u^2-1^2)}{4!}\Delta^4_{y_{-2}}$$

$$+\frac{u(u^2-1^2)(u^2-2^2)}{5!}\frac{\Delta^5_{y_{-2}}+\Delta^5_{y_{-3}}}{2}$$

$$+\frac{u^2(u^2-1^2)(u^2-2^2)}{6!}\Delta^6_{y_{-3}}$$

$$\vdots$$

$$+\frac{u^2(u^2-1^2)(u^2-2^2)\ldots\{u^2-(n-1)^2\}}{(2n-1)!}\times$$

$$\frac{\Delta^{2n-1}_{y_{-(n-1)}}+\Delta^{2n-1}_{y_{-n}}}{2}$$

$$+\frac{u^2\left(u^2-1^2\right)\left(u^2-2^2\right)\ldots\left\{u^2-(n-1)^2\right\}}{(2n)!}\Delta^{2n}_{y_{-n}}$$

where,

$$u=\frac{x-x_0}{h} \text{ and } h=x_1-x_0=x_2-x_1=\ldots$$

Q. 31 Expound Bessel's formula. Let there be a set of $(2n + 1)$ values as given in Q. 30 of the function $y = f(x)$. Bessel's formula is applicable only when the values of the argument x are at equal intervals. The formula expounded by Bessel to interpolate y for a given value of x which lies in the central part of the series is,

$$y_x=\frac{y_0+y_1}{2}+\frac{v}{1!}\Delta y_0+\frac{v^2-\frac{1}{4}}{2!}\,\frac{\Delta^2 y_0+\Delta^2 y_{-1}}{2}$$

$$+\frac{v\left(v^2-\frac{1}{4}\right)}{3!}\Delta^3 y_{-1}+\frac{\left(v^2-\frac{1}{4}\right)\left(v^2-\frac{9}{4}\right)}{4!}\times$$

$$\frac{\Delta^4 y_{-1}+\Delta^4 y_{-2}}{2}+\frac{v\left(v^2-\frac{1}{4}\right)\left(v^2-\frac{9}{4}\right)}{5!}\Delta^5 y_{-2}$$

$$+\frac{\left(v^2-\frac{1}{4}\right)\left(v^2-\frac{9}{4}\right)\left(v^2-\frac{25}{4}\right)}{6!}\,\frac{\Delta^6 y_{-2}+\Delta^6 y_{-3}}{2}$$

$$\vdots$$

$$+\frac{\left(v^2-\frac{1}{4}\right)\left(v^2-\frac{9}{4}\right)\ldots\left\{v^2-\frac{(2n-1)^2}{4}\right\}}{(2n)!}\times$$

$$\frac{\Delta^{2n} y_{-(n-1)}+\Delta^{2n} y_{-n}}{2}$$

$$+\frac{v\left(v^2-\frac{1}{4}\right)\left(v^2-\frac{9}{4}\right)\ldots\left\{v^2-\frac{(2n-1)^2}{4}\right\}}{(2n+1)!}\Delta^{2n+1} y_{-n}$$

where

$$v=\frac{x-x_0}{h}-\frac{1}{2}=u-\frac{1}{2}$$

and $$h=x_1-x_0=x_2-x_1=\ldots$$

Q. 32 Make a comparative statement on Stirling's and Bessel's formulae of interpolation.

Ans. Following points can be specified while comparing Stirling's and Bessel's formulae of interpolation.

1. Both the formulae are applicable for equidistant arguments.
2. Both are preferable for interpolating values near the middle of the series.
3. Stirling used u as in Newton-Gauss formulae whereas Bessel used v which is equal to $\left(u-\frac{1}{2}\right)$.
4. For both the formulae, choose x_0 which makes u and v as small as possible. Preferably u and v should lie between -0.5 and 0.5.
5. It has been experienced that
   - (i) Stirling's formula provides more accurate results if the interpolating value lies near the beginning or the end of the central interval and also u lies from -0.25 to 0.25.
   - (ii) Bessel's formula yields more accurate results when the interpolating value lies near the middle of the central interval and v ranges from -0.25 to 0.25, *i.e.*, $0.25 \le u \le 0.75$.

Q. 33 What do you understand by inverse interpolation?

Ans. When $y = f(x)$ and for a given values of arguments x and corresponding entries y, a value of x is to be estimated for a given value of y, the interpolation is known as inverse interpolation. In other words, if the value of the independent variable x is to be estimated for a given value of the dependent variable y within the series, it is called inverse interpolation.

Q. 34 Name different methods of inverse interpolation.

Ans. A few methods of inverse interpolation are:

(i) Lagrange's method
(ii) Iterative method
(iii) Successive approximation method.

Q. 35 Give a brief account of Lagrange's method for inverse interpolation.

Ans. Lagrange's method of interpolation can be used for inverse interpolation by writing the formula similar to interpolation replacing y by x and x by y. In this way, the interpolated value x_y for a given value y can be obtained by the formula.

$$x_y = \frac{(y-y_1)(y-y_2)(y-y_3)\dots(y-y_n)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)\dots(y_0-y_n)}x_0$$

$$+\frac{(y-y_0)(y-y_2)(y-y_3)\dots(y-y_n)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)\dots(y_1-y_n)}x_1$$

$$+\frac{(y-y_0)(y-y_1)(y-y_3)\dots(y-y_n)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)\dots(y_2-y_n)}x_3$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$+\frac{(y-y_0)(y-y_1)(y-y_2)\dots(y-y_{n-1})}{(y_n-y_0)(y_n-y_1)(y_n-y_2)\dots(y_n-y_{n-1})}x_n$$

This formula can be applied in exactly the same manner as for interpolating y.

Q. 36 Explain iterative method of inverse interpolation.

Ans. This method obtains an interpolated value in stages. This approach can be adopted for any polynomial formula suitable for a particular problem. Here it is explained by taking Newton's forward polynomial only. We know that Newton's forward polynomial can be written as,

$$y = y_0 + \binom{u}{1}\Delta y_0 + \binom{u}{2}\Delta^2 y_0 + \binom{u}{3}\Delta^3 y_0 + \dots$$

$$+\binom{u}{n}\Delta^n y_0$$

$$= y_0 + u\Delta y_0 + \sum_{r=2}^{n}\binom{u}{r}\Delta^r y_0$$

$$u = \frac{y-y_0}{\Delta y_0} - \frac{1}{\Delta y_0}\sum_{r=2}^{n}\binom{u}{r}\Delta^r y_0$$

The above formula is an iteration formula in u. If we know u, it is trivial to know x as $u = \frac{x-x_0}{h}$.

Suppose,

$$u_1 = \frac{y-y_0}{\Delta y_0}$$

and $$I(u) = \frac{1}{\Delta y_0}\sum_{r=2}^{n}\binom{u}{r}\Delta^r y_0$$

$$\therefore \qquad u = u_1 - I(u)$$

Let the value of u so obtained be denoted by u'_1 as a first approximation. Use u'_1 to obtain second approximation u'_2 by the formula,

$$u'_2 = u_1 - I(u'_1)$$

Again use u'_2 to get third approximation. Continue the process till almost same value of u is obtained in two successive iterations.

Q. 37 Explicate the method of successive approximation for inverse interpolation.

Ans. Under this method choose a polynomial deemed fit for interpolation. Once we have written the polynomial, the successive approximation method can be operated in the following manner. Now we take Newton's forward polynomial to explain the inverse interpolation. The polynomial is,

$$y = y_0 + \binom{u}{1}\Delta y_0 + \binom{u}{2}\Delta^2 y_0 + \binom{u}{3}\Delta^3 y_0 + \dots$$

$$+\binom{u}{n}\Delta^n y_0$$

After algebraic manipulation we can write the above polynomial in the form,

$$u = \frac{y-y_0}{\Delta y_0} - \frac{1}{\Delta y_0}\binom{u}{2}\Delta^2 y_0 - \frac{1}{\Delta y_0}\binom{u}{3}\Delta^3 y_0$$

$$\dots - \frac{1}{\Delta y_0}\binom{u}{n}\Delta^n y_0$$

As a first approximation, take

$$u_1 = \frac{y - y_0}{\Delta y_0},$$

neglecting all terms at right hand side involving u. As a second approximation, take $u = u_1$, the value of u obtained under first approximation and include one more term of the right hand side involving second order finite difference. Thus, the second approximated u is,

$$u_2 = \frac{y - y_0}{\Delta y_0} - \frac{1}{\Delta y_0}\binom{u_1}{2}\Delta^2 y_0$$

Repeat the process taking $u = u_2$ and include two terms of right hand side involving $\Delta^2 y_0$ and $\Delta^3 y_0$. Continue the process till all terms of right hand side are involved or two equal values of u under successive approximations are obtained.

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s) in the blanks:

1. Interpolation and extrapolation are the parts of ________ analysis.
2. Interpolation and extrapolation approaches are ________.
3. Interpolation and extrapolation formulae assume ________ in the data of the series.
4. For interpolation or extrapolation, the two variables should have ________ relationship.
5. Interpolation is ________ frequently used than extrapolation.
6. Non-algebraic method of interpolation is ________ method.
7. Binomial method for interpolation is applicable when the independent variable x advances with ________.
8. If n values of dependent variable y are known, we take ________ finite difference zero.
9. Parabolic method can be used for ________ as well as ________.
10. Parabolic method of interpolation is applicable in ________.
11. Interpolation helps to estimate the ________ value in a series of data.
12. The estimation for a value beyond the given series of data is called ________.
13. With the help of interpolation and extrapolation, two series can be made ________.
14. The differences between two consecutive dependent variate values are called ________ differences.
15. The finite differences $\left(\Delta^2_{y_2} - \Delta^2_{y_1}\right)$ is called ________ order finite difference.
16. The difference between two consecutive dependent variate values divided by the interval between the corresponding independent variate values is called ________ difference.
17. The independent variate values in interpolation are termed as ________.
18. The dependent variate value in interpolation and extrapolation is called ________.
19. In diagonal difference table, the ________ argument of the series is taken as origin.
20. ________ types of difference tables are usually used in interpolation and extrapolation.
21. In a central difference table, the origin lies in the ________ of the series.
22. Newton's formula for advancing differences is also known as ________ forward formula.
23. Newton's formula for advancing differences is applicable if the interpolating value lies in the ________ of the series.
24. Newton's formula for advancing differences

utilises ________ finite difference of each column of the difference table.

25. In Newton's forward formula, the restriction on arguments is that they should advance with ________.

26. Newton's backward formula is used when the ________ value lies at the end of the series.

27. Newton-Gauss forward formula is also known as ________ polynomial formula.

28. The origin x_0 in difference table in the Newton's-Gauss forward formula is the ________ value of x to the given value of x.

29. The relation between operators E and Δ within its usual sense in interpolation is ________.

30. Newton's backward polynomial formula utilises the ________ leading difference of each column.

31. In Newton's backward formula, the origin is the ________ value of the argument in the series.

32. Newton's method of divided differences takes care of the ________ spaced arguments.

33. All the Newton-Gauss formulae are ________ formulae.

34. The $(n + 1)^{\text{th}}$ order finite difference of a n^{th} order polynomial is ________.

35. Lagrange's interpolation formula has ________.

36. Lagrange's polynomial is suitable for ________ as well as ________.

37. Lagrange's formula can be used for ________ interpolation also.

38. Lagrange's polynomial for n given entries has ________ terms.

39. Each term of a Lagrange's formula involving n arguments is a polynomial of degree ________.

40. The demerit of Lagrange's formula is that it involves ________ computations as compared to other formulae.

41. If for a given functional relation $y = f(x)$, one estimates x for a given y, it is called ________ interpolation.

42. Lagrange's formula for inverse interpolation given that $y = f(x)$ is a polynomial in ________.

43. Iterative method of inverse interpolation is not confined to ________ formula.

44. Iteration is continued till a ________ value of $u = (x - x_0)/h$ is obtained.

45. In successive approximation, terms are added ________ at each stage.

46. Inverse interpolation is not as accurate as ________.

47. Given $y_0 = 3$, $y_1 = 12$, $y_2 = 10$, $y_3 = 8$ the value of $\Delta^3 y_0$ is equal to ________.

48. Binomial expansion method is based on the theme that if n entries are known, then ________.

49. Given the data,

x :	1	3	5
y :	2	?	15

the missing y-value is ________.

50. Given the following values of x and $f(x)$,

x :	30	31	32	33	34
y_x :	0	1	2	3	4

Newton-Gregory forward polynomial is ________.

51. If l_x represents the number living at the age x in a life table, given that

x :	10	15	20	25
l_x :	42	34	25	18

l_x for $x = 22$ by Newton's backward formula is ________.

52. Divided differences are symmetric ________ of their arguments.

53. Divided differences are independent of the ________ of the arguments.

54. Five pairs of values of arguments and entries

(d) all the above

Q. 8 Very many formulae of interpolation and extrapolation are given by:
(a) Newton
(b) Gregory
(c) Gauss
(d) all the above

Q. 9 Most general formula for interpolation and extrapolation is due to:
(a) Newton and Gauss
(b) Newton and Gregory
(c) Lagrange
(d) all the above

Q. 10 If n entries are known for $(n + 1)$ argument at equal intervals having a missing entry, a non-polynomial estimation method is:
(a) Lagrange's method
(b) Newton's formula
(c) binomial expansion method
(d) none of the above

Q. 11 Binomial expansion method is based on the principle that with n known entries:
(a) n^{th} finite difference $\Delta^n y_0 = 0$
(b) we take $(E - 1)^n y_0 = 0$
(c) $y_n - \binom{n}{1} y_{n-1} + \binom{n}{2} y_{n-2} \cdots + (-1)^n y_0 = 0$
(d) all the above

Q. 12 In binomial expression method we always get:
(a) n equations
(b) as many equations as the number of unknowns
(c) two equations
(d) $(n + 1)$ equations

Q. 13 Graphical method of estimation is:
(a) always very accurate
(b) sometimes very accurate
(c) never accurate
(d) none of the above

Q. 14 Graphical method can be used for:
(a) interpolation only
(b) extrapolation only
(c) interpolation and extrapolation both
(d) none of the above

Q. 15 Graphical method of interpolation is:
(a) simple
(b) non-algebraical
(c) not fully reliable
(d) all the above

Q. 16 For the given $(n + 1)$ paired values, parabolic method means fitting of a polynomial:
(a) of degree n
(b) of degree 2
(c) of degree $(n + 1)$
(d) of degree 3

Q. 17 Parabolic method of interpolation is also known as:
(a) method of orthogonal polynomials
(b) method of simultaneous equations
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 18 Parabolic method of estimation is good for:
(a) interpolation
(b) extrapolation
(c) interpolation as well as extrapolation
(d) none of the above

Q. 19 A polynomial of degree n is known as:
(a) a parabola of degree n
(b) a parabola of degree $(n + 1)$
(c) a parabola
(d) none of the above

Q. 20 Interpolation is helpful in estimating:
(a) missing value(s) of a series
(b) an intermediary value for a given argument
(c) the argument for a given entry
(d) all the above

Q. 21 Which of the following relation amongst finite differences is not correct?
(a) $\Delta^3 y_{-1} - \Delta^3 y_{-2} = \Delta^4 y_{-2}$
(b) $\Delta^2 y_{-1} - \Delta^2 y_0 = \Delta^3 y_0$
(c) $\Delta y_2 - \Delta y_1 = \Delta^2 y_1$
(d) $y_2 - y_1 = \Delta y_1$

(c) $y_x = y_0 + \sum_{r=1}^{n} \binom{u+r-1}{r} \nabla^r y_0$

(d) none of the above

Q. 33 Newton's method of divided differences is preferred when:

(a) the arguments are not equally spaced

(b) when the interpolating value of the argument lies in the upper half of the series

(c) both (a) and (b)

(d) none of (a) and (b)

Q. 34 Standard notation for divided difference is:

(a) Δ

(b) ∇

(c) $\not\Delta$

(d) D

Q. 35 Divided difference method can be used when the given independent variate values are:

(a) at equal intervals

(b) at unequal intervals

(c) not well defined

(d) all the above

Q. 36 Lagrange's formula is useful for:

(a) interpolation

(b) extrapolation

(c) inverse interpolation

(d) all the above

Q. 37 Lagrange's polynomial for interpolation can be used even if:

(a) the given arguments are not equally spaced

(b) extrapolation is to be done

(c) inverse interpolation is to be done

(d) all the above

Q. 38 If $(n + 1)$ pairs of arguments and entries are given, Lagrange's formula is:

(a) a polynomial of degree n in x

(b) a polynomial of degree n in y

(c) a polynomial in x in which each term has degree n

(d) a polynomial with highest degree 1

Q. 39 The method of inverse interpolation is:

(a) iterative method

(b) Lagrange's method

(c) successive method of approximation

(d) all the above

Q. 40 Iterative method of inverse interpolation utilises:

(a) Newton's forward formula only

(b) Newton's backward formula only

(c) any suitable polynomial formula

(d) none of the above

Q. 41 In iterative method of inverse interpolation:

(a) the value of u is estimated successively

(b) the value of x is directly estimated

(c) the value of y is directly estimated

(d) all the above

Q. 42 Method of successive approximation utilises:

(a) a polynomial formula as a whole

(b) terms of a polynomial formula successively

(c) only the terms independent of u

(d) only the terms involving u

Q. 43 Given the following data,

Income per day not exceeding (Rs.) :	10	18	20	28	40
Workers :	12	32	68	80	100

To interpolate number of workers for income not exceeding Rs. 30 per day, the suitable method is:

(a) Newton's backward formula

(b) Lagrange's formula

(c) binomial expansion method

(d) Gauss backward formula

Q. 44 Given the profits of a firm in lakh rupees and the number of firms as follows:

Profits	*No. of firms*
100-150	25
150-200	30
200-250	28
250-300	16

The appropriate formula for estimating the number of firms with profit below 180 lakh is:

(a) Newton's formula of advancing differences
(b) Lagrange's formula
(c) Newton's-Gauss forward formula
(d) all the above

Q. 45 Given the following frequency distribution of marks:

Marks	*No. of students*
20-30	3
30-40	10
40-50	?
50-60	15
60-70	8

An appropriate method of estimating the missing frequency is:

(a) Newton's-Gauss formula
(b) binomial expansion formula
(c) Lagrange's formula
(d) all the above

Q. 46 The missing value for the following data,

x:	5	10	15	20
y:	2	5	?	8

by the binomial expansion method is:

(a) 7
(b) −7
(c) 3
(d) 25/3

Q. 47 Given the profits for the last three years as,

Years	1991	1992	1993
Profits (lakh Rs.)	15	18	24

the expected profit during the year 1994 by parabolic method is:

(a) 27
(b) 48
(c) 33
(d) 21

Q. 48 If the temperature of three dates of June, 1994 were as follows:

Dates:	1	10	25
Temp (°C):	33	38	46

The estimated temperature for 20th June, 1994 by divided difference method is:

(a) 43.37
(b) 42.37
(c) 43.73
(d) 39.0

Q. 49 If $y = f(x)$ and the values of $f(x)$ for given x are, $f(1) = 14, f(2) = 12, f(5) = 6$ and $f(8) = 21, f(7)$ is:

(a) 2
(b) 12
(c) −8
(d) none of the above

Q. 50 If the observed values of x and function u_x are:

x :	2	6	8	9
u_x :	198	150	102	93

The interpolating function u_x is:

(a) $x^3 - 4x^2 + 80x + 102$
(b) $x^3 - 18x^2 + 80x + 294$
(c) $x^3 - 18x^2 + 80x + 102$
(d) none of the above

Q. 51 The most suitable formula for estimating a value lying in the central part of a series is:

(a) Lagrange's formula
(b) Stirling's formula
(c) Newton-Gauss forward formula
(d) Newton-Gauss backward formula

Q. 52 Interpolation formulae are based on the fundamental assumption that the data can be expressed as:

(a) a linear function
(b) a quadratic function
(c) a polynomial function
(d) none of the above

Q. 53 The relationship between u of Stirling's formula and v in Bessel's formula for interpolation is:

(a) $u = v + 1$
(b) $u = v - 1$

(c) $u = v - \frac{1}{2}$

(d) $u = v + \frac{1}{2}$

Q. 54 Interpolation provides good estimates of missing values if and only if:

(a) the change of values is consistent
(b) the series does not refer to abnormal periods
(c) the arguments are equidistant
(d) all the above

Q. 55 Bessel's and Stirling's interpolation formulae yield good estimates if the values of u and v in general lie between:

(a) −1 and +1
(b) −0.5 and 1
(c) −0.5 and 0.5
(d) 0 and 1

Q. 56 If there are consecutive missing values in a series, their estimation is:

(a) not possible
(b) not reliable
(c) a complicated problem
(d) all the above

Q. 57 Bessel's interpolation formula is most appropriate to estimate for a value in a series which lies:

(a) at the end
(b) in the beginning
(c) in the middle of the central interval
(d) outside the series

Q. 58 The problems of interpolation are simpler than prediction because:

(a) interpolation has fewer restrictions than prediction
(b) interpolation is based on more stringent restrictions than prediction
(c) there are no restriction in case of interpolation
(d) all the above

Q. 59 Stirling's and Bessel's interpolation formulae for interpolation are applicable in case of:

(a) equidistant arguments
(b) for all types of series
(c) arguments increasing by one only
(d) arguments decreasing by one only

Q. 60 If the arguments in a series are not at equal interval, the proper formula that can be used for interpolation is:

(a) Bessel's formula
(b) Lagrange's formula
(c) Stirling's formula
(d) Newton's formula

Q. 61 If Δy_x is constant, then y_x may be:

(a) constant
(b) at equal intervals
(c) both (a) and (b)
(d) none of the above

Q. 62 The third differences of a cubic $\Delta^3 y$ function are:

(a) constant
(b) not constant
(c) variables
(d) none of the above

Q. 63 Which formula is appropriate for central interpolation?

(a) Bessel's formula
(b) Stirling's formula
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 64 If the interpolating value lies near the middle of the central interval, then the most exact formula is:

(a) Bessel's interpolation formula
(b) Lagrange's interpolation formula
(c) Stirling's interpolation formula
(d) Newton-Gauss interpolation formula

Q. 65 If the interpolating value lies near the beginning or the end of the central interval, then the most exact formula is:

(a) Newton-Gauss formula
(b) Newton-Gregory formula
(c) Bessel's formula
(d) none of the above

ANSWERS

SECTION-B

(1) Numerical (2) non-probabilistic (3) no fluctuations (4) definite (5) more (6) graphical (7) equal jumps (8) n^{th} (9) interpolation; extrapolation (10) all situations (11) missing (12) extrapolation (13) comparable (14) finite (15) third (16) divided (17) arguments (18) entry (19) first (20) Three (21) midst (22) Newton-Gregory (23) beginning (24) first (25) equal increment (26) interpolating (27) Gauss forward (28) just lower (29) $E-1=\Delta$ (30) last (31) last (32) unequally (33) polynomial (34) constant (35) no restrictions (36) interpolation; extrapolation (37) inverse (38) n (39) $n-1$ (40) heavy (41) inverse (42) y variable (43) single (44) stable (45) one by one (46) interpolation (47) 11 (48) $\Delta^n y_0 = 0$ (49) 6.5 (50) 30 + x (51) 19.63 (52) functions (53) order (54) fourth (55) Bessel; Stirling (56) different (57) equidistant (58) $v = u - \frac{1}{2}$ (59) Stirling's (60) –0.5 and 0.5 (61) Bessel's (62) zero (63) interpolation (64) difference (65) second

SECTION-C

(1) a	(2) b	(3) a	(4) d	(5) d	(6) c
(7) a	(8) d	(9) c	(10) c	(11) d	(12) b
(13) b	(14) c	(15) d	(16) a	(17) b	(18) c
(19) a	(20) d	(21) b	(22) b	(23) a	(24) c
(25) a	(26) c	(27) a	(28) b	(29) d	(30) b
(31) d	(32) c	(33) c	(34) c	(35) b	(36) d
(37) d	(38) a	(39) d	(40) c	(41) a	(42) b
(43) b	(44) d	(45) b	(46) a	(47) c	(48) a
(49) b	(50) c	(51) b	(52) c	(53) d	(54) d
(55) c	(56) a	(57) c	(58) a	(59) a	(60) b
(61) c	(62) a	(63) c	(64) a	(65) d	

Suggested Reading

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Chapter **16**

Time Series Analysis

SECTION-A

Short Essay Type Questions

Q. 1 What is a time series?

Ans. A time series is a set of observations measured at time or space intervals arranged in chronological order. For instance, the yearly demand of a commodity, weekly prices of an item, food production in India from year to year, etc. Many economists and statisticians have defined time series in different words. Some of them are quoted below:

Wessel and Wellet: When quantitative data are arranged in the order of their occurrence, the resulting statistical series is called a time series.

Moris Hamburg: A time series is a set of statistical observations arranged in chronological order.

Patterson: A time series consists of statistical data which are collected, recorded or observed over successive increments.

Ya-lun-Chou: A time series may be defined as a collection of magnitudes belonging to different time periods, of some variable or composite of variables such as production of steel, per capita income, gross national product, price of tobacco or index of industrial production.

Cecil H. Meyers: A time series may be defined as a sequence of repeated measurements of a variable made periodically through time.

Werner Z. Hirsch: A time series is a sequence of values of the same variate corresponding to successive points of time.

Spiegel: A time series is a set of observations taken at specified times, usually at equal intervals. Mathematically, a time series is defined by the values $Y_1, Y_2, \ldots$ of a variable Y (temperature, closing price of a share, etc.) at time $t_1, t_2 \ldots$ Thus, Y, is a function of t symbolised by $Y = F(t)$.

Q. 2 What purpose is served by time series analysis?

Ans. The analysis of time series has seen found useful to economists and business persons, in particular, and also to scientists, so~~c~~iologist, etc. It has also found its utility in meteorology, seismology, oceanography, geomorphology, etc., in earth sciences; electrocardiograms, electroencephotograms in medical sciences and problem of estimating missile trajectories. Time series analysis helps in understanding the following phenomena.

(i) It helps in knowing the real behaviour of the past.

(ii) It helps in predicting the future behaviour like demand, production, weather conditions, prices, etc.

(iii) It helps in planning the future operations.

(iv) Analysis of time series helps to compare the present accomplishments with the past performances.

(v) Two or more times series can be compared belonging to the same reference period.

Q. 3 Throw light on the main drawbacks of the time series analysis.

Ans. The drawbacks of the time series analysis can be summarised as follows:

(i) The conclusions drawn on the basis of time series analysis are not cent per cent true.

(ii) Time series analysis is unable to fully adjust the influences affecting a time series like customs, climate, policy changes, etc.

(iii) The complex forces affecting a time series existing at certain period may not be having the same complex forces in future. Hence, the forecasts may not hold true.

Q. 4 What is the need of editing of data before time series analysis?

Ans. The data have to be critically examined and adjusted for various factors before the analysis of time series, otherwise many discrepancies are likely to arise leading to wrong conclusions. For example, the production for January could be more than February. In reality, it may be due to more number of days in January then in February.

Q. 5 Give various adjustments usually practised during editing of data meant for analysis of a time series.

Ans. Various adjustments normally incorporated in a times series data are as follows:

(i) *Calendar variation.* One should be wary of the type of variable dealing with before implementing the adjustments for calendar variation. Production of an unit per month has to be adjusted for 30 days a month for comparability. But series should not be adjusted for salary as employees are paid on monthly basis irrespective of the number of days in a month.

(ii) *Price variation.* Production or sales are to be adjusted for price variation by the formula.

$$q = v/p$$

where, q – quantity of sales or production in a specified period.

v – sales in terms of amount.

p – price per unit in the reference period.

If this adjustment is not done, increased prices will lead to the conclusion that production or sales have increased (decreased), though in reality it is not so.

(iii) *Population variation.* Demand or consumption of a commodity is directly related to the population of a particular area. If the demand doubles and population also doubles, it should not be taken that demand has increased. It is simply the effect of the number of consumers and not that of increased demand. As a matter of fact, demand is the same.

(iv) *Miscellaneous changes.* For comparison of two time series, it is necessary that the data pertaining to certain period should have been measured in the same units. If they are not, they should be converted to the same unit of measurements. Also, the articles, which are to be compared in terms of prices, should be adjusted for their standard and durability. For example, synthetic fibres clothes are more durable than cotton fibre clothes. Hence, their prices be adjusted for their durability in terms of time period.

Q. 6 Name various components of a time series.

Ans. There are four components or elements of a time series, namely:

(i) Secular Trend-T

(ii) Seasonal Variation-S

(iii) Cyclical Variation-C

(iv) Irregular Variation-I

Q. 7 What do you understand by secular trend?

Ans. Term trend implies *secular trend.* It measures long-term changes occurring in a time series without bothering about short-term fluctuations occurring in between. In short, secular trend measures smooth and regular long-term movements of a time series delineating the increasing, decreasing or stagnant trend over a long span of time. The graph showing trend is a straight line running from left bottom to right top, left top to right bottom or parallel to abscissa depicting growth, decline or stagnation respectively. In some situations curvilinear trend is also studied.

Q. 8 Give the idea of seasonal variation.

Ans. Short-term fluctuations observed in a time series data, particularly in a specified period usually within a year, are called seasonal variations. For instance, certain items have more sale in a particular season like ice cream in summer, rain coats in rainy season and woollens in winter season. Similarly, first week of a month records greater sale of grocery than the last week of a month. Certain items have tremendous sale on festivals only in a particular month. All such variation in a time series come under seasonal variation. Seasonal variations are more akin to climatic and weather conditions.

Q. 9 What is meant by cyclic variation?

Ans. Cyclic variation relates to periodic changes, particularly in business. A cycle consists of more than a year period. The cycles in a time series depict the prosperity and recession, ups and downs, booms and slumps of a business. A complete cycle usually has four constituents namely, *prosperity, recession, depression and recovery*. Cycles related to business are termed as *business cycles or trade cycles*. The length of business cycles varies from one business to the other. The graph of cyclic variation is a curve having alternately convexity and concavity.

Cycles are never regular in periodicity and amplitude. Hardly any time series has strict cycles. Hence, in practice statisticians and economists often use the term *undulations or oscillations* instead of cycles.

Since a cycle covers a long span of time, the data required for the depiction of cycles should be recorded for a large number of years (periods).

Q. 10 What factors are generally responsible for the occurrence of cycles?

Ans. A large number of factors are responsible for the occurrence of cycles. But a few important ones are given below:

(i) Likes and dislikes of the people change after a certain period and they cause cycles in business phenomenon.

(ii) Production of certain items is stopped and new items are produced. Again old items are adopted. Such changes form cycles.

(iii) Social customs change from time to time resulting into business cycles.

(iv) New scientific and technological developments affect the production and consumption of items which create cycles.

Q. 11 Discuss irregular variation in the context of time series.

Ans. Irregular variations or the so-called *random variations* are irregular in the sense that it is not possible to think of their time of occurrence, direction and magnitude. These variation usually occur due to epidemics, earthquakes, floods, wars, accidents, etc. Another name given to irregular variations is *residual variations*. This name is derived in the sense that all those variations which cannot be subsumed in trend, seasonal and cyclic variations, are assigned to irregular variations.

Q. 12 Discuss mathematical models for a time series analysis.

Ans. In a traditional or classical time series analysis, the most commonly assumed mathematical model is the *multiplicative model*. Here it is assumed that any particular observation Y at time t is as a result of the product of the effect of the four components of a time series namely. Trend (T), seasonal variation (S), cyclic variation (C) and the irregular variation (I), *i.e.*,

$$Y = T \times S \times C \times I$$

Further the multiplicative model does not assume the independence of the four components of the time series. It is appropriate for projections.

Some people believe that the observation Y is as a result of additive effect of the components T, S, C and I, *i.e.*,

$$Y = T + S + C + I$$

The additive model is based on the assumption that the four components are independent of each other. Additive model is rarely used as it is not appropriate for future events.

Some people have also advocated the use of mixed models. A mixed model is a mathematical relation which is expressed as a combination of multiplicative and additive components of a time series. They may be combined in a number of ways. Such types of models are hardly used. Some of the examples of mixed models are given below:

$$Y = T + S \times C + I$$
$$Y = T + S \times C \times I$$
$$Y = T + S + C \times I$$
$$Y = T \times C + S \times I$$

Q. 13 What are the essential requirements for proper analysis of a time series?

Ans. The essential requirements for proper analysis of a time series are:

(i) Data should be available for a long period.

(ii) The value should have been available as far as possible at equal interval of time. If not, they have to be adjusted.

(iii) The time periods should be definite according to calendar.

(iv) The data should consist of a homogeneous set of values in respect of units of measurements and time scale.

Q. 14 Give the names of different methods of measuring trend.

Ans. Various methods of measuring trend are:

(i) Free-hand or graphic method

(ii) Semi-average method

(iii) Moving average method

(iv) Least square method

Q. 15 How to find out the trend and trend value by the free-hand method?

Ans. The free-hand method is the simplest method of ascertaining trend. In this method, points (t_1, y_1) (t_2, y_2) ..., (t_n, y_n) are plotted on a graph paper taking time t on abscissa and variate values y on the ordinate by choosing suitable scales. Then a free-hand straight line is drawn in between the points with the help of a transparent scale such that half of the points are above the line and half below it. The angle of the line gives the idea about increasing, decreasing or stagnant trend of the phenomenon under consideration. Also the y-coordinate of a point on the line at any point of time gives the *trend value*.

The main drawback of free-hand method is that it solely depends on the judgement of the investigator or the scientist.

Q. 16 Explicate semi-average method of determining trend.

Ans. In this approach, the series is divided into halves. Then average is found out for each half of the series. The average values are plotted on the graph paper against the mid-points of the corresponding each half series. The line joining these two plotted points gives trend line. The direction of line indicates about rising, falling or constant trend of business movements.

If the number of years in a series is odd, the middle year (period) is excluded at the time of dividing the series into halves and then either included in both the series or excluded totally depending on whether the left over half series contains an even or odd number of years respectively.

The semi-average method gives good results when the trend is almost linear.

Q. 17 What are the merits and demerits of the semi-average method?

Ans.

Merits:

(i) It has no subjectively,

(ii) For one series, there is only one trend line.

Demerits:

(i) It is affected by extreme values, if present in time series data.

(ii) It does not ensure the elimination of seasonal and cyclic variations.

(iii) This method is appropriate, if the data are given for a long period.

Q. 18 Discuss in brief the moving average method for ascertaining the trend.

Ans. The moving average method is an improvement over semi-average method as short-term fluctuation are eliminated by it. Different steps of moving average method are as follows:

Step-1. To obtain the moving average, a group of beginning years (periods), which constitute a business cycle, is chosen for calculating the average. This average is placed in front of the middle of the years (periods) of the group averaged.

Step-2. Now delete first year (period) value from the group and add a succeeding year value in the group. Find the average of the reconstituted group and place it in front of the middle year of this group.

Step-3. If the number of years (periods) in a group is odd, there is no problem of locating the middle year. But if the number of years (periods) in the group is even, no single year is middle year. Hence, to overcome this difficulty, the average of the averages in pairs is calculated and placed against the mid-year of the two. In this way, moving averages are set against years.

Step-4. Keep on repeating step-2 till all years data are exhausted.

Step-5. These moving averages themselves constitute a time series.

Step-6. Plot the moving averages on a graph paper taking years (periods) along abscissa and moving averages along ordinate by choosing the proper scales.

Step-7. Join all the plotted points in the sequence of time periods. The resulting graph provides the trend.

Q. 19 What are the advantages and disadvantages of the moving average method?

Ans.

Advantages:

(i) The greatest advantage of this method is that it eliminates the short-term fluctuations.

(ii) It reduces the effect of extreme values.

(iii) Extending the series by some more ensuing years does not require to redo the whole computations but can add some more trend values by calculating additional moving averages.

Disadvantages:

(i) A few years in the beginning and at the end of the series are left over without the moving averages being entered against them. Hence, no points are plotted against these years causing a loss of information.

(ii) Since there are no points plotted for a few beginning and end years, the moving-average method is not appropriate for projections.

(iii) There is hardly any time series having regular cycles. But they are always taken to be regular. This introduces an error.

(iv) Normally, a small number of years (periods) in a group for moving average is preferred. But many a times greater number of years are included in groups which deflates the magnitude of oscillations in a time series. Greater the degree of deflation, less realistic is the resulting trend. In such a situation, moving averages anticipate faster changes than they actually occur.

(v) This method is not fully mathematical.

(vi) There is no strict rule through which one decides the number of years (periods) in a group. Hence, the element of subjectivity is introduced in this method. So, for the same time series there can be a number of trends.

Q. 20 How do you fit a trend line by the method of least squares?

Ans. Let us consider the linear trend equation as,

$$Y = \alpha + \beta X + \varepsilon$$

As per the least square principle, we will estimate the parameters α and β in such a manner that the error ε is minimised. In this endeavour, we minimise the quantity,

$$Q = \sum_i \varepsilon_i^2 = \sum_i (Y_i - \alpha - \beta X_i)^2$$

where i runs over all periods of time series 1 to n. Differentiating Q partially with respect to α and β and equating to zero, we get two normal equations. Also we replace α and β by a and b, their least square estimates.

Thus we have,

$$\sum_i Y_i = na + b\sum_i X_i$$

$$\sum_i X_i Y_i = a\sum_i X_i + b\sum_i X_i^2$$

Note. If the trend equation is computed from the annual monthly averages, there has to be no change in a and single operation divisor has to be used for b.

Q. 26 Comment on curvilinear trend.

Ans. The trend of the dependent variable Y on time X is not always linear, *i.e.*, the increase or decrease in Y does not occur at a constant rate with the elapse of time. Hence, one has to look for a curvilinear trend. Some of the frequently encountered curves are, (i) second or third degree parabola; (ii) exponential curve, and (iii) compertz curve, etc.

Q. 27 How to fit in a second degree trend equation?

Ans. The equation of a second degree parabola (quadratic trend) is,

$$Y = a + bX + cX^2$$

The curve can be fitted by the method of least squares. Following the usual procedure, we get three normal equations

$$\Sigma Y = \Sigma a + b\Sigma X + c\Sigma X^2$$

$$\Sigma XY = a\Sigma X + b\Sigma X^2 + c\Sigma X^3$$

$$\Sigma X^2 Y = a\Sigma X^2 + b\Sigma X^3 + c\Sigma X^4$$

Summation (Σ) runs over all pairs of observations.

Solving the three equations, we obtain the values of a, b and c, substituting the values of the constants a, b and c, we get the second degree parabola.

The shape of the second degree curve depends on the values of b and c.

If $b > 0$, $c > 0$, the curve is concave from left bottom to right top.

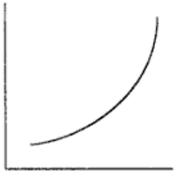

Fig. 16.1

If $b > 0$, $c < 0$, the curve is convex from left bottom to right top.

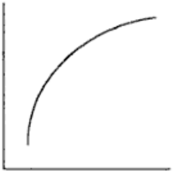

Fig. 16.2

If $b < 0$, $c < 0$, the curve is convex from left top to right bottom.

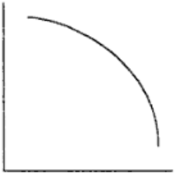

Fig. 16.3

If $b < 0$, $c > 0$, the curve is concave from left top to right bottom.

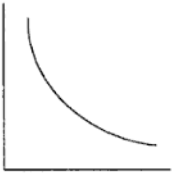

Fig. 16.4

Q. 28 Give the equation of an exponential curve and method for its fitting.

Ans. Exponential curve is also called *geometric trend*. The equation of the curve is,

$$Y = ab^X$$

If we plot the time series data as such, it will be a curve with upward trend provided $b > 1$ and with downward trend for $b < 1$. Since X occurs in power of b, it is called exponential curve.

It is difficult to fit in the equation as such but by taking logarithm of the equation, it becomes trivial to fit it. The log transformed equation is,

$$\log Y = \log a + X \log b,$$

which is a linear equation in log Y and X. The values of log a and log b can be computed by the formulae,

$$\log a = \frac{1}{n}\Sigma \log Y - \overline{X} \log b$$

$$\log b = \frac{\Sigma X \log Y - (\Sigma X)(\Sigma \log Y)/n}{\Sigma X^2 - (\Sigma X)^2/n}$$

where summation (Σ) runs over all pairs of values. Taking antilog of log a and log b, we obtain the values of a and b. On substituting the values of a and b in the equation of the exponential curve, the trend equation of the curve is obtained.

Q. 29 Write a short note on compertz curve with its relevance in time series analysis.

Ans. The curve is useful in actuarial studies and sometimes in business forecasting and population projections. The equation of the compertz curve is,

$$Y = ab^{c^X}$$

Taking logarithm of the equation, it is transformed to,

$$\log Y = \log a + c^X \log b$$

The equation can be fitted by Cowden's method with a little modification.

Q. 30 How can a best model for a time series be selected?

Ans. For a time series, many linear or curvilinear models may appear to be suitable. Now the question arises which one should be taken as best fitted model. The easiest way to adjudge is to compute the quantity $Q = \sum_i (y_i - \hat{y}_i)^2$ for each model, where y_i the actual value in the time series and y_i is its estimated value.

As a matter of fact, the quantity Q is residual sum of squares. The model yielding the minimum value of Q is the best and should be selected.

Q. 31 What methods are generally used for measuring the seasonal variation?

Ans. To isolate seasonal variation, one must eliminate trend, cyclic and irregular variations from the time series data and he is left with seasonal variation, *i.e.*,, $S = Y/T \times C \times I$ or $S = Y - T - C - I$.

Various methods of measuring seasonal variations are based on this very idea. Different methods are:

(i) Simple average method

(ii) Ratio to trend method

(iii) Ratio to moving average method

(iv) Link relative method.

Q. 32 Discuss the simple average method for measuring seasonal variation.

Ans. The simple average method of measuring seasonal variation is suitable when trend and cycles are not present, if any, have negligible effect on the time series.

We all know that a season is a part of the year, may the seasons be half-yearly, quarterly, monthly, weekly, etc. To calculate seasonal indices, first find out the average of each period (season) and then the average of the averages (grand average). The seasonal index for any period (season) in case of multiplicative model is calculated by the formula,

$$\frac{\text{Average of a season (period)}}{\text{Grand average}}$$

Calculate seasonal index for all the periods (seasons). If $\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_k$ are the seasonal averages based on the periodic data for all years and $\overline{X}$ is the grand average, then the seasonal indices for multiplicative model are, $\frac{\overline{X}_1}{\overline{X}} \times 100, \frac{\overline{X}_2}{\overline{X}} \times 100, \ldots, \frac{\overline{X}_k}{\overline{X}}$ × 100.

Also for additive model they are, $(\overline{X}_1 - \overline{X}), (\overline{X}_2 - \overline{X}), \ldots, (\overline{X}_k - \overline{X})$.

As a check to the calculations, one should ensure that the sum of the seasonal indices in multiplicative model is 100 × k. If k is the number for quarterly data, it is 400. Also for additive model, the sum of the seasonal indices in zero. If it is suspected that the trend is present, the seasonal indices are to be adjusted for trend through regression approach.

Q. 33 Give the merits and demerits of the simple average method.

Ans.

Merits:

(i) It is the easiest method to compute seasonal indices.

(ii) It gives good results if trend and cyclic variation have negligible or no effect on the time series. Which is rarely true. Hence this method is seldom used.

Demerits:

(i) It is not appropriate as it does not remove the cyclic and irregular variations.

(ii) Simple average method becomes quite cumbrous if the seasonal indices so obtained are to be adjusted for trend.

Q. 34 Delineate the procedure of computing seasonal indices by ratio to trend method.

Ans. This method provides seasonal indices free from trend as it assumes that seasonal variation for a given period is a constant fraction of trend. Different steps in ratio to trend method are:

Step-1. Estimate the trend values for each period (quarter or month) by establishing a trend line (or parabola) by the method of least squares.

Step-2. Divide each originally given seasonal (quarterly or monthly) value by the corresponding trend value and multiply it by 100 to convert it into percentage. The indices so obtained are free from trend.

Step-3. Now to obtain the seasonal indices free from cyclic or irregular variations, we proceed in this manner. Find the median (mean) of ratio to trend values for each season for any number of years. The median is preferred over mean if there are some extreme values. In this way irregular variations are removed. These median (mean) values represent seasonal indices. Mean removes the random effects. Mean removes the random effects better than median. Hence, one should choose median or mean after a careful examination of the data.

Step-4. If the seasons are quarters, the sum of seasonal indices in case of multiplicative model should be 400, and if months, it should be 1,200. But often the sum is not exactly what it should be. Hence the seasonal indices are adjusted by multiplying each of them by (400/sum of seasonal indices) or (1200/sum of seasonal indices) as the case may be.

Q. 35 Give plus points and limitations of ratio to trend method.

Ans.

Plus points:

(i) The method is based on sound and logical footings.

(ii) It utilises the complete information.

(iii) If periods are of short duration, it gives very good results.

(iv) It is easy to compute and understand.

Limitations:

(i) If there are pronounced cyclical swings in the time series data, a linear or curvilinear trend does not give good trend values. In such a situation ratio to moving average method performs better than ratio to trend method.

Q. 36. How can the ratio to moving average method be applied for computing the seasonal indices?

Ans. The ratio to moving average method is also known as *percentage to moving average method*. This method is the most popular and best on the ground that on taking the 12-month moving average in a monthly data or 4-quarter moving average in a

C.R. for I period = 100

C.R. for i^{th} period

$$= \frac{\text{L.R for } i^{th} \text{ period} \times \text{C.R. for } (i-1)^{th} \text{ period}}{100}$$

where i varies over all periods (month or quarters).

Step-4. The chain relative for the first season (*I* month or *I* quarter) is calculated on the basis of the last season by the formula,

C.R for *I* period =

$$\frac{\text{Median for } I \text{ period} \times \text{C.R. for the last period}}{100}$$

The chain relative for *I* period from the above formula will have a different value as it would have been by the previous formula. Hence it needs some adjustment. The adjustment factor c is worked out by the formula,

$$c = \frac{100 - \text{C.R for I period}}{\text{number of periods}}$$

For monthly data, no. of periods = 12 and for quarterly data, no. of periods = 4. The corrections for chain relatives for *I, II, III*, ..., periods are $0 \times c$, $1 \times c$, $2 \times c$, ..., etc. The quantities are added to *I, II, III*, ..., periods chain relatives, respectively. In this way, we obtain the adjusted chain relatives. Find the average of the adjusted chain relatives by the formula,

$$\text{Av. of adjusted C.R} = \frac{\text{Sum of adj. C.R.}}{\text{no. of periods}}$$

Seasonal index by the link-relative method is,

$$\text{Seasonal index} = \frac{\text{Adjusted C.R}}{\text{Average of Adj. C.R's}} \times 100$$

Please check that the sum of the seasonal indices is equal to 100 × no. of seasons.

Q. 40 What are the methods of measuring cyclic variations?

Ans. Six methods of measuring cyclic variations are as follows:

(i) Residual method

(ii) First difference method

(iii) Percentage ratio method

(iv) Direct method

(v) Reference cycle analysis method

(vi) Harmonic analysis method.

Q. 41 How can one isolate cyclic variations by residual method?

Ans. The name of the method is itself indicative of the procedure. First we remove seasonal variation from the data and then trend. In this way the values obtained contain only cyclic and irregular variations. As a matter of fact, the two variations are almost inseparable. In this way, the cyclic and irregular variations are obtained as the residuals of the observed values in terms of percentages. So the name follows. Symbolically,

For an additive model,

$$Y = T + S + C + I$$
$$Y - T - S = C + I$$

and for a multiplicative model,

$$\frac{Y}{S \times T} = C \times I$$

This method yields good results only if the trend and seasonal are perfectly measurable. The irregular fluctuations can be identified by an examination of the periods and corresponding observed values. If irregular fluctuations are present, they may be removed logically or applying the moving average method on $C \times I$ values.

Q. 42 How can the first difference method be applied to identify and measure cyclic variations?

Ans. The first difference method is applicable only when the yearly data are given. We know that yearly data are devoid of seasonality. Hence one needs to remove secular trend from the given time series data to obtain cycles.

In this method the differences of a year from its preceding year are calculated with signs. These differences are available for second year and onwards. The differences are plotted on the graph paper taking years on abscissa and differences along ordinate.

Joining the points we get the cycles present in the time series. The graph provides a clear picture of cycles through troughs and crests.

Q. 43 Discuss the percentage ratio method for measurement of cyclic variations.

Ans. This method is applicable for yearly data only. In this method, divide each year's observed value by its preceding year value and multiply it by hundred to get the values in percentages. Plot these values on the graph paper against their corresponding years. The convex and concave portions of the graph give clear picture of the cycles.

This method is equivalent to the first difference method in the sense that here we find relative changes in values, whereas in the first difference method we work out actual changes. Both the methods leads to almost the same results. In both the methods, one misses the value for the first year of the time series.

Q. 42 How do you indentify cycles in a time series by direct method?

Ans. This method is founded on calculating the per cent variation each month or quarter with respect to preceding year's same month or quarter. These percentages show upward changes in case of rising cycles and downward changes in case of declining cycles. Some businessmen adopt this techniques because of its simplicity.

Q. 45 Discuss in brief the reference cycle analysis method for the measurement of cyclic variations.

Ans. The reference cycle analysis method for identifying cycles had been developed by National Bureau of Economic Research, USA and used for more than 1000 time series analysis. This method is appropriate for analysing past series only.

Under this method, the index of variation in each series is computed with regard to a given reference date, whereas the reference dates are the dates of the peaks and troughs of business cycles. Obviously, if the reference date is a peak year, the economic series will show a downward falling index till the cycle takes a turn and vice-versa.

Q. 46 Give advantages and disadvantages of the reference cycle analysis method.

Ans.

Advantages:

(i) The main advantage of this method is that it facilitates a comparative assessment of the changes in various economic time series.

(ii) It is also possible through this analysis to find out which series has a lead or lag in the process of upward or downward swing, as the case may be. This method is frequently used in business forecasting.

(iii) Though the method appears to be cumbersome, it has proved to be simple and effective for comparing the cyclic variations of the individual series with those of general business.

(iv) This method is devoid of the errors which could have been introduced, in case the trend is not properly estimated.

Disadvantages: The foremost shortcoming of this method is that it cannot be applied to the current time series because no cycles can be studied in this way until it is completed.

Note: The above discussion gives only the approach of the method. The details are kept out.

Q. 47 Express succinctly the harmonic analysis method of determining the cyclic component of a time series.

Ans. The harmonic analysis method is a sophisticated mathematical device of determining cyclic component of a time series. This method is based on expressing any function Y_t in the form of a Fourier series, a series of sums of sines and cosines of the angles $2\pi/\lambda$ where λ is the period of oscillations. λ can better be determined by periodogram analysis.

Note: The mathematical details of the method are omitted.

Q. 48 How can the irregular influences be measured?

Ans. The name, irregular influences implies that there is nothing definite about their occurrence and amplitude. Hence, no mathematical methods have

92. Given the exponential curve $Y = 31.5\ (1.5)^X$, the equation of the curve by shifting the origin backward by two years is ________.

93. Given the trend equation $\hat{Y} = 122.5 + 7.2\,X$ with 1985 as origin and yearly data, the trend equation after shifting the origin to 1980 is ________ and to 1988 is ________.

94. Given the monthly trend equation $\hat{Y} = 9.4 + 2.4X$ with January as origin, the annual trend equation is ________.

95. If the actual sales of an item for the month of October 1990 is Rs. 785 and the seasonal index for October is 128.4, the estimated sales for October 1990 is ________ and based on this information, the estimate of the annual sales is ________.

96. Given the parabolic trend equation as $Y = 25 + 10X + 3X^2$ based on year to year data and 1980 as origin, the equation of the parabola with 1984 as origin will be ________.

97. Given the trend equation $\hat{Y} = 96 + 4X$ with origin 1987 and X unit = 1 year, the trend equation after shifting the origin to 1985 is ________ and to 1st Jan. 1990 is ________.

98. If the annual trend equation with 1984 as origin is $\hat{Y} = 112.8 + 6.48X$, the monthly trend equation is ________.

99. Given the equation $\hat{Y} = 54 + 3.6X$ with 1981 as origin and $X = \frac{1}{2}$ year and Y units in terms of annual production, the monthly trend equation is ________.

100. All seasonal variations are periodic but all periodic variations are not ________.

101. The monthwise seasonal indices of different years are called ________.

102. Cyclical fluctuations helps the business executives in framing ________ and establishing ________.

103. Appropriateness of various models can be decided by comparing ________.

104. The names in which the periodic movements can be classified are ________ and ________ fluctuations.

105. The averages of specific seasonals for a number of years are known as ________.

SECTION-C

Multiple Choice Questions

Select the correct alternative out of given ones:

Q. 1 A time series is a set of data recorded:
(a) periodically
(b) at time or space intervals
(c) at successive points of time
(d) all the above

Q. 2 The time series analysis helps:
(a) to compare the two or more series
(b) to know the behaviour of business
(c) to make predictions
(d) all the above

Q. 3 A time series is unable to adjust the influences like:
(a) customs and policy changes
(b) seasonal changes
(c) long-term influences
(d) none of the above

Q. 4 A time series consists of:
(a) two components
(b) three components
(c) four components
(d) five components

Q. 5 The forecasts on the basis of a time series are:
(a) cent per cent true
(b) true to a great extent
(c) never true

(d) none of the above

Q. 6 The component of a time series attached to long-term variations is terms as:
(a) cyclic variation
(b) secular trend
(c) irregular variation
(d) all the above

Q. 7 The component of a time series which is attached to short-term fluctuations is:
(a) seasonal variation
(b) cyclic variation
(c) irregular variation
(d) all the above

Q. 8 A lock-out in a factory for a month is associated with the component of a time series:
(a) irregular movement
(b) secular trend
(c) cyclic variation
(d) none of the above

Q. 9 The general decline in sales of cotton clothes is attached to the component of the time series:
(a) secular trend
(b) cyclical variation
(c) seasonal variation
(d) all the above

Q. 10 The sales of a departmental store on Dushera and Diwali are associated with the component of a time series:
(a) secular trend
(b) seasonal variation
(c) irregular variation
(d) all the above

Q. 11 The consistent increase in production of cereals constitutes the component of a time series:
(a) secular trend
(b) seasonal variation
(c) cyclical variation
(d) all the above

Q. 12 Secular trend is indicative of long-term variation towards:
(a) increase only
(b) decrease only
(c) either increase or decrease
(d) none of the above

Q. 13 Linear trend of a time series indicates towards:
(a) constant rate of change
(b) constant rate of growth
(c) change in geometric progression
(d) all the above

Q. 14 Method of least squares to fit in the trend is applicable only if the trend is:
(a) linear
(b) parabolic
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 15 Seasonal variation means the variations occurring within:
(a) a number of years
(b) parts of a year
(c) parts of a month
(d) none of the above

Q. 16 Salient factors responsible for seasonal variation are:
(a) weather
(b) social customs
(c) Festivals
(d) all the above

Q. 17 Cyclic variations in a time series are caused by:
(a) lockouts in a factory
(b) war in a country
(c) floods in the states
(d) none of the above

Q. 18 Irregular variations in a time series are caused by:
(a) lockouts and strikes
(b) epidemics
(c) floods
(d) all the above

Q. 19 Trend in a time series means:
(a) long-term regular movement
(b) short-term regular movement
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 20 The terms prosperity, recession, depression

(a) multiplicative model only
(b) additive model only
(c) multiplicative as well as additive model
(d) none of the above

Q. 49 In ratio to trend method for seasonal indices, the indices become free from trend component of the time series by:
(a) subtracting the trend value for each corresponding value
(b) taking the ratio of each seasonal value to the corresponding trend value
(c) taking the ratio of each trend value to the corresponding seasonal value
(d) none of the above

Q. 50. In ratio to trend method the median of the trend free indices for each period represents:
(a) the seasonal indices
(b) cyclic variation
(c) irregular variation
(d) none of the above

Q. 51 Ratio to trend method for seasonal indices provides good results if:
(a) the periods are of long duration
(b) the periods are given six monthly
(c) the periods are of short duration
(d) all the above situations

Q. 52 The best method for finding out seasonal variation is:
(a) simple average method
(b) ratio to moving average method
(c) ratio to trend method
(d) none of the above

Q. 53 In ratio to moving average method for seasonal indices, the ratio of an observed value to the moving average remove the influence of:
(a) trend
(b) cyclic variation
(c) trend and cyclic variation both
(d) none of these

Q. 54 The moving averages in a time series are free from the influences of:
(a) seasonal and cyclic variations
(b) seasonal and irregular variations
(c) trend and cyclical variations
(d) trend and random variations

Q. 55 Link relatives in a time series remove the influence of:
(a) the trend
(b) cyclic variation
(c) irregular variations
(d) all the above

Q. 56 Cyclic variations are interwoven with:
(a) trend
(b) seasonal variations
(c) irregular variations
(d) all the above

Q. 57 Residual method for measuring cycles in a time series consists of:
(a) removing the trend from the series
(b) removing the seasonal variation from the series
(c) removing the influences of trend, seasonal and irregular variations
(d) none of the above

Q. 58 First difference method for isolating cycles is applicable if observations pertain to:
(a) yearly data
(b) quarterly data
(c) monthly data
(d) any data

Q. 59 Graphically cycles of a time series are identifiable through:
(a) troughs and crests
(b) concave and convex portions
(c) cups and crests
(d) all the above

Q. 60 In percentage ratio method of measuring cyclic variations one finds:
(a) actual changes
(b) relative changes
(c) per cent ratio changes
(d) all the above

Q. 61 Reference cycle analysis method of measuring cyclic variations was developed by:
(a) Delhi school of economics
(b) National Bureau of economic Research, U.S.A

(b) trend is curvilinear only
(c) the value Y is not a function of time t
(d) none of the above

Q. 77 To which component of the time series, the term recession is attached?
(a) trend
(b) seasonals
(c) cycles
(d) random variation

Q. 78 If the slope of the trend line is positive, it shows:
(a) rising trend
(b) declining trend
(c) stagnation
(d) any of the above

Q. 79 The equation of the parabolic trend is,

$$Y = 46.6 + 2.4X - 1.3X^2$$

If the origin is shifted backward by three years the equation of the parabolic trend will be:
(a) $Y = 27.7 - 5.4X - 1.3X^2$
(b) $Y = 51.1 - 5.4X - 1.3X^2$
(c) $Y = 27.7 + 10.2X - 1.3X^2$
(d) none of the above

Q. 80 If the equation of exponential trend with 1989 as origin is

$$Y = 15\ (1.8)^X,$$

the equation of the exponential trend with 1991 as origin will be:
(a) $Y = 15\ (1.8)^{X^2}$
(b) $Y = 48.6\ (1.8)^X$
(c) $Y = 4.62\ (1.8)^X$
(d) $Y = 15/(1.8)^X$

Q. 81 Out of a number of models fitted to a time series data, the best model can be adjudged by:
(a) the estimates of the parameters
(b) the value of residual sum of squares
(c) the shape of the curves
(d) all the curves

Q. 82 The seasonal indices for each month or quarter of different years are called:
(a) chain relatives
(b) link relatives
(c) typical seasonals
(d) specific seasonals

Q. 83 Time series analysis helps to:
(a) understand the behaviour of a variable in the past
(b) predict the future behaviour of a variable
(c) plan future operations
(d) all the above

Q. 84 The averages of the specific seasonals for months or quarters for a number of years of a time series are known as:
(a) erratic fluctuations
(b) mean seasonals
(c) typical seasonals
(d) all the above

Q. 85 In spite of merits of least square method for trend, the limitation is that:
(a) predictions based on trend ignore other components of time series
(b) this method is not applicable for a number of growth curves which follow business trends
(c) both (a) and (b)
(d) neither (a) nor (b)

ANSWERS

SECTION-B

(1) Chronological (2) Patterson (3) Werner Z. Hirsch (4) real behaviour (5) predict (6) time series (7) not absolutely (8) influences (9) four (10) secular trend (11) editing (12) calendar (13) prices (14) population (15) seasonal (16) seasonal (17) cycles (18) 30.417/No. of days in the month (19) 30.5/No. of days in the month (20) prosperity; recession (21) four (22) cycles (23) irregular (24) three (25) $Y = T \times S \times C \times I$ (26) $Y = T + S + C + I$ (27) multiplicative (28) hardly (29) additive (30) free-hand (31) semi-average (32) seasonal; cyclic (33) short-term fluctuations (34) straight line (35) information is lost (36) projections (37) cycles (38) mathematical (39) regression (40) most exact (41) interpolated (42) linear; curvilinear (43) n^{th} degree

polynomial (44) geometric trend (45) $Y = ab^X$ (46) $Y = ab^{c^X}$ (47) trend; cycles (48) 100 × No. of seasons (49) zero (50) cyclic; irregular (51) periodic average; grand average (52) mean; median (53) inferior (54) short duration (55) best (56) seasonal; irregular (57) trend; cyclic (58) seasonal × irregular (59) median; mean (60) flexibility (61) Karl Pearson (62) preceding (63) chain relatives (64) irregular (65) trend; seasonals (66) seasonality (67) preceding (68) relative changes (69) same month (70) National Bureau of Economics Research, U.S.A (71) current time series (72) forecasting (73) Fourier series (74) T, S and C (75) linear function (76) yearly data (77) seasonal component (78) yearly (79) secular trend (80) business cycles (81) S, C and I (82) parabola (83) slope (84) origin (85) line of best fit (86) seasonal indices (87) irregular (88) trend (89) cycles (90) $\hat{Y} = 9 + 0.24X$ (91) 122.40 (92) $Y = 14\ (1.5)^X$ (93) $\hat{Y} = 86.5 + 7.2X$; $\hat{Y} = 144.1 + 7.2X$ (94) $\hat{Y} = 271.2 + 345.6X$ (95) Rs. 611.37; 7336.44 (96) $\hat{Y} = 113 + 34X + 3X^2$ (97) $\hat{Y} = 88 + 4X$; $\hat{Y} = 106 + 4X$ (98) $\hat{Y} = 9.4 + 0.045X$ (99) $\hat{Y} = 4.5 + 0.5X$ (100) seasonal variations (101) specific seasonals (102) policies; level of business activity (103) residual sum of squares (104) seasonal; cyclical (105) typical seasonals

SECTION-C

(1) d	(2) d	(3) a	(4) c	(5) b	(6) b
(7) d	(8) a	(9) a	(10) b	(11) a	(12) c
(13) a	(14) c	(15) b	(16) d	(17) d	(18) d
(19) a	(20) c	(21) d	(22) d	(23) d	(24) c
(25) d	(26) b	(27) b	(28) c	(29) a	(30) c
(31) b	(32) b	(33) d	(34) a	(35) d	(36) b
(37) a	(38) b	(39) c	(40) a	(41) b	(42) b
(43) c	(44) a	(45) c	(46) a	(47) c	(48) c
(49) b	(50) a	(51) c	(52) b	(53) c	(54) b
(55) a	(56) c	(57) c	(58) a	(59) d	(60) c
(61) b	(62) d	(63) b	(64) c	(65) d	(66) a
(67) c	(68) d	(69) c	(70) a	(71) d	(72) b
(73) c	(74) c	(75) a	(76) d	(77) c	(78) b
(79) a	(80) c	(81) b	(82) d	(83) d	(84) c
(85) c					

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Chapter **17**

Index Numbers

SECTION-A

Short Essay Type Questions

Q. 1 What is the rationale behind index numbers?

Ans. Economic phenomenon is dynamic in nature. Hence, one wants to know the state of economic activity at a particular time, space or situation as compared to some other time period, place or situation, particularly in respect of the value of products and services. In this endeavour, one finds a special type of average which provides a measure of relative changes from time to time or place to place. Index numbers are also known as economic barometers because they reveal the state of inflation or deflation.

Q. 2 Define index numbers.

Ans. An index number is a measure of relative change in the value added by a variable or a group of related variables over time or space. Many economists and statisticians have defined index numbers in their own way. Some of them are quoted below:

Irving Fisher: The purpose of index number is that it shall fairly represent, so far as one single figure can, the general trend of the many diverging ratios from which it is calculated.

John I. Griffin: An index number is a quantity which by reference to a base period, shows by its variations, the changes in the magnitude over a period of time. In general, index numbers are used to measure changes over time in magnitudes which are not capable of direct measurement.

Wessel, Willet and Simone: An index number is a special type of an average that provides a measurement of relative change from time to time or place to place.

Clark and Schkade: An index number is a percentage relative that compares economic measure, in a given period with those some measures at a fixed time period in the past.

A.M. Tuttle: An index number is a single ratio (usually in percentages) which measures the combined (*i.e.*, averaged) change of several variables between two different times, places or situations.

L.R. Conor: In its simplest form it represents a special case of an average, generally a weighted average, compiled from a sample of items judged to be representative of the whole.

A.L. Bowley: Index numbers are used to measure the changes in some quantity which we cannot observe directly.

M.M. Blair: Index numbers are the signs and guide posts along the business highway that indicate to the businessman how he should drive or manage his affairs.

Q. 3 What are the characteristics of an index number?

Ans. Following characteristics of index numbers are observed almost in all cases:

(i) Index numbers are expressed in percentages which make it feasible to compare any two or more index numbers.

(ii) They are of comparable nature at any two timings or places or any other situation.

(iii) Index numbers are a sort of averages, usually weighted averages, which always pertain to two periods, one is known as the base period and the other, the current period. The two are always comparable with each other.

(iv) Index numbers measure changes in some quantities which cannot be observed directly.

Q. 4 What are the uses of index numbers?

Ans. Various uses of index numbers are as follows:

(i) ***An aid to framing of policies:*** Fixing of wages and dearness allowance is mainly based on consumer price index. Many other economic policies are guided by index numbers like volume of trade, fixing of wholesale and retail prices, etc.

(ii) ***To find trend:*** Index numbers measure the changes from time to time which enable us to study the general trend of the economic activity under consideration. As a measure of average change in an specified group, the index number may be used for forecasting.

(iii) ***To assess the purchasing power of money:*** The consumer price index helps in computing the real wages of a person. If a person was getting Rs. 300 per month in 1960 and Rs. 3000 per month in 1990, his real wages have increased or decreased - can be assessed with the help of index number.

(iv) ***For adjusting national income:*** Index numbers are used for deflating the net national product (NNP) or net national income (NNI) converted at current prices. The deflated net national product or income represent the NNP or NNI at constant prices under inflationary conditions.

(v) ***A measure of comparative changes:*** The main purpose of index number is to measure the relative temporal or cross-sectional changes at a point of time over some previous time. All those phenomena which cannot be measured directly like consumer price index, price index, price level, etc., are measured with the help of index numbers.

Q. 5 Delineate the limitations and/or lecunae of index numbers.

Ans. Index numbers are extensively used in economic analysis. They are not devoid of lecunae and limitations. The some are discussed below:

(i) ***The errors of sampling:*** Since index numbers are based on sample data, all those errors which are involved in sampling procedure creep in the construction of index numbers. So the index numbers may not present the true picture.

(ii) ***Subjectivity in the construction of index numbers:*** There is no definite rule which makes one to decide about the base year, number of commodities, weights, sampling procedure, etc. All this depends on the purpose of index numbers and the person making decisions. So, an element of subjectivity is involved in the construction of index numbers.

(iii) ***Adjustment for changes:*** With rapid scientific advancement, change in outlook, taste and quality of material, it is difficult to make exact adjustments in the construction of indices. Hence, index numbers may not be the true measure of change.

(iv) ***Formula error:*** All formulae known so far for index numbers suffer from one or the other deficiency. For example, Laspeyre's formula generate an upward bias in index number, whereas Paasche's formula a downward bias. So the choice of a formula is a problem as it may introduce bias in index number.

(v) ***Choice of the type of average:*** Index numbers are a special kind of averages. Since various averages possess different virtues and limitations, it is not possible to say that a particular average is absolutely good for index number. Of course, arithmetic and geometric means are frequently used.

(vi) ***Errors in collection of data:*** After all index numbers are calculated from the data collected through surveys. If the data are not collected accurately regarding prices, consumption, production, etc., the index number will definitely be misleading.

Q. 6 What type of index numbers are usually calculated?

Ans. Index numbers are constructed in economic activity covering a wide range of aspects. Different kinds of usually constructed index numbers are:

(i) ***Price index numbers:*** These are the mostly used index numbers which measure the general change in the retail or wholesale prices of a commodity or a group of commodities at current period as compared to some previous period known as base year.

(ii) ***Quantity index numbers:*** This is another important measure of index number which measures the changes occurring in the quantity of goods demanded, consumed, produced, imported or exported, etc.

(iii) ***Consumer price index:*** In common parlance it is also known as *cost of living index*, though the two are not exactly the same. Consumer price index is a special kind which is constructed for the prices of only the essential items. Such a list of items is known as *basket*.

(iv) ***Value index:*** This compares the total value of certain item(s) at a point of time as compared to a base period. We know that the total value is the product of the price and quantity. This type of index is used in sales of a company, foreign trade, etc.

(v) ***Diffusion index:*** It reveals the changes in a group of time series indicating the turning point of an economic cycle.

Q. 7 Discuss various problems involved in the construction of index numbers.

Ans. No index number is an all-purpose index number. Hence, there are many problems involved in the construction of index numbers, which are to be tackled by an economist or statistician. They are briefly discussed below:

(i) ***Purpose of the index number:*** The first and foremost objective is to clearly delineate the purpose of index number for which it is going to be constructed. All other factors involved in the construction of index numbers mostly depend on the purpose. If the purpose is not stated clearly and unambiguously, the index number will do no good as an economic barometer.

(ii) ***Selection of base period:*** All index numbers are constructed in reference to a period against which the comparisons are to be made. Such a reference period is known as *base period*. The index for base period is always taken to be 100. As a matter of fact, the base primarily depends upon the purpose of the index number. Still there are certain requirements for an ideal base period.

(a) It should be a normal period in the sense that it should be free from epidemics, earthquakes, war, etc. A perfect normal period is not easy to obtain, hence 2-3 years average is sometimes taken as a base year.

(b) The base period should not be a distant past as the technology and circumstances changes with lapse of time.

(iii) ***Collection of data:*** Data are to be collected on the items which are to be included in the construction of index numbers. The choice of items totally depends on the purpose of index number. The information – usually the prices, consumption or demand – should be collected carefully from the units selected in the sample.

(iv) ***Selection of weights:*** All items do not carry the same importance with regard to their consumption or requirement. For instance, butter is less important than milk, fruits are less important than vegetables, etc. Hence, the items are to be weighted according to their relative importance. Sometimes people use unweighted indexes but rarely so.

(v) ***Choice of the average:*** An index number is a special type of average. Hence, which type of average should be used, need thoughtful consideration. Usually three types of averages are used given in order of their priority:

(1) Geometric mean (G.M.)
(2) Arithmetic mean (A.M.)
(3) Median

(1) *Geometric mean* is the most preferred one because:

(a) index numbers deal with ratios and proportions.

(b) index numbers give equal weights to equal ratio of changes,

(c) extreme values do not receive undue weights,

(d) geometric mean based indices are reversible.

(2) *Arithmetic mean* is unduly affected by extreme values. Still it is widely used because of casiness of computations.

(3) *Median ignores* completely the extreme values. But it is seldom used.

(vi) ***Selection of formula*:** The choice of a formula for index number depends on the purpose of index number and the data available. Anyhow, Irving Fisher has expounded a formula, which is considered as an ideal one and satisfies time reversal and factor reversal tests. Still no formula can be regarded as best because a formula may be very good in one situation and deficient in the other.

Q. 8 Discuss in general the basis of formulae evolved for index numbers.

Ans. Two types of formulae are in existence for index numbers namely, (i) unweighted formulae, (ii) weighted formulae.

Unweighted formulae have very little use because of their limitations. Hence, weighted formula basically requires three things:

(i) The data regarding the items (for which the index number is to constructed) for the base and current periods. Usually the data are with regard to prices and quantities.

(ii) The information necessitated for the weights.

(iii) The kind of average to be used.

Q. 9 Explain the notations commonly used in index number formulae.

Ans. Commonly we use the following notations:

p_{ji} – the price of the i^{th} commodity (item) in the j^{th} year.

q_{ji} – the quantity of the i^{th} commodity (item) in j^{th} year.

v_{ji} – the value of the i^{th} commodity (item) in the j^{th} year which is equal to $p_{ji} \times q_{ji}$.

In general $j = 0, 1, 2, ..., k$ and $i = 1, 2, ..., n$ (the number of commodities under consideration).

For a base year, $j = 0$ and current year, $j = 1$

P_{01} – The price index of the current period 1 as compared to based period 0.

Q_{01} – The quantity index for the current period 1 as compared to base period 0.

Q. 10 Give the unweighted price index number.

Ans. The unweighted price index number for n items for the current year 1 and base year 0 is given by the formula,

$$P_{01} = \frac{\sum\limits_i p_{1i}}{\sum\limits_i p_{0i}} \times 100$$

for $i = 1, 2, ..., n$

This formula has hardly got any utility because:

(i) different items generally have different units of measurements, *e.g.*, wheat Rs. per kg, cloth Rs./m, milk Rs. per litre and hence cannot be pooled.

(ii) the relative importance of different items is totally ignored.

Q. 11 Briefly discuss the unweighted quantity index.

Ans. An unweighted quantity index for n commodities can be computed by the formula,

$$Q_{01} = \frac{\sum\limits_i q_{1i}}{\sum\limits_i q_{0i}} \times 100$$

for $i = 1, 2, ..., n$.

This index suffers from the drawbacks that:

(i) different items included in the quantity index having different units of measurements like petrol in litres, cereals in quintal, electric bulbs in numbers, etc., and hence cannot be combined to get an aggregated quantity.

(ii) all items carry equal importance (weights) in this formula which is not admissible.

Q. 12 Explicate Laspeyre's price and quantity index numbers.

Ans. Laspeyre's index is also known as *base year method* index. In Laspeyre's price index, the base year quantities (consumption, demand, production

$$P_{01}^{ME} = \frac{\sum_i p_{1i}(q_{0i} + q_{1i})}{\sum_i p_{0i}(q_{0i} + q_{1i})} \times 100$$

for $i = 1, 2, ..., n$

Q. 18 Give geometrically crossed-weight formula.

Ans. In the Marshall-Edgeworth price index formula, one may even use the geometric mean of the quantities of the base and current years as weights. Hence the price index formula is,

$$P_{01}^{ME} = \frac{\sum_i p_{1i}\sqrt{q_{0i}\, q_{1i}}}{\sum_i p_{0i}\sqrt{q_{0i}\, q_{1i}}} \times 100$$

for $i = 1, 2, ..., n$

Q. 19 Discuss Kelly's fixed weight formula.

Ans. Truman L. Kelly preferred the fixed weights for price index as it is not necessary to use the base year and/or current year quantities as weight. These may be any logically ascertained quantities. But to be more sound, the average of any two or more year's quantities may be taken as weight. The choice of the type of average (A.M. or G.M.) lies with the investigator.

The greatest advantage of Kelly's fixed weight system is that the change in base year does not require to determine the new weights. The fixed weight index is quite popular in U.S.A. in the construction of whole-sale price index of the Bureau of Labour.

Note: Formulae for quantity index numbers can conveniently be obtained from price index numbers by replacing p by q and q by p.

Q. 20 What is meant by average of price relatives?

Ans. The average of the price indices calculated for each individual commodity at a given year '1' relative to a base year '0' is known as the average of the price relatives. Usually the average is either arithmetic mean or geometric mean. Price index based on arithmetic mean is,

$$P_{01} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{p_{1i}}{p_{0i}}\right) \times 100$$

Price index based on geometric mean is,

$$P_{01} = \left[\prod_{i=1}^{n}\left(\frac{p_{1i}}{p_{0i}}\right)\right]^{1/n} \times 100$$

F.Y. Edgeworth pleaded for the use of harmonic mean as well. But it is seldom used.

Q. 21 Give the formulae for weighted average of price relatives.

Ans. Commonly, the indices are obtained through weighted price relatives. If $w_1, w_2, ..., w_n$ are the weights for n items price relatives, the formula for price index based on arithmetic mean of weighted price relatives is,

$$P_{01} = \frac{1}{\sum_i w_i}\sum_i w_i\left(\frac{p_{1i}}{p_{0i}}\right) \times 100$$

for $i = 1, 2, ..., n$

If w_i is taken as the base year value, *i.e.*, $w_i = p_{0i}\, q_{0i}$, the above formula reduces to Laspeyre's formula and if current year values, *i.e.*, $w_i = p_{0i}.q_{1i}$, the above formula is changed to Paasche's formula.

Again the formula for price index based on geometric mean of weighted price relatives is,

$$P_{01} = \left[\prod_i\left(\frac{p_{1i}}{p_{0i}}\right)^{w_i}\right]^{1/\sum_i w_i} \times 100$$

for $i = 1, 2, ..., n$

Q. 22 How can one judge the adequacy of a formula.

Ans. All formulae stand at equal footing based on their own logic. Now the question arises how to choose one out of many. To have some definite idea about their exactness, Irving Fisher and some other statisticians and economists put forth some tests or criteria which conform to their reasonalability. A test satisfying all the tests can be considered as best. But no formula satisfies all the tests.

Q. 23 What is formula error?

Ans. The difference between the price (quantity) indices due to Laspeyre's and Paasche's is known as

We can also write,

$$\frac{P_{01} \times Q_{01}}{V_{01}} = 1$$

If any formula does not satisfy this relation, the error

$$E_2 = \frac{P_{01} \times Q_{01}}{V_{01}} - 1$$

is called the joint error.

It can easily be verified that only Fisher's ideal formula satisfies factor reversal test and none else. That is why Fisher's formula is an ideal one.

Q. 29 What is circular test and what does is signify?

Ans. This test is an extension of time reversal test and is based on the shiftability of the base period. In this test we find the price indices by taking the preceding year as base year, and the first year price index is worked out by taking the last year as base. If we have data for $(k + 1)$ years at hand and calculate price indices as $P_{01}, P_{12}, P_{23}, \ldots, P_{(k-1)k}$ and P_{k0}, then for the circular test to hold true, the following relation be satisfied.

$$P_{01} \times P_{12} \times P_{23} \times \ldots \times P_{(k-1)k} \times P_{k0} = 1$$

or $$P_{01} \times P_{12} \times P_{23} \times \ldots \times P_{(k-1)k} = P_{0k}$$

The circular test is satisfied only by the indices:

(i) based on geometric mean of price relatives,

(ii) obtained from Kelly's fixed weight method.

Even Fisher's ideal formula does not satisfy circular test.

Q. 30 What do you understand by test of proportionality?

Ans. If all sub-indices are equal to P, then total index should also be equal to P.

Q. 31 What is meant by test of definiteness?

Ans. If one of the sub-indices is zero or infinity, then the total index must not be zero or infinity or indeterminate.

Q. 32 What is being ensured through test of commensurability?

Ans. If the unit of measurement of commodity is changed, the value of the index must not change.

Q. 33 How can one find indices by the chain-base method?

Ans. If there are k consecutive year's data at hand and each time we consider the same n items to be included in the construction of indices, the chain-base method consists of calculating the price index for each year taking the preceding year as base. This brings the homogeneity error to almost zero level.

In this method, we can use any appropriate index number formula. All such year-to-year indices are called *link relatives*. The beauty of this method is that the index for any year to zero base year can be obtained by multiplying all the indices worked out so far. Notationally, a link relative for the j^{th} year is,

$$P_{(j-1)j} = \frac{\sum_i p_{ji}\, q_{(j-1)i}}{\sum_i p_{(j-1)i}\, q_{(j-1)i}}$$

for $i = 1, 2, \ldots, n$

$j = 1, 2, \ldots, k$

Putting $j = 1, 2, \ldots, k$, we get the formula for all link relatives.

Various indices of the series by chain base method can be obtained by the following relations:

$$P_{01} = P_{01} \qquad \text{(as a first link)}$$

$$P_{02} = P_{01}\, P_{12}$$

$$P_{03} = P_{01}\, P_{12}\, P_{23} = P_{02}\, P_{23}$$

$$\vdots$$

$$P_{0k} = P_{01}\, P_{12}\, P_{23} \ldots P_{(k-1)k} = P_{0(k-1)}\, P_{(k-1)k}$$

with the help of the above relations, a general formula can be given as,

$$\text{Chain index} = \frac{\text{Previous year chain index} \times \text{Current year link relative}}{100}$$

Q. 34 Give the relation of converting the chain index number to fixed-base index number.

Ans. The formula for converting the chain-base

Table Contd.

(i) *Year*	*(ii)* *Index No. base 1970-71*	*(iii)* *Index No. base 1981-82*	*(iv)* *Forward spliced Index No.*	*(v)* *Backward spliced Index No.*
1981-82	264	100	$264 \times \frac{100}{100} = 264$	100
1982-83		107	$264 \times \frac{107}{100} = 282$	107
1983-84		118	$264 \times \frac{118}{100} = 312$	118
1984-85		126	$264 \times \frac{126}{100} = 333$	126
1985-86		126	$264 \times \frac{126}{100} = 333$	126
1986-87		137	$264 \times \frac{137}{100} = 362$	137
1987-88		153	$264 \times \frac{153}{100} = 404$	153
1988-89		160	$264 \times \frac{160}{100} = 422$	160

Q. 40 What is meant by deflating the index numbers and how can it be done?

Ans. Deflating the index number means, making allowance in indices for the effect of changes in price levels. If the present price of a commodity is doubled as compared to a base year, the purchasing power for that commodity is reduced to half. In this way money value of our earning changes with the rise or fall in prices of commodities or consumer price index. Hence using deflation technique, the real wages, money income index number and real income index number can be calculated by the following formulae.

Real wage or real income

$$= \frac{\text{Income of the year (money wage)}}{\text{Price index of the current year}} \times 100$$

Money Income Index No.

$$= \frac{\text{Real Income}}{\text{Income of base year}} \times 100$$

Real Income Index No.

$$= \frac{\text{Money Income Index No.}}{\text{Consumer Price Index No.}} \times 100$$

Real income is also known as deflated income. This technique is widely used in deflating value series or value indices, rupee sales, inventories, income wages, etc. Also,

$$\text{Purchasing power of money} = \frac{1}{\text{price index}} \times 100$$

Q. 41 Per capita income of a person from 1980-81 to 1986-87 and the consumer prices index with 1980-81 as base were as follows:

Year	*Income per capita (Rs.)*	*Index Nos.*
1980-81	1627	100
1981-82	1851	103.5
1982-83	1993	103.4
1983-84	2290	109.4
1984-85	2494	110.9
1985-86	2735	113.8
1986-87	2970	115.6

Find real wages and real income indices.

index. Because it is a special purpose price index, it needs some specific considerations. The same are highlighted below:

(i) ***Selection of items:*** The items which are most commonly used by the section of the people, for whom CPI is required, are included in the basket. The basket of goods for middle class people consists of the following items:

(a) *Food items*: Wheat, rice, pulses, meat, milk, edible oils, condiments and spices, sugar, tea, etc..

(b) *Smoking and intoxicants*: Cigarettes, bidi, betels, liquors, tobacco, etc.

(c) *Fuel and light*: Firewood, coke, cole, kerosene, electricity, cooking gas, etc.

(d) *Services*: medical, barber charges, entertainment, transport, postal expenses, etc.

(e) *House rent*: House rent or rental value of the self occupied house.

(f) *Miscellaneous*: Soaps, hair oil, cream, powder, crockery, utensils, etc.

Presently the total number of items included in the basket is 110. While including the commodities, their quality or brand has to be specified.

(ii) ***Adjustment for quality changes:*** Due to technological advancement and new inventions, new products of better quality enter the market. Hence, adjustments should be made in the prices of such items. This makes the comparison of prices at two occasions more realistic.

(iii) ***Collection of price data:*** Price data should be collected from the retailers who are patronised by the majority of consumers. Also the prices be adjusted to the same units at two periods under reference. For instance, price of cloth per yard in base period be adjusted to the price of cloth per meter in current period, rate of rice per ser in the base year be adjusted to Rs. per kg, etc.

(iv) ***Fixing of weights for CPI:*** Assigning weights to prices is the usual practice in calculating the price indices. In consumer price index, the weights are the percentages of expenditure of each item of the goods and services of the basket to the total expenditure.

In the calculation of consumer price index, weights are assigned to each individual item of a group first and then to the groups like food items, clothing, house, fuel, etc. The whole structure of weights is known as *weighting diagram.*

(v) ***Selection of base period:*** Generally the base period for consumer price index is the year declared by the government. The weighting diagram is based on the budget enquiry conducted in the declared base year. In practice, the weighting diagram determined once remains fixed for 10 to 15 years, depending on the life of a series.

(vi) ***Imputation of expenditure:*** There are many items on which the expenditure is incurred in a family but those items do not belong to the basket. The expenditure due to such items are distributed amongst the items of the basket so that the total articles are taken into account. For instance, curd and cheese are included with milk, maize, jowar and bajra are added to cereals. This process of adjusting the weights is known as *imputation of weights.*

(vii) ***Selection of formula for consumer price index:*** The computation of CPI is based on Laspeyre's formula. There are two approaches which are discussed below.

(a) *Weighted aggregate expenditure method:* In this method, the prices of various items are weighted with the respective quantities consumed in the base year. Thus, with usual notations

$$\text{CPI} = \frac{\sum_i p_{1i}\, q_{0i}}{\sum_i p_{0i}\, q_{0i}}$$

for $i = 1, 2, \ldots, n$.

This formula faces the problem that quantitative weights cannot be assigned for the services utilised. Hence, this formula can be used when the basket includes only consumable articles.

(b) *Family budget method:* This method is also known as *method of weighted relatives.* The computation of CPI under family budget method runs into two steps. As a first step, the price index is calculated for each group separately. In the second step, the weighted average of the group indices is computed which is nothing but CPI.

In the first step, we calculate price relatives in percentages for individual items of a groups, *i.e.*, $p_{1i}/p_{0i} \times 100$. Each price relative is weighted by the weight w_i, which is usually in terms of percentage of expenditure. So we calculate the quantity, $\left(\frac{p_{1i}}{p_{0i}} \times 100\right) w_i$. Then find the weighted average for a group say, j^{th} group G^j, over all the items of the group for i running from 1 to n_j. So the CPI for the j^{th} group is,

$$G_{01}^{j} = \frac{\sum_{i}\left(\frac{p_{1i}}{p_{0i}} \times 100\right) w_{ij}}{\sum_{i} w_{ij}}$$

As a second step, the group index numbers $\left(G_{01}^{j}\right)$ are multiplied by the respective group weights. The sum of these products divided by the sum of group weights provides the consumer price index. Generally the CPI is obtained on monthly basis and is known as *general index* for the mid of that month. The formula for general CPI is,

$$P_{01} = \frac{\sum_{j} G_{01}^{j} w_j}{\sum_{j} w_j}$$

where j varies over all groups.

Q. 45 What does a consumer price index reflect?

Ans. A consumer price index does not reflect on a particular family budget but it tells about the price changes experienced by families on an average. To be more specific, if the effect of price changes experienced by all the families of the population are pooled and averaged, the average experience would be reflected by consumer price index.

Q. 46 Differentiate between explicit and implicit weights.

Ans. Explicit weights are those weights which can be expressed with definiteness, e.g., the quantities consumed or produced. In most of the studies explicit weights are used.

Sometimes items are assigned weights according to their importance. Such weights are known as implicit weights. Suppose, two varieties of wheat are marketed. One variety is four times in demand than the other. So it will receive four times the weight than the weight for other variety. But such a weighting system is seldom used.

Q. 47 In what respects the wholesale price index differs from the general price index?

Ans. The wholesale price index is basically not much different from the general price index except for some special considerations given below:

(i) The wholesale price index gives an indication of price movement in all markets except retail markets.

(ii) The wholesale price index quotations are taken only from wholesale dealers on each Friday.

(iii) The price quotations are collected from officials as well as non-official sources.

(iv) The wholesale price index does not include items pertaining to services like barber charges, repairing, etc.

(v) In the wholesale price index, the commodity weights are determined by the estimates of commodity value of domestic production and the value of imports inclusive of import duty during the base year. In general, price index, weighting system and collection of price data are not exactly so.

Q. 48 How is the wholesale price index worked out?

Ans. Once the data required for prices and weights are collected, the commodity indices are obtained as the average of the price relatives; sub-groups or groups indices are calculated as the weighted arithmetic mean of commodity indices. Finally, the general wholesale price index or *all commodities* index is obtained as the weighted average of the group indices.

Q. 49 What series of the wholesale price indices are issued so far?

Ans. The first series of the wholesale price index was issued by the Economic Advisor, Ministry of Commerce and Industry in January, 1947 with the year ending August, 1939 as base year.

The second wholesale price index series was issued in April 1956 with 1952-53 as base year. This series was based on 112 commodities classified into six groups namely, food; liquor and tobacco; fuel and power, light and lubricants; industrial raw material; intermediate products; finished products. For this series, 555 price quotations were collected from 183 official and non-official markets. In this series, the base year was 1952-53, but weights were based on 1948-49 information.

On the recommendation of Wholesale Price Index Revision Committee, a new series of wholesale price indices with base 1961-62 were issued. This series covered 139 commodities and price data were collected through 774 quotations. This series added two new groups namely, chemicals; machinery and transport equipments.

The next series was issued in January 1977 with 1970-71 as base year. This series was comprised of 360 commodities and 1295 price quotations.

A new series of wholesale price indices with 1981-82 as base year had been released in July, 1989 to replace old series with 1970-71 as base. This series comprised of 447 items and 2371 price quotations.

It is a continuous process so long as the needs, trends and prices continue to change.

Q. 50 What do you understand by the index of industrial production?

Ans. The index of industrial production (IIP) measures the changes in the level of industrial production in a given period as compared to a base period. Here it should be kept in mind that it measures the changes in the quantum of production and not in values. The data for IIP includes the production of private and public sectors.

The formula for computing IIP for the current year 1 as compared to a base year 0 is,

$$IIP_{01} = \frac{\left(\sum_i \frac{q_{1i}}{q_{0i}}\right) w_i}{\sum_i w_i} \times 100$$

where i varies over all items of industrial production.

w_i's represent the weights based on the relative importance of different outputs.

The index of industrial production throws light on the industrial development of a country and also the availability of industrial goods.

Q. 51 What series of index of industrial productions have been issued?

Ans. The revised series of index of industrial production were constructed by the Central Statistical Organisation, Government of India, New Delhi with base 1980-81. The industries were classified into four categories namely, basic industries; capital goods industries; intermediate goods industries and consumer goods industries. The weights were assigned to different industrial groups according to the reports published by the *Annual Survey of Industries.*

Q. 52 How index numbers are related to gross national product?

Ans. The gross national product (GNP) at factor cost is the value of the product at factor cost attributable to the factors of production supplied by the normal residents of the country prior to deduction of the consumption of the fixed capital. The gross national product is equal to the value of the gross domestic product (GDP) at factor cost plus the factor income from abroad.

The gross national product enables one to determine the degree by which the physical goods and services have grown (or deflated) over time. The increase in the value of goods and services due to price rise should be eliminated. If this is not done, there can be a deceleration in the production of quantity of goods and services but the value of GNP gets higher due to inflation. Hence to determine the real economic growth, one should use price index number to deflate to GNP value. This is achieved by dividing GNP for the current year by an adjustment factor, known as *deflator index number*. This is how price index number is attached with gross national product.

Q. 53 What is the difference between the net national product and the gross national product?

Ans. The Net national product is the value of the product at factor cost attributable to the factors of production by the normal residents of a country after deducting the consumption of the fixed capital. Whereas in the gross national product it is prior to the deduction of the consumption of the fixed capital.

Q. 54 Given the following information,

Groups	*Weights*	*Group indices*
Food	48	320
Fuel and lighting	12	180
Clothing	10	210
House rent	20	250
Miscellaneous	10	150

find the consumer price index.

Ans. The consumer price index.

$$P_{01} = \frac{\sum\limits_j G_{01}^j w_j}{\sum\limits_j w_j}$$

$$= \frac{320 \times 48 + 180 \times 12 + 210 \times 10 + 250 \times 20 + 150 \times 10}{48 + 12 + 10 + 20 + 10}$$

$$= \frac{26120}{100}$$

$$= 261.2$$

Q. 55 Given the sum of the products of prices and quantities for the current year 1 and base year 0 for five items as,

$$\Sigma p_0 q_0 = 782, \Sigma p_0 q_1 = 1008, \Sigma p_1 q_0 = 1084, \Sigma p_1 q_1 = 1329$$

On the basis of the given information show that the data satisfies time reversal test.

Ans. For Fisher's ideal formula,

$$P_{01} = \sqrt{\frac{1084}{782} \times \frac{1329}{1008}} \times 100$$

$$= 135.19$$

and $$P_{10} = \sqrt{\frac{1008}{1329} \times \frac{782}{1084}} \times 100$$

$$= 73.97$$

For the time reversal test,

$$P_{01} \times P_{10} = \frac{135.19 \times 73.97}{100 \times 100}$$

$$= 1.00$$

Hence the given data satisfies the time reversal test.

Q. 56 For the information given in Q. No. 55, show that the data satisfies factor reversal test.

Ans. Fisher's ideal index number,

$$P_{01} = \sqrt{\frac{1008}{782} \times \frac{1329}{1008}} \times 100$$

$$= 135.19$$

and Fisher's ideal quantity index number

$$Q_{01} = \sqrt{\frac{1008}{782} \times \frac{1329}{1084}} \times 100$$

Thus,

$$P_{01} Q_{01} = \sqrt{\frac{1084}{782} \times \frac{1329}{1008} \times \frac{1008}{782} \times \frac{1329}{1084}} \times 100$$

$$= \frac{1329}{782}$$

$$= \frac{\sum\limits_i p_{1i} q_{1i}}{\sum\limits_i p_{0i} q_{0i}}$$

$$= V_{01}$$

Hence, the given data satisfies factor reversal test.

Q. 57 In the section of middle class people in cities *A* and *B*, a survey revealed that the expenditure in city *A* for food and other items was in the ratio of 60:40 and that in the city *B* it was in the ratio of 50:50. If the consumer price index for city *A* in 1991 is 350 and that of city *B* 300, then find the price indices for the groups (i) food, and (ii) other items.

Ans. Let the CPI for food group is *x* and other items groups *y*.

Making use of the formula for CPI, for city *A*,

$$\frac{60x+40y}{60+40}=350$$

and for city *B*,

$$\frac{50x+50y}{50+50}=300$$

Solving the two equations we get, $x = 550$ and $y = 50$. It means that the price index for food is 550 and other items 50.

Q. 58 The prices and quantities consumed for three items *A*, *B* and *C* during the base year 0 and current year 1 were as follows:

Commodity	*Base year*		*Current year*	
	Price p_0	*Quantity* q_0	*Price* p_1	*Quantity* q_1
A	2	6	3	4
B	3	5	5	6
C	5	4	6	7

For the given data prove that,

$$\frac{L(p)}{L(q)}=\frac{P(p)}{P(q)}$$

Ans. From the given data we get,

$$\sum_i p_{0i}\,q_{0i}=47,\ \Sigma\,p_{0i}\,q_{1i}=51, \Sigma\,p_{1i}\,q_{0i}=67,$$

$$\Sigma\,p_{1i}\,q_{1i}=84$$

$$L(p)=\frac{\sum_i p_{1i}\,q_{0i}}{\sum_i p_{0i}\,q_{0i}}=\frac{67}{47}$$

$$L(q)=\frac{\sum_i q_{1i}\,p_{0i}}{\sum_i q_{0i}\,p_{0i}}=\frac{51}{47}$$

$$P(p)=\frac{\sum_i p_{1i}\,q_{1i}}{\sum_i p_{0i}\,q_{1i}}=\frac{84}{51}$$

$$P(q)=\frac{\sum_i q_{1i}\,p_{1i}}{\sum_i q_{0i}\,p_{1i}}=\frac{84}{67}$$

Now,

$$\frac{L(p)}{L(q)}=\frac{67/47}{51/47}=\frac{67}{51}$$

$$\frac{P(p)}{P(q)}=\frac{84/51}{84/67}=\frac{67}{51}$$

Therefore $$\frac{L(p)}{L(q)}=\frac{p(p)}{p(q)}$$

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s) in the blanks:

1. Index numbers are known as ________.
2. Index numbers measure changes over time in magnitudes which are not capable of ________.
3. An index number is a special type of ________.
4. Index numbers are expressed in ________.
5. Index numbers always involve ________ periods or places.
6. Index numbers help in framing of ________.
7. The trend of economic activity can be studied with the help of ________.
8. Index numbers can be used for ________.
9. Index numbers help to assess the ________.
10. Price indices are ________ used.
11. Consumer price index is also known as ________.

54. The condition for the circular test to hold good with usual notations is ________.

55. Circular test is satisfied only by the indices based on ________.

56. Indices base on Kelly's fixed weight method satisfy ________ test.

57. Circular test ________ satisfied by Fisher's ideal formula.

58. The relation between price indices of the type $P_{02} = P_{01}\, P_{12}$ comes from ________ method.

59. In the chain-base method we find indices in respect of ________ as base period.

60. The indices obtained sequentially with preceding year as base are known as ________.

61. In the chain-base method, addition and deletion of items is ________.

62. In the chain-base method, weights can ________ be adjusted on any stage.

63. Indices calculated by the chain-base method are free from ________.

64. The chain-base indices are not suitable for ________ comparisons.

65. Base shifting is required to make the two series of indices ________.

66. Combining of two series of indices with different base periods into one series with common base period is known as ________.

67. If an old series is connected with a new series of index numbers, it is known as ________.

68. If a new series of index numbers is connected with an old one, it is called ________.

69. Deflation of index numbers helps to determine ________.

70. Consumer price index indicates ________ in prices of items of a basket at a point of time as compared to a base period.

71. Cost of living index is same in general sense as ________.

72. Purchasing power of money can be assessed through ________.

73. Consumer price index is mostly used for framing ________.

74. Dearness allowance of a certain cadre of people is fixed on the basis of ________.

75. Cost of living at two different cities can be compared with the help of ________.

76. Consumer price index helps to evaluate ________ of money.

77. The list of items included in the computation of consumer price index is known as ________.

78. For consumer price index, the price data should be collected from ________.

79. Weights used in the computation of consumer price index are usually the ________ on each item and groups.

80. Inclusion of expenditure on items not included in the basket in the calculation of consumer price index is known as ________.

81. The most popular method of computing consumer price index as ________.

82. Consumer price index tells about ________ experienced by the population on an average.

83. Weights which can be expressed with definiteness are called ________.

84. The price movement in all the markets except retailers is indicated by ________.

85. The industrial development of a country is reflected by ________.

86. The gross national product value is deflated through ________.

87. The adjustment factor used to deflate the gross national product is known as ________.

88. The production of a country at factor cost by the residents of a country after deducting the consumption of fixed capital is known as ________.

89. Family budget method is also known as ________.

90. An upward bias is given by ________ formula.

91. Paasche's index number gives ________ bias.

92. Unit test ensures the independence of index numbers from ________.

93. The consumer price index for April, 1985 was 125. The food index was 120 and for other items 135. The percentage of total weight given to food is ________.

94. If the salary of a person in the base year is Rs. 2000 per annum and in the current year Rs. 5000. The CPI is 325, then the allowance required to maintain the same standard of living is Rs. ________.

95. If the percentage of expenditure on five groups of commodities is 30, 15, 25, 20 and 10 and group indices 190, 180, 140, 120 and 100, then the consumer price index is ________.

96. The total expenditure of a family is Rs. 2500 per month and he spends Rs. 1000 on food, Rs. 300 on clothing, Rs. 800 per month on rent and has no record of expenditure on light and miscellaneous groups. If the group indices are 190, 180, 140, 120 and 100 and CPI as 160, the family expenditure on light and miscellaneous items are Rs. ________ and Rs. ________ respectively.

97. Fisher's ideal formula does not satisfy ________ test.

98. Weighted geometric mean of relatives with fixed weights does not satisfy ________ test.

99. Weighted arithmetic mean of price relatives with fixed weights satisfies ________ of the time reversal, factor reversal and circular tests.

100. Simple arithmetic mean of price relatives does not satisfy ________ tests.

101. If all the sub-indices are equal to a value *P* and total index is also equal to *P*, then it is satisfying the test of ________.

102. Unit test is not satisfied by ________ index.

103. If on changing the unit of measurement of commodities, the index number remains same, then is said to fulfil the requirement of the test of ________.

104. Splicing is very useful for comparison between ________ and ________ index numbers.

105. Test of definiteness ensures that if a sub-index is zero or infinity, then the total index must not be ________ or ________.

SECTION-C

Multiple Choice Questions

Select the correct alternative out of given ones:

Q. 1 Index numbers reveal the state of:
(a) inflation
(b) deflation
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 2 Index numbers are also known as:
(a) economic barometers
(b) signs and guide posts
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 3 Index number is a:
(a) measure of relative changes
(b) a special type of an average
(c) a percentage relative
(d) all the above

Q. 4 Index numbers are expressed:
(a) in percentages
(b) in ratios
(c) in terms of absolute value
(d) all the above

Q. 5 Index numbers help:
(a) in framing of economic policies
(b) in assessing the purchasing power of money
(c) for adjusting national income

(d) all the above

Q. 6 The error(s) involved in the construction of index numbers is/are:
(a) error of sampling
(b) formula error
(c) error in collected data
(d) all the above

Q. 7 Element of subjectivity is involved in index numbers due to:
(a) choice of base year
(b) selection of weights
(c) choice of commodities
(d) all the above

Q. 8 Most commonly used index number is:
(a) Diffusion index number
(b) price index number
(c) value index number
(d) none of the above

Q. 9 One of the limitations in the construction of index numbers is:
(a) the choice of the type of average
(b) choice of investigators
(c) choice of variables to be studied
(d) all the above

Q. 10 Consumer price index number is constructed for:
(a) a well defined section of people
(b) all people
(c) factory workers only
(d) all the above

Q. 11 Diffusion index reveals the changes in:
(a) elite
(b) industrial production
(c) a group of time series
(d) none of the above

Q. 12 The first and fore most step in the construction of index numbers is:
(a) choice of base period
(b) choice of weights
(c) to delineate the purpose of index numbers
(d) all the above

Q. 13 Base period for an index number should be:
(a) a year only
(b) a normal period
(c) a period at distant past
(d) none of the above

Q. 14 Data for index numbers should be collected from:
(a) the retailers
(b) the wholesale dealers
(c) the selected group of persons
(d) none of the above

Q. 15 Most preferred type of average for index numbers is:
(a) arithmetic mean
(b) geometric mean
(c) harmonic mean
(d) none of the above

Q. 16 Most frequently used index number formulae are:
(a) weighted formulae
(b) unweighted formulae
(c) fixed weight formulae
(d) none of the above

Q. 17 The unweighted price index formula based on n items is:

(a) $\sum_{i=1}^{n} p_{1i}/p_{0i}$

(b) $\sum_{i=1}^{n} \frac{p_{1i}}{p_{0i}} \times 100$

(c) $\frac{\sum_{i=1}^{n} p_{1i}}{\sum_{i=1}^{n} p_{0i}} \times 100$

(d) none of the above

Q. 18 Unweighted price index formula is:
(a) most frequently used
(b) seldom used
(c) the best
(d) all the above

Q. 19 Laspeyre's index formula uses the weights of the:
(a) base year
(b) current year

(c) $P_{01}/P_{10} = 1$
(d) $P_{01} + P_{10} = 1$

Q. 46 The discrepancy ($P_{01} \times P_{10} - 1$) is termed as:
(a) joint error
(b) homogeneity error
(c) formula error
(d) none of the above

Q. 47 Factor reversal test was invented by:
(a) Walsh
(b) A.L. Bowley
(c) John I. Griffin
(d) Irving Fisher

Q. 48 The condition for the factor reversal test to be satisfied with usual notations is:
(a) $P_{01} \times Q_{01} = V_{01}$
(b) $\dfrac{P_{01} \times Q_{01}}{V_{01}} = 1$
(c) $\dfrac{P_{01} \times Q_{01}}{V_{01}} - 1 = 0$
(d) all the above

Q. 49 The condition for the price indices to satisfy the circular test for four years data is:
(a) $P_{01}\, P_{12}\, P_{23}\, P_{30} = 1$
(b) $P_{01}\, P_{12}\, P_{23}\, P_{34} = 1$
(c) $P_{01} + P_{12} + P_{23} = P_{43}$
(d) $P_{12} + P_{23} + P_{34} = 1$

Q. 50 Circular test for prices indices is satisfied by the formula:
(a) based on geometric mean of price relatives
(b) obtained by Kelly's fixed weight method
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 51 Fisher's ideal formula does not satisfy:
(a) time reversal test
(b) circular test
(c) factor reversal test
(d) unit test

Q. 52 Year-to-year indices in the chain-base method are called:
(a) chain indices
(b) link relatives
(c) fixed base indices
(d) all the above

Q. 53 The relation between fixed base indices P_{01}, P_{02}, P_{03} and the chain-base indices P_{01}, P_{12}, P_{23} is:
(a) $P_{03} = P_{01}\, P_{12}\, P_{23}$
(b) $P_{03} = P_{02}\, P_{23}$
(c) $P_{01}\, P_{12} = P_{02}\, P_{23}$
(d) all the above

Q. 54 Indices calculated by the chain-base method are almost free from:
(a) homogeneity error
(b) seasonal variations
(c) rigidity of weights
(d) all the above

Q. 55 When the indices given for a number of years are to be worked out to a new base period, this phenomenon is known as:
(a) splicing
(b) base shifting
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 56 Combining of two index number series having different base periods into one series with common base period is known as:
(a) splicing
(b) base shifting
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 57 If the old series is connected, with the new series of index numbers, it is known as:
(a) base sifting
(b) backward splicing
(c) forward splicing
(d) none of the above

Q. 58 If the new series is connected with the old series, it is known as:
(a) base shifting
(b) backward splicing

(c) GNP > NNP
(d) GNP ≥ NNP

Q. 72 Factor reversal test permits the interchange of:
(a) base periods
(b) price and quantity
(c) weights
(d) none of the above

Q. 73 The consumer price index in 1990 increases by 80 per cent as compared to the base 1980. A person in 1980 getting Rs. 60,000 per annum should now get:
(a) Rs. 1,08000 per annum
(b) Rs. 72,000 per annum
(c) Rs. 54,000 per annum
(d) none of the above

Q. 74 If a family spends on food, housing and clothing in the ratio of 5:3:2 and experiences the rise in prices of these heads by 40, 30 and 20 per cent respectively, the family budget will be increased by:
(a) 33 per cent
(b) 30 per cent
(c) 27 per cent
(d) none of the above

Q. 75 If the index number for 1990 to the base 1980 is 250, the index number for 1980 to the base 1990 is:
(a) 4
(b) 40
(c) 400
(d) none of the above

Q. 76 If Laspeyre's price index is 324 and Paasche's price index 144, then Fisher's ideal index is:
(a) 234
(b) 180
(c) 216
(d) none of the above

Q. 77 If the consumer price index for 1994 is 800, then the purchasing power of a rupee is:
(a) 0.125 paise
(b) 12.5 paise
(c) 8 paise
(d) none of the above

Q. 78 The consumer price index numbers for 1981 and 1982 to the base 1974 are 320 and 400 respectively. The consumer price index for 1981 to the base 1982 is:
(a) 125
(b) 80
(c) 128
(d) none of the above

Q. 79 The index number for 1985 to the base 1980 is 125 and for 1980 to the base 1985 is 80. The given indices satisfy:
(a) time reversal test
(b) factor reversal test
(c) circular test
(d) all the above

Q. 80 The price relatives for three commodities are 125, 120 and 130 with their respective weights 5, *w* and 8. If the price index for the set is 1125.25, the value of *w* is:
(a) 6
(b) −7
(c) 7
(d) none of the above

Q. 81 If the group indices are 80, 120 and 125 and their respective group weights are 60, 20 and 20, the consumer price index is:
(a) 108.33
(b) 97.00
(c) 98.49
(d) none of the above

Q. 82 Which index satisfies factor reversal test?
(a) Paasche's index
(b) Laspeyre's index
(c) Fisher's ideal index
(d) Walsh price index

Q. 83 If a sub-index is zero, then the total index must:
(a) be zero
(b) be infinity
(c) indeterminate
(d) not be zero

Q. 84 If the unit of measurement of a commodity changes, the value of index number:
(a) also changes
(b) remains same

(c) increases
(d) decreases

Q. 85 If all the sub-indices are equal to *P*, then the total index will be:
(a) equal to *P*
(b) equal to 100
(c) equal to 1
(d) equal to 0.

Q. 86 The property, that on changing the unit of measurement of a commodity, the index number does not change, is called:
(a) test of equality
(b) test of homogeneity
(c) test of commensurability
(d) test of proportionality

Q. 87 The property that a sub-index is zero or infinity but the total index is not zero or infinity is known as:
(a) test of randomness
(b) factor reversal test
(c) test of definiteness
(d) none of the above

Q. 88 The property that in case of all sub-indices being *P* resulting into total index equal to *P*, is termed as:
(a) test of randomness
(b) test of proportionality
(c) test of definiteness
(d) test of commensurability

Q. 89 Index numbers are the special type of:
(a) averages
(b) percentage relatives
(c) ratios
(d) all the above

Q. 90 The index that indicates the progress or recession of an economic cycle in a group of time series is:
(a) value index
(b) quantitative index
(c) diffusion index
(d) all the above

ANSWERS

SECTION-B

(1) economic barometers (2) direct measurement (3) average (4) percentages (5) two (6) policies (7) index numbers (8) forecasting (9) real wages (10) frequently (11) cost of living index (12) time series (13) sales of company (14) normal period (15) better representative (16) relative importance (17) geometric mean (18) arithmetic mean (19) equal (20) reversible (21) weighted (22) Irving Fisher (23) little (24) $\left[\sum_i q_{1i} / \sum_i q_{0i}\right] \times 100$ (25) base year (26) base year method (27) 1871 (28) French economist (29) German statistician (30) Current period (31) given year method (32) 1874

(33) $\left[\sum_i p_{1i}\, q_{1i} / \sum_i p_{0i}\, q_{1i}\right] \times 100$

(34) $\left[\sum_i q_{1i}\, p_{1i} / \sum_i q_{0i}\, p_{1i}\right] \times 100$

(35) arithmetic mean (36) $\sqrt{L \times P}$ (37) geometric mean (38) combined quantities (39) Marshall and Edgeworth (40) Kelly's (41) formula error (42) sampling error (43) homogeneity error (44) 0; 1 (45) no unique (46) no binary (47) Lesser (48) poorer (49) time reversal (50) joint error (51) time reversal; factor reversal (52) $P_{01} \times Q_{01} = V_{01}$ (53) joint error (54) $P_{01}\, P_{12}\, P_{23} \ldots P_{(k-1)\,k}\, P_{k0} = 1$ (55) geometric mean (56) circular (57) is not (58) chain base (59) preceding period (60) link relatives (61) easily feasible (62) easily (63) seasonal variations (64) long range (65) comparable (66) splicing (67) forward splicing (68) backward splicing (69) real wages (70) rise or fall (71) consumer price index (72) consumer price index (73) wage policy (74) consumer price index (75) consumer price index (76) purchasing power (77) basket (78) retailers (79) percentages of expenditure (80) imputation of expenditure (81) family budget method (82) price changes (83) explicit weights (84) wholesale price index (85) index of industrial production (86) price index number (87) deflator index number (88) net national product (89)

method of weighted relatives (90) Laspeyre's (91) downward (92) units of measurements

(93) 66.67% $\left[\text{Hint: } \dfrac{120x+(100-x)135}{100}=125\right]$

(94) 1500 $\left[\text{Hint: } \dfrac{325}{100}\times 2000 = 6500;\ \text{Amt. reqd.} = (6500-5000)\right]$

(95) 153 per cent (96) 200; 200 (97) circular (98) factor reversal (99) none (100) time and factor reversal (101) proportionality (102) simple aggregative index (103) commensurability (104) new; old (105) zero; infinite

SECTION-C

(1) c	(2) c	(3) d	(4) a	(5) d	(6) d
(7) d	(8) b	(9) a	(10) a	(11) c	(12) c
(13) b	(14) d	(15) b	(16) a	(17) c	(18) b
(19) a	(20) c	(21) b	(22) c	(23) b	(24) c
(25) c	(26) b	(27) a	(28) d	(29) b	(30) c
(31) d	(32) b	(33) c	(34) c	(35) d	(36) c
(37) a	(38) b	(39) c	(40) c	(41) b	(42) a
(43) b	(44) c	(45) a	(46) a	(47) d	(48) d
(49) a	(50) c	(51) b	(52) b	(53) d	(54) d
(55) b	(56) a	(57) c	(58) b	(59) d	(60) c
(61) c	(62) b	(63) a	(64) b	(65) b	(66) a
(67) d	(68) a	(69) b	(70) d	(71) c	(72) c
(73) a	(74) a	(75) b	(76) c	(77) b	(78) b
(79) a	(80) c	(81) b	(82) c	(83) d	(84) b
(85) a	(86) c	(87) c	(88) b	(89) d	(90) c

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Chapter **18**

Business Forecasting

SECTION-A

Short Essay Type Questions

Q. 1 What do you understand by business forecasting?

Ans. The success of any business depends on its future estimates. On the basis of these estimates, a businessman plans his production, stocks, selling markets, expansion of plants, arrangement of additional funds, curtailment of loans, etc. Forecasting is different from prediction and projections. Regression analysis, analysis of time series, extrapolation, index numbers are some of techniques through which the predictions and projections are made. Whereas forecasting is a method of foretelling the course of business activity based on the analysis of past and present data mixed with the consideration of ensuing economic policies and circumstances. Forecasting means forewarning. Forecasts based on statistical analysis are much more reliable than a mere guess work.

Many economists and statisticians defined forecasting according to their own perceptions. Some of them are quoted below.

(i) **Charles W. Elliot**: All business proceed on beliefs or judgements of probabilities and not on certainties.

(ii) **H.J. Wheldon**: Business forecasting is not so much the estimation of certain figures of sales, production, profits, etc., as the analysis of known data, internal and external, in a manner which will enable policy to be determined to meet probable future conditions to the best advantage.

(iii) **Leo Barness**: The aim of forecasting is to establish, as accurate as possible, the probable behaviour of economic activity based on all data available and to set policies in terms of these probabilities.

(iv) **Neter and Wasserman**: Business forecasting refers to the statistical analysis of the past and current movements in a given time series, so as to obtain clues about the future pattern of the movements.

(v) **T.S. Lewis and R.A. Fox**: Forecasting is using the knowledge we have at one time to estimate what will happen at some future movement of time.

Q. 2 What are the main aspects of business forecasting and their role?

Ans. Two main aspects of business forecasting are:

(i) Historical analysis

(ii) Analysis of current economic conditions.

(i) ***Historical analysis:*** To understand the future course of business activity, it becomes necessary to analyse the past data and circumstances. Analysis of past data is very well carried through analysis of time series which separately tell about the four components of time series namely, trend, seasonals, cyclic variation and irregular variation.

Trend clearly reveals the happenings of the past and indicate towards the future trend of business activity. Study of seasonal variations discloses the likely changes in near future whereas the study of cyclic variations depicts the state of business, *i.e.*, whether the business is passing through the state of prosperity, decline, depression or recovery.

(ii) ***Analysis of current economic conditions:*** The forecasting is very much affected by the changes in governmental economic policies, industrial policies, taxation, traditions, fashion, import and export, war, etc. *Hence, the study of all such factors is necessary for* forecasting.

For a reliable business forecast, it becomes essential to study the above two aspects.

Q. 3 Give the names of different methods of business forecasting.

Ans. There are three basic categories of business forecasting methods:

(i) Naive method
(ii) Barometric methods
(iii) Analytical methods

Each of these categories covers a number of methods which are used for business forecasting.

(i) ***Naive method:*** It contains only the economic rhythm theory.

(ii) ***The barometric methods cover:***
   (a) Specific historical analogy
   (b) Lead-lag relationship
   (c) Diffusion index
   (d) Action-reaction theory.

(iii) ***The analytical methods contain:***
   (a) The factor listing method
   (b) Cross-cut analysis theory
   (c) Exponential smoothing
   (d) Econometric methods.

Q. 4 How are the forecasts made under economic rhythm theory?

Ans. The economic rhythm theory implies the forecasts made by the manufacturer himself regarding production, sales, dividends, etc. In this method, the manufacturer analyses the time series data of his own firm or company and forecasts on the basis of projections so obtained. The forecasts based on this method are applicable only for the individual firm for which the data are analysed. The forecasts under this method are not very reliable as no subjective matters are being considered.

Q. 5 Name the forecasting agency which believes in economic rhythm theory and delineate its functioning.

Ans. Standard and Poor's Trade and Securities Service, New York, believes in economic rhythm theory. The institute analyses data for forecasting according to the existing situations with a fresh outlook.

Q. 6 What is the theme behind specific historical analogy method of business forecasting?

Ans. The method is based on the famous dictum, 'history repeats itself'. Under this method, a period from past bearing similarity to the present economic situation is selected and a study is made how the business movement took place under the circumstances. The same sort of business movements are considered to occur now, of course with certain amount of adjustment looking to some special circumstances.

Q. 7 How does the lead-lag relationship theory help in business forecasting?

Ans. The Lead-Lag theory of forecasting is also known as *sequence theory*. This theory of forecasting is based on the principle that changes in business occur in succession, not simultaneously. For instance, the decrease in exchange rate causes the increase in wholesale prices which eventually increase the retail prices and consequently the salaries and wages. Thus, the impact of inflation on wholesale and retail prices and wages is not simultaneous but in phases one after the other with certain lag of time. This shows that the variables governing the business activity have a lead-lag relationship. Under this theory, an effort is made to determine the lag of time between the movement of business cycles.

The lead-lag relationships are usually developed by the critical inspection of graphs of various series and the correlation studies.

Q. 8 In what manner did Harward index of general business conditions consist of three prediction curves, (a) speculation, the leading series, (b) business, the coincident series and (c) money, the lagging series.

Ans. The rise and fall of the speculation curves was used to forecast the movement of business cycles. It is worth pointing out that the speculation curve and money curve have movements in opposite directions. It means that the upturn of speculation curve is accompanied by the downturn of the money curve. Such a situation marks the boom in business within few months while a reverse situation is an indication of the recession in business.

Q. 9 What are the indicators involved in lead-lag approach of business forecasting?

Ans. Three types of indicators are involved in lead-lag approach of business forecasting:

First, the lead indicators are price indices of equity shares, bank reserves, exchange rate of currency, borrowing from financial agencies, creation of new companies, etc.

Secondly, the coincident indicators are employment, industrial production index, gross national product at current prices, business profits, total freight traffic, etc.

Third, the lag indicators are personal income payments, total sales from retail stores, business loans (quarterly), interest rates, inventories of finished products, etc.

Q. 10 What are the limitations of lead-lag approach of business forecasting?

Ans. There are many limitations in lead-lag method of business forecasting which are as follows:

(i) It is not always possible to correctly interpret the movement of indicator variables because of the variability of timing and presence of irregular movements in the series.

(ii) Selection of indicators is also a problem as no thumb-rule is available. Many times some indicators fail to signal at all.

(iii) It is difficult to forecast the amplitude of the cycles.

(iv) The lead-lag method is a supplement to the methods of forecasting, not beyond it.

Q. 11 Which forecasting agencies make use of the lead-lag method?

Ans. *The National Bureau of Economic Research, Massachusetts* has carried out most careful and comprehensive work on the lead-lag relationship as initiated by the Harward Committee of Economic Research in the early twenties of twentieth century. It publishes a weekly report.

The Econometric Institute and the Index Number Institute, New York, analyse data and measure trends, cycles, their lead and lag about each other or phase differences, the systematic patterns of residuals and econometric relationships.

The institute publishes a forecasting bulletin, *economic measures.*

Q. 12 Discuss the diffusion index method of business forecasting.

Ans. The diffusion index method is based on the principle that different factors, affecting business, do not attain their peaks or troughs simultaneously. There is always a time lag between them. This method has the convenience that one has not to identify which series has a lead and which a lag. The diffusion index depicts the movement of broad group of series as a whole without bothering about the individual series. The diffusion index shows the percentage of a given set of series as expanding in a time period. It should be carefully noted that the peaks and troughs of the diffusion index are not the peaks and troughs of the business cycles. All series do not expand or contract concurrently. Hence, if more than 50 per cent are expanding at a given time, it is taken that the business is in the process of booming and *vice-versa.*

The graphic method is usually employed to work out the diffusion index. The diffusion index can be constructed for a group of business variables like prices, investments, profits, etc.

Q. 13 In what manner the action-reaction theory is applied in business forecasting?

Ans. Newton's third law of motion has been applied to the theory of economics. Enough confidence has been posed in action-reaction theory as it has been observed that in business activity, there is always a recession after prosperity and prosperity after recession. This process continues eternally.

Babson found that the area covered by a time series or index of activity curve above the line is approximately as much as it is below the line of normal activity. Of course, the span of the period of prosperity and recession are usually not the same.

Q. 14 Which business forecasting organisation utilises action-reaction theory and what are its publications?

Ans. *The Business Statistics Organisation*, Washington, one of the oldest organisations of the USA founded by *Roger W. Babson* formerly known as *Babson's Statistical*, Organisation, utilises action-reaction theory. It has its affiliated organisation in Canada named as Babson's Canadian Reports Ltd., Toronto, Ontario.

The publications of this organisation are:

(i) Investment and Barometer (Weekly)

(ii) Confidential Barometer Letter (Weekly)

(iii) Business Inventory – Commodity Price Forecasts (Monthly).

(iv) Babson's Washington Forecasts.

Q. 15 What is the basis of business forecasting by the factor listing method?

Ans. Under the factor listing method, various factors which are likely to influence the business are identified by the analyst. Each factor is analysed to assess whether the probable impact of the factor upon aggregate business activity is favourable or not. No mathematical tool is applied for the analysis of data. Inferences are drawn regarding the effect of each factor on the business and then aggregated, merely by his own judgement, to forecast the likely state of business in near future.

Q. 16 What are the merits and demerits of the factor listing method?

Ans.

Merits:

(i) This method is not dependent on any mathematical treatment.

(ii) There is no restriction on the number of variables to be included in the process of forecasting.

(iii) Due weightage can be given to each factor according to its importance in explaining the business condition without any hassles.

(iv) A judgement of likely turning points through dominating variables is always easy and handy.

Demerits:

(i) The main drawback of the factor listing method is that it is totally subjective. Hence, a decision about the state of business condition will vary from person to person.

(ii) The results are not precise as no mathematical analysis of data is involved.

Q. 17 Delineate the cross-cut analysis method of business forecasting.

Ans. The cross-cut analysis theory is just opposite to historical analogy theory. Cross-cut analysis theory claims that past cycles cannot be the guide for future cycles as the factors like technological advancement, government policies, demand, availability of inputs, styles, fads, etc., change with the lapse of time. Hence, a thorough analysis of all the factors under present situations has to be done and an estimate of the composite effect of all factors is being made. This method takes into account the views of managerial staff, economists, consumers, etc., prior to the forecast. The forecast about future state of business is made on the basis of the overall assessment of the effect of all the factors.

Q. 18 What are the drawbacks of cross-cut analysis method?

Ans. There are two main drawbacks of the cross-cut analysis method:

(i) The people do not express their views sincerely and plainly.

(ii) It is very difficult to analyse each variable separately and then to pool their effect for a business forecast.

Q. 19 Which forecasting agency follows the principle of cross-cut analysis and how it operates?

Ans. The Brookmire Economic Service, New York emphasises the theory of cross-cut (cross-section) analysis. Their belief is that domination of one cycle may have little or no effect on the next cycle. The forecasts of this service are based on this principle. The Brookmire Economic Service has the following publications:

(i) *Brookmire bulletin*

(ii) *Brookmire special report* (Weekly).

Q. 20 How is the method of opinion polling used for the purpose of business forecasting?

Ans. The device of opinion poll for business forecasting makes use of the survey reports and opinions obtained through surveys. The theme behind this method is that the present attitudes of the people towards business can be real guide for evaluating the real change in business conditions in near future. For this purpose, opinions are invited from business experts, persons in strategic positions, the sales forces and consumers. The information so obtained is analysed for business forecasts.

The forecasts made by the opinion polling method are simple and least expensive. Opinion polling method is suitable for short-term business forecasts.

Q. 21 How does the method of exponential smoothing help in business forecasting?

Ans. It is a mathematical device for improving trend by moving average method. Under exponential smoothing, the weights assigned to various items are in geometric progression. Greater weights are assigned to recently observed values and smaller weights to distant past observations.

Suppose the weights in geometric series are:

$$1, (1-w), (1-w)^2, \ldots, (1-w)^{n-1}$$

where $0 < w < 1$

and n = No. of observations.

The weighted average till the current period t for n observations $X_t, X_{t-1}, \ldots, X_{t-(n-1)}$ is,

$$\overline{X}_t = \frac{1\cdot X_t + (1-w)X_{t-1} + (1-w)^2 X_{t-2} + \ldots + (1-w)^{n-1} X_{t-(n-1)}}{1 + (1-w) + (1-w)^2 + \ldots + (1-w)^{n-1}}$$

Similarly,

$$\overline{X}_{t+1} = \frac{1\cdot X_{t+1} + (1-w)X_t + (1-w)^2 X_{t-1} + \ldots + (1-w)^{n-1} X_{t-(n-2)}}{1 + (1-w) + (1-w)^2 + \ldots + (1-w)^{n-1}}$$

Taking n large and neglecting higher powers of w and $(1 - w)$, doing certain algebraic manipulations, the following relationship between the next period forecast value $\overline{X}_{t+1}$ and current forecast value $\overline{X}_t$ can be established.

$$\overline{X}_{t+1} = wX_{t+1} + (1-w)\overline{X}_t$$

i.e., New forecast = w × observation + $(1 - w)$ × old forecast $\overline{X}_{t+1}$, the smoothed value at the next period is itself an average smoothed value. Now to make a forecast for each successive period, these smoothed values are used to find out a change in each period. w is called the *smoothing* coefficient and $w/(1 - w)$ the *trend factor*.

Since only one w is used as smoothing coefficient in the above procedure, it is called *single parameter exponential* smoothing.

The forecast for the first period is usually taken from some old forecast if available and if not, it is often assumed.

The quantity $\left(\overline{X}_t - \overline{X}_{t-1}\right)$ is called the error. Thus, the forecast for the period t is the preceding average $\overline{X}_{t-1}$ plus w times the error. The trend coefficient which is required for preparing the forecast is calculated by the formula.

(Trend coeff.) θ_1 = w × change in smoothed value + $(1 - w)$ × preceding trend coefficient

The forecast F_t is obtained by the relation, F_t = *Smoothed value + Trend factor × Trend coeff.*

Also error in forecast,

$$E_t = X_t - F_t$$

Q. 22 How to choose the value of weight *w* in exponential smoothing procedure for business forecasting?

Ans. There is no rule which governs to choose the value of *w*. It is usually chosen arbitrarily. Of course, some tips may be given to choose the value of *w*. If the fluctuations in production, sales, demand, etc., appear to be random, a smaller value of *w* be chosen. Also in case the actual value turns down but the forecast does not turn down, a larger value of *w* is more suitable and *vice-versa.*

Q. 23 Can there be more than one parameter *w* in the exponential smoothing method of forecasting?

Ans. It has been observed that whenever there is a pronounced upward trend in the actual value of a time series, the forecast resulting from the single parameter exponential smoothing procedure is consistently low. To overcome this difficulty, a second smoothing constant is chosen from trend itself. Even more than two smoothing constants can be chosen.

Q. 24 What is the role of econometric methods in business forecasting?

Ans. Econometric methods involve economics, statistics and mathematics. The econometric methods are used to analyse an econometric system which are useful in economic theory. Usually, in econometric methods an inter-relationship is established between a number of variables in an economic system.

Q. 25 What type of variables are involved in economic methods?

Ans. Two types of variables are involved in economics system namely, (i) endogeneous variables, and (ii) exogeneous variables.

(i) *Endogenous variables* are those which belong to the economic system itself. For example, the production, stocks, interest, rent, prices, money reserves, employment, wages, etc.

(ii) *Exogenous variables* are those which do not belong to the economic system but they do affect it. For instance, politics, customs, life style, nature, environment, etc.

Q. 26 Name three forecasting agencies which do not believe in any single theory for the purpose of forecasting.

Ans. Three forecasting agencies which do not believe in any single theory for the purpose of forecasting are:

(i) Standard and Poor's Trade and Securities Service, New York.

(ii) Moody's Investors Service, New York.

(iii) International Statistical Bureau, New York.

Q. 27 What are the publications of standard and Poor's Trade and Securities Services, New York.

Ans. Standard and Poor's Trade and Securities Service, New York, publishes the following weekly and monthly bulletins.

(i) *Industry Surveys – Trends and Projections*

(ii) *The Outlook*

(iii) *The stock guide*

(iv) *Basic Statistics* – a set of statistical bulletins

(v) *Basic Surveys* – irregular reports.

Q. 28 What bulletins are published by Moody's Investors Service?

Ans. Moody's Investors Service, New York publishes two regular bulletins as given below:

(i) *Moody's stock survey*

(ii) *Moody's Bond survey.*

Q. 29 Is there any regular publication of the International Statistical Bureau, New York?

Ans. Yes, The International Statistical Bureau, New York has a solo weekly publication entitled, *Business and Investment.*

Q. 30 What is the position of forecasting agencies in India?

Ans. There are no professional forecasting agencies as such in India. But some big companies have established separate corporate planning and forecasting cells. To name a few, the companies are;

Imperial Chemical Industries, Dunlop India, Indian Textile Corporation, Hindustan Levers, Metal Box, Texamco and Coal India Ltd.

Q. 31 In what sense does forecasting differ from prediction and projection?

Ans. A forecast is an estimate for some future period, partly based on past and present data and partly on subjective estimates arising out of experience and judgement of the forecaster. Whereas prediction is an estimate based on the analysis of past data for a point outside the series and projection is a prediction based on certain assumptions.

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s) in the blanks:

1. The success of a business depends on ________.

2. Forecasting is different from ________ and ________.

3. Forecasting involves ________ as well as ________ factors.

4. Forecasts based on statistical analysis are ________.

5. Forecasting means ________.

6. The statement, "All business proceeds on beliefs or judgement of probabilities and not on certainties" was given by ________.

7. Historical analysis is an aspect of ________.

8. Consideration of industrial policies, taxation, import and export for forecasting comes under ________.

9. Naive method of forecasting follows ________.

10. Forecasts made under the economic rhythm theory are applicable for ________.

11. Specific historical analogy for forecasting comes under the category of ________ methods.

12. Forecasts made under the economic rhythm theory are based on ________ data.

13. Specific historical analogy method for forecasting is based on the adage ________.

14. The lead-lag theory of forecasting is also known as ________.

15. The theory based on the principle that changes in business occur in succession is called ________.

16. Wages are increased in succession of ________.

17. The lead-lag theory for forecasting is utilised by ________.

18. The Harward index of general business conditions consisted of ________ series.

19. Speculation curve is used to forecast the ________.

20. Speculation and money curves move in ________ directions.

21. Upturn of speculations curve and downturn of money curves marks the ________ in business.

22. Price indices and bank reserves work as ________ in lead-lag approach of forecasting.

23. Employment and industrial production index are known as ________ indicators in lead-lag approach for forecasting.

24. Personal income and total sales are called ________ indicators in lead-lag approach for forecasting.

25. In the lead-lag approach, it is difficult to forecast the ________ of the cycles.

26. The lead-lag relationship approach of forecasting is followed by ________.

27. Diffusion index depicts the movement of ________ as a whole without bothering about the individual series.

28. In diffusion index, one finds the _________ of series of a set which are expanding at a given time.

29. If more than 50 per cent series out of a set of series are expanding at a given time, it is considered that the business is in the process of _________.

30. _________ method is used to construct the diffusion index for a group of business variables.

31. In action-reaction theory as applied to business forecasting, prosperity follows _________ and vice-versa.

32. The span of the period of prosperity and recession in business activity are usually _________.

33. The action-reaction theory for forecasting is followed by _________.

34. The factor listing method means identifying _________ which are likely to influence the business activity.

35. The factor listing method involves _________.

36. Main drawback of the factor listing method is _________.

37. The cross-cut analysis theory is just opposite to _________.

38. Under the cross-cut analysis forecasts are made on the basis of overall assessment of _________.

39. In the cross-cut analysis method it is _________ to pool the effect of all factors for forecasting.

40. The cross-cut analysis method of forecasting is adopted by _________.

41. The opinion polling method for forecasting relies on the _________ of the people.

42. The opinion polling method is suitable for _________ business forecasts.

43. Exponential method for forecasting is to improve _________ by _________.

44. _________ weights are assigned to recent observations and _________ weight to distant past observations.

45. The difference between the forecast values $\overline{X}_{t+1}$ and $\overline{X}_t$ at the time periods $(t + 1)$ and t is called _________.

46. If w is the smoothing coefficient in exponential smoothing method of forecasting, the quantity $w/(1 - w)$ is called _________.

47. Whenever there is a pronounce upward trend in the actual values of a time series, the forecast resulting from the single parameter exponential smoothing procedure is consistently _________.

48. In econometric methods an _________ is established between a number of variables in an economic system.

49. The variables which belong to the economic system itself are called _________.

50. The variables like customs, politics which do not belong to economic system but do affect it are called _________.

SECTION-C

Multiple Choice Questions

Select the correct alternative out of given ones:

Q. 1 Business forecasts are made on the basis of:
(a) present data
(b) past data
(c) policies and circumstances
(d) all the above

Q. 2 Forecasting enables a businessman:
(a) to set policies for future business
(b) to know his future
(c) to be certain about profits
(d) none of the above

(c) lead-lag theory
(d) none of the above

Q. 18 The forecasts made by Moody's Investors Service, New York, are based on:
(a) the action-reaction theory
(b) specific historical analogy
(c) lead-lag relationship
(d) none of the above

Q. 19 The diffusion index method of forecasting makes use of:
(a) the movement of individual series
(b) broad group of series
(c) some selected series
(d) none of the above

Q. 20 Under the diffusion index method, business is considered in the process of booming iff:
(a) more than 50 per cent series are expanding
(b) less than 50 per cent series are expanding
(c) exactly 50 per cent series are expanding
(d) none of the above

Q. 21 The business forecasts under the action-reaction theory are based on the principle that:
(a) prosperity follows recession
(b) recession follows prosperity
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 22 The action-reaction theory for business forecasting was expounded by:
(a) Roger W. Babson
(b) Brookmire
(c) Harward
(d) none of the above

Q. 23 Under the factor listing method for forecasting, one has to identify:
(a) lead factor
(b) all factors which are likely to influence the business
(c) past time series
(d) none of the above

Q. 24 The factor listing method makes forecasts by utilising:
(a) index numbers
(b) extrapolation
(c) no mathematical tool
(d) none of the above

Q. 25 The merit(s) of the factor listing method of business forecasting is/are:
(a) it is independent of mathematical juglery
(b) it has no restriction on the number of variables to be studied
(c) it gives due weightage to different factors
(d) all the above

Q. 26 The demerit(s) of the factor listing method of business forecasting is/are:
(a) it is fully subjective
(b) it is fully mathematical
(c) it is easy to indentify dominating variables
(d) none of the above

Q. 27 The cross-cut analysis of business forecasting makes use of the:
(a) past series
(b) present situation
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 28 The cross-cut analysis method of forecasting faces the difficulty that:
(a) the people do not express their views clearly
(b) all variables cannot be analysed individually and then pooled
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 29 The method of cross-cut analysis for forecasting is followed by the agency:
(a) The Brookmire Economic Service, New York
(b) Standard and Poor's Trade and Securities Service, New York.
(c) Babson's Statistical Organisation, Washington
(d) all the above

Q. 30 Forecasts made by the method of opinion polling are suitable for:

Chapter **19**

Statistical Techniques in Quality Control

SECTION-A

Short Essay Type Questions

Q. 1 What is meant by quality of a product?

Ans. Every article or product is required for a specific purpose. If it fully serves the purpose, it is of good quality otherwise not. It means that if an article or material meets the specifications required for its rightful use, it is good quality, and if not, then the quality of the article is considered to be poor.

Q. 2 What role does statistical quality control play in maintaining the quality of a product?

Ans. There is hardly any control on the quality of products produced by the nature and hence the statistical quality control remains confined to articles produced by the industry. Variation in items produced in any manner is inevitable. This variation occurs due to two types of causes namely (i) chance factors and (ii) assignable causes.

(i) ***Chance factors:*** Some deviations from the desired specification is bound to occur in the articles produced, howsoever efficient, the production process may be. If the variations occurs due to some inherent pattern of variation and no causes can be assigned to it, it is called chance or random variation. Chance variation is tolerable and does not materially affect the quality of a product. In such a situation, the process is said to be under statistical control.

(ii) ***Assignable causes:*** If the articles show marked deviation from the given specifications of a product, the utility of art'cles is in jeopardy. In that situation, one has to make a search for the causes responsible for the large variation in the product. The causes due to faulty process and procedure are known as *assignable causes*. The variation due to assignable causes is of non-random nature.

Hence, the role of statistical quality control is to collect and analyse relevant data for the purpose of detecting whether the process is under control or not. If not, what can possibly be the reason for the fault(s).

Q. 3 What do you understand by control charts in statistical quality control?

Ans. Control charts are the devices to describe the patterns of variation. The control charts were developed by the physicist, Dr. Walter A. Shewhart of Bell Telephone Company in 1924.

Control Chart delimits the range or band in which the basic variability is within tolerance limits and beyond this range or band, the variability is due to some assignable causes. The range or band ascertained by the control chart is little broader than the natural tolerance range. The most frequently used control charts are for mean $\overline{X}$, the range R, the root mean square σ, number of defects c, etc. In general, a control chart consists of three lines namely, (i) central line (CL) depicting the desired standard or level of the process, (ii) upper control limit (UCL) and (iii) the lower control limit (LCL) as shown in Fig. 19.1.

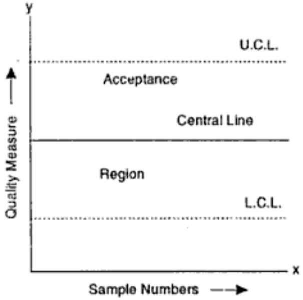

Fig. 19.1. Control chart in general.

Q. 4 What benefits are expected out of the use of control charts?

Ans. Control charts are meant to uncover whether the basic variability of a manufacturing process is within its natural tolerance or beyond tolerance range due to assignable causes.

If the variability is due to assignable causes, the causes can be discovered and faults can be removed. This brings the process under control. Once the process is under control, one can feel confident that the product will meet the specifications.

Q. 5 Delineate the main tools for statistical quality control.

Ans. There are four closely associated tools for statistical quality control as given below:

(i) ***Shewhart's control charts for variables:*** Such charts consider measurable data on quality characteristics which are usually continuous in nature. Such type of data utilises $\overline{X}$, R and σ charts.

(ii) ***Shewhart control charts for fraction defectives:*** Such charts are used when the units are classified as defective or non-defective. The control charts meant for fraction defectives are known as p-charts.

(iii) ***Shewhart control charts for number of defects per unit:*** In this situation one deals with discrete variables which are the counts of the number of defects per unit. Control charts used in this situation are called c-charts.

(iv) ***Acceptance sampling procedure:*** Acceptance inspection has to be carried out by the manufacturer or by the buyer. It is not only difficult but rather impossible in many cases to inspect all the items produced or purchased. Hence, sampling inspection is the only device left over at hand. Moreover, it has many advantages over 100 per cent inspection. Some acceptance sampling procedures are evolved which are superior than traditional sampling procedure.

Q. 6 What is the rationale behind setting of control limits?

Ans. Let μ and σ be the population mean and standard deviation respectively. By the property of normal distribution curve we know that 99.73 per cent units full within the $\mu \pm 3\,\sigma$ limits. Hence, 3σ limits are usually used for control charts. If the population constants are not known, their estimated values are used to set-up the control limits.

Suppose we have taken k samples of size n. Let the mean of the i^{th} sample is $\overline{X}_i$ and standard deviation s_i for $i = 1, 2, ..., k$. The process is considered to be under control if the mean values $\overline{X}_i$ fall within $\mu \pm 3\,\sigma$ limits.

Q. 7 How are the control limits set-up for mean?

Ans. In many situations the specifications of the units produced are known and also the variation which can be tolerated. Suppose μ', the mean and σ', the standard deviation are the known specifications. In this situation, the control limits for mean are given by the formula,

$$\mu' \pm 3\frac{\sigma'}{\sqrt{n}}$$

$$\text{U.C.L.}_s = (c_2 + 3c_3)\frac{\overline{S}}{c_2}$$

$$= \left(1 + 3\frac{c_3}{c_2}\right)\overline{S}$$

$$= B_4\overline{S}$$

$$\text{C.L.}_s = c_2\frac{\overline{S}}{c_2} = \overline{S}$$

$$\text{L.C.L.}_s = (c_2 - 3c_3)\frac{\overline{S}}{c_2} = \left(1 - \frac{3c_3}{c_2}\right)\overline{S} = B_3\,\overline{S}$$

where $B_4 = \left(1 + \frac{3c_3}{c_2}\right) = \frac{B_2}{c_2},\ B_3 = \left(1 - \frac{3c_3}{c_2}\right) = \frac{B_1}{c_2}$

Values of B_1, B_2, B_3 and B_4 and c_2 have already been tabulated and can be seen in appendix table XV of *Basic Statistics* by B.L. Agarwal.

Q. 9 What is the theme behind the use of *R*-chart in statistical quality control?

Ans. Statisticians experience has revealed that in case of small samples, the range and standard deviation vary concurrently in the same direction. Generally, small samples are used in quality control. Hence, the control limits for standard deviation (σ) can be replaced by the control limits for range (R). Thus, the charts constructed by making use of range are known as *R*-charts.

The main advantage of *R*-charts is that it is easier to calculate R than σ. Of course, there is a little loss of efficiency in replacing σ by R. But looking to the convenience, a little loss in efficiency is acceptable.

Q. 10 How do you set the control limits for *R*-charts in statistical quality control?

Ans. We know that,

$$E\,(R) = d_2\,\sigma$$
$$S.D.\ (R) = d_3\,\sigma$$

If $\overline{R}$ is the mean of R_i's, the ranges of k samples and σ_R is the standard error of R for $i = 1, 2, \ldots, k$. Also if σ' is known value of σ, then $d_2 = \overline{R}/\sigma'$.

Thus, $\overline{R} = d_2\sigma'$ or $\sigma' = \overline{R}/d_2$.

Case (i): When the range R' and standard error of R', $\sigma_{R'}$ are known values of R and σ_R respectively, the control limits for R' are given as,

$$\text{U.C.L.}_{R'} = E(R') + 3S.D.(R')$$
$$= d_2\sigma_{R'} + 3d_3\sigma_{R'}$$
$$= (d_2 + 3d_3)\sigma_{R'}$$
$$= D_2\sigma_{R'}$$
$$\text{C.L.}_{R'} = d_2\,\sigma_{R'}$$

Similarly, $\text{L.C.L.}_{R'} = (d_2 - 3d_3)\sigma_{R'} = D_1\,\sigma_{R'}$

where, $D_1 = d_2 - 3d_3$ and $D_2 = d_2 + 3d_3$

Case (ii): When the population range R is not known, we make use of the sample mean range $\overline{R}$.

For $\overline{X}$ chart, the control limits are,

$$\text{U.C.L.}_{\overline{X}} = \overline{X} + 3\sigma_{\overline{X}}$$
$$= \overline{X} + 3\sigma/\sqrt{n}$$
$$= \overline{X} + 3\frac{\overline{R}}{d_2\sqrt{n}}$$
$$= \overline{X} + A_3\overline{R}$$
$$\text{C.L.}_{\overline{X}} = \overline{X}$$

Also, $\text{L.C.L.}_{\overline{X}} = \overline{X} - A_3\overline{R}$

where, $A_3 = 3/\left(d_2\sqrt{n}\right)$

For R chart, the control limits are,

$$\text{U.C.L.}_R = E(R) + 3S.D.(R)$$
$$= d_2\,\sigma + 3d_3\,\sigma$$
$$= \left(1 + 3\frac{d_3}{d_2}\right)\overline{R}$$
$$= D_4\,\overline{R}$$
$$\text{C.L.}_R = \overline{R}$$

Also, $\text{L.C.L.}_R = \left(1 - 3\frac{d_3}{d_2}\right)\overline{R}$

$$= D_3\,\overline{R}$$

though the process is under control. Such a pattern occurs due to material and machines.

Q. 15 A dry-cells producing factory wanted to test the life of cells produced daily. The cells will be considered satisfactory if their life is 25 hours. For this, a sample of 5 cells was drawn on 12 consecutive days. The results were as follows:

Days (Subgroups)	*Life of cells (in hours)*				
	1	*2*	*3*	*4*	*5*
1.	27.0	28.0	25.5	26.5	23.0
2.	23.5	27.5	26.0	27.0	29.0
3.	27.5	27.0	28.0	26.5	24.5
4.	28.0	26.5	27.5	28.5	27.0
5.	27.5	24.5	25.0	26.0	27.5
6.	26.5	26.0	27.0	27.5	26.0
7.	21.0	22.0	28.0	26.5	25.0
8.	25.5	24.5	25.0	27.5	27.5
9.	28.0	26.5	30.0	29.5	27.0
10.	25.0	27.0	26.5	24.5	23.0
11.	22.0	26.5	27.5	23.5	25.5
12.	26.0	28.0	27.0	30.0	29.0

(i) Calculate 3 σ-limits of control chart for $\overline{X}$ when the value of mean of the universe is 25 hours and standard deviation is 2 hours.

(ii) Set-up the control limits for $\overline{X}$-chart when the standard values for mean and S.D. are not known. Also prepare the control chart and comment whether the process is under control or not.

(iii) Construct σ-chart when the specified values for standard deviation are not known.

(iv) Draw a range chart when standard values are not specified.

[Given that for $n = 5$; $A = 1.342$, $A_1 = 1.596$, $A_2 = 0.577$, $c_2 = 0.8407$, $B_1 = B_3 = 0$, $B_2 = 1.756$, $B_4 = 2.089$, $d_2 = 2.326$, $D_1 = D_3 = 0$, $D_2 = 4.918$, $D_4 = 2.115$]

Ans.

(i) Given that $\mu' = 25$ and $\sigma' = 2$, 3-sigma limits for $\overline{X}$ - chart are,

$$\mu' \pm 3\sigma'/\sqrt{n}$$

$$\text{U.C.L.}_{\overline{X}} = 25 + \frac{3 \times 2}{\sqrt{5}}$$

$$= 27.68$$

$$\text{C.L.}_{\overline{X}} = 25$$

$$\text{L.C.L.}_{\overline{X}} = 25 - 2.68$$

$$= 22.32$$

(ii) To construct trial control limits for $\overline{X}$, σ and R charts, we prepare the following calculation table.

Days *(Subgroups)*	Total	$\overline{X}$	s	R
1.	130	26.0	1.90	5.0
2.	133	26.6	2.04	5.5
3.	133.5	26.7	1.35	3.5
4.	137.5	27.5	0.79	2.5
5.	130.5	26.1	1.39	3.0
6.	133.0	26.6	0.65	1.5
7.	122.5	24.5	2.96	7.0
8.	130.0	26.0	1.41	3.0
9.	141.0	28.2	1.52	3.5
10.	126.0	25.2	1.60	4.0
11.	125.0	25.0	2.24	5.5
12.	140.0	28.0	1.58	4.0
Total	1582.0	316.4	19.43	48.0
Mean	26.37	26.37	1.62	4.0

The control limits for $\overline{X}$-chart, when standard values not known, are

$$\overline{X} \pm A_1 \overline{S}$$

From the above table $\overline{X} = 26.37$ and $\bar{s} = 1.62$.

Also given that $A_1 = 1.596$. Thus,

$$\text{U.C.L.}_{\overline{X}} = 26.37 + 1.596 \times 1.62$$

$$= 26.37 + 2.586$$

$$= 28.956$$

$$\text{C.L.}_{\overline{X}} = 26.37$$

$$\text{L.C.L.}_{\overline{X}} = 26.37 - 2.586$$
$$= 23.784$$

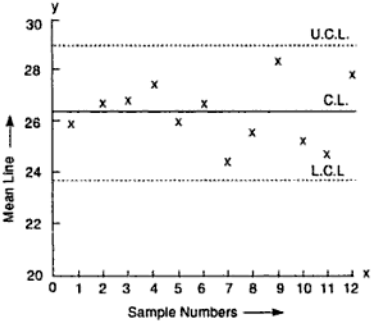

Fig. 19.2. $\overline{X}$-chart

Since no point falls outside the control limits, we conclude that the process is under control.

(iii) When the specific value for standard deviation is not given, the control limits are,

$$\text{U.C.L.}_{\overline{X}} = B_4\overline{S}$$
$$= 2.089 \times 1.62$$
$$= 3.38$$

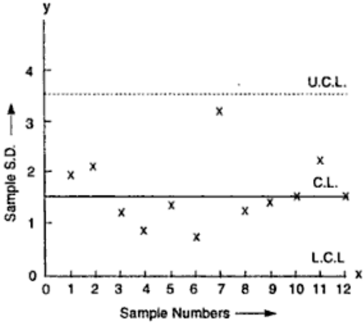

Fig. 19.3. σ-chart

$$\text{C.L.}_{s} = \overline{S} = 1.62$$
$$\text{L.C.L.}_{s} = B_3\overline{S} = 0$$

(iv) The control limits for R-chart, when the standard value of range is not known, are

$$\text{U.C.L.}_{R} = D_4\overline{R}$$
$$= 2.115 \times 4.0$$
$$= 8.46$$
$$\text{C.L.}_{R} = \overline{R} = 4.0$$
$$\text{L.C.L.}_{R} = D_3\overline{R} = 0$$

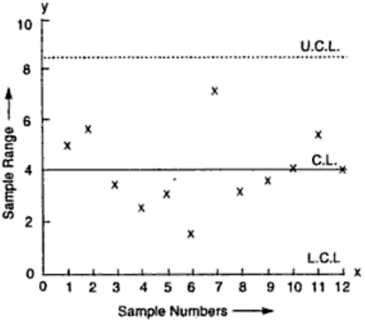

Fig. 19.4. R-chart

Since the mean ranges of all the subgroups lie within the control limits and no particular pattern is observed, it is inferred that the process is very well under control.

Q. 16 When should the control charts for fraction defectives be prepared?

Ans. In many situations, one is interested in the units possessing certain desired specifications. If the units do not possess all those specifications, they are classified as defective. In this way, the variable is dichotomised and thus follows binomial distribution. Hence, the mean and standard deviation of a binomial distribution are applicable.

If the lot contains a proportion of defectives which is acceptable to the producer, the process is under control, otherwise not. It safeguards against falling of reputation of the firm. So the control charts are to be prepared for fraction defectives.

Q. 17 How will you prepare the control charts for fraction defectives?

Ans.

Case (i): The charts prepared for fraction defectives are known as *p*-charts. Let *P* be the proportion of defectives in the population and *p'* be its given standard value. If we have selected samples from each lot of size *n*, then 3-sigma control limits can be given as,

$$\text{U.C.L.}_{p'} = p' + 3\sigma_{p'}$$

$$= p' + 3\sqrt{\frac{p'q'}{n}}$$

$$\text{C.L.}_{p'} = p'$$

$$\text{L.C.L.}_{p'} = p' - 3\sqrt{\frac{p'q'}{n}}$$

The sample values of *p* are plotted against sample numbers. If the points lie within control band, then the process is considered to be under control, otherwise not.

Case (ii): When the standard value of *P* is not given, it has to be estimated through sample fraction defectives. Let *k* samples be drawn at regular intervals from the lots in an specified period. Suppose d_i is the number of defectives in the i^{th} sample of size n_i for $i = 1, 2, ..., k$.

The fraction defective p_i in the i^{th} sample is,

$$p_i = \frac{d_i}{n_i}$$

Also the estimate of the population fraction defective *P* is,

$$\bar{p} = \sum_{i=1}^{k} d_i \Big/ \sum_{i=1}^{k} n_i$$

Generally, a sample of equal size is preferred in statistical quality control. Therefore, taking $n_1 = n_2 = ... = n_k = n$,

$$\bar{p} = \frac{1}{nk}\sum_{i=1}^{k} d_i$$

$$= \frac{1}{k}\sum_{i=1}^{k} p_i$$

It is trivial to verify that $\bar{p}$ is an unbiased estimate of *P*.

Again 3-sigma control limits for fraction defective by using $\bar{p}$, the estimated value of *P* based on the samples of equal size *n* are,

$$\text{U.C.L.}_p = \bar{p} + 3\sqrt{\frac{\bar{p}\,\bar{q}}{n}}$$

$$\text{C.L.}_p = \bar{p}$$

$$\text{L.C.L.}_p = \bar{p} - 3\sqrt{\frac{\bar{p}\bar{q}}{n}}$$

where $\bar{q} = 1 - \bar{p}$

Sample *p*-values are plotted on the graph against sample numbers. If any plotted point lies outside the control limits, the process is considered to be out of control, otherwise not.

The points lying above the upper control limit are called *high spots*, and below the lower control limit *low spots*. High spots show a deterioration in the process, whereas low spots may be due to slackness.

Q. 18 How can one decide the sample size for fraction defective charts (*p*-charts)?

Ans. To make an effective use of *p*-chart, some fraction defectives be observed in the sample which can be rejected. If the quality of the product is good, there are meagre chances of defective items and hence to get defective item(s), a large sample is to be drawn. For instance, for 0.1 per cent defectives to be detected in the process at least a sample of 1,000 items has to be selected. If this is not done and we take a sample of 10 items and a defective appears in this sample, it will lead to 10 per cent of the items rejectable which is not really the case.

In short, better is the quality, the larger must be the size of the sub-samples. In other words, smaller the sample sizes, less sensitive is the *p*-chart to detect the changes in quality level or to indicate the assignable causes of variation.

Q. 19 Formulate control limits for number of defectives.

Ans. If p_i is the proportion of defectives in the i^{th} sample of size n for $i = 1, 2, ..., k$, the number of defectives,

$$d_i = np_i$$

and
$$\bar{p} = \frac{1}{nk}\sum_{i=1}^{k} d_i$$

3-sigma trial control limits for number of defectives are,

$$\text{U.C.L.}_{d} = d + 3\sigma_d = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$\text{C.L.}_{d} = n\bar{p}$$

$$\text{L.C.L.}_{d} = d - 3\sigma_d = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

If the standard value p' of p is given, the control limits for number of defectives are,

$$\text{U.C.L.}_{d'} = d' + 3\sigma_{d'} = np' + 3\sqrt{n\,p'\,q'}$$

$$\text{C.L.}_{d'} = d' = n\,p'$$

$$\text{L.C.L.}_{d'} = d' - 3\sigma_{d'} = n\,p' - 3\sqrt{n\,p'\,q'}$$

where $d' = np'$ and $p' = 1 - q'$

Here the points are plotted for the number of defectives and sample number. The decision about process control is taken in the usual manner.

Q. 20 When should the control charts for number of defects be constructed?

Ans. The control charts for number of defects per unit are constructed in situations where the chances of defects are large and the actual occurrence tends to be small. For instance, the number of woven defect in per sq. m. of cloth, blemishes in a fool-scap sheet of paper, number of defects per fan, etc. In such situations, the variable C, denoting the number of defects, follows Poisson distribution.

An item is defective even for the minutest deviation from its specifications or standardised fabrication. But mere classification whether an item is defective or non-defective is not enough. It also matters how many defects per unit occur. Hence, it seems germane to construct control charts for number of defects.

Q. 21 Give control limits for C-charts, *i.e.*, the number of defects charts in case of samples of equal size.

Ans. Here we consider first the number of defects c per sample which follow Poisson distribution.

If the Poisson variate c is distributed with parameter λ and λ' is its given specified value, then the control limits are,

$$\text{U.C.L.}_c = \lambda' + 3\sqrt{\lambda'}$$

$$\text{C.L.}_c = \lambda'$$

$$\text{L.C.L.}_c = \lambda' - 3\sqrt{\lambda'}$$

If the value of λ is not given, it has to be estimated through samples. If C_i is the number of defects in the i^{th} sample for $i = 1, 2, ..., k$, the estimated value of λ say $\bar{C}$ is,

$$\hat{\lambda} = \bar{C} = \frac{1}{k}\sum_{i=1}^{k} C_i$$

Let the mean of the defects counted in samples usually of size 25 or more be $\bar{C}$. Obviously its variance will also be $\bar{C}$. Hence 3-sigma trial control limits for number of defects are:

$$\text{U.C.L.}_c = \bar{C} + 3\sqrt{\bar{C}}$$

$$\text{C.L.}_c = \bar{C}$$

$$\text{L.C.L.}_c = \bar{C} - 3\sqrt{\bar{C}}$$

The values of $\bar{C}$ for various samples are plotted against the sample numbers. The conclusions about process control are taken in the usual manner, *i.e.*, if any point lies outside the control limits, it is inferred that the process is not under control, otherwise it is under statistical quality control.

If we consider 'C' as the number of defects per unit and its established value C' is known, then 3-sigma control limits will be,

$$\text{U.C.L.}_{c'} = C' + 3\sqrt{C'/n}$$

(i) Number of missing rivettes in the assembly of an aircraft.

(ii) Number of weak points in a coil of insulated electric wire.

(iii) Number of air bubbles observed per square metre in a sheet of glass.

(iv) Number of defects per television set.

(v) Number of defects per metre in a hand woven cloth.

Q. 24 What conspicuous advantage is there having an established value in advance of the parameter used in a control chart?

Ans. The practical aspect of a given standard value and an estimated value used in a chart is that no control chart can be prepared for the estimated value(s) until the estimated value is known, *i.e.*, not before the end of the period. Whereas a standard value established in advance enables one to compute the control limits everyday and plot the points on the control chart. Now it is trivial to check whether the process is under control or not.

So, a value established in advance provides a day-to-day checking whereas the estimated values enable to decide about the process control after a fixed period only.

Q. 25 A TV voltage stabilizer manufacturer checks the quality of 50 units of his product daily for 15 days and finds the fraction of non-conforming units and number of defectives as follows:

Days	*Fraction defectives*	*No. of defectives*
1	0.10	5
2	0.20	10
3	0.06	3
4	0.04	2
5	0.16	8
6	0.02	1
7	0.08	4
8	0.06	3
9	0.02	1
10	0.16	8
11	0.12	6
12	0.14	7
13	0.08	4
14	0.10	5
15	0.06	3

(i) Construct 3-sigma trial control limits for fraction defectives.

(ii) Construct 3-sigma trial control limits for *np*-chart.

Ans.

(i) 3-sigma trial control limits for fraction defectives can be established in the following manner:

The estimated value,

$$\bar{p} = \frac{1}{k}\sum_{i=1}^{15} p_i$$

$$= \frac{1.4}{15} \qquad \text{since } \sum_{i=1}^{15} p_i = 1.4$$

$$= 0.093$$

and $1 - \bar{p} = 0.907$

3-sigma control limits for fraction defectives are,

$$\text{U.C.L.} = \bar{p} + 3\sqrt{\frac{\bar{p}\,\bar{q}}{n}}$$

$$= 0.093 + 3\sqrt{\frac{0.093 \times 0.907}{50}}$$

$$= 0.093 + 0.123$$

$$= 0.216$$

$$\text{C.L.} = 0.093$$

$$\text{L.C.L.} = 0.093 - 0.123$$

$$= -0.03 = 0$$

No sample value of fraction defectives is beyond control limits and hence the product conforms to statistical quality control.

(ii) Now we find 3-sigma trial control limits for np-chart.

Total number of defectives = 70

Average fraction defective,

$$\bar{p} = \frac{70}{50 \times 15}$$

$$= 0.093$$

$$n\bar{p} = 50 \times 0.093 = 4.65$$

$$3\hat{\sigma}_{np} = 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$= 3\sqrt{4.65 \times 0.907}$$

$$= 6.161$$

3-sigma trial control limits for np-chart are,

$$\text{U.C.L.}_{np} = n\bar{p} + 3\hat{\sigma}_{np}$$

$$= 4.65 + 6.161$$

$$= 10.811$$

$$\text{C.L.}_{np} = 4.65$$

$$\text{L.C.L.}_{np} = 4.65 - 6.161$$

$$= -1.51 = 0$$

Control limits for np-chart lead to the result that the process is under statistical quality control.

Q. 26 What are the advantages of statistical quality control?

Ans. Statistical quality control methods are extensively used in industrial production process because of number of advantages as given below:

(i) ***Reduction in cost:*** Control charts are based on sampling inspection. Hence, there is reduction in cost. Also a lot of time is saved.

(ii) ***Greater efficiency:*** Because of the availability of time due to little inspection time, a greater efficiency is maintained in the process of inspection.

(iii) ***Timely detection of faults:*** Since the samples are inspected quickly, there is timely detection of faults. Hence, if the process is out of control, the faults can be removed. This protects against heavy losses.

(iv) ***Adherence to specifications:*** Statistical quality control enables a manufacturer to adhere to quality specification of a product as the assignable causes are soon detected and the process is brought under control.

(v) ***The only possibility:*** In situations where the units are destroyed or perished during the process of inspection, cent per cent inspection of items is not possible. Hence, the samples from the lots can only be inspected to adjudge the quality of the product. For example, the life of electric bulbs, life of cells, tensile strength of wire, etc.

(vi) ***Basis for specifications:*** Process control provides the basis of deciding the specifications of a product. There is no use of fixing the specifications which cannot be maintained.

(vii) ***To ascertain the impact of change in process or personnel:*** Statistical quality control enables the manufacturer to ascertain whether the changes brought in the process or technical persons have brought an improvement in the product or not.

(viii) ***Basis for satisfaction:*** The engineers may expect a total adherence to specifications whereas the operators may emphasise their performance satisfactory in spite of large variation. But the controversy is resolved if the process is under control.

(ix) ***Protection against losses:*** Statistical quality control provides protection against losses to the manufacturers as well as consumers.

Q. 27 What are the limitations of statistical quality control?

Ans. Statistical quality control should not be considered a panacea against all evils of production process. Many factors are involved which are not covered by statistical quality control. A production is not a part of statistical quality control but it is the other way round.

Q. 28 Briefly differentiate between natural, specification and modified control limits.

Ans. If μ and σ are the process mean and standard deviation respectively, the limits $\mu \pm 3\sigma$ are called the *natural tolerance limits.* The width of tolerance band is 6σ and is known as natural tolerance. If μ and σ are not known, they are estimated by $\hat{\mu} = \overline{X}$ and $\hat{\sigma} = \dfrac{\overline{R}}{d_2}$ or $\hat{\sigma} = \dfrac{\overline{S}}{c_2}$ and the control limits are constructed.

The *specification limits* are those which are desired by the consumer. Let the upper specification limit (U.S.L) and lower specification limit (L.S.L) be denoted by $X_{\max}$ and $X_{\min}$ respectively for some quality characteristic. A comparison of tolerance limit with the specification limits leads to the following inferences:

(a) If $X_{\max} - X_{\min} > 6\,\sigma$, there is a likelihood of better product. In this case specification limits lie outside the natural tolerance limits.

(b) If $X_{max} - X_{min} = 6\,\sigma$, this is an ideal situation. In this situation, the product will meet the specifications.

(c) If $X_{max} - X_{min} < 6\sigma$, a process under statistical control does not ensure that the product will meet the specifications.

Modified control limits give the relationship between specification limits and $\overline{X}$ values in $\overline{X}$-chart used to allow a shift in process levels within permissible limits.

For the samples (sub-groups) of size n, the maximum and minimum values of upper and lower control limits for $\overline{X}$ which are known as upper rejection limit (U.R.L) and lower rejection limit (L.R.L) respectively are given as,

$$\text{URL}_{\overline{X}} = \text{USL} - 3\sigma' + \frac{3\sigma'}{\sqrt{n}}$$

$$\text{LRL}_{\overline{X}} = \text{LSL} - 3\sigma' - \frac{3\sigma'}{\sqrt{n}}$$

where σ' is the specified known value of standard deviation.

The upper and lower rejection limits are known as *modified control limits*.

In case of rejection limits we do not have a line but a central band in which the product will conform to specifications. The upper and lower edges of the central band are given by USL $-3\sigma'$ and LSL $+3\sigma'$.

Q. 29 What do you understand by acceptance sampling plan?

Ans. It looks germane to inspect each and every item produced by a manufacturing unit and make sure about the quality specifications before releasing it for sale. But cent per cent inspection has its own weaknesses.

Firstly, due to fatigue of checking a large number of items, the efficiency of inspection goes down and hence one cannot expect that no defective or non-conforming item will not be left out after inspection.

Secondly, cent per cent inspection is an impossibility in cases where the produce or items are perished under inspection such as inspection for life of electric bulbs, life of battery cells, combustibility of coke, etc.

Thirdly, the time and cost are other two important economic factors which deter a manufacturer from 100 per cent inspection. So the acceptance or rejection of a lot is usually based on the inspection of samples drawn from the lots at regular intervals during the manufacturing process. Usually people call it *acceptance sampling plan*. Thvs, under acceptance sampling plan one takes the decision whether a lot is to be accepted or rejected.

Q. 30 When should the cent per cent inspection be preferred?

Ans. In spite of many factors against cent per cent inspection, it is still preferred under special situations such as:

(i) a defective item may cause danger to life.

(ii) a defect may stop the whole function of a system.

(iii) the lot size is small.

(iv) the incoming quality is very poor.

Q. 31 State the situations when no inspection is called for quality control.

Ans. There are many situations which call for no inspection. If the process is such that the items are of high quality and there is hardly any chance of defective item, the product may be sold without inspection.

Q. 32 What purposes are served by sampling inspection plans?

Ans. Broadly, three purposes are served by sampling inspection plans.

(i) It enables to know whether the process is producing the product which meets the quality specification or not.

(ii) It reveals whether the finished product is good for marketing or not.

(iii) It minimizes the risk of the consumer and protects the producer from future losses.

Q. 33 Compare sampling inspection by attributes with inspection by variables.

Ans. The acceptance or rejection of a lot is based on inspection of a sample drawn from a submitted lot. Usually the decision is based on the number or proportion of defectives present in the sample according to attributes under consideration. But the item can also be classified as defective or non-defective on the basis of measurable quality characteristics as well. For instance, an item heavier than a fixed weight or shorter than a fixed length can be classified as defective. Hence, the inspection by attributes contrasted with inspection by variables is given below:

(i) Inspection by variables has one major advantage over inspection by attributes that it needs fewer items for inspection for a given degree of accuracy.

(ii) The decision about the acceptance or rejection of a lot is more reliable if based on inspection by variable than by attributes for the fixed sample size.

(iii) The main advantage of inspection by attributes over inspection by variables is that it requires less accuracy of measurement, less skill, less time and less sophisticated instrument for measurements.

(iv) The calculations involved in deciding about the lot are much less in case of inspection by attributes as compared to inspection by variables.

(v) In case of inspection by attributes, selection and installation of sound sampling plans is easier than by variables.

(vi) Sampling inspection plan for variables do exists but are not so prevalent as for attributes.

(vii) Inspection plans by attributes are more economical than for variables. Thus, the lower cost of inspecting each item by attributes offsets the increase in sample size.

(viii) Per cent defective is usually an appropriate measure and its appropriateness is not affected by shifting between variables and attributes. Hence, inspection by attributes has been in vogue.

Q. 34 What types of decision procedures are usually used for acceptance or rejection of a lot? Give a brief description.

Ans. Two types of sampling inspection procedures are normally used, (i) sampling inspection by attributes and (ii) sampling inspection by variables.

Both the inspection types lead to the same conclusion. But the sampling inspection by attributes is mostly practised. Hence, we discuss only acceptance sampling by attributes. In this type of inspection, items are classified as defective or non-defective. Also the acceptance or rejection of a lot depends on fraction defectives in the sample. The decision about the lot can be of two types, (i) acceptance-rejection type, (ii) acceptance-rectification type. In the first type, the lot is either accepted or rejected. If the lot is accepted, then the defective items in the sample are replaced by non-defectives and then the lot is passed for marketing. Whereas if the lot is rejected, it will cause heavy losses to the manufacturer. Hence, to reduce losses, the second decision procedure, *i.e.*, acceptance-rectification plan is adopted. In this plan, each and every unit of the lot is inspected and the defectives found in the lot are replaced by the non-defectives and the lot is released for sale.

Q. 35 How will you distinguish between a major and minor defect?

Ans. A defect of an item is called *major* if the defect will cause a failure of the item to function for which it is meant. Again, a defect of an item is called *minor* if the defect impairs the efficiency or shorten the life of the item.

Q. 36 Define a producer and producer's risk.

Ans. A producer is a person who produces goods for the purpose of consumption by others.

Producer's risk lies in rejecting a lot even though this is of desirable quality. Suppose the process is set for a fraction defective $\bar{p}$. The *average fraction defective* $\bar{p}$ is known as *producer's process average*. The probability of rejecting a lot having 100 $\bar{p}$ as the process average per cent defectives is known as producer's risk P_p. Symbolically,

$$P_p = \text{Prob (rejecting a lot having } 100\ \bar{p} \text{ defectives)}$$
$$= \alpha$$

Q. 37 Define a consumer and consumer's risk.

Ans. A consumer is a person who purchases goods for his own consumption and not for sales to others directly or indirectly.

There is always a risk that a person gets a lot having an undesirable proportion of defectives. Such a risk is known as *consumer's risk*. If P_t is the proportion of defectives in the lot which can be tolerated, the probability of accepting a lot with fraction defectives P_t is known as consumer's risk and is denoted by P_c. Thus,

$$P_c = \text{Prob (accepting a lot having } P_t \text{ defectives)}$$
$$= \beta$$

Dodge and Roming suggested that β can be taken as 10 per cent, *i.e.*, 0.1.

Q. 38 What do you understand by acceptance quality level (A.Q.L)?

Ans. The percentage of defective items, in an inspection lot which under sampling plan leads to the acceptance of 95 per cent of the submitted lots for inspection, having that percentage of defectives, is known as A.Q.L.

Let p_1 be the small fraction defective in the lot which is such that one is not to reject the lot more than a small number of times, then p_1 is known as the *acceptance quality level*. Usually,

$$P \text{ (rejecting a lot of quality level } p_1) = 0.05$$
$$P \text{ (accepting a lot of quality level } p_1) = 0.95$$

A lot having this quality is considered satisfactory.

Q. 39 What is meant by lot tolerance percentage defective (L.T.P.D)?

Ans. Lot tolerance percentage defective say, p_t denotes the quality of the lot which is not acceptable to the consumer. A lot, having p_t or more proportion of defectives, is not acceptable to the consumer. Thus, 100 p_t, is called *lot tolerance percentage defective*. In a way, p_t is the quality level at which the lot is rejected by the consumer and is also called *rejecting quality level* (R.Q.L). Customarily this percentage defective is about 10 per cent.

Q. 40 Explain the meaning of average outgoing quality (A.O.Q).

Ans. Average percentage of defectives remaining in the product finally accepted is known as *average outgoing quality*. Let it be denoted by p. Also the maximum average percentage of defective items in a product finally accepted is known as average outgoing quality limit (A.O.Q.L).

Suppose, a sample of n items is drawn from a lot of size N and P_a is the probability of accepting a lot of average quality level p, then

$$\text{A.O.Q.L} = \frac{p(N-n)P_a}{N}$$

If the sampling fraction n/N is negligible, then,

$$\text{A.O.Q.L} = p \cdot P_a$$

Q. 41 What does average total inspection mean?

Ans. Average total inspection relates to rectification of inspection plan. Average number of items inspected per inspection lot under a particular sampling plan is known as *average total inspection* (A.T.I).

Q. 42 What is blind sampling?

Ans. Drawing items from a lot or process without any regard to their quality is called *blind sampling*.

Q. 43 Give the definitions of average sample number (*ASN*) and *ASN* curve.

Ans. Average sample number is the expected size of the sample required to arrive at a decision about the acceptance or rejection of a lot under sampling inspection plan. This depends on p, the actual proportion of defectives present in the lot.

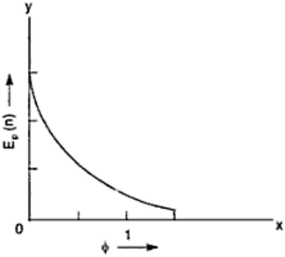

Fig. 19.6. ASN curve

The graph of the average sample numbers '$E_p(n)$' against proportion 'p' of defectives is known as *ASN curve*.

A sampling plan resulting into the lowest *ASN* curve is considered batter than any other sampling plan provided all other factors are kept constant.

Q. 44 Delineate operating characteristic (*OC*) curve of a sampling plan.

Ans. Let a large number of lots containing a given fraction defective p are submitted to a given plan and out of these lots, a proportion of lots, 'L_p' is accepted. Obviously, this proportion differs from one plan to the other. Now it becomes necessary to know what proportion of lots under each plan will be accepted for each possible quality p of submitted lots. It is evident that a plan is unsuitable if it accepts too many lots of unsatisfactory quality or rejects too many lots of satisfactory quality. This information is very well represented by *OC* curve of the plan. The *OC* curve is plotted by taking p along abscissa and L_p along ordinate. A typical *OC* curve is shown below:

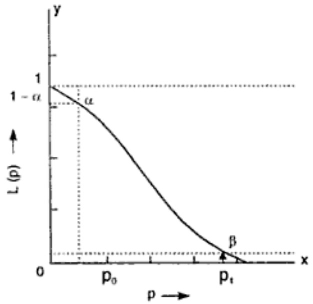

Fig. 19.7. Operating characteristic curve

Here it is emphasised that the *OC* curve should not be taken to show the probability that an accepted lot will be of quality p but it gives the probability of accepting a lot of quality p.

In short, a curve showing the proportion (percentage) of submitted lots that will be accepted on the basis of sampling plan for each percentage of defectives items in the lots under consideration is known as *OC* curve.

More steep is the *OC* curve, better it is as this provides greater protection to consumer. In nutshell, it can be said that the *OC* curve of an acceptance sampling plan reveals the ability of the plan to distinguish between good and bad lots.

In the Fig. 19.7, p_0 is the *limiting acceptable quality and* p_1 *the limiting admissible* quality. α is the producer's risk and β is the consumer's risk.

Q. 45 Discuss sampling inspection plans in reference to statistical quality control.

Ans. There are various sampling inspection plans which can be grouped into three classes, viz. – single, double and sequential sampling plans. A plan which is good for some product and/or a supplier may not be good for the other. So there is great need for selecting a plan best suited for a product and supplier in question. A plan is suitable in the sense that it holds certain properties as given below:

(i) The sampling plan ensures that the lots of high quality will be accepted and those of poor quality will be rejected.

(ii) The amount of inspection required under the plan is neither very high nor too low.

(iii) The plan encourages the producer to improve the quality of the product if it is not up to the mark.

(iv) The plan is not too much complicated so that it cannot be administered and operated with ease.

Q. 46 Delineate single sampling inspection plan and its implications.

Ans. A sampling plan in which a decision about the acceptance or rejection of a lot is based on one sample that has been inspected.

Suppose that the lot consists of N items having a proportion of defectives as P and in all D defectives, *i.e.*, $D = NP$. A sample of size n is drawn from the submitted lot which contains d defectives. Let c be the maximum allowable number of defectives in the sample. c is known as *acceptance number*. The quantities n and c can either be determined by *lot quality*

$x_i = 1$ if the item is found to be defective,

and $x_i = 0$ if the item is found to be non-defective.

Suppose the sequential sampling continues till the m items are inspected. So we have variates $x_1, x_2, ..., x_m$ which are independently and identically distributed with probability mass function $f(x, p)$. So the probability mass function,

$$f(x, p) = \prod_{i=1}^{m} p^{x_i}(1-p)^{1-x_i}$$

Under H_1, the probability mass function (p.m.f) for the sample $x_1, x_2, ..., x_m$ is,

$$p_{1m} = \prod_{i=1}^{m} p_1^{x_i}(1-p_1)^{1-x_i}$$

and under H_0, the p.m.f is,

$$p_{0m} = \prod_{i=1}^{m} p_0^{x_i}(1-p_0)^{1-x_i}$$

Thus, the likelihood ratio λ_m is given as,

$$\lambda_m = \frac{\prod_{i=1}^{m} f(x_i, p_1)}{\prod_{i=1}^{m} f(x_i, p_0)}$$

$$= \frac{p_1(m)}{p_0(m)}$$

$$= \frac{\prod_{i=1}^{m} p_1^{x_i}(1-p_1)^{1-x_i}}{\prod_{i=1}^{m} p_0^{x_i}(1-p_0)^{1-x_i}}$$

$$= \frac{p_1^{\Sigma x_i}(1-p_1)^{m-\Sigma x_i}}{p_0^{\Sigma x_i}(1-p_0)^{m-\Sigma x_i}}$$

Supposing that out of m units inspected, d_m are defective. Then,

$$\lambda_m = \frac{p_1(m)}{p_0(m)}$$

$$= \frac{p_1^{d_m}(1-p_1)^{m-d_m}}{p_0^{d_m}(1-p_0)^{m-d_m}}$$

The decision about the acceptance, rejection or continuance of sampling process is taken according to the following rules.

Accept the lot if

$$\lambda_m \le \frac{\beta}{1-\alpha}$$

Reject the lot if

$$\lambda_m \ge \frac{1-\beta}{\alpha}$$

Continue sampling if

$$\frac{\beta}{1-\alpha} < \lambda_m < \frac{1-\beta}{\alpha}$$

Now substituting for λ_m in terms of the p.m.f., the inequalities can be expressed as:

Accept the lot provided,

$$\left(\frac{p_1}{p_0}\right)^{d_m}\left(\frac{1-p_1}{1-p_0}\right)^{m-d_m} \le \frac{\beta}{1-\alpha}$$

Taking logarithm we get

$$d_m\left\{\log\frac{p_1}{p_0} - \log\frac{1-p_1}{1-p_0}\right\} \le m\log\frac{1-p_0}{1-p_1} + \log\frac{\beta}{1-\alpha}$$

or $$d_m \le \frac{m\log\frac{1-p_0}{1-p_1} + \log\frac{\beta}{1-\alpha}}{\log\frac{p_1}{p_0} - \log\frac{1-p_1}{1-p_0}}$$

$$d_m \le \frac{\log\frac{\beta}{1-\alpha}}{\log\frac{p_1}{p_0} - \log\frac{1-p_1}{1-p_0}} + m\frac{\log\frac{1-p_0}{1-p_1}}{\log\frac{p_1}{p_0} - \log\frac{1-p_1}{1-p_0}}$$

Suppose R.H.S. of the inequality is denoted by a_m. Thus, accept the lot if

$$d_m \le a_m$$

Similarly reject the lot if

$$d_m \ge r_m$$

where,

$$r_m = \frac{\log \dfrac{1-\beta}{\alpha}}{\log \dfrac{p_1}{p_0} - \log \dfrac{1-p_1}{1-p_0}} + m \frac{\log \dfrac{1-p_0}{1-p_1}}{\log \dfrac{p_1}{p_0} - \log \dfrac{1-p_1}{1-p_0}}$$

Continue sampling if

$$a_m < d_m < r_m$$

Q. 51 How can sequential sampling plan be implemented graphically under S.P.R.T.?

Ans. We know a_m and r_m represent straight lines which are parallel to each other as the coefficient of m in both lines is same. In other words, both the lines have slope equal to

$$\left\{\log \frac{1-p_0}{1-p_1}\right\} \Big/ \left\{\log \frac{p_1}{p_0} - \log \frac{1-p_1}{1-p_0}\right\}$$

On a graph paper, represent m on abscissa and cumulative number of defectives d_m on ordinate choosing suitable scales. Draw the lines a_m and r_m in the first quadrant of the graph as shown in the figure below.

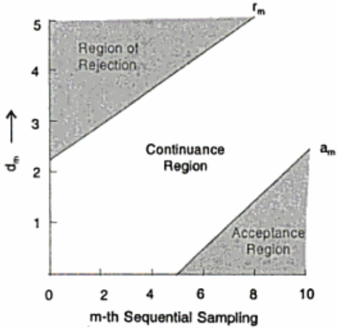

Fig. 19.8. Decision space diagram

Start from plotting the first sample point. If it lies on or below the line a_m, accept the lot. If the plotted point lies on or above the line r_m, reject the lot. In either situation terminate the sampling process as a decision is reached. If the point lies between a_m and r_m, the process of sampling and inspection continues. Again select another item and inspect it. Plot the point and see where does this point lies. The process is continued till a decision is taken about the acceptance or rejection of the lot.

Q. 52 Give *OC* function of a sequential probability ratio test.

Ans. The *OC* function of a S.P.R.T. for testing H_0: $p = p_0$ vs. $H_1 : p = p_1$, is given by

$$L(p) = \frac{\left(\dfrac{1-\beta}{\alpha}\right)^h - 1}{\left(\dfrac{1-\beta}{\alpha}\right)^h - \left(\dfrac{\beta}{1-\alpha}\right)^h} \qquad (1)$$

The value of h is obtained by the equation,

$$p\left(\frac{p_1}{p_0}\right)^h + (1-p)\left(\frac{1-p_1}{1-p_0}\right)^h = 1 \qquad (2)$$

It is cumbersome to solve equation (2) for h. Hence, we rearrange the equation (2) for p in terms of h, *i.e.*,

$$p = \frac{1-\left(\dfrac{1-p_1}{1-p_0}\right)^h}{\left(\dfrac{p_1}{p_0}\right)^h - \left(\dfrac{1-p_1}{1-p_0}\right)^h} \qquad (3)$$

Now to draw an *OC* curve, we calculate p for different arbitrary chosen values of h. For these values of h, calculate $L(p)$ from (1). Plot the points $[p, L(p)]$ on graph and draw *OC* curve. As a norm, at least five points be plotted to have correct shape of the curve.

It is now trivial to verify that when there is no defective, *i.e.*, $p = 0$, then $L(0) = 1$. Also when all are defective items, *i.e.*, $p = 1$, then $L(1) = 0$.

Q. 53 What do you understand by expected sample size of a sequential sampling plan and how to determine it?

Ans. We know that in sequential sampling plan, the sample size n is a random variable and hence it can be determined with the help of the probability function $f(x, p)$. Expected sample size is also known as average sample number (A.S.N.). The expected sample size or A.S.N. of a S.P.R.T. for testing H_0: $p = p_0$ vs. $H_1 : p = p_1$, where A.Q.L. = p_0 and L.T.D.P = p_1, can be determined by the formula,

$$E(n) = \frac{L(p)\log\dfrac{\beta}{1-\alpha} + \{1 - L(p)\}\log\dfrac{1-\beta}{\alpha}}{E(z)} \qquad (1)$$

where, $$E(z) = p\log\frac{p_1}{p_0} + (1-p)\log\frac{1-p_1}{1-p_0} \qquad (2)$$

Formula (1) is known as A.S.N. function. A good idea for A.S.N. curve can be had by suitably choosing five points which are easy to locate corresponding to the values $p = 0, 1, p_0, p_1$ and

$$\left[\left\{\log\frac{1-p_0}{1-p_1}\right\} \Big/ \left\{\log\frac{p_1}{p_0} + \log\frac{1-p_0}{1-p_1}\right\}\right]$$

Q. 54 Make comparative statement on single, double and sequential sampling plans.

Ans. Single and double sampling are equally good in respect of operating characteristic and A.S.N. But, double sampling results into 25-33 per cent saving in sampling whereas sequential sampling on the average requires 33-50 per cent less inspection of sampling units as compared to single sampling.

Q. 55 What problems are faced in case of sampling inspection for variables as compared to fraction defectives?

Ans. The methods developed for fraction defectives are also applicable for variables.

But the defects will be due to different measurements like length, width, diameter, chemical composition, etc. Hence, different charts have to be prepared for each variable separately and then the results are to be pooled to arrive at a conclusion about the acceptance or rejection of a lot. When the results are contradictory to one another, it is very difficult to reach a single decision. The ideas can be summarised as follows:

(i) For a given sample size, better quality protection is usually obtained in case of inspection by variables than by attributes.

(ii) Smaller samples may be used with variables than with attributes.

(iii) The variables criterion is given more weight to attain the desired quality protection.

(iv) Errors of measurements are better surfaced with variables information.

(v) The use of variables as a decision criteria about the acceptance and rejection of a lot is more cumbersome and time consuming than with attributes.

SECTION-B

Fill in the Blanks

Fill in the suitable word(s) or phrase(s) in the blanks:

1. Variation in the measurements of items produced under any system is _________.
2. The variation due to _________ factors is tolerable.
3. Statistical quality control takes care of the variation due to _________ causes.
4. Through statistical quality control, one finds whether the process is under _________ or not.
5. Whether the variability in the manufactured items is within tolerance limits or not can be ascertained through _________.
6. Control charts _________ the _________ within which the variability is tolerable.
7. A control chart contains _________ lines.
8. Most frequently used five control charts are _________ charts.
9. In control charts we establish _________ limits.
10. Control limits utilise the constant factors given by _________.

47. The decision based on inspection by variables is _________ reliable than by attributes.

48. Sampling plans for variables do _________.

49. The average fraction defective $\overline{p}$ is known as _________.

50. The probability of rejecting a lot $\overline{p}$ defectives is known as _________.

51. The probability of accepting a lot with fraction defectives p_t is known as _________.

52. Inspection by attributes is _________ and _________ as compared to inspection by variables.

53. Inspection by attributes is in _________.

54. The per cent defectives in a lot below which only the lot is acceptable is known as _________.

55. The minimum quality level at which the lot is rejected is called _________.

56. Average percentage of defectives remaining in an outgoing lot is known as _________.

57. The percentage of maximum defective items finally accepted in a lot is known as _________.

58. Drawing units from a lot or process irrespective of their quality is known as _________.

59. The expected sample size required to arrive at a decision about the lot is called the _________.

60. The average sample number depends on the _________ present in the lot.

61. The graph drawn for proportion defectives and average sample number is known as _________.

62. The sampling inspection plan resulting into the lowest A.S.N. curve is _________.

63. Operating characteristic (*OC*) curve depicts the probability of _________ a lot of quality *p*.

64. More steep is the *OC* curve, _________ it is.

65. OC curve is a device which reflects on the quality of a sampling plan to differentiate between _________.

66. A sampling inspection plan is good if it can correctly decide about the _________ or _________ of the lot.

67. A good sampling inspection plan requires an _________ number of inspection of items.

68. A sampling inspection plan is considered satisfactory if it is not _________.

69. The probability of obtaining *d* defectives in a sample of size *n* from a lot having *N* items with *p* as proportion of defectives is _________.

70. The curve depicting the probabilities of different fraction defectives as a function of the lot quality of finite lots is called _________.

71. The curve giving the probability of acceptance of a lot as a function of the product quality of infinite lots is categorised as _________.

72. Type *A OC* curve helps to evaluate _________ of individual lots.

73. Type *B OC* curve usually evaluate _________.

74. Type *A OC* curve utilises _________ probabilities.

75. Type *B OC* curve is based on _________ probabilities.

76. Type *B OC* curve always lies _________ the Type *A OC* curve if plotted on the same graph.

77. If the number of defectives in a sample lies between the acceptance and rejection limits, we make use of _________.

78. In sequential sampling decision about the acceptance or rejection of a lot is taken after the selection of _________.

79. Sequential sampling plan requires _________ amount of inspection.

80. The theory of sequential sampling plan was originally given by _________.

81. Sequential analysis was developed in the year _________.

82. In a sequential decision problem, the total space is divided into _________ regions.

83. The lines dividing the space into region of acceptance, rejection and continuance are obtained through ________.

84. The lines dividing the space into three regions in S.P.R.T. are always ________.

85. The graph of the points [p, L (p)] under S.P.R.T. provides ________.

86. In sequential sampling plan, the sample size is a ________.

87. The expectation of the sample size n in sequential sampling is known as ________.

88. In double sampling plan, there is a ________ per cent saving in sampling inspection as compared to single sampling plan.

89. In sequential sampling, the sampling inspection is reduced by ________ per cent as compared to single sampling plan.

90. Sampling inspection by variables provides ________ protection than by attributes.

91. Errors of measurements are more prominent in sampling inspection by ________ than by ________.

92. To ensure that the proportion of defective items in the manufactured product is not beyond certain limits is called ________.

93. Control on the quality of the product by critical examination at strategic points is called ________.

94. The control charts help to achieve ________.

95. Sampling inspection plans are meant for ________.

96. Sampling inspection plans were pioneered by ________.

97. The inspection of 25 aircrafts revealed that there are 350 missing rivets in all. The appropriate control chart in this situation which can be prepared is ________.

98. A list shows the number of non-conforming items in each of the 25 samples, each sample consisting of 40 items. The appropriate statistical control chart in this situation is ________.

99. The number of mistakes committed by a mechanic in 20 samples of assembled radios are 25. The lower control limit for c-chart from the given data is ________.

100. A factory produces 300 articles per day. After inspecting 3,000 articles on 30 consecutive days, 270 articles were non-conforming to the specification. The upper control limit for p-chart is ________.

SECTION-C

Multiple Choice Questions

Select the correct alternative out of given ones:

Q. 1 Variation in the items produced in a factory may be due to:

(a) chance factors
(b) assignable causes
(c) both (a) and (b)
(d) none of the above

Q. 2 Chance or random variation in the manufactured product is:

(a) controlable
(b) not controlable
(c) both (a) and (b)
(d) none of the above

Q. 3 Chance variation in respect of quality control of a product is:

(a) tolerable
(b) not effecting the quality of a product
(c) uncontrolable
(d) all the above

Q. 4 The causes leading to vast variation in the specifications of a product are usually due to:

(a) random process
(b) assignable causes

(c) non-traceable causes
(d) all the above

Q. 5 Variation due to assignable causes in the product occurs due to:
(a) faulty process
(b) carelessness of operators
(c) poor quality of raw material
(d) all the above

Q. 6 The faults due to assignable causes:
(a) can be removed
(b) cannot be removed
(c) can sometimes be removed
(d) all the above

Q. 7 Control charts in statistical quality control are meant for:
(a) describing the pattern of variation
(b) checking whether the variability in the product is within the tolerance limits or not
(c) uncovering whether the variability in the product is due to assignable causes or not
(d) all the above

Q. 8 Control charts consist of:
(a) three control lines
(b) upper and lower control limits
(c) the level of the process
(d) all the above

Q. 9 Main tools of statistical quality control are:
(a) shewhart charts
(b) acceptance sampling plans
(c) both (a) and (b)
(d) none of the above

Q. 10 Trial control limits for mean with usual notations are:
(a) U.C.L. = $\overline{X} + A_1\overline{S}$, C.L. = $\overline{X}$ and L.C.L. = $\overline{X} - A_1\overline{S}$
(b) U.C.L. = $\overline{X} + A_1\overline{S}$, C.L. = $A_1\overline{S}$ and L.C.L. = $\overline{X} - A_1\overline{S}$
(c) U.C.L. = $\overline{X} + A_2\overline{S}$, C.L. = $\overline{X}$ and L.C.L. = $\overline{X} - A_2\overline{S}$
(d) none of the above

Q. 11 The trial control limits for σ-chart with $\overline{S}$ as mean standard deviation and usual constant factors are:
(a) U.C.L. = $\overline{S} + B_1\overline{S}$, C.L. = $\overline{S}$ and L.C.L. = $\overline{S} - B_4\overline{S}$
(b) U.C.L. = $B_4\overline{S}$, C.L. = B_4 and L.C.L. = $B_3\overline{S}$
(c) U.C.L. = $B_4\overline{S}$, C.L. = $\overline{S}$ and L.C.L. = $B_3\overline{S}$
(d) U.C.L. = $B_3\overline{S}$, C.L. = $\overline{S}$ and L.C.L. = $B_4\overline{S}$

Q. 12 The relation between expected value of R and S.D. σ with usual constant factors is:
(a) $E(R) = d_1\sigma$
(b) $E(R) = d_2\sigma$
(c) $E(R) = D_1\sigma$
(d) $E(R) = D_2\sigma$

Q. 13 The control limits for R-chart with a known specified range R' and usual constant factors are:
(a) $\text{U.C.L.}_{R'} = (d_2 + 3d_3)\sigma_{R'}$, $\text{C.L.}_{R'} = d_2\sigma_{R'}$ and $\text{L.C.L.}_{R'} = (d_2 - 3d_3)\sigma_{R'}$
(b) $\text{U.C.L.}_{R'} = D_2 R'$, $\text{C.L.}_{R'} = d_2\sigma_{R'}$ and $\text{L.C.L.}_{R'} = D_1\sigma_{R'}$
(c) either (a) or (b)
(d) neither (a) nor (b)

Q. 14 When the value of the population range R is not known, then for $\overline{X}$-chart, the trial control limits with usual constant factors are:
(a) U.C.L. = $\overline{X} + A_3\overline{R}$, C.L. = $\overline{X}$ and L.C.L. = $\overline{X} - A_2\overline{R}$
(b) U.C.L. = $\overline{X} + A_3\overline{R}$, C.L. = $\overline{X}$ and L.C.L. = $\overline{X} - A_3\overline{R}$

(c) $\text{U.C.L.} = A_3\,\overline{R}$, $\text{C.L.} = \overline{X}$ and $\text{L.C.L.} = A_2\overline{R}$

(d) $\text{U.C.L.} = \overline{X} + A_3\,\overline{R}$, $\text{C.L.} = \overline{X}$ and $\text{L.C.L.} = \overline{X} - A_3\overline{R}$

Q. 15 The trial control limits for *R*-chart with usual constant factors are:

(a) $\text{U.C.L.} = D_4\,R$, $\text{C.L.} = R$ and $\text{L.C.L.} = D_3R$

(b) $\text{U.C.L.} = D_4\,\overline{R}$, $\text{C.L.} = \overline{R}$ and $\text{L.C.L.} = D_3\overline{R}$

(c) $\text{U.C.L.} = D_4\,\overline{R}$, $\text{C.L.} = \overline{R}$ and $\text{L.C.L.} = D_4\overline{R}$

(d) all the above

Q. 16 *R*-charts are preferable over σ-charts because:

(a) *R* and S.D. fluctuate together in case of small samples

(b) *R* is easily calculable

(c) *R*-charts are economical

(d) all the above

Q. 17 The Shewhart control charts are meant:

(a) to detect whether the process is under statistical quality control

(b) to find the assignable causes

(c) to reflect the selection of samples

(d) all the above

Q. 18 3-sigma control limits of defectives having a given value of fraction defectives p' are:

(a) $\text{U.C.L.} = p' + \sqrt{\dfrac{3\,p'\,q'}{n}}$, $\text{C.L.} = p'$ and

$\text{L.C.L.} = p' - \sqrt{\dfrac{3\,p'\,q'}{n}}$

(b) $\text{U.C.L.} = p' + \dfrac{1}{3}\sqrt{\dfrac{p'\,q'}{n}}$, $\text{C.L.} = p'$ and

$\text{L.C.L.} = p' - \dfrac{1}{3}\sqrt{\dfrac{p'\,q'}{n}}$

(c) $\text{U.C.L.} = p' + 3\sqrt{\dfrac{p'\,q'}{n}}$, $\text{C.L.} = p'$ and

$\text{L.C.L.} = p' - 3\sqrt{\dfrac{p'\,q'}{n}}$

(d) all the above

Q. 19 3-sigma trial control limits with p' as mean number of defectives based on a sample of size *n* are:

(a) $\text{U.C.L.} = n\,\overline{p} + \sqrt{n\overline{p}(1-\overline{p})}$, $\text{C.L.} = \overline{p}$

and $\text{L.C.L.} = n\,\overline{p} - \sqrt{n\,\overline{p}(1-\overline{p})}$

(b) $\text{U.C.L.} = n\overline{p} + 3\sqrt{n\overline{p}(1-\overline{p})}$, $\text{C.L.} = n\overline{p}$

and $\text{L.C.L.} = n\,\overline{p} - 3\sqrt{n\,\overline{p}(1-\overline{p})}$

(c) $\text{U.C.L.} = n\,\overline{p} + 3\sqrt{n\overline{p}(1-\overline{p})}$, $\text{C.L.} = \overline{p}$

and $\text{L.C.L.} = \overline{p} - 3\,\sqrt{n\,\overline{p}(1-\overline{p})}$

(d) none of the above

Q. 20 2-sigma trial control limits for *c*-chart for equal size samples are given as:

(a) $\text{U.C.L.} = \overline{C} + 3\sqrt{\overline{C}}$, $\text{C.L.} = \overline{C}$ and

$\text{L.C.L.} = \overline{C} - 3\sqrt{\overline{C}}$

(b) $\text{U.C.L.} = \overline{C} + \sqrt{2\overline{C}}$, $\text{C.L.} = 2\overline{C}$ and

$\text{L.C.L.}\ \overline{C} - \sqrt{2\overline{C}}$

(c) $\text{U.C.L.} = \overline{C} + 2\sqrt{\overline{C}}$, $\text{C.L.} = \overline{C}$ and

$\text{L.C.L.} = \overline{C} - 2\sqrt{\overline{C}}$

(d) $\text{U.C.L.} = \overline{C} + 2\sqrt{\overline{C}}$, $\text{C.L.} = C$ and

$\text{L.C.L.} = \overline{C} - 2\sqrt{\overline{C}}$

Q. 21 A control chart based on known parameter values is:

(a) more advantageous than the one based on estimated values

(b) complicated than that of the control chart based on estimated values
(c) less reliable than the control chart based on estimated values
(d) all the above

Q. 22 If μ and σ are the process mean and S.D., then the control limits $\mu \pm 3\sigma$ are known as:
(a) modified control limits
(b) natural control limits
(c) specified control limits
(d) none of the above

Q. 23 The control limits delimited by the consumer are called:
(a) modified control limits
(b) natural control limits
(c) specified control limits
(d) none of the above

Q. 24 For the subgroups of size *n*, the upper and lower control limits for rejection of a lot are termed as:
(a) modified control limits
(b) natural control limits
(c) specified control limits
(d) none of the above

Q. 25 Acceptance sampling plans are preferable due to:
(a) the economy in inspection
(b) protection to perishable items
(c) increased efficiency in the inspection of items
(d) all the above

Q. 26 Cent per cent inspection is preferable when:
(a) a defective item may cause danger to life
(b) a defective item may stop the function as a whole
(c) the incoming items are of very poor quality
(d) all the above

Q. 27 One may require no inspection of items if:
(a) the demand of item is too much
(b) the items produced are of very high quality
(c) the inspection cost is very high
(d) all the above

Q. 28 The sampling inspection procedures adopted in statistical quality control are of:
(a) three types
(b) two types
(c) mixed types
(d) none of the above

Q. 29 The decision about the lot under sampling inspection prodecures can be of:
(a) two types
(b) three types
(c) no use
(d) none of the above

Q. 30 Sampling inspection procedure by variables as compared to by attributes is:
(a) more prevalent
(b) not practised
(c) less prevalent
(d) all the above

Q. 31 A defect in an item is classified as minor if:
(a) it stops the function of the process
(b) it is easily detectable
(c) it impairs the life of the system
(d) all the above

Q. 32 A defect in an item is classified as major if:
(a) its stops the function of the process
(b) it is not detectable
(c) it shortens the life of the system
(d) all the above

Q. 33 The probability of rejecting a lot having $\bar{p}$ as the process average defectives is known as:
(a) consumer's risk
(b) type II error
(c) producer's risk
(d) all the above

Q. 34 The probability of accepting a lot with fraction defective P_t is known as:
(a) consumer's risk
(b) type I error
(c) producer's risk
(d) none of the above

Q. 35 The decision about the acceptance or rejection of a lot by variables is:
(a) less reliable than by attributes

(b) more reliable than by attributes
(c) not feasible
(d) none of the above

Q. 36 Sampling inspection by variables requires:
(a) fewer inspection than by attributes
(b) greater inspection than by attributes
(c) equal inspection to that of by attributes
(d) none of the above

Q. 37 Inspection by attributes over inspection by variables requires:
(a) less time
(b) less skill
(c) less calculations
(d) all the above

Q. 38 The small fraction of defectives p_1, on the basis of which a lot is not rejected except for a small number of times, is called:
(a) lot tolerance percentage defective (LTPD)
(b) rejecting quality level (RQL)
(c) acceptance quality level (AQL)
(d) none of the above

Q. 39 If the percentage defective p_t or more in a lot is not acceptable to the consumer, then it is known as:
(a) let tolerance percentage defective (LTPD)
(b) rejecting quality level (RQL)
(c) both (a) and (b)
(d) none of the above

Q. 40 The maximum limit of percentage defectives in a finally accepted product is called:
(a) acceptance quality level (AQL)
(b) average outgoing quality limit (AOQL)
(c) lot tolerance percentage defective (LTPD)
(d) all the above

Q. 41 Drawing items from a lot without giving any heed to their quality is known as:
(a) random sampling
(b) purposive sampling
(c) systematic sampling
(d) blind sampling

Q. 42 The expected sample size required to arrive at a decision about the lot is called:
(a) a random variable
(b) average sample number (ASN)
(c) both (a) and (b)
(d) none of the above

Q. 43 The graph of the proportion of defectives in the lot against average sample number is:
(a) *OC* curve
(b) A.S.N. curve
(c) power curve
(d) all the above

Q. 44 The lowest A.S.N. curve of a sampling plan as compared to any other sampling plan under similar conditions is considered:
(a) better
(b) inferior
(c) useless
(d) none of the above

Q. 45 A curve showing the probability of accepting a lot of quality p is known as:
(a) *OC* curve
(b) A.S.N. curve
(c) Compertz curve
(d) none of the above

Q. 46 *OC* curve reveals the ability of the sampling plan to distinguish between:
(a) good and bad lots
(b) good and bad sampling plans
(c) good and bad product
(d) all the above

Q. 47 A sampling plan is good for use provided:
(a) it ensures correct decision about the acceptance or rejection of a lot
(b) it requires an adequate number of inspections
(c) it is not complicated
(d) all the above

Q. 48 The decision about the acceptance or rejection of a lot through a single sampling plan is reached by considering:
(a) number of defectives in the sample and acceptance number
(b) the acceptance quality level
(c) lot tolerance percentage defective
(d) all the above

Q. 49 Type *A* and Type *B OC* curves differ from one another in respect of:

(a) hypergeometric and binomial probabilities
(b) finite and infinite sizes of the lots
(c) consumer's and producer's risks
(d) all the above

Q. 50 In a double sampling plan, a decision about the acceptance or rejection of a lot:

(a) will never reach
(b) will always reach
(c) will sometimes reach
(d) none of the above

Q. 51 The decision in a sequential sampling scheme is taken:

(a) after inspecting the sample as a whole
(b) after selection and inspection of items one by one
(c) both (a) and (b)
(d) none of the above

Q. 52 In a decision problem under sequential sampling scheme, the total decision space is divided into:

(a) two regions namely, acceptance and rejection regions
(b) *n* equally spaced regions consisting of alternating acceptance and rejection regions
(c) three regions namely, the regions of acceptance, rejection and continuance
(d) none of the above

Q. 53 If p is the unknown proportion of defectives in the lot and p_0 and p_1 are two values such that $p_0 < p_1$. Also

$$\alpha = P\text{ (reject the lot } | p \le p_0)$$

$$\beta = P\text{ (accept the lot } | p \ge p_1)$$

Then the operating characteristic function $L(p)$ takes the values as:

(a) $L(p) = 1 - \alpha$ when $p \le p_0$ and $L(p) = \beta$ when $p = p_1$
(b) $L(p) = 1 - \alpha$ when $p < p_0$ and $L(p) = \beta$ when $p \ge p_1$
(c) $L(p) = 1 - \alpha$ when $p \ge p_0$ and $L(p) = 1 - \beta$ when $p \ge p_I$
(d) $L(p) = 1 - \alpha$ when $p \le p_0$ and $L(p) = 1 - \beta$ when $p \ge p_1$

Q. 54 In a sequential probability ratio test, the criterion for acceptance of the lot with usual notations is:

(a) $\lambda_m \le \frac{\beta}{1-\alpha}$

(b) $\lambda_m \ge \frac{\beta}{1-\alpha}$

(c) $\lambda_m \le \frac{1-\beta}{\alpha}$

(d) $\lambda_m \ge \frac{1-\beta}{\alpha}$

Q. 55 In sequential probability ratio test, the lot is rejected if (with usual notations), the following inequality holds:

(a) $\lambda_m \le \frac{1-\beta}{\alpha}$

(b) $\lambda_m \ge \frac{1-\beta}{\alpha}$

(c) $\lambda_m \le \frac{\beta}{1-\alpha}$

(d) $\lambda_m \ge \frac{\beta}{1-\alpha}$

Q. 56 An inspector continues his inspection of items selected one after the other so long as the following inequalities hold:

(a) $\lambda_m > \frac{\beta}{1-\alpha}$ and $\lambda_m < \frac{1-\beta}{\alpha}$

(b) $\frac{\beta}{1-\alpha} < \lambda_m$ and $\frac{1-\beta}{\alpha} > \lambda_m$

(c) $\frac{\beta}{1-\alpha} < \lambda_m < \frac{1-\beta}{\alpha}$

(d) all the above

Q. 57 In sequential probability ratio test, the lines dividing the total space into regions are:

(a) perpendicular to each other
(b) passing through the origin
(c) parallel to each other
(d) all the above

Q. 58 When there is no defective in the lot, the *OC* function for $p = 0$ is:
(a) $L(0) = 0$
(b) $L(0) = 1$
(c) $L(0) = \infty$
(d) none of the above

Q. 59 When the lot contains all defectives, the *OC* function for $p = 1$ is:
(a) $L(\mathrm{p}) = 0$
(b) $L(\mathrm{p}) = 1$
(c) $L(\mathrm{p}) = \infty$
(d) none of the above

Q. 60 If P (reject a lot | p_0) = α and P (accept a lot | p_1) = β in case of S.P.R.T. for testing $H_0 : p = p_0$ vs. $H_1 : p = p_1$, the *O.C.* function:
(a) $L(p_0) = 1 - \beta$
(b) $L(p_0) = \alpha$
(c) $L(p_0) = \beta$
(d) $L(p_0) = 1 - \alpha$

Q. 61 In a sequential sampling plan, the sample size is:
(a) a discrete random variable
(b) a continuous random variable
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 62 Expected sample size of S.P.R.T. is called:
(a) discrete random variable
(b) continuous random variable
(c) average sample number
(d) all the above

Q. 63 Which of the following statement is correct?
(a) Single sampling plan requires maximum inspection
(b) In doubling sampling plan, there is 25-33 per cent saving in sampling inspection as compared to single sample plan.
(c) In double sampling plan there is 33-50 per cent saving in sampling inspection as compared to single sampling plan
(d) all the above

Q. 64 A sequential sampling plan is:
(a) an infinite process
(b) the process requiring much more sampling units than a fixed size sample
(c) a process in which sampling terminates with probability one
(d) all the three

Q. 65 Which of the following statement is/are correct?
(a) Sampling inspection by variables provides better quality protection than by attributes
(b) Sampling inspection by variables require less inspection than by attributes
(c) Errors of measurements are better surfaced in sampling inspection by variables than by attributes
(d) all the above

ANSWERS

SECTION-B

(1) inevitable (2) chance (3) assignable (4) control (5) control charts (6) delimit; control band (7) three (8) $\overline{X}$, R, σ, p and c (9) 3-sigma (10) Shewhart (11) A, A_1 and A_2 (12) c_2, B_1, B_2, B_3 and B_4 (13) standard deviation (14) d_2, D_1, D_2, D_3 and D_4 (15) between (16) within (17) R-chart (18) σ-charts (19) σ (20) assignable causes (21) material; machines (22) dichotomous (23) binomial (24) larger (25) large (26) Poisson (27) money; time (28) greater efficiency (29) losses (30) sampling (31) manufacturing processes (32) specification (33) natural (34) fatal (35) high quality (36) purchaser (37) future losses (38) variables (39) two (40) major; minor (41) major (42) minor (43) producer's process (44) Type I (45) Type II (46) lesser (47) more (48) exist (49) producer's process average (50) producer's risk (51) consumer's risk (52) convenient; economical (53) vogue (54) L.T.P.D. (55) R.Q.L. (56) A.O.Q. (57) A.O.Q.L. (58) blind sampling (59) Average sample number (60) proportion of defectives (61) A.S.N. curve (62) best (63) accepting (64) better (65) good and bad lots (66) acceptance; rejection (67) adequate

Chapter 20

Vital and Population Statistics

SECTION-A

Short Essay Type Questions

Q. 1 What events are covered under vital statistics?

Ans. Vital statistics maintains the records of marriage, divorce, adoption, legitimation, recognition, annulment and legal separation, live births, deaths, foetal deaths, still births, sickness, etc. It is a part of demography.

Q. 2 Which records are maintained under population statistics?

Ans. Population statistics usually mean the maintenance of records of population of countries, regions or community and distribution of population at a point of time. It also encompasses the population growth, immigration, emigration, fecundity, education, etc.

Q. 3 Are vital statistics and population statistics totally different?

Ans. No, vital statistics and population statistics in general sense are interchangeable and cover all the topics under either head. In practice, no clear-cut distinction is made between the two headings.

Q. 4 Define vital statistics.

Ans. Vital statistics mean the data pertaining to the vital events of a population under reference, especially with regard to births, deaths, marriages, health, migration, etc. In the present times, vital statistics not only consider the number of people but also the quality of human life. Here the vital statistics defined by certain authors are quoted.

1. ***Arthur Newsholme***: The branch of biometry which deals with data and the laws of human mortality, morbidity and demography.
2. **Arthur Newsholme**: Vital statistics may be interpreted in two ways—in a broader sense it refers to all types of population statistics collected by whatever mode, while in a narrower sense if refers only to the statistics derived from the registration of births, deaths and marriages.
3. **B. Benjamin**: Vital statistics are conventionally, numerical records of marriage, births, sickness and deaths by which the health and growth of a community may be studied.

Q. 5 What are the differences between population census and vital statistics?

Ans. The differences between census and vital statistics are delineated below:

(i) Census consists of the complete enumeration of the population of a country or a region under reference and collecting information about the individuals with regard to age, sex, marital status, religion, occupation and other socio-economic aspects. While vital statistics collects information about special events like births, deaths, marriages, health, annulment, divorce, etc.

(ii) Census is like a still picture of the population of a country whereas vital statistics presents a motion picture of the vital events of a population in space and time.

(iii) Population census is a sort of human inventory, while vital statistics provide the analysis of a population with regard to deaths, births, fecundity, growth rates, death rates, etc.

(iv) Population census is conducted at a definite interval of time, mostly ten years, whereas the collection of vital statistics is a continuous process.

(v) Information for population census is to be supplied necessarily by legislation. But in many countries, information about vital events is not compulsorily registered under the act.

Q. 6 What are various uses of vital statistics for a country?

Ans. Various uses of vital statistics for a country are as follows:

(1) ***Analysis of demographic trends.*** Vital statistics reflect on the changing pattern of the population pertaining to the intercensual years. It reveals regarding death rates, birth rates, fecundity, virility of races, etc. The vital rates enable to measure the growth of population.

(2) ***Legal value.*** The records regarding births, marriages, deaths, citizenship, etc., are legal documents. They help to protect their rights in property, insurance, etc.

(3) ***Help in planning of health services.*** On the basis of vital statistics, governments decide about family planning, number of maternity homes and hospitals, etc.

(4) ***Planning of medical research and eradication of diseases.*** The data regarding the causes of deaths enables the governments and social bodies to plan for medical research for the treatment and eradication of diseases.

(5) ***Use in public administration.*** Population estimation and projection, birth and mortality rates are the basic tools in the hands of the administration for maintaining the regular supply of foodgrains, light and water, etc. These data also help to prepare electoral rolls, to create jobs, etc.

(6) ***Useful to actuaries.*** Mortality rates help to estimate life expectancy which is the back bone of various insurance schemes of all life insurance companies.

(7) ***Basis for social reforms.*** The data with regard to marriages, separation, divorce, legitimation, annulment, etc., reveal the existing state of society. On the basis of this information social evils can be reformed so that social security, social justice, economic independence can be created for each section of the society.

Q. 7 Give a brief historical background of collection of vital statistics.

Ans. Before Christ, ecclesiastical societies in Egypt, Greek and Rome used to collect data regarding births, deaths and marriages. In the year 720, Japan made arrangements for the registration of births, deaths and marriages, etc. The registration of vital information also started in the European countries particularly America, Canada and England.

In India, vital statistics are registered and collected under Births, Deaths and Marriages Registration Act, 1886. But under this Act, the supply of information of the vital events was not a binding. But some States made it compulsory under their own legislation like the governments of Tamil Nadu and West Bengal.

The central government of India started a central registration system under an Act known as Birth-Death Registration Act, 1969. In villages, collection of vital statistics is the responsibility of the Head of villages, Patwaris, Lekhpals and chowkidars. In cities, registration work is taken care of by the municipalities.

In spite of the act and administrative support, there is a lot of incompleteness in the information which has made the estimates and results dubious.

Q. 8 What are the weaknesses of registration of vital statistics in India?

Ans. The registration system of vital events in India suffers with many lacunae which are enunciated below:

(i) ***Incompleteness of reporting.*** First, in spite of the Act, the registration of births, deaths and marriages is incomplete due to slackness of registering authorities, *Secondly*, the Act has not been implemented sincerely. *Thirdly* most of the marriages in India are sacramental. Hence, hardly any of them are being entered in the registration offices. *Fourthly*, people are not serious about reporting of the events in registration offices.

(ii) ***Lack of uniformity.*** Different systems and rules for registration of vital statistics exist in different States of India. This has created many discrepancies.

(iii) ***Voluntary registration.*** As per the Births, Deaths and Marriages Registration Act of India, the registration of births, deaths and marriages is voluntary. Hence, a large number of births, deaths and marriages are not registered creating a big gap in information. Some States have made the registration compulsory but it has not been implemented effectively.

(iv) ***Incomplete coverage.*** About $\frac{3}{4}$ th part of the population of India is covered under the registration scheme. So the information is obviously incomplete.

(v) ***Lack of accuracy.*** The information supplied to the registration authorities with regard to births, deaths and marriages is often inadequate due to carelessness of the reporting personnel and registration authorities. Also there is no system to check the correctness of information.

(vi) ***Lack of training.*** The reporting and registration authorities are not fully trained. So they are not sincere to their jobs. This has brought in lot of inaccuracy and incompleteness in vital statistics.

Q. 9 What steps have been taken to improve the registration system of vital statistics?

Ans. Since pre-Independence many steps had been taken to improve the quality and completeness of vital statistics. They are summarily presented below:

(i) Bhor Committee in 1946 recommended that:

(a) a office of the 'Registrar General of Vital and Population statistics' be created at the centre and Registrar's offices at the state level.

(b) The registration of vital events be made compulsory.

(c) Educated and trained personnel be engaged in the job of registration.

(d) The area allotted to them be such that they can visit their areas at least once in a week.

(ii) In 1948, the government of India formed a vital statistics committee to consider the recommendations of the Bhor committee. Further, it suggested that the information about the age of the mother on the day of delivery, order of birth, age on the day of death of a person, reason for death, etc., should also be registered.

(iii) In 1952, a national register of citizens was opened to have comprehensive list of citizens of India.

(iv) To have an estimate of births and death rates in the States, a sample census system was implemented.

(v) A number of demographic surveys are being conducted in different parts of the country to reveal many interesting aspects of vital statistics.

(vi) In 1969, the government of India passed Registration of Births and Deaths Act. This Act has a number of provisions.

Q. 10 What are the provisions of Registration of Births and Deaths Act, 1969?

Q. 16 Explain sampling registration system for collecting vital statistics.

Ans. Sample registration of births and deaths is carried out in almost all parts of the country by selecting a fairly large sample under the aegis of Registrar General and Census Commissioner. Sampling registration system came into operation in July, 1968 in urban areas and in 1967 in rural areas. Under this scheme, a large number of census blocks are randomly selected in rural and urban areas separately as sampling units. The sample covers less than one per cent of the population. Regular information about births, marriages and deaths is collected by the appointed recorders. Also periodic enumeration is done for each family in the selected blocks by a team of investigators. The anomalies found in the daily records and periodic (half-yearly) enumeration are removed by revisiting the families in doubt by a new team of investigators.

Presently, the scheme of permanent house marking has been in operation so as to maintain full coverage at the time of half-yearly checking. Once the survey is complete, all old sampling units are replaced by new ones based on the frame of the latest census.

Following factors are studied under sampling registration system:

At all-India level

(a) Infant mortality.

(b) Age specific mortality rates in rural areas.

(c) Sampling variability of vital rates.

At state level

(a) Differences in birth rates in respect of education, religion and parity.

(b) Extent of institutional and domiciling event.

(c) Sex ratio of vital statistics.

(d) Seasonality in birth and death rates.

The main advantage of S.R.S. is that it provides estimates for rural and urban areas separately. A series of estimates every six month enable the planners to revise or continue their programmes accordingly.

Q. 17 What are the lacunae of sample registration system?

Ans. The sampling registration system fails to record:

(i) volume of migration in the sample areas.

(ii) age and sex composition

(iii) errors in matching

Q. 18 What do you understand by analytical methods for estimating vital statistics or rates?

Ans. Ad-hoc surveys cannot be conducted as soon as we desire to know the vital statistics or rates for any period in between two censal years. All the more such surveys are time consuming and too expensive. Also many errors creep in due to sampling and non-sampling errors. Many a times, the information becomes obsolete by the time sample estimates are available. Hence, analytical methods, which are nothing but the mathematical devices to get the estimates from the available data, are generally useful. These methods are based on certain assumptions and thus provide good estimates provided the assumptions hold true. The most prominent of all assumptions is that population grows at a constant rate.

Q. 19 Give the formula for estimating the population of region or place in a given intercensal year.

Ans. Knowing well the following information,

P_0 – population of the region in the previous census

P_1 – population of the region in the succeeding census

N – number of gap years between two census

n – number of gap years between the given year and the previous census

The interpolation formula for the population estimate $\hat{P}_t$ in the year t is,

$$\hat{P}_t = P_0 + \frac{n}{N}(P_1 - P_0)$$

The above formula provides very good estimate provided the population grows at a constant rate throughout the intercensal years.

Q. 20 What algebraic formula will be applied to estimate the population for an intercensal year '*t*' having known the number of births, deaths and migrants?

Ans. If the information about,

P_0 – the population in the previous census say 0 year

B – number of births between 0 and t

D – number of deaths between 0 and t

I – number of immigrants between 0 and t

E – block of emigrants from 0 to t

is available, the estimate P_t of the population for the year

'*t*' is obtained by the formula,

$$\hat{P}_t = P_0 + (B - D) + (I - E)$$

The estimate from the above formula is very good provided the figures for P_0, B, D, I and E are exact.

Q. 21 How can one estimate population by compound interest formula?

Ans. Malthus believed that population increases in geometric progression if unchecked. Hence, the population of any year '*t*' after the previous census year 0 can be estimated if we know the population P_0 and growth rate r. The population estimate $\hat{P}_t$ can be computed by the formula,

$$\hat{P}_t = P_0 (1 + r)^n$$

where n is the gap between 0 and t years. If r is not known, but the population P_0 for the previous census and P_n for the succeeding census are known, r can be calculated by the formula,

$$r = \sqrt[n]{\frac{P_n}{P_0}} - 1$$

where n represents the number of gap year between two censuses.

Q. 22 Give the formula to estimate the population for the mid-period of the two censuses?

Ans. If P_1 and P_2 are the populations at the two consecutive censuses, the estimated population P_{1-2} for the mid-period is easily calculable by the formula,

$$P_{1-2} = \sqrt{P_1 \times P_2}$$

Q. 23 What is meant by vital rates in general?

Ans. All vital events of life in a population under reference are of prime importance and are thus enumerated in the form of frequencies such as, n_1 births have taken place in a year, n_2 persons have died in a year, n_3 persons have married in a year, etc. These frequencies are not well interpretable from statistical point of view. Hence, the data are analysed statistically and expressed in the form of vital rates. In general vital rates can be expressed as,

$$\text{Vital rate of an event} = \frac{\text{No. of persons in a population covered under the event}}{\text{Total number of persons in the population under reference}}$$

vital rates are usually expressed on the basis of per thousand (‰). Hence the above ratio is multiplied by 1000.

Q. 24 What are the measures of mortality to express death rates?

Ans. Broadly there are three types of death rates which can be used to know about depletion of population. They are:

(i) Crude death rate (C.D.R.)

(ii) Specific death rate (S.D.R.)

(iii) Standardised death rate ($S_T D.R.$)

Q. 25 How can one calculate the crude death rate (C.D.R.)?

Ans. The Crude death rates are the simplest type of death rates. This is defined as the ratio of the number of deaths occurred in a specified period to the population under reference in the same period multiplied by 1000. In practice, the period is usually one year. So the formula can be given as,

$$\text{C.D.R.} = \frac{\text{No. of deaths in the year}}{\text{Annual mean population}} \times 1000$$

C.D.R., if not expressed as per thousand person but as simple ratio, it is equivalent to the probability of death of a person during the year under consideration.

The death rate so calculated is crude in the sense that it gives equal weightage to all age group persons which is not true.

Q. 26 In what sense specific death rates are better than the crude death rates?

Ans. The experience tells that the deaths are not uniformly distributed in all sections of the population. They do differ in different age groups or according to sex, occupation or social status. Hence, it becomes necessary to know the mortality rates for various age groups or in women of child bearing age, etc. As an instance, we know that infant deaths are more than deaths in adolescence.

The child welfare societies will be interested in knowing mortality rates in children of 1 to 10 years age. A women welfare organisation would be interested to know the death rate of women in the reproductive age group of 15 to 45 years and so on. Hence, specific death rates reveal categorically more than that what crude death rates do.

Q. 27 How can you calculate the specific death rate for a specific section of the population?

Ans. The death rate for a particular segment of the population is known as specific death rate. It can, in general, be calculated by the formula,

$$\text{S.D.R.} = \frac{\begin{array}{c}\text{No. of deaths in the specified section}\\ \text{of a population in a given period}\end{array}}{\begin{array}{c}\text{Average total population of the}\\ \text{specified section in the same period}\end{array}}$$

$$= \frac{D_s}{P_s} \times 1000$$

Usually the given period is one year.

Q. 28 Define and discuss age specific death rates.

Ans. The death rates calculated for the population consisting of the specified age group is called age specific death rate (age—S.D.R.). Let the number of deaths in the age group x to $(x + n)$ in a given period $= {}_nD_x$.

Total number of persons in the same period in the age group x to $(x + n) = {}_nP_x$.

Then the age specific death rate (Age – S.D.R.) per thousand person for the age x to $(x + n)$ is,

$$\text{Age S.D.R.} = \frac{{}_nD_x}{{}_nP_x} \times 1000$$

(i) If $x > 0$ and $(x + n) < 1$ year, then, the Age – S.D.R. is known as infant mortality rate. Thus, infant mortality is the number of child deaths under one year of age per 1000 live births.

(ii) The age – S.D.R. of the children under one month of age are called neo-natal mortality rate.

(iii) The death rate due to child birth among the women of child bearing age, *i.e.*, 15-49 years during a given year in a region is known as *maternal mortality* rate.

(iv) If the death rates are calculated for male and female populations separately, they represent the specific death rates.

(v) Age specific death rate per thousand males can be calculated by a similar formula as above for male population only in the age group x to $x + n$ as,

$$\text{Age} - \text{S.D.R}_m = \frac{{}^m_nD_x}{{}^m_nP_x} \times 1000$$

Similarly for females,

$$\text{Age} - \text{S.D.R}_f = \frac{{}^f_nD_x}{{}^f_nP_x} \times 1000$$

The main drawback of age specific death rates is that they are not capable to throw light on mortality conditions prevailing in two different regions.

Q. 29 What purpose is served by standardised death rates, and how are they calculated?

Ans. We know that many socio-economic factors do effect the death rates. Hence, to compare the death rates of two regions or communities, heterogeneity factors such as educational level, income, occupation, etc., be removed. In this way, the population is standardised. If the death rates are calculated by pooling death rates of different categories of populations of a region, they are known as *standardised death rates.*

Age group (Years)	*1988*						*1992*					
	Population			*No. of deaths*			*Population*			*No. of deaths*		
	Male	*Female*	*Total*	*Male*	*Female*	*Total*	*Male*	*Female*	*Total*	*Male*	*Female*	*Total*
< 1	12680	10255	22935	1864	1569	3433	14720	11358	26078	2208	1840	4048
1-4	12711	10284	22995	750	977	1727	14633	11351	25984	717	840	1557
5-14	12523	10200	22723	551	510	1061	14878	11479	26357	610	551	1161
15-24	12690	10327	23017	533	568	1101	14638	11318	25956	688	747	1435
25-34	12706	10353	23059	648	528	1176	14830	11338	26168	875	805	1680
35-44	12600	10347	22947	756	507	1263	14838	11404	26242	1009	593	1602
45-59	12696	10322	23018	1003	609	1612	14713	11338	26051	2001	907	2908
≥ 60	12668	10277	22945	6562	5015	11577	14749	11364	26113	6637	5091	11728
Total	101274	82365	183639	12667	10283	22950	117999	90950	208949	14745	11374	26119

To calculate the standardised death rate, firstly the death rate for each group or category obtained separately for each region is multiplied by the respective population of that group or category of the standard region and summed up. This sum is divided by the total population of the standard groups. The formula for standardised death rate can be given as,

$$S_T.D.R.=\frac{\Sigma_x p_x^s D_x}{\Sigma_x p_x^s}\times 1000$$

for $x = 1, 2, \ldots$

where, p_x^s – population of standard group x

D_x – S.D.R. for the group x.

Standardised death rates for each region are calculated on the basis of standard population. In this way, $\Sigma_x p_x^s$ for each region remains the same for S_T. D.R. Standardised death rates are logically suitable for comparing the death rates of two different regions.

Q. 30 The population and number of deaths according to age and sex during the years 1988 and 1992 were as follows:

(i) Calculate the crude death rate for the year 1988 and 1992 separately.

(ii) Calculate specific death rates for males and females for the two years 1988 and 1992 separately.

(iii) Calculate age specific death rates for the years 1988 and 1992 separately.

(iv) Calculate standardised death rate taking the population of 1988 as standardised population and compare the death rates of the two years.

Ans. From the given data, the crude death rate for 1988 is,

$$\text{C.D.R.}=\frac{22950}{183639}\times 1000$$

$$= 125‰$$

The crude death rate for the year 1992 is,

$$\text{C.D.R.}=\frac{26119}{208949}\times 1000$$

$$= 125‰$$

(ii) Specific death rates for males and females in the year 1988 are,

$$\text{S.D.R}_M = \frac{12667}{101274} \times 1000$$
$$= 125 \text{ males ‰}$$

$$\text{S.D.R}_F = \frac{10283}{82365} \times 1000$$
$$= 124.85 \text{ female ‰}$$

Specific death rates for males and females in the year 1992 are,

$$\text{S.D.R}_M = \frac{14745}{117999} \times 1000$$
$$= 124.96 \text{ males ‰}$$

$$\text{S.D.R}_F = \frac{11374}{90950} \times 1000$$
$$= 125.06 \text{ females ‰}$$

(iii) Age specific death rates for the year 1988 and 1992 are calculated and displayed in the table given below:

Age group (Years)	1988	1992
	Age Specific death rate $\left(D_x^a\right)$ *(per thousand)*	*Age Specific death* $\left(D_x^b\right)$ *rate (per thousand)*
<1	$\frac{3433}{22935} \times 1000 = 149.68$	$\frac{4048}{26078} \times 1000 = 155.22$
1 - 4	$\frac{1727}{22995} \times 1000 = 75.10$	$\frac{1557}{25984} \times 1000 = 59.92$
5 - 14	$\frac{1061}{22723} \times 1000 = 46.69$	$\frac{1161}{26357} \times 1000 = 44.05$
15 - 24	$\frac{1101}{23017} \times 1000 = 47.83$	$\frac{1435}{25956} \times 1000 = 55.28$
25 - 34	$\frac{1176}{23059} \times 100 = 51.00$	$\frac{1680}{26168} \times 1000 = 64.20$
35 - 44	$\frac{1263}{22947} \times 1000 = 55.04$	$\frac{1602}{26242} \times 1000 = 61.05$
45 - 59	$\frac{1612}{23018} \times 1000 = 70.03$	$\frac{2908}{26051} \times 1000 = 111.63$
≥ 60	$\frac{11577}{22945} \times 1000 = 504.55$	$\frac{11728}{26113} \times 1000 = 449.12$

(iv) The calculation table for obtaining the standardised death rates is prepared first. The standardised death rate for the year 1988 is, (see table on page 528)

$$\text{S}_\text{T}.\text{D.R.} = \frac{22949537.45}{183639}$$
$$= 124.97$$

The standardised death rate for the year 1992 is, (see table on page 528)

$$\text{S}_\text{T}.\text{D.R.} = \frac{22967018.90}{183639}$$
$$= 125.07 \text{ ‰}$$

Comparison of the standardised death rates for the years 1988 and 1992 reveals that the death rate in 1992 is higher than in 1988.

Q. 31 Define crude birth rate.

Ans. Crude birth rate (C.B.R.) is the ratio of the number of live-births during the year to the total mean population during the same year. To express it per thousand persons, this ratio is multiplied by 1000. As a formula,

$$\text{C.B.R.} = \frac{\text{Total No. of live-births during a year}}{\text{Total mean population during the same year}} \times 1000$$

Crude birth rate is hardly able to reveal any exhumating facts about fertility. Hence, one has to resort to some specific fertility rates.

Q. 32 Name different types of fertility rates.

Ans. Normally, six types of fertility rates have been defined.

(i) General fertility rate (G.F.R.).

(ii) Age specific fertility rate (A.S.F.R.).

(iii) General marital fertility rate (G.M.F.R.).

(iv) Age specific marital fertility rate (A.S.M.F.R.).

(v) Total marital fertility rate (T.M.F.R.).

(vi) Total fertility rate (T.F.R.).

Q. 33 What is general fertility rate, and how can it be determined?

Age group (Year)	*Standard population*	*1988*		*1992*	
	P_x^a	*S.D.R.* D_x^a	$P_x^a \times D_x^a$	*S.D.R.* D_x^b	$P_x^a \times D_x^b$
< 1	22935	149.68	3432910.80	155.22	3559970.70
1-4	22995	75.10	1726924.50	59.92	1377860.40
5-14	22723	46.69	1060936.87	44.05	1000948.15
15-24	23017	47.83	1100903.11	55.28	1272379.76
25-34	23059	51.00	1176009.00	64.20	1480387.80
35-44	22947	55.04	1263002.88	61.05	1400914.35
45-59	23018	70.03	1611950.54	111.63	2569499.34
≥ 60	22945	504.55	11576899.75	449.12	10305058.40
Total	183639		22949537.45		22967018.90

Ans. General fertility rate gives the rate of births per thousand women of child bearing age of a country or region in a given year without giving any cognizance to any other factor. In India, the child bearing age is 15-49 years. G.F.R. is obtained as the annual number of births divided by the mid-year population of women of child bearing age. In the formulaic form,

$$\text{G.F.R.} = \frac{\text{Annual number of births}}{\text{Mid-year population of women of age 15-49 of the region in the given year}} \times 1000$$

General fertility rates do not provide adequate basis for effective planning with regard to reducing the population growth and family welfare schemes.

Q. 34 In what sense are age-specific fertility rates better than general fertility rate, and how to calculate them?

Ans. Population explosion is the most horrifying problem today before the world. Hence, to bring down the population growth, one has to study it very minutely. We know that the fertility of women is not the same for all age groups. Hence, it seems germane to find out the marital status and age-specific fertility rates.

Age-specific fertility rate may be defined as, the number of births during a given period (usually a calender year) to women of a specified age or age group divided by the average number of women of that age or age group living during that period (usually by the mid-year). In the formulaic form,

$$\text{A.S.F.R.} = \frac{\text{No. of live births to the women of age group } x \text{ to } (x+n) \text{ during a year}}{\text{Average No. of women in the same age group during that year}}$$

Q. 35 What information do we gather from general marital fertility rates?

Ans. The births are confined to married women only barring exceptions of births to unmarried or widow women. Hence, for family planning, emphasis has to be laid to married women only. For this it looks very logical to study the fertility rates among married women only.

General marital fertility rate can be defined as the number of offsprings born alive during a period (usually a year) per thousand married women of child bearing age. It can be formulated as,

$$\text{G.M.F.R.} = \frac{\text{No. of births during a year to married women}}{\text{Mid-year population of married women during that year of age 15-49 years}} \times 1000$$

G.M.F.R. do not provide classified fertility rates of married women, particularly, in respect of age. Hence, the information through G.M.F.R. is not complete.

Q. 36 How are age specific marital fertility rates more helpful in family planning programmes, and how are they calculated?

Ans. It is a known fact that fecundity varies with age. For example, in Indian women fecundity is maximum in the age group 25-29 and is little less in the age group 20-24 also after 29 year of age it goes down. Hence, for the success of family planning schemes, it becomes necessary to study age-specific marital fertility rates.

A.S.M.F.R. is defined as, the number of children born alive during a given period (usually a calender year) per thousand married women of a particular age or age group. As a formula,

$$\text{A.S.M.F.R.} = \frac{\text{No. of live births to married women during a year in the age group } x \text{ to } (x+n)}{\text{Mid-year population of married women during that year in the same group}} \times 1000$$

Q. 37 What is total marital fertility rate, and how can it be calculated?

Ans. Total marital fertility rate gives the total number of live births that would have taken place per thousand married women, had the current schedule of age-specific marital fertility rates been applicable for the entire child bearing period. For the age group x to $(x + n)$ years, the total marital fertility rate,

$$\text{T.M.F.R.} = \text{A.S.M.R.R.} \times n$$

where, n is the interval in years.

Q. 38 Define and formulate total fertility rate.

Ans. Total fertility rate is a measure which gives approximately the magnitude of 'complete family size' that is the total number of children, a women would bear on an average in the lifetime assuming no mortality and no adoption of family planning measures. In short, the sum of age-specific fertility rates over all ages of the child bearing period. Formula for total fertility rate in the age group x to $(x + n)$ is,

$$\text{T.F.R.} = \text{Sum of A.S.F.R.} \times n$$

To obtain the total fertility rate per woman, the above value of T.F.R. has to be divided by 1000.

Q. 39 What do you understand by measurement of population growth?

Ans. Population growth mainly depends on the sex of the newly born children, *i.e.*, the population growth rate increases if the majority of births consists of girls. Thus, fertility rates fail to reflect on the rate of population growth because they do not take into consideration the sex of the newly born children. Therefore, to measure the rates of population growth, it becomes necessary to take into account the female births and their mortality before they reach the child bearing age. The rate of population growth are measured in terms of reproduction rates.

Q. 40 Name different measures of population growth.

Ans. Various measures of population growth are as follows:

(i) Crude rate of national increase
(ii) Vital index
(iii) Gross reproduction rate (G.R.R.)
(iv) Net reproduction rate (N.R.R.)
(v) Replacement index.

Q. 41 Explicate the method of obtaining the crude rate of natural increase.

Ans. Crude rate of natural increase is equal to the difference between crude birth rate (C.B.R.) and crude death rate (C.D.R.). This difference is also known as *survival rate*. Symbolically,

$$\text{Crude rate of natural increase} = \text{C.B.R.} - \text{C.D.R.}$$

In case, the total number of births (ΣB), the total number of deaths (ΣD) and the mid-year population (ΣP) for a given year are known, then

$$\text{Crude rate of natural increase} = \frac{\Sigma B - \Sigma D}{\Sigma P}$$

Q. 42 What is meant by vital index of population, and how can it be measured?

Ans. Vital index of population was propounded by Dr Rayond Pearl. Vital index is measured as the ratio of births to deaths in a given year expressed in terms of percentages. In the formulaic form,

$$\text{Vital Index} = \frac{\text{Births in a year}}{\text{Deaths in the same year}} \times 100$$

Vital index is not a good measure of population growth as it does not take into consideration the number of female births and deaths in the year under reference.

Q. 43 Discuss gross reproduction rate.

Ans. Population growth is dependent on the number of female births who are actually the future mothers. Hence, population growth is certainly a function of the fertility rates restricted to the female births. According to the 'demographic book', published the United Nations in 1954, "*The gross reproduction rate indicates the average number of daughters who would be borned to a group of girls beginning life together in a population where none died before the upper limit of child bearing age and where the given set of fertility rates was in operation.*"

The gross reproduction rate is based on the assumptions:

(i) No female children die till they attain the upper limit of child bearing age which is 49 years in India.

(ii) Female population remains stagnant in spite of migration.

(iii) The current fertility rates continue to hold during the entire period of child bearing age.

The above assumptions are not relevant in real life. Hence, gross reproduction rate does not depict a true picture.

Q. 44 Give the formula for gross reproduction rate.

Ans. Gross reproduction rate can be calculated in either of the following ways.

(a) If S_{ix} is the fertility rate at age x restricted to female births $\sum_{x=0}^{n} S_{ix}$ is equal to G.R.R. where n is the upper child bearing age.

(b) Another formula for gross reproduction rate is,

$$\text{G.R.R.} = \frac{\text{No. of female live births}}{\text{Total number of live births}} \times \text{Total fertility rate}$$

(c) Also, the gross reproduction rate can be calculated by making use of the current age specific fertility rate, under the assumption of no mortality, by the formula,

$$\text{G.R.R.} = \frac{\text{No. of daughters expected to be borned to 1000 newly borned girls}}{1000}$$

Gross reproduction rates are not accurate because mortality of females and migration in all age groups are inevitable.

Q. 45 What improvement is brought out by net reproduction rate over gross reproduction rate?

Ans. Gross reproduction rate gives the idea about the replacement of one generation to the next. But it overestimates the next generation because it omits the mortality of newly born female infants till they attain their maximum child bearing age. This makes G.R.R. quite artificial. The other factors like migration, divorce, unmarried women do affect the reproduction rate but are negligible as compared to the extent of mortality. The United Nations Demographic Year Book 1954 defined net reproduction rate as, "*The net reproduction rate may be interpreted as the number of daughters that would be produced by women throughout their lifetime if they were exposed at each age to the fertility and mortality rates on which the calculation is based.*"

From the above discussion it is apparent that NRR gives the number of females produced per woman who continue to survive till their maximum reproduction age.

Q. 46 In what ways, can the net reproduction rate be calculated?

Ans. Various approaches to calculate net reproduction rate are as follows:

(a) The calculation of net reproduction rate by making use of life-tables. If B_g is the number of girls borned to P_g women of the age group x to $(x + n)$ and S is the survival rate (p_x) per woman which is obtained by the life table for each age group, then

$$\text{N.R.R.} = \sum_{\substack{\text{all age}\\\text{group}}} \left(\frac{B_g}{P_g} \times S\right) \times \begin{array}{c}\text{class interval of}\\\text{the age group}\end{array}$$

(b) Net reproduction rate can directly be calculated by the formula,

$$\text{N.R.R.} = \frac{\begin{array}{r}\text{No. of female babies expected to be}\\\text{borned to 1000 newly born females}\\\text{on the basis of current fertility and}\\\text{mortality rates}\end{array}}{1000}$$

(c) George Barclay gave the formula for net reproduction rate in terms of life table notations as,

$$\text{N.R.R.} = \overset{49}{\underset{x=15}{S}} b_x \frac{L_x}{L_0}$$

where,

S – Stands for summation just as Σ.

b_x – rate of female births per person at age x.

L_x – No. of persons-years lived at age x per woman borned to the original cohort

L_0 – Original cohort

x – varies from 15 to 49 years of age, *i.e.*, the child bearing age.

(d) In common parlance,

$$\text{N.R.R.} = \frac{\begin{array}{c}\Sigma(\text{No. of female births}\\\times\ \%\ \text{survival rate})\end{array}}{100}$$

Q. 47 Comment on the values of net reproduction rate.

Ans. The experience gives some ideas about the values of N.R.R. and their importance as follows:

(i) Normally N.R.R. varies from 0 to 5.

(ii) N.R.R. can not exceed G.R.R. as N.R.R. takes into account the mortality.

(iii) N.R.R. will be equal to G.R.R. if all the newly borned female children survive till their maximum child bearing age.

(iv) If N.R.R. = 1, the female population will exactly replace itself into new generation and population remains constant.

(v) If N.R.R. < 1, this will result into the reduction in the number of mothers and will thus cause reduction in population.

(vi) If N.R.R. > 1, there will be a greater number of mothers in the next generation which will tend to increase the population.

Q. 48 What are the drawbacks of net reproduction rate?

Ans. Beside many good qualities of N.R.R. over G.R.R., it is not devoid of drawbacks such as:

(i) N.R.R. are based on the constant rates of fertility and mortality over generation which is not true to the real life phenomenon.

(ii) N.R.R. do not take into account the number of emigrants or immigrants. Many times, the number is so large that it affects the reproduction rate.

(iii) A.S.F.R.'S are also not so constant as they are taken to be for the purpose of N.R.R.

Q. 49 The population and its distribution by sex and number of births in a tehsil in 1991 and survival rates are given in the table below:

From the given data, calculate

(i) General fertility rate

(ii) Age specific fertility rate

(iii) Total fertility rate

(iv) Gross reproduction rate

(v) Net reproduction rate

assuming no mortality.

Q. 50 What is replacement index, and what purpose does it serve?

Ans. Replacement index is defined as, the proportion of observed child-woman ratio of actual population to the child-woman ratio from life-table stationary population.

Sometimes the trend of population growth is indicated by replacement index R. Replacement index is seldom used in practice.

Q. 51 Formulate replacement index.

Ans. In this index, child-woman ratio is the ratio of the number of children of age less than 5 years (0-4 years), *i.e.*, $\sum_{x=0}^{4} P_x$ to the number of women of child bearing age (15-49 years), *i.e.*, $\sum_{15}^{49} P_f$.

Thus the replacement index,

$$R = \frac{\sum_{x=0}^{4} P_x}{\sum_{15}^{49} P_f}$$

$$= R_a / R_s$$

where,

R_a – child-woman ratio in actual population

R_s – child-woman ratio in stationary population.

Q. 52 Express net migration rate and its importance.

Ans. Net migration rate (N.M.R.) is the ratio of the annual net migration to the annual mean population. When this ratio is multiplied by 1000, it gives the net migration rate per thousand. Thus,

$$\text{N.M.R.} = \frac{\text{Annual net migration}}{\text{Annual mean population}} \times 1000$$

$$= \frac{\text{overseas immigration} \\ - \text{overseas emigration}}{\text{Annual mean population}} \times 1000$$

Net migration rate conveys about the population augmentation and depletion through net migration during a given year.

Q. 53 Give the concept of a life-table.

Ans. A life-table comprises of a set of values showing how a group of infants born on the same day and living under similar conditions would gradually die out. In other words, a life-table summarises the mortality or longevity of any cohort.

George Barclay expressed life-table as, *"The table is a life history of a hypothetical group or cohort or people, as it is diminished gradually by deaths. The record begins at birth of each member and continues till all have died."*

Q. 54 What is revealed by a life-table?

Ans. A life-table reveals the following aspects:

(i) Beside death rate, life-table is another device of describing mortality in a population under consideration. That is why some people call life-table as *mortality table.*

(ii) As a matter of fact, a life-table exhibits the numbers living and dying at each age on the basis of experience of a cohort.

(iii) It also gives the probability of dying and living separately at each age. The probability to dying manifests the mortality rate. Obviously, the probability of living evinces survival rate.

(iv) The life-table provides the expectation of life at any age x. In other words, a life-table essentially presents the mortality at all ages in a tabular form.

Q. 55 On what assumptions or factors is the construction of life-table based?

Ans. The factors or assumptions in the construction of life-table are as given below:

(i) Mortality rates are not same in all age groups. Hence, life-tables utilise only the age specific mortality rates.

(ii) The deaths are uniformly distributed between two consecutive birth days.

(iii) The mortality rates differ widely for male and female populations. Hence, life-tables for males and females are constructed separately. Not only this, the experience shows that the pattern of mortality for different races, occupational groups, regions, etc., are different. Hence, life-tables are prepared separately for different cohorts.

(iv) The cohort is closed to migration. In other words, the change in population occurs only due to deaths.

(v) The cohort originates with a standard number of births say 1,00,000, 10,000, 1,000, etc. This number is known as the *radix* of the table.

Q. 56 What is the historical background of life-tables?

Ans. Ulpians life-tables consisting of a series of values diminishing with increasing age appeared first. But the manner in which they were constructed and their purpose was not clear.

Life-table in the real sense came into existence and popularity at the end of eighteenth century for the purpose of life assurance. This enabled the acturies to calculate the insurance risks and premium rate. In recent years, life-table approaches are being increasingly utilised even to follow up studies of chronic diseases of hospital patients, chalking out welfare programmes for different cohorts, etc.

Q. 57 What are various uses of life-table?

Ans. A life-table mainly displays the death rate between two consecutive birth days and expectation of life at any age of a cohort. The information revealed by a life-table has applications in many fields and hence its uses can be summarised as given below:

(i) Life-tables are of great use to actuaries to estimate the insurance risks and premiums.

(ii) Life-tables help to construe the population projections by age and sex.

(iii) Life-table is used to calculate accurate fertility rates.

(iv) Life-table clearly depicts the distribution of people according to sex which helps a great deal in planning of education, employment and workforce.

(v) Life-table helps to check the accuracy of census figures, registration of deaths, births, etc.

(vi) The computation of net reproduction rate and true rate of natural increase is easily made by the use of life-tables.

(vii) It helps to assess the impact of family planning programmes on population growth.

(viii) How far, the better medical aid, high standard of living and new scientific inventions have increased the span of life, can be evaluated through life-tables.

Q. 58 How can a life-table be constructed?

Ans. A life-table starts with a convenient cohort size like 1,00,000 or 10,000 known as radix. The record of a life-table begins at the birth of each member and continues till all have died. It is worth pointing out that a life-table diminishes gradually.

A life-table consists of eight columns which are as given below:

Columns:

(1) x = The age in years

(2) l_x = No. of persons living at the age x.

(3) d_x = No. of persons dying between the ages x and $(x + 1)$.

$$= l_x - l_{x+1}$$

(4) q_x = Prob. of dying of a person between the ages x and $(x + 1)$

(5) p_x = Prob. of living of a person between the ages x and $(x + 1)$

(6) L_x = No. of person living between x and $(x + 1)$.

$$L_x = \frac{l_x + l_{x+1}}{2}$$

$$= l_x - \frac{1}{2} dx$$

(7) T_x = No. of persons living above the age x

$$l_x + l_{x+1} + \ldots = L_x + T_{x+1}$$

(8) e_x^0 = Expectation of life at age x.

$$. = T_x / l_x$$

Q. 59 Given the age returns for the two ages $x = 9$ years and $x + 1 = 10$ years with a few life-table values as, $l_9 = 75{,}824$, $l_{10} = 75{,}362$, $d_{10} = 418$ and $T_{10} = 49{,}53{,}195$. Give the complete life-table for two ages of persons.

(iii) q_x – the probability of dying of a person in age group x to $(x + n)$.

$$_nq_x = \frac{l_x - l_{x+n}}{l_x}$$

$$= 1 - \frac{l_{x+n}}{l_x}$$

(iv) $_nd_x$ – the number of deaths in the interval x to $(x + n)$, *i.e.*,

$$_nd_x = l_x \times {_nq_x}$$

(v) $_np_x$ – the probability of living of a person between the age x and $(x + n)$ and can be given as,

$$_np_x = \frac{l_{x+n}}{l_x}$$

$$= 1 - {_nq_x}$$

(vi) $_nL_x$ – the number of persons of the life-table stationary population in the interval x to $(x + n)$ years and can be obtained as,

$$_nL_x = \int_0^n l_{x+t}\,dt$$

(vii) T_x – the number of persons living after the age x.

(viii) e_x^0 – the expectation of life at the age x and is given as,

$$e_x^0 = \frac{T_x}{l_x}$$

Q. 69 In what way, does the construction of an abridged life-table differ from a complete life-table?

Ans. The main difference lies in finding the value of $_nq_x$ because $_nq_x$ is often not a smooth function of x and shows irregular variation because of the 'arger interval instead of one year. Hence, a standardised value of $_nq_x$ is determined for usage. The value of $_nq_x$ is standardised either by Compertz law or Makeham's law or by some other rule. Also a suitable interval n should be chosen and proper grouping of age be done.

In an abridged life-table, the class interval n is more than one year and is usually 5 to 10 years.

Q. 70 Discuss the term, 'central mortality rate', and give the formula for its calculations.

Ans. The central mortality rate is the probability of dying of a person, whose exact age is not known but lies between the age x and $(x + 1)$, within one year following the attainment of that age. Let us denote,

m_x – the central mortality rate.

d_x – the number of deaths between x and $(x + 1)$ years.

L_x – the average size of the cohort in the interval x to $(x + 1)$.

l_x – the cohort size at the age x.

The central mortality rate,

$$m_x = \frac{\text{No. of deaths in the interval } x \text{ to } (x+1)}{\text{Average size of the cohort in the interval}}$$

$$= \frac{d_x}{L_x}$$

$$= \frac{d_x}{l_x - \frac{1}{2}d_x}$$

$$= \frac{2d_x/l_x}{2 - d_x/l_x}$$

$$= \frac{2q_x}{2 - q_x}$$

or $\quad q_x = \dfrac{2m_x}{2 + m_x}$

Q. 71 Discuss force of mortality.

Ans. In the construction of life-tables, we have confined to initial population l_x at an age x where x is an integral number. But death does not occur only at the end of a year alone but at any time during the year. Thus, l_x is a continuous function of x. Hence, the rate of decrease in l_x can be given by the differential, $\frac{d}{dx}l_x$.

47. Age specific fertility rates create better ground for ________.

48. General marital fertility rate is confined to birth rate of children borned to ________ women only.

49. Most fertile period of women can be ascertained by calculating ________.

50. The total number of children that could have borned to 1000 married women during their entire child bearing period is known as ________.

51. The sum of age specific fertility rates multiplied by n in the age interval x to $(x + n)$ provides the estimate of ________.

52. The rate of increase of population mainly depends on the ________ of the new-born.

53. Measures of population growth are the function of ________ and their ________.

54. Population growth is measured in terms of ________.

55. The ratio of births to deaths in a year is called ________.

56. Gross reproduction rate is a ________ number.

57. Gross reproduction rate is based on fictitious assumption that ________ till the age of 49 years.

58. Number of daughters expected to be borned to 1000 newly borned girls is equivalent to ________ per thousand.

59. The number of females produced per woman who survive till their full reproduction age is nothing but ________.

60. When the mortality of newly borned girls is zero, the net reproduction rate is same as ________.

61. Gross reproduction rate cannot be ________ net reproduction rate.

62. If NRR = 1, then the female population will exactly ________.

63. If NRR < 1, the population will in general tend to ________.

64. If NRR > 1, the population of a country will very likely ________.

65. The assumption of constant fertility and mortality in the calculation of NRR is practically ________.

66. NRR based on current fertility and mortality rates ________ the future growth in reality.

67. Migrants are ________ in the calculation of net reproduction rates.

68. Replacement index is ________ used in practice as a measure of population growth.

69. The ratio of the total number of births to the total deaths in a given region during a given year is called ________.

70. The range of vital index is ________ to ________.

71. The value of vital index greater than 1 is indicative of ________.

72. Vital index less than 1 shows a ________ in population.

73. The measure for population change merely due to migration is ________.

74. The number living and dying at each age on the basis of the experience of a cohort are exhibited by ________.

75. The probability of dying manifests the ________.

76. The probability of living reveals the ________ rate.

77. The expectation of life at any age can be obtained from a ________.

78. Life-tables are prepared ________ for male and female populations.

79. Age-specific mortality rates are the back bone of ________.

80. Life-tables are ________ of the consideration of migrations.

81. The standard number of births originating a cohort is called ________ of the life-table.

82. A life-table shows the pattern in which the population ________ gradually.

83. The life- table continues till all people _________.

84. The life-table appearing first are due to _________.

85. The extensive use of life-tables is made by _________.

86. Life-table can also be utilised in medical science for follow up studies of _________.

87. Accuracy of census figures can be checked with the help of _________.

88. The impact of family planning programmes can be assessed through _________.

89. The overall impact of developed medical aid on life expectancy can be evaluated from _________.

90. A life-table contains _________ columns in all.

91. The calculation of unemployment rates can be done parallel to _________.

92. A population of constant size having the same sex composition over time is called a _________ population.

93. If the births and deaths, immigrants and emigrants are equal in number, the size of population remains _________.

94. A population with varying size but having a constant rate of growth or depletion is called a _________ population.

95. A stable population has a fixed _________ ratio over time.

96. A stable population is closed for _________.

97. If the birth and death rates over time are equal and no migration takes place, the _________ and _________ populations are equal.

98. An abridged life-table usually consists of ages at distance of _________ years.

99. In an abridged life-table, the distance between ages is _________ one year.

100. The main difference in the construction of a complete life-table and an abridged life-table lies in determining _________.

101. Central mortality rate is the probability of _________ of a person whose _________ age is not known but lies in the interval x and $(x + 1)$.

102. Merrel's method of construction of abridged life-table is mainly based on _________.

103. Greville mainly utilises _________ in the construction of abridged life-table.

104. Greville's approximation to ${}_nL_x$ for abridged life-tables is based on _________.

105. King estimated the population and number of deaths for construction of abridged life-table for the central age in the interval $\{x, (x + n)\}$ by approximating to _________.

106. Out of Merrell's, Greville's and King's methods of construction of abridged life-tables, maximum approximations are involved in _________ method.

107. A life-table based on actual cohort is called _________ life-table.

108. The death rate of 15.4 of a posh city A and the death rate of 12.9 of a suburb B is _________ of salubrity of the later.

SECTION-C

Multiple Choice Questions

Select the correct alternative out of given ones:

Q. 1 Vital statistics is mainly concerned with:
(a) births
(b) deaths
(c) marriages
(d) all the above

Q. 2 Population statistics mainly display the records pertaining to:
(a) foetal deaths

(b) population of regions
(c) morbidity
(d) all the above

Q. 3 Complete count of the heads of people of a country is known as:
(a) census
(b) vital statistics
(c) demography
(d) none of the above

Q. 4 Vital statistics throws light on:
(a) changing pattern of the population during intercensal period
(b) virility of races
(c) growth of population
(d) all the above

Q. 5 The registration of births, deaths and marriages are:
(a) a fancy of society
(b) a part of medical research
(c) a legal document
(d) all the above

Q. 6 Vital statistics is greatly utilised by:
(a) acturies
(b) planners
(c) social reformers
(d) all the above

Q. 7 In India, the collection of vital statistics started for the first times in:
(a) 720
(b) 1886
(c) 1969
(d) 1946

Q. 8 The registration of vital statistics in India suffers from:
(a) incomplete reporting
(b) incomplete coverage
(c) lack of accuracy
(d) all the above

Q. 9 To improve upon the registration of vital statistics in India, the central government appointed a committee in 1948 known as:
(a) Bhor Committee
(b) Rath Committee
(c) Arthur Committee
(d) none of the above

Q. 10 Registration of vital statistics is organised at the apex by:
(a) Director General
(b) Registrar General
(c) Census Commissioner
(d) all the above

Q. 11 At state level, the registration of vital statistics is carried by:
(a) Director of Economics & Statistics
(b) Chief Returning Officer
(c) Chief Registrar
(d) none of the above

Q. 12 In post-Independence India, the registration of Births and Deaths Act was passed in:
(a) 1948
(b) 1959
(c) 1969
(d) 1979

Q. 13 Vital statistics is obtained through:
(a) census operation
(b) registration system
(c) survey method
(d) all the above

Q. 14 Sampling registration system of births and deaths came into operation in rural areas in the year:
(a) 1967
(b) 1968
(c) 1969
(d) none of the above

Q. 15 Sampling registration system for recording births and deaths in urban areas started in the year:
(a) 1967
(b) 1968
(c) 1969
(d) none of the above

Q. 16 The advantage of sampling registration system is that:
(a) it has full coverage
(b) it is more accurate
(c) it provides the estimate for rural and urban areas separately
(d) all the above

Q. 17 The sampling registration system fails to record:

(a) age and sex composition

(b) birth rates

(c) death rates

(d) all the above

Q. 18 The most important assumption, on which the analytical methods are based, is that:

(a) the population is stagnant

(b) the population grows at a constant rate

(c) there is no time lag

(d) none of the above

Q. 19 Having known the population of the two consecutive censuses, the formula for population estimate $\hat{P}_t$ in the intercensal year t with usual rotations is:

(a) $\hat{P}_t = P_0 + \frac{N}{n}(P_1 - P_0)$

(b) $\hat{P}_t = P_1 + \frac{n}{N}(P_1 - P_0)$

(c) $\hat{P}_t = P_0 + \frac{n}{N}(P_1 - P_0)$

(d) $\hat{P}_t = P_0 + \frac{N}{n}(P_0 - P_1)$

Q. 20 If P_1 and P_2 are the populations at two censuses conducted at an interval of five years, then the rate of population growth 'r' can be estimated by the formula:

(a) $r = \sqrt[10]{\frac{P_2}{P_1}} - 1$

(b) $r = \sqrt[5]{\frac{P_1}{P_2}} - 1$

(c) $r = \sqrt[5]{\frac{P_2}{P_1}} - 1$

(d) $r = \sqrt[n]{\frac{P_2}{P_1}} - 1$

Q. 21 If we have the last census population, migration, births and deaths data for a region in a given period, the population at the time t can be estimated by the formula (using usual notations) as:

(a) $\hat{P}_t = P_0 + (B - D) + (I - E)$

(b) $\hat{P}_t = (B - D) + (I - E)$

(c) $\hat{P}_t = P_0 \{(B - D) + (I - E)\}$

(d) none of the above

Q. 22 Having known the last census population 'P_0' and growth rate 'r', the population after n years based on compound interest formula will be:

(a) $\hat{P}_t = P_0 \, (1 + r)^n$

(b) $\hat{P}_t = P_0 \, (1 + n)^r$

(c) $\hat{P}_t = P_0 / (1 + r)^n$

(d) $\hat{P}_t = P / (1 + r)^n$

Q. 23 If P_1 and P_2 are the population at an interval of 10 years, the population just after five years will be:

(a) $\frac{1}{2}(P_1 + P_2)$

(b) $\sqrt{P_1 \times P_2}$

(c) $\frac{1}{2}\left(\frac{1}{P_1} + \frac{1}{P_2}\right)$

(d) $\sqrt{P_1 + P_2}$

Q. 24 Vital rates are customarily expressed as:

(a) percentages

(b) per thousand

(c) per million

(d) per trillion

Q. 25 Crude death rate, expressed simply as a ratio, provides:

(a) the probability of babies borned and died during the year under reference

(b) the probability of a foetal death during the year under reference

(c) expectation of life at each age
(d) all the above

Q. 55 A life-table is a compendium which:
(a) foretells about each individual
(b) forecasts the year of death of each individual
(c) provides the age of each individual of the population
(d) none of the above

Q. 56 Construction of life-tables is based on the assumption that:
(a) age specific death rates are constant at all ages
(b) death rates are uniformly distributed between two birth days
(c) mortality rates are same for male and female populations
(d) all the above

Q. 57 Life-tables are usually constructed:
(a) jointly for male and female populations
(b) separately for male and female populations
(c) both (a) and (b)
(d) neither (a) and (b)

Q. 58 The standard number of births 10,000 originating a life-table is known as:
(a) a cohort
(b) initial population
(c) radix
(d) all the above

Q. 59 A life-table is most utilised by:
(a) life insurance companies
(b) general insurance companies
(c) employment exchanges
(d) all the above

Q. 60 Life-table is a mean of:
(a) monitoring the family planning programmes
(b) checking the census figures
(c) giving population projections
(d) all the above

Q. 61 Normally a life-table is constructed for an age interval of:
(a) five years
(b) five to 10 years
(c) one year
(d) none of the above

Q. 62 Which leader amongst the following attained the maximum age of a life-table?
(a) Mao Tse-tung of China
(b) Karl Marx of Germany
(c) Morarji Desai of India
(d) Macmillan of U.K.

Q. 63 A life-table consists of:
(a) seven columns
(b) eight columns
(c) nine columns
(d) none of the above

Q. 64 Unemployment rates are similar to:
(a) death rates
(b) survival rates
(c) migration rates
(d) none of the above

Q. 65 A population have constant size and composition is called a:
(a) stable population
(b) stationary population
(c) continuous population
(d) discrete population

Q. 66 A population maintaining a constant growth rate is said to be a:
(a) stable population
(b) stationary population
(c) mobile population
(d) none of the above

Q. 67 The probability of dying of a person of age between x and $(x + 1)$ years is known as:
(a) age-specific death rate
(b) infant mortality rate
(c) central mortality rate
(d) none of the above

Q. 68 A life-table constructed for an age interval of 5 to 10 years is specifically known as:
(a) grouped life-table
(b) interval life-table
(c) abridged life-table
(d) none of the above

Q. 69 An abridged life-table can be constructed by the method suggested by:

(a) Reed Merrel
(b) Greville
(c) G. King
(d) all the above

Q. 70 Reed Merrel method of construction of abridged life-tables utilises:

(a) age specific mortality rates
(b) central mortality rates
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 71 Greville's method for estimating death rate in the age interval of x to $(x + n)$ years utilises:

(a) Compertz law
(b) exponential law
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 72 G. King's abridged life tables are based on the calculation of:

(a) central mortality rate
(b) the number of persons and deaths for the central age in the interval $\{x, x + n\}$
(c) both (a) and (b)
(d) neither (a) and (b)

Q. 73 A life-table based on the experience of actual cohort is called:

(a) generation life-table
(b) fluent life-table
(c) both (a) and (b)
(d) neither (a) and (b)

Q. 74 The probability of living of a person in the age group x to $(x + n)$ can be obtained by the formula:

(a) l_{x+n}/l_x
(b) $(l_x - l_{x+n})/l_x$
(c) $(l_x - l_{x+n})/l_{x+n}$
(d) l_x/l_{x+n}

Q. 75 The central mortality rate 'm_x' in terms of q_x is given by the formula:

(a) $2q_x/(2 + q_x)$
(b) $2q_x/(2 - q_x)$
(c) $q_x/(2 + q_x)$
(d) $q_x/(2 - q_x)$

Q. 76 If l_x is the number of persons living at the age x and L_x the number of persons living in the mid of x and $(x + 1)$ years, then the relation between l_x and L_x is:

(a) $L_x = \frac{1}{2}(l_x + l_{x+1})$

(b) $L_x = \frac{x}{2} + l_x$

(c) $L_x = l_{x+\frac{1}{2}}$

(d) none of the above

Q. 77 The relation between the central mortality rate 'm_x' and force of mortality 'μ_x' is:

(a) $m_x = \mu_x + \frac{1}{2}$

(b) $m_x = \frac{1}{2}\mu_{x+1}$

(c) $m_x = \frac{1}{2}(\mu_x + \mu_{x+1})$

(d) none of the above

Q. 78 The ratio of the rate of decrease in initial population l_x at age x to l_x is known as:

(a) nominal annual rate of mortality
(b) force of mortality
(c) central mortality rate
(d) all the above

Q. 79 The probability q_x of dying of a person between the age interval x and $(x + 1)$ and m_x, the central mortality rate are related as:

(a) $q_x = 2m_x/(2 - m_x)$
(b) $q_x = m_x/(2 + m_x)$
(c) $q_x = 2m_x/(2 + m_x)$
(d) none of the above

Q. 80 If l_x, the initial population at age x in an abridged life-table vanishes at the age $w + n$, then with usual notation:

(a) ${}_nL_w = l_x/m_x$
(b) ${}_nL_x = l_w/{}_nm_w$
(c) ${}_nL_w = l_x/m_x$
(d) ${}_nL_w = l_x/{}_nm_x$

ANSWERS

SECTION-B

(1) demography (2) interchangeable (3) vital statistics (4) Arthur Newsholme (5) Benjamin (6) human inventory (7) analysis (8) ten years (9) continuing process (10) acturies (11) social reforms (12) vital statistics (13) vital statistics (14) 1969 (15) 1886 (16) sacramental (17) three-fourth (18) Bhor (19) 1952 (20) sample census (21) Registrar General (22) Comprehensive profile (23) census year (24) vital (25) programes (26) 1967 (27) 1968 (28) census blocks (29) all India level (30) rural; urban (31) migration (32) is not (33) $p_0 + \frac{n}{N}(P_1 - P_0)$ (34) at a constant rate (35) $\sqrt[5]{P_n/P_0} - 1$ (36) $\sqrt{P_1 \times P_2}$ (37) per thousand (38) probability; death (39) equal weightage (40) more informative (41) mortality conditions (42) pooled death (43) exhumating (44) 15-49 (45) all ages (46) family planning (47) family planning (48) married (49) A.S.F.R. (50) T.M.F.R. (51) T.F.R. (52) sex (53) female births; mortality (54) reproduction rates (55) vital index (56) normative (57) no female child dies (58) gross reproduction rate (59) net reproduction rate (60) gross reproduction rate (61) less than (62) replace itself (63) decrease (64) increase (65) not valid (66) fail to estimate (67) not involved (68) rarely (69) vital index (70) 0 ; ∞ (71) population growth (72) depletion (73) net migration rate (74) life-table (75) mortality rate (76) survival (77) life-table (78) separately (79) life-tables (80) devoid (81) radix (82) diminishes (83) have died (84) Ulpian (85) acturies (86) chronic diseases (87) life-tables (88) life-tables (89) life-tables (90) eight (91) death rate (92) stationary (93) constant (94) stable (95) sex (96) migration (97) stationary; stable (98) 5 to 10 (99) more than (100) ${}_nq_x$ (101) dying; exact (102) central mortality rate (103) age specific central mortality rate (104) numerical quadrature (105) second degree parabola (106) King's (107) generation or fluent (108) no evidence.

SECTION-C

(1) d	(2) b	(3) a	(4) d	(5) c	(6) d
(7) b	(8) d	(9) a	(10) b	(11) c	(12) c
(13) d	(14) a	(15) b	(16) d	(17) a	(18) b
(19) c	(20) c	(21) a	(22) a	(23) b	(24) b
(25) c	(26) c	(27) c	(28) a	(29) b	(30) a
(31) c	(32) d	(33) b	(34) c	(35) b	(36) d
(37) c	(38) b	(39) a	(40) c	(41) b	(42) d
(43) d	(44) a	(45) d	(46) b	(47) d	(48) b
(49) a	(50) d	(51) d	(52) b	(53) a	(54) d
(55) d	(56) b	(57) b	(58) c	(59) a	(60) d
(61) c	(62) c	(63) b	(64) a	(65) b	(66) a
(67) c	(68) c	(69) d	(70) b	(71) a	(72) c
(73) c	(74) a	(75) b	(76) c	(77) a	(78) b
(79) c	(80) b				

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Chapter **21**

Basic Experimental Designs

SECTION-A

Short Essay Type Questions

Q. 1 What is an experimental design?

Ans. Any person has often total or partial ignorance about certain factors or treatments underlying a phenomenon and is desirous to confirm it by way of certain experimentation or trial. The effect of factors or treatments can be ascertained or compared only if they are set completely free to show their effects on the subjects while all other factors which are likely to affect the treatments or factors under study, are kept under control.

Thus, the plan in which an experiment has to be conducted so that all situations, except that of treatments or factors are kept under control as much as possible, are known as experimental design. These designs may be for engineering, chemical industry, crop experiments, etc. For instance, an experiment may be conducted to see the effect of different fertilisers. In this situation, the field has to be divided into a number of plots of equal size and same shape, homogeneous in respect of soil fertility, irrigation, crop, seed, plant protection umbrella, crop management, etc., and each plot receives the fertilizer treatment randomly. The variability in crop yield or any other criterion measure can enable the investigator to distinguish between fertilizers. Such a plan of experiment is known as *design of experiment.*

Q. 2 How do you define an experimental unit?

Ans. A subject or a group of objects or the total material to which a treatment is applied in a trial in a single replication is known as an experimental unit. For instance, a plot in field trial in agricultural experiments, a rat or a rabbit in a biological experiments, a cow or a horse in an experiment in animal sciences, a group of insects in entomological experiment, etc.

Q. 3 Define a treatment in reference to an experiment.

Ans. A treatment is a substance or a known factor which is administered or allocated to one or more experimental units to estimate its effect pertaining to certain characters or for comparing it with others. Application of *fertilizers* to field plots, testing *feeds* on cows, administering *medicines* to patients are a few of the examples of treatments which are applied to experimental units. Whereas *dates of sowing, crop geometries* to see their effect on the yield of crop, *breed* of poultry birds for their meat quality, etc., are the treatments which are attached to the experimental units. An experimental unit may receive a simple treatment or a combination of treatments.

Q. 4 What are the requirements of a good experimental design?

Ans. There are three requirements of a good design namely:

(a) Randomization
(b) Replication
(c) Local control.

Q. 5 What do you understand by randomization in experimental designs?

Ans. The allocation of treatments to experimental units in such a manner that an experimental unit has equal chance of receiving any of the treatments is called randomization. The total number of units in an experiment is always equal to the sum of the replicates of all the treatments selected.

Randomization is appropriately implemented with the help of random number tables.

Q. 6 What is the role of randomization in the process of experimentation?

Ans. There are definitely many advantages of assigning the treatments randomly to the experimental units. They are summarily discussed below:

(i) Randomization eliminates the human biases. May these biases be introduced advertently or inadvertently.

(ii) Randomization makes the experiment free from any systematic influences of environment.

(iii) The process of randomization enables to apply mathematical theories which would have otherwise not been possible.

(iv) One of the important assumption for the analysis of variance of data is that the experimental errors are independent. If this assumption is violated, the validity of F-test or t-test is always doubtful. But a proper randomization introduces the independence in the assignment of treatments to the experimental units which in turn creates independence amongst the observations. *Thus, randomization may be considered a device to effectively make the experimental errors independent.*

In short, randomization makes tests valid in the analysis of experimental data.

(v) R.A. Fisher in 1949 stated that randomization in experimental designs is one of the required conditions to attach a probability statement to the estimated treatment difference.

Q. 7 What is meant by replication, and what purpose dose it serve in experimental designs?

Ans. Repetition of a treatment on a number of experimental units under similar conditions is called the *replication* of the treatment. As a matter of fact, many purposes are served by the use of replications in an experiment.

(i) Replications are essential to obtain a valid estimate of the experimental error variance.

(ii) Replications are necessarily required to attach a probability statement with estimated treatment differences.

(iii) The larger number of replications reduces the standard error of the treatment mean(s). But one cannot take more than certain number of replications because of the management problems and limitation of resources.

(iv) Replications enable the experimenter to infer whether the differences in treatment means are actually more than the sampling fluctuations. If the variation among treatments is more by a certain quantity as compared to sampling fluctuation, then they are considered to be significantly different, otherwise not.

(v) Replications of a treatment also compensate for the inadvertent favour or disfavour received by a particular treatment from one unit to the other. For example, a treatment may be favoured due to better soil fertility in one plot and be handicapped in the other plot due to poor soil fertility. In this way, average performance of the treatment is better assessed by way of replications.

Q. 8 What do you understand by local control and in what way does it increase the efficiency of an experimental designs?

Ans. Local control is a device to maintain greater homogeneity of experimental units within a block of an experiment or as a whole. This is achieved by considering the natural factors likely to influence the treatment effects inherent within the experimental units. For instance, soil fertility of field is a factor which affects the plant growth and yield. So all the neighbouring plots having the same soil fertility should constitute a block. The soil fertility of land can be assessed by conducting an uniformity trial on the field prior to actual field experiment and making fertility contour map on the basis of fertility gradient. These contours help to form blocks. In animal experiments, local control means the animals of the same litter, breed, age, weight, lactation, etc., be grouped together to constitute a block.

Local control is also called as *error control* by some workers. It contributes a lot in increasing the efficiency of an experiment. A few points in favour of local control are highlighted below:

(i) With the help of fertility map one can form blocks of contiguous plots which are homogeneous in the real sense.

(ii) Local control reduces the experimental error.

(iii) Local control is meant to make designs more efficient.

(iv) It makes any test of significance more sensitive and powerful.

(v) A reduction in experimental error consequently helps the investigator to detect the small real difference between treatment means.

Q. 9 Discuss and define experimental error.

Ans. It is always an endeavour of the experimenter to control all factors which can effect the independent performance of the treatments and the choice of the design depends which can most likely fulfil this objective. But, in spite of all efforts, there are always certain extraneous factors which are beyond the control of the experimenter. *Thus, the error caused by the extraneous factors which are beyond the control of human approach is known as experimenter error.* For instance, germination of seed, plant growth, number of pods or ears per plant, response to feed by the animal, taste of a person, etc., are the factors beyond human control. All such factors add towards experimental error.

The reduction in experimental error provides smaller standard error for a treatment mean or for the difference between two treatment means which results in detecting small differences between treatment means. In this way the efficiency of the experiment increases.

Q. 10 What factors are responsible for determining the number of replications?

Ans. There are mainly nine factors which are responsible for determining the number of replications.

(i) Extent of precision required.

(ii) Heterogeneity of experimental material.

(iii) Availability of resources.

(iv) Size of experimental units.

(v) Required degrees of freedom of experimental error.

(vi) The relative cost of experimental units.

(vii) The extent and nature of competition among experimental units.

(viii) The number and nature of the treatments.

(ix) The fraction to be sampled.

Q. 11 How does the extent of precision work as a criterion for ascertaining the number of replications in an experiment?

Ans. The extent of precision desired by an investigator means to spell out the magnitude of the treatment differences which are expected to be detected through the experiment. As a general rule, if the differences between treatment means are expected to be large, then a low degree of precision is required. Hence, in this situation, a small number of replications are enough. On the contrary, if the differences between treatment means are likely to be small, a large number of replications are to be taken.

Keeping in view the above ideas, if we know the magnitude '*d*' of the difference between two treatment means to be detected, an estimate of the error means square '*s*' based on some previous experiment or some empirical study, then the number of replications can be determined by the formula,

$$r = \frac{2t_\alpha^2 s^2}{d^2}$$

more. Hence, to have a stable information, the number of replications should be such that the available degrees of freedom for error variance are at least 12.

Q. 16 How does the cost of the experimental units play its role in deciding the number of replications?

Ans. Obviously, when the cost per unit is high, one would like to take minimum number of replications and if low, a large number of replications may preferably be taken keeping all other factors responsible for replications constant. In this situation, two designs with unequal number of replications may yield the same amount of information as the cost for additional replications in one experiment may be compensated by lesser per unit cost and vice-versa on a per unit information basis. Thus, if s^2 is the error mean square based on n df taking r replications in the design and c, the cost per replicate, the information available by the experimental data is,

$$\left(\frac{c}{s^2}\right)\left(\frac{n+1}{n+3}\right)$$

Also the relative efficiency of design 1 over design 2 for fixed cost is,

$$E_{12} = \left(\frac{c_1}{s_1^2}\right)\left(\frac{n_1+1}{n_1+3}\right) \Big/ \left(\frac{c_2}{s_2^2}\right)\left(\frac{n_2+1}{n_2+3}\right)$$

Since n_1 and n_2 depend on the number of replications one can choose r keeping in view the cost such that $E_{12} = 1$.

Q. 17 Define relatively efficiency of one design over the other:

Ans. The relative efficiency of design 1 over design 2 is the ratio of the inverse of the error mean square of design 1 to the inverse of the error means square of design 2, *i.e.*,

$$E_{12} = \frac{1/\sigma_1^2}{1/\sigma_2^2} = \frac{\sigma_2^2}{\sigma_1^2}$$

Q. 18 How does sampling in experimental units have bearing on the number of replications?

Ans. If the plot size in an experiment is large and it is not possible to harvest the entire plot for the study of certain characters, a sample from experimental units is often selected. Therefore, an experiment should be planned in such a way that the plot size and required number of replicates should result in the required degree of precision in spite of sampling. Yates and Zacopanay suggested that for the cereal crops, the sampling rate should be about 6 per cent of the plot. Also they worked out that the loss of information due to sampling as compared to complete harvesting was about one-third.

Q. 19 What is meant by analysis of variance of experimental data?

Ans. The measurements are taken on each experimental unit pertaining to character of interest and its variance is calculated. This total variance is due to various factors involved in the experiment. The purpose of analysis of variance is to split the total variance into component variances and to test the hypothesis about these component factors. The main interest lies in estimating and comparing pairwise treatment or testing contrasts (comparisons) among treatments.

The component variance are always displayed and tested in an analysis of variance table abbreviated as ANOVA table. An analysis of variance table contains five columns as depicted below.

The sources of variance are various components *A*, *B*, *C*, ... and also the experimental error which are responsible for total variation.

The skeleton ANOVA table is as given below:

Source of variation	*Degrees of freedom*	*Sum of squares*	*Mean sum of squares*	*F-value*
		OR		
Source of variation	*d.f.*	*S.S.*	*M.S.*	*F-value*
A				
B				
C				
⋮				
Error				

The analysis of variance enables one to know whether the variance due to a component factor is significantly more than the variance due to experimental error or not. Here it should be kept in mind that mean sum of square due to component is nothing but the variance due to that component. Also the ratio of two component variances is distributed as *F* with corresponding d.f. Also often the interest lies in particular comparisons (contrasts) of treatments and to test their significance. This can also be tested by analysis of variance. For comparing pairwise treatments, further tests have to be carried out.

Q. 20 How do we decide the component factors responsible for variation in an experiment?

Ans. The measurement, generally the yield, in crop experiments or likewise any other character measured is as a result of many factors which are usually taken to be of additive nature. For instance, the yield of a cereal in an experiment having *k* treatments applied in each of the *r* blocks of *k* plots which are homogeneous within themselves but likely to be heterogeneous between themselves can be represented by following linear statistical model,

$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij}$$

for $i = 1, 2, ..., k$

$j = 1, 2, ..., r$

where

y_{ij} – yield of the plot receiving i^{th} treatment of j^{th} block.

μ – true mean effect, *i.e.*, the mean yield which would have been even without the i^{th} treatment and j^{th} block effects.

τ_i – true effect of the i^{th} treatment.

β_j – true effect of the j^{th} block.

ε_{ij} – error, *i.e.*, true effect of the experimental unit receiving i^{th} treatment in j^{th} block.

In the above model it is assumed that μ is a constant and ε_{ij} are identically and independently distributed normal variates with mean zero and variance σ_e^2, *i.e.*, $\varepsilon_{ij} \sim N\left(0, \sigma_e^2\right)$. But the specification of the model is based on the nature of treatment τ_i. Thus, the components of variance in the ANOVA table come from statistical model. Here the factors are μ, τ, β and ε.

Q. 21 Is there any method of testing the additiviy of a statistical model?

Ans. Yes, if there is any doubt about the addivity of components in a statistical model, it can be tested by a test suggested by J.W. Tukey in 1949. The procedure is a part of error sum of squares and has one degrees of freedom. This sum of square tend to be inflated if there is non-additivity. This sum of square is tested against error mean square by *F-test.* If *F*-value is significant, it is indicative of non-additivity. In this situation, the investigator should think of alternative model for it.

Q. 22 What are different types of statistical model for experimental designs?

Ans. Statistical models for experimental designs are classified into three types namely,

(i) fixed effect model (Model-I)

(ii) random effect model (Model-II)

(iii) mixed effect model (Model-III)

The specification of the model emerges with the idea that either the effect of treatments (factors) is of fixed or random nature. Also there are some experiments in which certain effects are of fixed type and others of random nature. Such experiments lead to mixed effect model.

Q. 23 Explicate fixed effect model.

Ans. A fixed effect model is also known as *analysis of variance model* or *Model I.* In this model, the investigator is concerned to draw inferences about *t* treatments involved in the experiment. If we take the model for a design in which *k* treatments are randomly assigned to *n* homogeneous units whereas i^{th} treatment is replicated r_i times such that $\sum_{i=1}^{k} r_i = n$. Then the statistical model with usual notations is,

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij} \qquad (1)$$

for $i = 1, 2, ..., k$

$j = 1, 2, ..., r_i$

In case, the main interest likes only in estimating significance of differences between all pairs of treatment means involved in an experiment.

Ans. When the F-value for treatments comes out to be significant, it reveals that treatment means are not equal. But the question remains unanswered, which of them differ significantly with one another and which do not. Various tests have been evolved for comparing differences among treatment means by a number of statisticians as named below. These tests are known as *multiple range tests.*

(i) Least significant difference (lsd) test or multiple t-test.

(ii) Student-Newman-Kuel test.

(iii) Duncan's multiple range test.

(iv) Tukey's test.

Q. 27 How can one compare the differences in a set of paired treatment means with the help of least significant difference?

Ans. Firstly a standard value is calculated which is the minimum difference required to be significant between any two treatment means in a set of treatments. In principle, the formula for least significant difference is,

$$lsd = \sqrt{\frac{2s_e^2}{r}} \times t_{.05,\nu}$$

where s_e^2 is the error mean square associated with ν d.f. and r, the replication of treatments and $t_{.05,\nu}$ is the t-value at 5 per cent level of significance and ν d.f. Compare the absolute differences between the treatment means with the value of *lsd.* If the difference between any two means remains less than *lsd,* it is non-significant and as soon as a difference comes out greater than or equal to *lsd*, it becomes significant.

Least significant difference is considered appropriate for small number of treatment means. As a matter of fact, type I error increases as the number of treatment means increases. Type I error associated with the largest and smallest treatment mean pair for five treatments is 27 per cent, for ten treatments it is 59 per cent and for twenty treatments, it is 86 per cent.

Q. 28 What is Student-Newman-Kuel's test for comparing the ordered treatment means?

Ans. This test takes care of the distance between two means in an ordered set of treatment means. This test is similar to *lsd* test except that for every pair of treatment means, a separate value of 'q' for percentage points depending on the level of significance α and the distance between the ordered means and ν d.f. for error is used instead of t.

The value,

$$q_{\alpha,n,\nu} = \frac{\bar{x}_{\max} - \bar{x}_{\min}}{S_{\bar{x}}}$$

where $$s_{\bar{x}} = \sqrt{\frac{s_e^2}{r}}$$

and r is the number of replications for each treatment.

The difference between two extreme treatment means in the ordered set $(\bar{x}_n - \bar{x}_1)$ is compared with W_n where,

$$W_n = q_{\alpha,n,\nu}.\sqrt{2}\ s_{\bar{x}}$$

Similarly for $\bar{x}_{n-1}$ vs. $\bar{x}_1$,

$$W_{n-1} = q_{\alpha,n-1,\nu}\sqrt{2}\ s_{\bar{x}}$$
$$\vdots$$

and for $\bar{x}_2$ vs. $\bar{x}_1$,

$$W_2 = q_{\alpha,2,\nu}.\sqrt{2}\ s_{\bar{x}}$$

and so on.

The values of $q_{\alpha,n,\nu}$ can either be calculated or can be read from Table-29 of *Biometrika Tables for Statisticians*, Vol. 1, Cambridge Univ. Press, 1954 by E.S Pearson and H.O. Hartley.

The test can easily be performed by preparing a two way difference table by taking the treatment means along rows in descending order and along columns in ascending order. If $W_n \geq q_{\alpha,n,\nu}\sqrt{2}\ s_{\bar{x}}$, then the difference between two means is significant, otherwise not.

Q. 29 Give Duncan's multiple range test for comparing the paired treatment means.

Ans. Duncan's multiple range test makes use of a least significant difference value for each pair of treatment means. The treatment means are arranged in order and the least significant value for comparing two treatment means situated at a distance n in an ordered set and α level of significance calculated by the formula,

$$D_{\alpha,n} = s_{\bar{x}} \times \begin{array}{l}\text{(Dunchan's significant range}\\ \text{value for } \alpha \text{ level, } n \text{ distance}\\ \text{and } \nu \text{ error d.f.)}\end{array}$$

Duncan's significant ranges can be seen in the table given in *Biometrics*, 11, 1-42, 1955 by D.B. Duncan.

For each pair situated at different distances, a different Duncan's least significant value is obtained and compared with the difference in the two treatment means. If this difference is greater than or equal to Duncan's least significant value $D_{\alpha, n}$, then the two treatments are taken to differ significantly in their effects. All possible pairs of Treatment means can conveniently be compared by preparing a two way difference table by taking the treatment means in ascending order along rows and in descending order along columns. In this table all differences are covered by the upper diagonal and diagonal elements.

The greatest advantage of Duncan's multiple range test lies in the fact that it allows the experimenter to commit fewer *Type II error* and more *Type I error* than *lsd* and Newman-Kuel's test.

D.B. Duncan showed that the level of significance for k-treatments changes according to the following formula.

$$\alpha_k = 1-(1-\alpha)^{k-1}$$

Q. 30 What is special with Tukey's test?

Ans. It is a multiple range test similar to lsd test except that Tukey utilised honestly significant difference (*hsd*) test or the *w*-procedure. This is one approach out of many discussed by Tukey. The value of honestly significant difference is equal to $W_n = k$ of Newman-Kuel multiple range test. Any two treatment means having a difference more than honestly significant difference (*hsd*) are said to be significantly different, otherwise not. *hsd* for n treatments is equal to $q_{\alpha,n} \cdot s_{\bar{x}}$, where $q_{\alpha,n}$ is same as Student-Newman-Kuels critical value.

Q. 31 Discuss Contrasts.

Ans. A researcher is not only interested in comparing pairwise treatment means but is often interested in certain specific functions of treatments known as *contrasts*. Here it is to point out that the term *comparison* is equally and frequently used for contrasts.

If there are k treatment totals, $T_1, T_2, \ldots, T_k$ where the treatment T_i is replicated r_i times for $i = 1, 2, \ldots, k$, then a linear combination Z_w of k treatment totals defined as,

$$Z_w = l_{w1}\, T_1 + l_{w2} T_2 + \ldots + l_{wk} T_k$$

is said to be a contrast iff,

$$r_1\, l_{w1} + r_2 l_{w2} + \ldots + r_k l_{wk} = 0$$

or
$$\sum_{i=1}^{k} r_i\, l_{wi} = 0$$

Since most of the theory of contrasts has been developed by taking equireplicate treatments, we also consider that each treatment is replicated r times. Hence, the linear combination z_w among k treatment totals is said to be a contrast iff

$$l_{w1} + l_{w2} + \ldots + l_{wk} = 0$$

or
$$\sum_{i=1}^{k} l_{wi} = 0$$

For instance, linear combinations among three quantities T_1, T_2 and T_3 like, $T_1 - 2T_2 + T_3$; $T_1 - T_3$; $T_3 - T_1$; $T_1 - T_2$ are contrasts. Similarly for treatments T_1, T_2, T_3 and T_4, the linear combinations like, $T_1 + T_2 - T_3 - T_4$; $-3T_1 - T_2 + T_3 + 3T_4$; $T_1 - T_2 - T_3 + T_4$, etc., are contrasts. In the same way, for five quantities T_1, T_2, T_3, T_4 and T_5, the linear combinations like, $-2T_1 - T_2 + T_4 + 2T_5$; $T_1 - T_3$; $T_1 - 2T_2 + T_5$, etc., are contrasts. An arbitrary contrast does not reveal anything worth-while to the researcher. Hence, the interest lies in a particular set of contrasts known as orthogonal contrasts or a contrast depicting a specially known effect of treatment combination.

Q. 32 Distinguish between pairwise and nonpairwise contrasts.

Ans. A linear combination of K (≥ 3) treatments, in which all coefficients are zero except two coefficients whose sum is also zero, is called a pairwise contrast. Any contrast among K treatments with more than two non-zero coefficients is a nonpairwise contrast.

Q. 33 Differentiate between *a priori* and *posteriori* contrasts.

Ans. A contrast which is constructed for testing its significance with specific purpose before the analysis of data is called *a priori* or *planned* contrast. On the other hand, a contrast constructed following a significant F-test is called *a posteriori* or *post hoc* or *unplanned* contrast.

Q. 34 What are orthogonal contrasts?

Ans. Two contrast Z_w and $Z_{w'}$ among k equireplicate treatment totals $T_1, T_2, ..., T_k$ say,

$$Z_w = l_{w1}T_1 + l_{w2}T_2 + \ldots + l_{wk}T_k \qquad (1)$$

and $$Z_{w'} = l_{w'1}T_1 + l_{w'2}T_2 + \ldots + l_{w'k}T_k \qquad (2)$$

are said to be orthogonal iff,

$$l_{w1}\, l_{w'1} + l_{w2}\, l_{w'2} + \ldots + l_{wk} l_{w'k} = 0$$

i.e., $$\sum_{i=1}^{k} l_{wi}\, l_{w'i} = 0 \qquad (3)$$

Out of k treatments, there can be utmost $(k-1)$ orthogonal contrasts. The idea of orthogonal contrasts emerged from orthogonal polynomial. If we have k treatments, an orthogonal polynomial of order $(k-1)$ can at most be fitted. For instance, if $k = 4$, we can at the maximum fit a cubic,

$$E(y) = a + bx + cx^2 + dx^3 \qquad (4)$$

between the response variable y and the treatment levels x of a treatment or factor which are equidistant.

Instead of fitting the equation (4), we fit in the polynomial,

$$E(y) = \beta_0\,\mu_0 + \beta_1\,\mu_1 + \beta_2\,\mu_2 + \beta_3\,\mu_3 \qquad (5)$$

In polynomial (5), $u_0 = 1$ and u_i for $i = 1, 1, 2, 3$, are polynomials of order i in x. Elaborately,

$$u_1 = a_1 + b_1\, x\,; u_2 = a_2 + b_2\, x + c_2 x^2;$$

$$u_3 = a_3 + b_3\, x + c_3\, x^2 + d_3\, x^3$$

These polynomials are orthogonal if $\Sigma u_i\, u_i' = 0$ for each pair (i, i') for $i \neq i'$. Now let us consider four levels of a factor as 30, 60, 90, 120. May it be fertilizer does, temperature, timings, concentrations of a chemical compound, etc. Now consider $u_1 = a_1 + b_1\, x$. The condition for u_1 to be an orthogonal polynomial is $\Sigma u_0\, u_1 = 0$. Since $u_0 = 1$, $\Sigma\, u_1 = 0$. Thus,

$$\Sigma u_1 = ka_1 + b_1\, \Sigma x \qquad (6)$$

$$0 = 4a_1 + 300\, b_1 \qquad (7)$$

Equation (7) is a single equation in two unknowns a_1 and b_1. The solution of (7) which results into the smallest integral values of u_1 for the four values of x are $a_1 = -5$, $b_1 = \frac{1}{15}$. Putting these values in $u_1 = a_1 + b_1\, x$

or $$u_1 = \frac{x - 75}{15}$$

when $x = 30$, $u_1 = -3$.

$x = 60$, $u_1 = -1$

$x = 90$, $u_1 = 1$

$x = 120$, $u_1 = 3$

Thus, the contrast,

$$-3T_1 - T_2 + T_3 + 3T_4 \qquad \text{(I)}$$

represents the linear effect.

Now we consider the quadratic,

$$u_2 = a_2 + b_2\, u_1 + c_2\, u_1^2$$

The orthogonality conditions are,

$$\Sigma\, u_0\, u_2 = 0 \text{ and } \Sigma\, u_1\, u_2 = 0$$

The condition $\Sigma\, u_0\, u_2 = 0$ gives the equation,

Q. 39 In what way are contrasts helpful in the analysis of experimental data?

Ans. Contrasts have great utility in the analysis of experimental data as given below:

(i) A researcher can see the effect of a specified combination of treatments conceived prior to analysis of the data.

(ii) We can calculate the linear, quadratic, cubic effects, etc., of treatments through contrasts.

(iii) All main effects and interactions in a 2^n factorial experiment can be represented by a set of orthogonal contrasts.

(iv) The treatment sum of square can be partitioned into component sum of squares each having one degree of freedom through orthogonal contrasts.

(v) Contrasts provide much more information about treatments than a simple F-test of equality of treatment means.

Q. 40 What is the classification made for experimental designs?

Ans. The experimental designs (plans) have been divided into two classes namely,

(i) systematic designs (ii) randomized designs.

Q. 41 What are systematic designs?

Ans. Experimental plans, which are devoid of randomization process, are categorised as *systematic designs*. In these type of designs all the treatments, being repeated a fixed number of times, are assigned together on adjacent experimental units sequentially one after the other in any direction. For instance, if there are four treatments *A, B, C,* and *D* and each treatment is replicated three times, then a systematic design may be set in any of the following manner.

A	*B*	*C*	*D*
A	*B*	*C*	*D*
A	*B*	*C*	*D*

Plan (i)

A	*A*	*A*
B	*B*	*B*
C	*C*	*C*
D	*D*	*D*

Plan (ii)

ABCD	*ABCD*	*ABCD*

Plan (iii)

The history of agricultural field research using systematic designs dates back to 1834.

Q. 42 What are the disadvantages of systematic designs?

Ans. The main disadvantages of systematic designs are:

(i) One cannot rule out the possibility of personal biases.

(ii) Systematic errors are introduced due to correlation among adjacent experimental plots.

(iii) It is not possible to have valid estimates of variances as no probability is involved. This makes the analysis of experimental data meaningless.

Due to the above disadvantages, systematic designs are out of context.

Q. 43 What are the advantages of systematic designs?

Ans. The advantages of systematic designs are delineated below:

(i) There is an ease of operation particularly in crop experiments.

(ii) It enables the experimenter to make an intelligible placement of various treatments to experimental units.

(iii) Systematic designs make sampling of experimental area easier and better.

(iv) The chances of drifting the influence of one treatment over the other are reduced.

Q. 44. When do you call an experimental design a randomized design?

Ans. Experimental designs in which the treatments are allocated randomly to the experimental units come under the category of randomized designs and are classified as completely randomized design, randomized block design, Latin square design, split plot design, cross over design, progeny row trial, family block design, etc.

Q. 45 Characterise a completely randomized design.

Ans. Any design in general is characterised by the nature of grouping of experimental units and the manner in which the treatments are randomly allocated to the experimental units.

Completely randomized design is the one in which all the experimental units are taken in a single group which are homogeneous as far as possible. For example, all the field plots constituting the group are having the same soil fertility, soil depth, soil texture, soil moisture, etc.; all the cows forming a group are of the same breed, same age, same weight, same lactation, etc. Suppose there are k treatments and the treatment T_i is replicated r_i times. So we require $\sum_{i=1}^{k} r_i = n$ units. Assign K treatments randomly to n experimental units such that the treatment T_i occupies r_i units or plots. Such a design is called a *completely randomised design* (CRD). This design is also seldom called a *non-restrictional design.* Completely randomized design is most suitable for laboratory experiments, pot experiments, green house experiments, and sometimes for poultry and animal experiments.

To specify more elaborately, suppose there are four treatments T_1, T_2, T_3 and T_4 which are replicated 4, 3, 3 and 5 times, respectively. Then the layout of the completely randomized design having 15 plots (units) and 4 treatments, using a one digited random number table is as displayed below:

T_2 (1)	T_2 (10)	T_2 (11)
T_3 (2)	T_4 (9)	T_3 (12)
T_4 (3)	T_4 (8)	T_1 (13)
T_1 (4)	T_1 (7)	T_1 (14)
T_3 (5)	T_4 (6)	T_4 (15)

Layout of a CRD

Q. 46 What are the merits of a completely randomized design?

Ans. A completely randomized design possesses many virtues which are delineated below:

(i) Its layout is very easy.

(ii) There is complete flexibility in this design, *i.e.*,, any number of treatments and replicates for each treatment can be taken.

(iii) Whole experimental material can be utilised in this design.

(iv) This design yields maximum degrees of freedom for experimental error.

(v) The analysis of data is simplest as compared to any other design.

(vi) Missing observation(s) creates no problem in analysis of data. The analysis is carried out in the usual manner neglecting the missing plot as if it was not there in the experiment. This does not break any assumption or rule of analysis of variance.

(vii) Completely randomized designs are specially suitable in situations where an appreciable fraction of the units is likely to be destroyed during experimentation or is likely to fail to respond.

Q. 47 What are the demerits of a completely randomized design?

Ans. Completely randomized design suffers from many lacunae which are as follows:

(i) The design is suitable for a small number of treatments.

(ii) It is difficult to find homogeneous experimental units in all respects.

(iii) Completely randomized design is seldom suitable for field experiments as compared to other experimental designs.

Q. 48 Give statistical model for completely randomized design with one observation per unit.

Ans. The appropriate statistical model for a completely randomized design with k treatments and r_i replications of the treatment T_i is,

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij} \qquad (1)$$

for $i = 1, 2, \ldots, k$

$j = 1, 2, \ldots, r_i$

Also $\sum_{i=1}^{k} r_i = n =$ total number of experimental units.

In the above model,

μ – true mean effect.

τ_i – true effect of the i^{th} treatment.

	Treatments						
	T_1	T_2	...	T_i	...	T_k	
	y_{11}	y_{21}	...	y_{i1}	...	y_{k1}	
	y_{12}	y_{22}	...	y_{i2}	...	y_{k2}	
	$\vdots$	$\vdots$		$\vdots$		$\vdots$	
	y_{1r_1}	y_{2r_2}	...	y_{ir_i}	...	y_{kr_k}	
Total	$y_{1\cdot}$	$y_{2\cdot}$	...	$y_{i\cdot}$	...	$y_{k\cdot}$	$y_{\cdot\cdot}$

$$\text{Treatment S.S.} = \frac{y_{1.}^2}{r_1} + \frac{y_{2.}^2}{r_2} + \ldots + \frac{y_{k.}^2}{r_k} - \frac{y_{..}^2}{n}$$

$$= \sum_{i=1}^{k} \frac{y_{i.}^2}{r_i} - \text{C.F.}$$

$$\text{Total S.S.} = \sum_{i=1}^{k} \sum_{j=1}^{r_i} y_{ij}^2 - \text{C.F.}$$

$$\text{Error S.S.} = \sum_{i=1}^{k} \sum_{j=1}^{r_i} y_{ij}^2 - \sum_{i=1}^{k} \frac{y_{i.}^2}{r_i}$$

Q. 51 Given the following data obtained from a completely randomized design with four treatments, analyse the given data and draw conclusion about the equality of treatment effects.

Treatments			
T_1	T_2	T_3	T_4
20.9	23.7	13.2	5.8
12.4	14.4	10.2	6.1
10.1	9.0	5.1	4.8
4.2			1.5

Ans. Here we test,

$$H_0: \tau_1 = \tau_2 = \tau_3 = \tau_4$$

vs. H_1: at least two of them are not equal.

Treatment Total,

$$T_1 = 47.6, T_2 = 47.1, T_3 = 28.5, T_4 = 18.2$$

and $G = 141.4$

Number of replications for treatments T_1, T_2, T_3 and T_4 are, $r_1 = 4$, $r_2 = r_3 = 3$ and $r_4 = 4$ respectively and $n = 14$.

$$y_{..} = 141.4, \text{ C.F.} = \frac{(141.4)^2}{14} = 1428.14$$

$$\begin{aligned} \text{Total S.S.} &= (20.9^2 + 12.4^2 + \ldots + 1.5^2) - \text{C.F.} \\ &= 1960.70 - 1428.14 \\ &= 532.56 \end{aligned}$$

$$\text{Treatment S.S.} = \frac{47.6^2}{4} + \frac{47.1^2}{3} + \frac{28.5^2}{3} + \frac{18.2^2}{4} - \text{C.F.}$$

$$\begin{aligned} &= 1659.47 - 1428.14 \\ &= 231.33 \\ \text{Error S.S.} &= 532.56 - 231.33 \\ &= 301.23 \end{aligned}$$

ANOVA Table

Source of Variation	*d.f.*	*S.S.*	*M.S.*	*F-value*
Treatments	3	231.33	77.11	2.56
Error	10	301.23	30.12	
Total	13	532.56		

Tabulated value of $F_{.05\,(3,10)} = 3.71$ which is greater than the calculated value of $F = 2.56$.

Hence, we accept H_0 which means that the four treatments are equally effective.

Q. 52 Is there any equivalence between the analysis of data in completely randomized design with two treatments and student's *t*-test?

Ans. Yes, in analysis of variance with two treatments we test H_0: $\tau_1 = \tau_2$ under Model-I by *F*-test and in student *t*-test, we test H_0: $\mu_1 = \mu_2$ where $\mu_1 = \tau_1 + \mu$ and $\mu_2 = \tau_2 + \mu$.

Q. 53 What do you understand by sub-sampling in experimental designs?

Ans. If more than one determinations are made on each experimental unit pertaining to the same character, such a process of collecting data is known as sub-sampling. For example, one may be interested in determining the nitrogen uptake in plants and

(i) We should confirm whether the postulated linear model is true to the existing situation or not.

(ii) Check whether the proper randomization had been adopted or not.

(iii) One must check whether observations follow the law of normality or not?

(iv) We must test whether the variances of sub-classes are homogeneous or not.

Q. 56 What do you understand by crossed classification?

Ans. If every level of a factor A can be used in combination with every level of the other factor B, then such a situation is known as *crossed classification.* Here the factors have crossed each other and their interactions are the sub-classes or cells wherein the data arise. Even if the data for some cell does not exist, even then it remains crossed classification as it does not rule out the feasibility of the existence of that cell. For instance, one may be interested to know the effect of 3 levels of fertilizers on 4 varieties of wheat. Here we have 12 cells and will be able to get the effects of fertilizer levels, varieties and their interactions.

Q. 57 When is a classification called nested or hierarchical classification?

Ans. When we have a set of sampled units and from each sampled unit, a sub-sample is taken and from sub-sampled unit, a number of determinations are taken. *Such a process of repeated sampling and sub-sampling within each sampling unit is called nested or hierarchical classification.* For example, we may select some plant from a field, from each plant we may select a few leaves and on each leave we may do three estimations of calcium content. Such a classification will lead to nested or hierarchical classification.

When we consider the factors in an experiment in which all levels of a actor occur within each single level of some other factor, we should call these factors as *nested factors.* As an instance, in a plant breeding experiment if a researcher takes 's' species and n_i strains within each specie and study r_{ij} individuals per strain, the factors involved in this manner are called nested factors. As an other example, if we may take s sires, each sire is mated to a set of d dams and study r_{ij} siblings of ds sire-dam combinations, the factors involved in this experiment are called nested factors. "In short, the effects of a factor which are restricted to a single level of the other factor are said to be nested within that factor.

Q. 58 Discuss the model for a two-factor nested design and its peculiarity and the analysis of variance table.

Ans. Consider a two-factor experiment in which the factor B is nested under factor A. Let there be a levels of a factor A and b levels of factor B. The statistical model for such a design with n determinations on each unit is as follows:

$$y_{iju} = \mu + \alpha_i + \beta_{j(i)} + \varepsilon_{iju} \tag{1}$$

for $i = 1, 2, ..., a$

$j = 1, 2, ..., b$

$u = 1, 2, ..., n$

where,

μ – true mean effect

α_i – true effect of the i^{th} level of A

$\beta_{j(i)}$ – true effect of the j^{th} level of B nested under i^{th} level of A.

The peculiar feature of this nested experiment is that the interaction effect of A and B cannot be evaluated. Analysis of variance table-6 for the above model of the nested design is given on page 568.

Mean square for A is to be tested against B or E will depend on the model specification.

Q. 59 Describe a randomized complete block design.

Ans. In common parlance, randomised complete block design is spoken as *randomized block design* (R.B.D.). The word *complete* is implicit.

If experimental units show one way variation, the effect of this kind of heterogeneity is taken care of by grouping the experimental units into blocks in such a way that the units are homogeneous within a block but there is every possibility of heterogeneity amongst blocks.

Each block consists of as many experimental units as the number of treatments.

Table - 6

Source of Variation	*d.f.*	*S.S.*	*M.S.*	*F-value*
Among A's	$a-1$	$\frac{1}{nb}\sum_i y_{i..}^2 - \frac{y_{...}^2}{abn} = A_{yy}$	$\frac{A_{yy}}{a-1} = A_y$	A_y/B_y or E_y
Among B's within A's	$a\,(b-1)$	$\sum_i \left\{\sum_j \frac{y_{ij.}^2}{n} - \frac{y_{i..}^2}{nb}\right\} = B_{yy}$	$\frac{B_{yy}}{a(b-1)} = B_y$	B_y/E_y
Experimental error (within determinations)	$ab\,(n-1)$	$\sum_i \sum_j \left\{\sum_u y_{iju}^2 - \frac{y_{ij.}^2}{n}\right\} = E_{yy}$	$\frac{E_{yy}}{ab(n-1)} = E_y$	
Total	$abn-1$	$\sum_i \sum_j \sum_u y_{iju}^2 - \frac{y_{...}^2}{abn}$		

The treatments are allocated randomly to the experimental units within each block independently such that each treatment occurs once. So each treatment is replicated as many times as the number of blocks or the number of blocks are chosen to be equal to the number of replications for the treatments.

In this design all treatments are equireplicated. Sometimes blocks are also termed as replications.

Randomized block designs are used in field experiments where the soil fertility is varying in one direction. Blocks are always formed in a direction perpendicular to fertility gradient. In animal experiments, litters, pens or cages may be taken as blocks each containing similar animals within the pen and they may differ from pen to pen. An oven may form one block maintained at one temperature, while other one may form the other block maintained at some other temperature, etc.

Randomized block design is a *one restrictional design.*

As a word of caution, one should form the groups or blocks in respect of the observable character or variable which is most likely to influence the character(s) under study.

The layout of a randomised block design with five treatments and four blocks can be of the kind given ahead:

Block-1	*Block-2*	*Block-3*	*Block-4*
A	*B*	*E*	*C*
C	*D*	*D*	*A*
D	*A*	*B*	*D*
E	*C*	*A*	*E*
B	*E*	*C*	*B*

Q. 60 Enumerate the advantages of a randomized complete block design over a C.R.D.

Ans. Randomized complete block design is superior to completely ramdomized design in many respects as delineated below:

(i) Blocking reduces the sum of squares due to experimental error. So it increases the efficiency of the design as compared to C.R.D.

(ii) The design is feasible with any number of treatments and blocks subject to the basic requirements of the design. For instance, there should be at least two blocks to provide an estimate of error variance and a test of significance.

(iii) The number of replicates should be enough to contribute at least 12 d.f. to error.

(iv) Control treatments can easily be included without causing any complications in the analysis of data.

(v) The statistical analysis is simple and less time consuming.

(ii) the error degrees of freedom are the product of the blocks and treatments degrees of freedom.

Q. 64 For the observations of a randomized block design with k treatments and b blocks presented below, give the formulae for calculating the sum of squares required for analysis of variance of data.

Table - 8

Treatments	*Blocks*						*Total*
	B_1	B_2	...	B_j	...	B_b	
T_1	y_{11}	y_{12}	...	y_{1j}	...	y_{1b}	$y_{1.}$
T_2	y_{21}	y_{22}	...	y_{2j}	...	y_{2b}	$y_{2.}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
T_i	y_{i1}	y_{i2}	...	y_{ij}	...	y_{ib}	$y_{i.}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
T_k	y_{k1}	y_{k2}	...	y_{kj}	...	y_{kb}	$y_{k.}$
Total	$y_{.1}$	$y_{.2}$	...	$y_{.j}$	...	$y_{.b}$	$y_{..}$

Ans. S.S. due to correction for mean, *i.e.*, correction factor (C.F.) = $y_{..}^2/bk$.

$$\text{Block S.S.} = \frac{1}{k}\{y_{.1}^2 + y_{.2}^2 + \ldots + y_{.b}^2\} - \frac{y_{..}^2}{bk}$$

$$= \frac{1}{k}\sum_{j=1}^{b} y_{.j}^2 - \frac{y_{..}^2}{bk}$$

$$\text{Treatment S.S.} = \frac{1}{b}\{y_{1.}^2 + y_{2.}^2 + \ldots + y_{k.}^2\} - \frac{y_{..}^2}{bk}$$

$$= \frac{1}{b}\sum_{i=1}^{k} y_{i.}^2 - \frac{y_{..}^2}{bk}$$

$$\text{Total S.S.} = \sum_{i=1}^{k}\sum_{j=1}^{b} y_{ij}^2 - y^2../bk$$

Error S.S. = Total S.S. – Block S.S. – Treat. S.S.

$$= \sum_{i=1}^{k}\sum_{j=1}^{b} y_{ij}^2 - \frac{1}{k}\sum_{j=1}^{b} y_{.j}^2 - \frac{1}{b}\sum_{i=1}^{k} y_{i.}^2 + y^2../bk$$

Q. 65 How can the relative efficiency of a randomized block design relative to a completely randomized design be calculated?

Ans. Consider a R.B.D. with k treatments and b blocks. Also let B and E be the block and Error M.S. respectively. The relative efficiency (R.E.) of R.B.D. relative to C.R.D. utilising the same experimental material is,

$$\text{R.E.} = \frac{\left(\dfrac{n_2+1}{n_2+3}\right)\dfrac{1}{\text{Error M.S. for RBD}}}{\left(\dfrac{n_1+1}{n_1+3}\right)\dfrac{1}{\text{Error M.S. for CRD}}}$$

$$= \frac{(n_1+3)(n_2+1)\times \text{Error M.S. for CRD}}{(n_1+1)(n_2+3)\times \text{Error M.S. for RBD}}$$

where $n_1 = k(b-1), n_2 = (k-1)(b-1)$.

Error mean square E_1 of a C.R.D. with the same experimental material as in R.B.D. is equal to,

$$E_1 = \frac{(b-1)B_y + b(k-1)E_y}{(bk-1)}$$

Thus,

$$\text{R.E.} = \frac{\{(k-1)(b-1)+1\}\{k\,(b-1)+3\}\;E_1}{\{(k-1)(b-1)+3\}\{k\,(b-1)+1\}\;E}$$

Q. 66 In what way can the analysis of data of a randomized block design having one missing value be carried out?

Ans. If during the process of experimentation, the response value from one experimental unit is lost for whatsoever reason, it cannot be neglected as the data do not conform to the pattern which was originally postulated. Hence, the missing value is estimated first and then analysis of variance is carried out in the usual manner with a little modification that one d.f. is reduced from total d.f. which results into the reduction of one d.f. for error.

F.E. Allan and J. Wishart were the first who proposed a formula in 1930 to estimate the missing value. The same formula was reaffirmed by F. Yates in 1933 proving that their formula is one which Yates obtained on minimising error sum of square. The method of estimating one missing value is very

(a) The standard Error of the difference between two-treatment means not involving the missing value,

$$\text{S.E}_d = \sqrt{\frac{2s_e^2}{r}} = \sqrt{\frac{2 \times 0.574}{3}}$$

$$= 0.619/\text{plant}$$

(b) Standard error of the difference between two treatment means, one with missing value and an other mean is,

$$\text{S.E}_d = \sqrt{s_e^2\left[\frac{2}{b} + \frac{k}{b(b-1)(k-1)}\right]}$$

$$= \sqrt{0.574\left(\frac{2}{3} + \frac{9}{3 \times 2 \times 8}\right)}$$

$$= 0.70/\text{plant}$$

Critical difference for the two treatment means not involving the treatment having missing value,

$$\text{C.D}_1 = \text{S.E}_d \times t_{0.05 \times 15}$$

$$= 0.619 \times 2.131$$

$$= 1.319$$

Critical difference for the two-treatment means out of which one-treatment (variety) has a missing value,

$$\text{C.D}_2 = 0.70 \times 2.131$$

$$= 1.492$$

There is a highly significant difference between varieties since the calculated value of $F = 343.61$ is greater than the tabulated value of F at $\alpha = 0.01$ and (8, 15) d.f., *i.e.*, 4.00. Now we find which of the pairs of treatment means differ significantly. The variety means are arranged in ascending order and displayed below. Now comparing the difference between variety means by the appropriate critical difference (C.D.) value, the varieties which do not differ significantly are underlined.

$$\bar{V}_2 \quad \bar{V}_3 \quad \bar{V}_1 \quad \bar{V}_7 \quad (\bar{V}_4) \quad \bar{V}_6 \quad \bar{V}_5 \quad \bar{V}_8 \quad \bar{V}_9$$

$$\underline{18.5\ \ 18.5}\ \ 20.17\ \ 25.63\ \ 27.13\ \ \underline{31.33\ \ 32.03}\ \ 39.17\ \ 39.17$$

Q. 69 Give statistical model and analysis of variance table for the analysis of data of a randomized block design under sub-sampling.

Ans. In many situations, two or more determinations are made for the same characteristic on each experimental unit. Such a process of obtaining several observations on each unit is termed as sub-sampling.

The appropriate statistical model for a randomized block design with k treatments and b blocks in which n observations are taken on each experimental unit is,

$$y_{iju} = \mu + \tau_i + \beta_j + \varepsilon_{ij} + \eta_{iju}$$

for $\quad i = 1, 2, \ldots, k$

$\quad\quad j = 1, 2, \ldots, b$

$\quad\quad u = 1, 2, \ldots, n$

The analysis of variance table for the above model adopting standard notations is as given in Table-9. The ANOVA table contains an extra column meant for depicting the expected mean squares under Model-I.

Expected mean squares in the above ANOVA table clearly reveal that experimental error is appropriate to test the hypothesis H_0: $\tau_i = 0$ and H_0: $\beta_j = 0$ for treatment effects and block effects respectively.

Note. (1) The sum of squares can be calculated in the usual manner.

(2) Under model-II, the expected mean square for blocks and treatments will be $\sigma_n^2 + n\sigma_e^2 + kn\sigma_\beta^2$ and $\sigma_n^2 + n\sigma_e^2 + bn\,\sigma_\tau^2$ respectively and all other terms remain the same.

Q. 70 How can be sampling error be utilised in the test of significance about treatments?

Ans. A.E. Paull in 1950 (Ann. Math. Stat.) initiated the idea of *preliminary test of significance* (P.T.S.) concerning the hypothesis H_0': $\sigma_e^2 = 0$. Here experimental error mean square is tested against

Table - 9

Source of variation	*d.f.*	*S.S.*	*M.S.*	*F-value*	*Expected M.S.*
Blocks	$b-1$	B_{yy}	$\frac{B_{yy}}{b-1}=B_y$	B_y/E_y	$\sigma_\eta^2+n\sigma_e^2+kn\sum_{j=1}^{b}\frac{\beta_j^2}{b}-1$
Treatments	$k-1$	T_{yy}	$\frac{T_{yy}}{k-1}=T_y$	T_y/E_y	$\sigma_\eta^2+n\sigma_e^2+bn\sum_{i=1}^{k}\frac{\tau_i^2}{k-1}$
Experimental Error	$(b-1)(k-1)$	E_{yy}	$\frac{E_{yy}}{(b-1)(K-1)}=E_y$		$\sigma_\eta^2+n\sigma_e^2$
Sampling Error	$bk(n-1)$	S_{yy}	$\frac{S_{yy}}{bk(n-1)}=S_y$		σ_η^2

sampling error mean square by F-test. In case H_0' is accepted, it is always preferable to pool the sum of squares due to experimental error and sampling error and dividing this pooled sum of square by their pooled degrees for freedom. Now use this new pooled mean square for testing H_0: $\tau_i = 0$. Again if H_0' is rejected, then do not pool the two error sum of squares and test H_0: $\tau_i = 0$ against experimental error M.S. In this way, we have performed a F-test for testing the significance of experimental error variance against sampling error variance prior to testing the final hypothesis. Such a test is referred to as *preliminary test of significance.*

Now one pertinent question arises: What would happen if we always pool the two errors or never pool them? Why depend on P.T.S.? To reply these questions, we will follow Paull. Supposed the researcher decides to always pool the two mean squares without performing P.T.S. This will work well if per chance $H_0': \sigma_e^2 = 0$ is true. In case, H_0' is not true and pooling is done, in that situation, the pooled mean square in final F-test tend to be too small and the final F-test produces a significant result even though H_0: $\tau_i = 0$ is true. Such a result is very misguiding. Quoting Paull, "a test which the research worker thinks is being made at the 5 per cent level might actually be at, say, the 47 per cent level." Thus a P.T.S. guards against such an eventuality.

Another advantage of a PTS is that it increases the power of the final F-test relative to the never pool test. So a P.T.S. is a good tool in the hands of a researcher. Of course it is preferable to perform a P.T.S. at some higher level of significance than 5 per cent as the situation may call for. However, a researcher will not go far wrong if he decides to follow the rule of never pooling rather than always pooling. So pooling be done only when P.T.S. permits, otherwise not.

Q. 71 Discuss a Latin square design.

Ans. A Latin square is a balanced two-way classification scheme. This arrangement was originally presented through Latin letters. So it is called a *Latin square.* When these letters represent some treatments of an experiment it is known as *Latin square design.* A Latin square design of a 3 × 3 arrangement can be of the type as,

A	*B*	*C*		*A*	*C*	*B*
C	*A*	*B*		*B*	*A*	*C*
B	*C*	*A*		*C*	*B*	*A*

Arrangement (*a*) Arrangement (*b*)

A Latin square design of order four exhibits the following type of balance of the letters (treatments).

A	*B*	*C*	*D*		*C*	*D*	*A*	*B*
B	*C*	*D*	*A*		*D*	*A*	*B*	*C*
C	*D*	*A*	*B*		*A*	*B*	*C*	*D*
D	*A*	*B*	*C*		*B*	*C*	*D*	*A*

Arrangement (*c*) Arrangement (*d*)

In the same way, Latin square arrangements of any order can be given.

(ii) A Latin square for less than five treatments does not provide adequate degrees of freedom for experimental error.

(iii) A Latin square design of order 2, 3, and 4 may not be as efficient as a C.R.D. or R.B.D. with same number of replications.

Q. 76 Elucidate mutually orthogonal Latin squares.

Ans. Two Latin squares are said to be orthogonal if one square is superimposed over the other, the same pair of symbols occur once and only once in the composite square. Consider the following two squares of order three.

A	*B*	*C*		α	β	γ
B	*C*	*A*		γ	α	β
C	*A*	*B*		β	γ	α

Square (*a*) Square (*b*)

If square (*a*) is superimposed over the square (*b*) or vice-versa, the composite square is as given below:

$A\alpha$	$B\beta$	$C\gamma$
$B\gamma$	$C\alpha$	$A\beta$
$C\beta$	$A\gamma$	$B\alpha$

Square (*c*)

Square (*c*) is such that each pair of Greek and Latin letters has occurred only once. Hence, squares (*a*) and (*b*) are mutually orthogonal. Square (*c*) is known as *Greaco-Latin square.*

In a 3 × 3 Greaco-Latin Square, the degrees of freedom will be partitioned as 2 d.f. each for rows, columns, Greek letters and Latin letters which totals to 8 d.f. Also there are nine experimental units in all leading to eight d.f. for total. So no more orthogonal Latin squares can exist. In this way for a 3 × 3 Latin square, there can only be two orthogonal Latin squares. Now we give a general rule for the existence of the possible number of mutually orthogonal Latin squares.

"*For a Latin square of order p, where p is a prime number or a power of the prime number, there can be at the most a set of (p – 1) mutually orthogonal Latin squares (MOLS)*".

In a set of MOLS, any two Latin squares are orthogonal to each other. For example, a complete set of orthogonal Latin squares of order 4 × 4 is given below:

I					*II*			
A	*B*	*C*	*D*		*A*	*B*	*C*	*D*
B	*A*	*D*	*C*		*C*	*D*	*A*	*B*
C	*D*	*A*	*B*		*D*	*C*	*B*	*A*
D	*C*	*B*	*A*		*B*	*A*	*D*	*C*

III

A	*B*	*C*	*D*
D	*C*	*B*	*A*
B	*A*	*D*	*C*
C	*D*	*A*	*B*

Q. 77 Give the layout of a 4 × 4 Greaco-Latin square with Greek letters α, β, γ, δ and Latin letters *A*, *B*, *C* and *D*.

Ans. A Greaco-Latin square of order 4 × 4 can be as given below:

$A\,\alpha$	$B\,\beta$	$C\,\gamma$	$D\,\delta$
$B\,\delta$	$A\,\gamma$	$D\,\beta$	$C\,\alpha$
$C\,\beta$	$D\,\alpha$	$A\,\delta$	$B\,\gamma$
$D\,\gamma$	$C\,\delta$	$B\,\alpha$	$A\,\beta$

The above Greaco-Latin square is one arrangement out of 144 possible arrangements.

Q. 78 Give statistical model and analysis of variance table for a Latin square of side *k*.

Ans. Statistical model for a Latin square of side *k* with one observation per unit based on the assumptions:

(i) the row, column and treatment effects are additive.

(ii) the treatment effects do not interact with the row or column effects.

(iii) the errors are independently and identically distributed normally with mean 0 and a constant variance.

$$y_{ij(u)} = \mu + \rho_i + \gamma_j + \tau_u + \varepsilon_{ij(u)}$$

for $i = 1, 2, ..., k$
$j = 1, 2, ..., k$
$u = 1, 2, ..., k$

where μ is a true mean, ρ_i, γ_j and τ_u are the true effect of the i^{th} row, j^{th} column and u^{th}

Table - 10

Source of variation	*d.f.*	*S.S.*	*M.S.*	*F-value*	*Expected mean square*	
					Model-I	*Model-II*
Rows	$k-1$	R_{yy}	$\frac{R_{yy}}{k-1}=R_y$	R_y/E_y	$\sigma_e^2+k\sum\limits_i\frac{\rho_i^2}{k-1}$	$\sigma_e^2+k\sigma_\rho^2$
Columns	$k-1$	C_{yy}	$\frac{C_{yy}}{k-1}=C_y$	C_y/E_y	$\sigma_e^2+k\sum\limits_j\frac{\gamma_j^2}{k-1}$	$\sigma_e^2+k\sigma_\gamma^2$
Treatments	$k-1$	T_{yy}	$\frac{T_{yy}}{k-1}=T_y$	T_y/E_y	$\sigma_e^2+k\sum\limits_u\frac{\tau_u^2}{k-1}$	$\sigma_e^2+k\sigma_\tau^2$
Experimental error	$(k-1)$ $\times(k-2)$	E_{yy}	$\frac{E_{yy}}{(k-1)(k-2)}=E_y$		σ_e^2	σ_e^2
Total	k^2-1	$\Sigma\Sigma y_{ij}^2-\text{C.F.}$				

treatment placed in (i, j)th position, respectively.

Under Model-I, the assumptions are,

$$\sum_{i=1}^{k}\rho_i=\sum_{j=1}^{k}\gamma_j=\sum_{u=1}^{k}\tau_u=0$$

Under Model-II, the assumptions are,

$$\rho_i\sim N\left(0,\sigma_\rho^2\right),\gamma_j\sim N\left(0,\sigma_\gamma^2\right),\tau_u\sim N\left(0,\sigma_\tau^2\right)$$

The analysis of variance table based on model (1) along with two extra columns for expected mean squares under Model-I and Model-II is as displayed above (Table-10).

Q 79 How can the sum of squares for analysis of variance of a Latin square design of order k be calculated?

Ans. Let R_i, C_j and T_u be the totals of the ith row, jth column and uth treatment for $i, j, u = 1, 2, ..., k$. Also suppose G is the total of all the k^2 observations. Thus, the expressions for the sum of squares are:

$$\text{Sum of square due to }\mu=\text{C.F.}=\frac{G^2}{k^2}$$

$$\text{Row S.S.}=\frac{1}{k}\left(R_1^2+R_2^2+\ldots+R_k^2\right)-\text{C.F.}=R_{yy}$$

$$\text{Column S.S.}=\frac{1}{k}\left(\text{C}_1^2+C_2^2+\ldots+C_k^2\right)-\text{C.F.}=C_{yy}$$

$$\text{Treatment S.S.}=\frac{1}{k}\left(T_1^2+T_2^2+\ldots+T_k^2\right)-\text{C.F.}=T_{yy}$$

$$\text{Total S.S.}=\sum_i\sum_j y_{ij}^2-\text{C.F.}$$

$$\text{Error S.S.}=\text{Total S.S.}-R_{yy}-C_{yy}-T_{yy}$$

Q. 80 How can the data with a single missing value of a $k\times K$ Latin square design be analysed?

Ans. To analyse the data of a $k\times k$ Latin square design having one missing value, we first estimate the missing value by minimizing the error mean square as suggested by F. Yates in 1933 (Emp. J. Exp. Agr.). Under this procedure, substitute x for the missing value and analyse the data as if all observations are present. Obviously, the error sum of square involves x. To minimize the error variance, differentiate the expression for it with respect to x and equate it to zero. Solving this equation for x, we obtain the estimated value of x. Supposing that R', C' and T' are the totals for the row, column and treatment having the missing value and G' is the total of (k^2-1) available values, the estimated value of x is,

$$\hat{x}=\frac{k(R'+C'+T')-2G'}{(k-1)(k-2)}$$

Substitute the missing value and analyse the data in the usual manner with the following modifications.

Critical difference for two-treatment means not having missing value = $S.E_d \times t_{.05, 5}$

$$\text{C.D.} = 3.01 \times 2.571$$
$$= 7.74$$

Critical difference for the treatment C versus any other treatment = $S.E_d' \times t_{.05, 5}$

$$\text{C.D.} = 3.48 \times 2.571$$
$$= 8.94$$

Comparing the treatment means by the appropriate C.D. values, non-significant differences are underlined.

D	*B*	*C*	*A*
14.25	20	21.5	27

Result: Car *D* is significantly better than Car A. Rest all other cars are at par.

Q. 84 What is meant by group of Latin squares and why are they required?

Ans. It has been observed that a single 3 × 3 Latin square provides only 2 degrees of freedom for error, which is much less than required. Hence, a group of Latin squares is often used to have a sufficient degrees of freedom for error. In groups of experiments, generally lower order Latin squares are used, *i.e.,* 2 × 2 or 3 × 3 Latin squares.

For a group of Latin squares, either we have to take two or more Latin squares at the same experimental site or one Latin square at several locations. In both the situations, the Latin squares considered together for the purpose of analysis of data are said to be a group of Latin squares.

Q. 85 Give the analysis of variance table for a group of *m* Latin squares of side *k*.

Ans. The analysis of variance table for a group of *m* Latin squares of side *k* with usual notations will be as follows (Table-11).

The sum of squares in the followig ANOVA table for various components can be calculated in the usual manner.

Q. 86 Elucidate cross-over design. :

Ans. Balanced designs are often required in situations where number of experimental units are less than the number of treatments. To fulfil the requirement of balancing in such a case, several

Table -11

Source of Variation	*d.f.*	*S.S.*	*M.S.*	*F-value*
Squares (Locations)	$(m-1)$	S_{yy}	$\frac{S_{yy}}{m-1} = S_y$	S_y/E_y
Rows within squares	$m\,(k-!)$	R_{yy}	$\frac{R_{yy}}{m(k-1)} = R_y$	R_y/E_y
Columns within squares	$m\,(k-1)$	C_{yy}	$\frac{C_{yy}}{m(k-1)} = C_y$	C_y/E_y
Treatments	$(k-1)$	T_{yy}	$\frac{T_{yy}}{k-1} = T_y$	T_y/E_y
Treat. × squares	$(m-1\}(k-1)$	T_{syy}	$\frac{T_{syy}}{(m-1)(k-1)} = T_{sy}$	T_{sy}/E_y
Residual within squares or Experimental Error	$m\,(k-1)\,(k-2)$	E_{yy}	$\frac{E_{yy}}{m(k-1)(k-2)} = E_y$	
Total	mk^2-1			

cycles in respect of periods are usually taken. The cross-over design is one such balanced design. Such balanced designs are covered under various names, *cross-over design, switch back design, reversal design, change over design,* etc. All these designs amount to same type of designs.

The cross-over designs are more appropriate for small number of treatments say, 2 or 3. For the construction of this design, the condition is that the number of replicates must be a multiple of the number of treatments.

The cross-over designs are suitable for the problems in which the treatments have carry-over or residual effect. Because this design is capable of measuring the direct effects of treatments by giving a proper rest period between two applications of treatments. Also with the help of this design one is able to measure direct as well as residual effects. In animal and biological experiments, the cross-over designs are frequently used. To remove inter-subject variability for the comparison of bio-availability due to drugs, the cross-over designs are often used.

Q. 87 Give the layouts and analysis of variance tables for the cross-over design.

Ans. Below we give two types of layouts of a cross-over design with two treatments and six subjects. In the layout (*a*), subjects are taken as replicates and in the layout (*b*), we make three Latin squares of order 2.

	Subjects (Replicates)					
Rows	*I*	*II*	*III*	*IV*	*V*	*VI*
Period 1	*B*	*A*	*B*	*B*	*A*	*A*
Period 2	*A*	*B*	*A*	*A*	*B*	*B*

(a) Cross-over design

	Columns					
	Square I		*Square II*		*Square II*	
Row 1	*B*	*A*·	*A*	*B*	*A*	*B*
Row 2	*A*	*B*	*B*	*A*	*B*	*A*

(b) 3 sets of 2 × 2 Latin squares

In both the layouts given above, each treatment occurs equal number of times in each row and a treatment is followed by the other treatment equal number of times. The difference between the two types of arrangements is apparent from the following break-up in ANOVA tables.

The cross-over design gives more degrees of freedom for error than a set of Latin squares. But the set of Latin squares gives an additional information. So, if the number of treatments is four or more, it is inadvisable to use a cross-over design in place of sets of Latin squares.

ANOVA: Cross-over design

Table - 12

Source of Variation	*d.f.*
Columns	5
Rows (Pd. 1 vs. Pd. 2)	1
Treatments	1
Error	4
Total	11

Three sets of Latin squares

Table - 13

Source of Variation	*d.f.*
Squares	2
Columns within squares	3
Rows within squares	3
Treatments	1
Error	2
Total	11

Q. 88 What is meant by a factorial experiment?

Ans. In many experiments, our interest does not confine in the effect of a factor alone but in its effect at various levels also. Further, the interest lies in estimating the effect of the levels of a factor at different levels of the other factor(s). The experiments, in which the effect of a number of levels of a factor is to be assessed in combination with levels of other factor(s) simultaneously are called *factorial experiments.* The name factorial experiment was given by F. Yates in 1926 (J. Ministry. Agr.). Before this, such experiments were called *'complex experiments'*. Here one should bear in mind that a factorial experiment is not a factorial design.

Q. 89 What are symmetrical and asymmetrical factorials?

Ans. Each factorial experiment involves the combinations of two or more factors each at two or more levels taken as treatments. If all factors have equal number of levels, they are called symmetrical factorial and otherwise asymmetrical factorial. For instance, an experiment with combination of 3 levels of nitrogen and 3 levels of phosphorous is a symmetrical, 3^2-factorial experiment and experiment with combinations of 3 levels of nitrogen and 2 levels of phosphorous is an asymmetrical, 3 × 2 factorial experiment.

A p^n-factorial means a symmetrical factorial experiment with n factors each at p levels. Also a $p_1 \times p_2 \times p_3 \times \ldots \times p_k$ factorial is an asymmetrical experiment with k factors each at the levels $p_1, p_2, p_3, \ldots, p_k$ respectively where all p's are not equal.

Q. 90 What effects are measured in factorial experiments?

Ans. Two types of effects are measured in factorial experiments namely (i) main effects (ii) interaction effect(s).

(i) The *main effect* of a factor may be defined as a measure of change in response due to change in the level of the factor averaged over all levels of all other factors in a factorial experiment.

(ii) The *interaction* between two factors may be defined as the failure of a factor to give the same response at various levels of the other factor(s).

In general, an interaction is an additional effect due to the combined effect of two or more factors.

Also the interaction AB between two factors A and B is same as BA. Further, the interaction between two factors is known as first order interaction, between three factors as second order interaction and so on.

Q. 91 Write the set of orthogonal contrasts for main effects and interactions in (i) 2^2 factorial, (ii) 2^3 factorial.

Ans. (i) Suppose A and B are two factors each at two levels 0 and 1. Let a_1 and a_0 denote the levels of factor A and similarly b_1 and b_0, the levels of factor B. For simplicity, we denote the higher level a_1 of A by a, lower case letter and its lower level a_0 by 1, *i.e.*, by its absence. In the same manner, b_1 by b and b_0 by, 1, *i.e.*, its absence. The contrasts can be specified for main effects and interaction as given below.

$$A = (a_1 - a_0)(b_1 + b_0)$$

$$= (a-1)(b+1)$$

$$= ab - b + a - (1)$$

Similarly,

$$B = (a_1 + a_0)(b_1 - b_0)$$

$$= (a+1)(b-1)$$

$$= ab + b - a - (1)$$

and

$$AB = (a_1 - a_0)(b_1 - b_0)$$

$$= (a-1)(b-1)$$

$$= ab - a - b + (1)$$

In a concised manner, the contrasts for main effects A, B and the interaction AB can be written as given below supposing that the factors with levels given in columns are attached with the signs for the corresponding factors.

Effects	*(1)*	*a*	*b*	*ab*
A	−	+	−	+
B	−	−	+	+
AB	+	−	−	+

Here it is worth pointing out that the coefficients for the contrast AB can be easily obtained by multiplying the coefficients of A and B contrasts.

(ii) In a manner similar to 2^2 factorials, the set of contrasts for main effects and interactions in 2^3 factorials can be given as follows:

Effects	*(1)*	*a*	*b*	*ab*	*c*	*ac*	*bc*	*abc*
A	−	+	−	+	−	+	−	+
B	−	−	+	+	−	−	+	+
AB	+	−	−	+	+	−	−	+
C	−	−	−	−	+	+	+	+
AC	+	−	+	−	−	+	−	+
BC	+	+	−	−	−	−	+	+
ABC	−	+	+	−	+	−	−	+

Q. 92 Explain simple effect in a 2^n factorial experiment.

Ans. Any factor at its lower level denoted by level 0 is called the first level of that factor and the higher level denoted by level 1 is called the second level of that factor.

11, 12, 20, 21 and 22. The factor combinations for responses at three levels reduced to modulo 3 are:

$(A)_0 = 00 + 01 + 02$ For $i = 0 \bmod 3$
$(A)_1 = 10 + 11 + 12$ $i = 1 \bmod 3$
$(A)_2 = 20 + 21 + 22$ $i = 2 \bmod 3$
$(B)_0 = 00 + 10 + 20$ $j = 0 \bmod 3$
$(B)_1 = 01 + 11 + 21$ $j = 1 \bmod 3$
$(B)_2 = 02 + 12 + 22$ $j = 2 \bmod 3$
$(AB)_0 = 00 + 12 + 21$ $i + j = 0 \bmod 3$
$(AB)_1 = 01 + 10 + 22$ $i + j = 0 \bmod 3$
$(AB)_2 = 02 + 11 + 20$ $i + j = 2 \bmod 3$

The sum of the responses at factor levels so obtained can be summarized in a 3 × 3 table as follows:

	b_0	b_1	b_2	*Total*
a_0	X_{00}	X_{01}	X_{02}	$X_{0.}$
a_1	X_{10}	X_{11}	X_{12}	$X_{1.}$
a_2	X_{20}	X_{21}	X_{22}	$X_{2.}$
Total	$X_{.0}$	$X_{.1}$	$X_{.2}$	$X_{..}$

The nine treatments represented by all possible combinations has 8 d.f. which has a break up as 2 d.f. for A, 2 d.f. for B and 4 d.f. for $A \times B$. The main effects A and B may be linear or quadratic effects with 1 d.f. each and the interaction $A \times B$ can be splitted into two parts AB and AB^2 with 2 d.f. for each.

Note: Here it should be noted that A^2B is same as AB^2 reduced to modulo 3, *i.e.*, $(A^2 B)^2 = A^4 B^2 = AB^2$. Also $A^2 B^2$ is equivalent to AB *i.e.* $(A^2 B^2)^2 = A^4 B^4 = AB$.

It is known that for three treatments (3 levels), the linear contrast will be represented by – 1, 0, 1 and quadratic contrast by 1, –2, 1.

Let us assume that in the above two way table each value X_{ij} $(i, j = 0, 1, 2)$ is a total of r individual responses for nine treatment combination. The linear and quadratic effects for the factors A, B, and their interaction effects can be calculated as explicated below:

Note. One is not required to prepare both parts of the table given below. One may prepare only the right half or left half table.

The divisors for the estimates of the effect will be $3r$. Moreover the divisors for the sum of squares will be as tabulated on page 586.

	A_l	A_q		B_l	B_q
	−1 0 1	*1 −2 1*		*−1 0 1*	*1 −2 1*
b_0	$X_{20} - X_{00}$	$X_{20} - 2X_{10} + X_{00}$	a_0	$X_{02} - X_{00}$	$X_{02} - 2X_{01} + X_{00}$
b_1	$X_{21} - X_{01}$	$X_{21} - 2X_{11} + X_{01}$	a_1	$X_{12} - X_{10}$	$X_{12} - 2X_{11} + X_{10}$
b_2	$X_{22} - X_{02}$	$X_{22} - 2X_{12} + X_{02}$	a_2	$X_{22} - X_{20}$	$X_{22} - 2X_{21} + X_{20}$
Total	$X_{2.} - X_{0.} = A_l$	$X_{2.} - 2X_{1.} + X_{0.} = A_q$	Total	$X_{.2} - X_{.0} = B_l$	$X_{.2} - 2X_{.1} + X_{.0} = B_q$
$(-1, 0, 1)\, B_l$	$(X_{22} - X_{02}) - (X_{20}$ $- X_{00}) = A_l\, B_l$	$(X_{22} - 2X_{12} + X_{02}) -$ $(X_{20} - 2X_{10} + X_{00})$ $= A_q\, B_l$	$(-1, 0, 1) A_l$	$(X_{22} - X_{20}) - (X_{02}$ $- X_{00}) = A_l\, B_l$	$(X_{22} - 2X_{21} + X_{20}) -$ $(X_{02} - 2X_{01} + X_{00})$ $= A_l\, B_q$
$(1, -2, 1) B_q$	$(X_{22} - X_{02}) -$ $2\,(X_{21} - X_{01}) +$ $(X_{20} - X_{00}) = A_l\, B_q$	$(X_{22} - 2X_{12} + X_{02}) -$ $2\,(X_{21} - 2X_{11} + X_{01})$ $+ (X_{20} - 2X_{10} + X_{00})$ $= A_q\, B_q$	$(1, -2, 1) A_q$	$(X_{22} - X_{20}) -$ $2\,(X_{12} - X_{10}) +$ $(X_{02} - X_{00}) = A_q\, B_l$	$(X_{22} - 2X_{21} + X_{20}) -$ $2\,(X_{12} - 2X_{11} + X_{10})$ $+ (X_{02} - 2X_{01} + X_{00})$ $= A_q\, B_q$

Effect	*Divisors*	*S.S.*
A_l	$3r\{(-1)^2+0^2+1^2\}=6r$	$A_l^2/6r$
B_l	$3r\{(-1)^2+0^2+1^2\}=6r$	$B_l^2/6r$
A_q	$3r\{1^2+(-2)^2+1^2\}=18r$	$A_q^2/18r$
B_q	$3r\{1^2+(-2)^2+1^2\}=18r$	$B_q^2/18r$
$A_l\,B_l$	$r\{(-1)^2+0^2+1^2\}\{(-1)^2+0^2$ $+1^2\}=4r$	$(A_l\,B_l)^2/4r$
$A_l\,B_q$	$r\{(-1)^2+0^2+1^2\}\{1^2+(-2)^2$ $+1^2\}=12r$	$(A_l\,B_q)^2/12r$
$A_q\,B_l$	$r\{1^2+(-2)^2+1^2\}\{(-1)^2+0^2$ $+1^2\}=12r$	$(A_q\,B_l)^2/12r$
$A_q\,B_q$	$r\{1^2+(-2)^2+1^2\}\{1^2+(-2)^2$ $+1^2\}=36r$	$(A_q\,B_q)^2/36r$

Q. 95 Give the method, of calculating sum of squares for factorial effects in general.

Ans. In general method, one prepares 2-way, 3-way, 4-way tables and calculate the sum of squares due to main effects and interaction(s) with the help of these tables.

For 2^2 factorial experiment say, laidout in randomized block design with r blocks, the total for four treatment combinations are as tabulated below:

	b_0	b_1	*Total*
a_0	X_{00}	X_{01}	$X_{0.}$
a_1	X_{10}	X_{11}	$X_{1.}$
Total	$X_{.0}$	$X_{.1}$	$X_{..}$

$$S.S. \text{ due to } A=\frac{1}{2r}\left(X_{0.}^2+X_{1.}^2\right)-\frac{X_{..}^2}{4r}$$

$$S.S. \text{ due to } B=\frac{1}{2r}\left(X_{.0}^2+X_{.1}^2\right)-\frac{X_{..}^2}{4r}$$

$$S.S. \text{ due to } AB=\frac{1}{r}\left(X_{00}^2+X_{01}^2+X_{10}^2+X_{11}^2\right)-\frac{X_{..}^2}{4r}$$

$$-\text{S.S.(A)}-\text{S.S.(B)}$$

Let us consider the data of a 2^3 factorial experiment. For this data prepare two way tables for *A* & *B*, *A* & *C* and *B* & *C*. From these tables calculate the sum of squares due to *A*, *B*, *C*, *AB*, *AC* and *BC* in the usual manner. The sum of square for ABC interaction is obtained by calculating treatment sum of square and subtracting from it the sum of squares due to all main effects and first order interactions.

For a $p_1 \times p_2 \times p_3$ asymmetrical factorial with factors *A*, *B* and *C* prepare two way tables for *A* and *B* of order $p_1 \times p_2$, *A* and *C* of order $p_1 \times p_3$ and *B* and *C* of order $p_1 \times p_3$. From these tables calculate the sum of squares for the main effects and interactions as we do for 2^2 factorials. The divisor in calculating sum of squares is equal to the number of individual experimental units in a value which is squared. The sum of square for *ABC* is obtained by calculating the treatment sum of square and subtracting from this the sum of squares due to main effects and first order interactions.

Q. 96 Give Yates' method of analysis of data of a 2^n factorial experiment.

Ans. Yates' method is a mechanical approach to calculate the effects and sum of squares due to various factors and their interactions in a 2^n factorial only. This method has emerged as a result of thorough study of the nature of contrasts.

Various steps involved in Yates' method are as follows:

(i) In column (1), write all treatment combinations in a sequence introducing the letters *a*, *b*, *c*, etc., in turn.

(ii) In column (2), give the total responses corresponding to the treatment combinations given in column (1).

(iii) In column (3), in the upper half write the sums of the values of column (2) taken in pairs and in the lower half write the difference of pairs even minus odd, *e.g.*, (2nd-1st), (4th-3rd) and so on.

(iv) Repeat step (iii) on the values of column (3) to obtained column (4).

(v) Process of pairwise adding and subtracting is continued in as many columns as n, the number of factors.

(vi) The last column containing sums and differences gives effect totals. One may verify that these effect totals are same as those obtained from contrasts.

(vii) To obtain factorial effect means, the effect total are divided by $2^{n-1} \times r$, whereas the effect total for (1) is divided by $2^n \times r$ giving grand mean.

(viii) To obtain the sum of squares due to factorial effects, square of each effect total is divided by $2^n \times r$. Yates' method does not require writing of any contrasts or formation of tables etc. The method is very quick and handy.

Q. 97 In an experiment with three fertilizers *N, P* and *K* each at two levels 0 and 1, the yield data (kg/plot) due to eight treatment combinations of a R.B.D. with three replications is give below:

Treats.	*Rep. I*	*Rep. II*	*Rep. III*
(1)	40	53	38
n	35	59	18
p	40	69	43
np	64	78	52
k	71	72	58
nk	26	31	19
pk	43	45	27
npk	51	62	40

Calculate the treatment mean effects and sum of squares by Yate's method.

Ans. To calculate the mean effects and sum of squares due to various effects by yate's method we prepare the following table.

Treats.	*Treat. Totals*			*Effect Totals*	*Effect means*	*S.S. due to*
Col. 1	*Col. 2*	*Col. 3*	*Col. 4*	*Col. 5*		*effects*
(1)	131	243	589	1134	94.50	53581.50
n	112	346	545	–64	–5.33	170.67
p	152	277	23	94	7.83	368.17
np	194	268	–87	224	18.67	2090.67
k	201	–19	103	–44	–3.67	80.67
nk	76	42	–9	–110	–9.17	504.17
pk	115	–!25	61	–112	–9.33	522.67
npK	153	38	163	102	8.50	433.50

The last two columns give the estimates of the effect means and sum of squares respectively.

Q. 98 Comment on the designs to be used in case of factorial experiments.

Ans. The layout of a factorial experiment can be done in any design namely, completely randomized design, randomized complete block design, Latin square design, split plot design and incomplete block designs. The analysis of data will be carried out in the usual manner. The only difference in analysis of variance table is that the treatment sum of square is further splitted up in main effects and interactions sum of squares. If all the contrasts are orthogonal, the total of the sum of squares due to main effects and interactions is equal to the treatment sum of square.

Q. 99 Write down the statistical model for a two-factor factorial experiment with a levels of A, b levels of B respectively laid out in completely randomized design. Also give analysis of variance table with expected mean squares.

Ans. The statistical model for a two factor factorial with a levels of A, b levels of B and r replications in a completely randomized design with one observation per experimental unit is,

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \qquad (1)$$

For $i = 1, 2, ..., a$
$j = 1, 2, ..., b$
$k = 1, 2, ..., r$

ANOVA Table - 14

(i)	*(ii)*	*(iii)*	*(iv)*	*(v)*	*(vi)*	*(vii)*	*(viii)*
Source of Variation	*d.f.*	*S.S.*	*M.S.*	*Expected M.S.* *Model-I* *(A and B fixed)*	*Expected M.S.* *Model-II* *(A and B random)*	*Expected M.S.* *Model-III* *(A fixed, B random)*	*Expected M.S.* *Model-III* *(A random, B fixed)*
Treatments	$ab-1$	T_{yy}	$\frac{T_{yy}}{ab-1}=T_y$				
A	$a-1$	A_{yy}	$\frac{A_{yy}}{a-1}=A_y$	$\sigma_e^2+rb\sum_{i=1}^{a}\alpha_i^2/(a-1)$	$\sigma_e^2+r\sigma_{\alpha\beta}^2+rb\sigma_\alpha^2$	$\sigma_e^2+r\sigma_{\alpha\beta}^2+rb\sum_{i=1}^{a}\frac{\alpha_i^2}{a-1}$	$\sigma_e^2+rb\,\sigma_\alpha^2$
B	$b-1$	B_{yy}	$\frac{B_{yy}}{b-1}=B_y$	$\sigma_e^2+ra\sum_{j=1}^{b}\beta_j^2/(b-1)$	$\sigma_e^2+r\sigma_{\alpha\beta}^2+ra\sigma_\beta^2$	$\sigma_e^2+ra\,\sigma_\beta^2$	$\sigma_e^2+r\sigma_{\alpha\beta}^2+ra\sum_{j=1}^{b}\frac{\beta_j^2}{b-1}$
AB	$(a-1)\times$ $(b-1)$	$(AB)_{yy}$	$\frac{(AB)_{yy}}{(a-1)(b-1)}=AB_y$	$\sigma_e^2+r\sum_{i=1}^{a}\sum_{j=1}^{b}\frac{(\alpha\beta)_{ij}^2}{(a-1)(b-1)}$	$\sigma_e^2+r\sigma_{\alpha\beta}^2$	$\sigma_e^2+r\sigma_{\alpha\beta}^2$	$\sigma_e^2+r\sigma_{\alpha\beta}^2$
Experimental error	$ab(r-1)$	E_{yy}	$E_{yy}/ab(r-1)=E_y$	σ_e^2	σ_e^2	σ_e^2	σ_e^2
Total	$abr-1$	$\sum_i\sum_j y_{ijk}^2$ $-C.F.$					

where μ is the true mean effect, α_i is the true effect of the i^{th} level of A, β_j is the true effect of the j^{th} level of B, $(\alpha\beta)_{ij}$ is the true interaction effect of the i^{th} level of A and j^{th} level of B and ε_{ijk} is the true effect of the k^{th} experimental unit subject to (i, j) treatment combination. Also $\varepsilon_{ij} \sim NID\left(0, \sigma_e^2\right)$.

The analysis of variance table with expected mean squares when the model (1) is fixed, random or mixed is displayed above (Table-14).

The expected mean squares in the above table reveal that in case of Model-II, the effects A and B should be tested against the interaction AB mean square by F-test and interaction AB should be tested against error mean square by F-test. Similarly for Model-III (A fixed, B random), M.S. for A be tested against AB M.S. and B M.S. and AB M.S. against error M.S. Again in Model-III (A random, B fixed), M.S. for B be tested against AB M.S. and A M.S. and AB M.S. be tested against error M.S.

Q. 100 Set-up statistical model for a two-factor factorial experiment in a randomized block design. Also give analysis of variance table with expected mean squares.

Ans. Suppose there are two factors A and B at levels a and b respectively and r blocks. The appropriate model for the two-factor factorial experiment in randomized block design is,

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \rho_k + \varepsilon_{ijk}$$

For $i = 1, 2, ..., a$

$j = 1, 2, ..., b$

$k = 1, 2, ..., r$

μ, α_i, β_j, $(\alpha\beta)_{ij}$ carry their usual meaning and ρ_k is the true effect of the k^{th} block and $\varepsilon_{ijk} \sim \text{NID}\left(0, \sigma_e^2\right)$.

The analysis of variance table for the above model is presented in table-15 with additional columns for expected mean squares under three models. (see in the table on the last page).

After making a critical view of the expected mean squares under various models, it is trivial to decide when should the main effects *A* and *B* be tested against experimental error and when against interaction *AB* mean square. Also interaction *AB* has to be tested against experimental error in all situations.

Q. 101 Give general rules for writing the expected mean squares in case of factorial experiments with one observation per unit.

Ans. There is no end to models for varying number of factors involved in factorial experiments and the respective ANOVA tables. Expected mean squares are of prime importance as they enable one to decide about an appropriate test for factors involved and their interactions. Hence, it looks pertinent to give general rules for writing expected mean squares.

Here we denote any main effect or an interaction by a letter *U* in general. The method of writing the expected mean squares is as follows:

(i) The expected mean square for any effect *U* always includes the term σ_e^2 (error mean square) with coefficient unity.

(ii) In fixed effect model (Model-I), the expected mean square due to a fixed effect *U* is σ_e^2 plus the sum of square due to the effect *U* divided by its degrees of freedom and this expression is multiplied by *r* times the levels of all factors except itself.

(iii) In case of random effect model, the expected mean square due to a factor *U* is σ_e^2 plus σ_u^2 and add all variance terms of the interactions with *U* whereas each variance except σ_e^2 is multiplied by *r* times the levels of the factors not present in *U*

(iv) In case of mixed effect model, expected mean square due to a fixed effect U contains σ_e^2, mean sum of square due to *U* multiplied by *r* times the levels of all factors except *U* and all variance terms of the interaction of *U* with random effects only each multiplied by *r* times the product of levels of factors that do not appear in the interaction under consideration.

(v) In case of mixed model, expected mean square due a random factor *U* contains σ_e^2, the variances due to *U* and its interactions with random effects multiplied by *r* times the product of the levels of factors that did not appear in suffix to σ^2.

Q. 102 What do you understand by confounding?

Ans. We know that in randomized block design, a block should usually not contain more than 20 plots of medium size and a Latin square design of order more than 10 is not commendable. Factorial experiments are usually conducted in randomized block design and seldom in Latin square and other designs. In factorial experiments, the treatment combinations increase rapidly as the number of factors increases or levels of factors increase. Large number of treatment combinations in factorial experiments become incompatible to be accommodated in randomized blocks as their homogeneity is in jeopardy. Hence, there arises the need of a device which can retain the feature of factorial arrangement and reduce the block size so as to maintain their homogeneity. To fulfil these objectives, there is a device known as *confounding*. In confounding, each block is subdivided into two or more blocks of suitable size known as incomplete blocks by sacrificing the information about a factorial effect usually higher order interaction(s).

Through confounding some factors, which are not confounded, are estimated and tested with high precision and some factor(s) which are confounded lose their identity as they are inextricably mixed with block differences. We know, second and higher order interactions are of little or no importance from their physical interpretation point of view, hence they are primarily chosen for confounding. Confounding is a device of reducing block size applicable only in factorial experiments.

Q. 103 Differentiate between complete and partial confounding.

Ans. If the same effect is confounded in all the replicates, it is known as *complete confounding.* This effect is generally of little or no value to the experimenter. For example, in a 3-factor factorial experiment, the second order interaction ABC is generally of least interest. So it is confounded in all replications.

Again if an experimenter is not prepared to sacrifice an effect totally, he chooses to confound one effect in one replicate and the other in other replicate(s). In this process total information is not lost about any effect. The effect which is confounded in one replicate is being estimated and tested on the basis of the replicates in which it is not confounded. This system of confounding is known as *partial confounding.* For example, in a 2^3 factorial, one confounds AB in first replicate, AC in second replicate and BC in the third replicate. The whole set may be repeated more than once.

Q. 104 When do you call the partial confounding as balance and unbalanced confounding?

Ans. When all effects of the same order are confounded with incomplete blocks differences equal number of times, the confounding is said to be *balanced confounding* or more specifically *balanced partial confounding.* For instance, AB, AC and BC are confounded in first, second and third replication respectively and there are three such sets of replications.

On the other hand if the effects of certain order are confounded an unequal number of times in the replications, this system of confounding is known as *unbalanced partial confounding.*

Q. 105 How can complete confounding be operated in a 2^n factorial to reduce the block size to half-replicate?

Ans. In a 2^n factorial, the block size in randomized block design can be reduced to half by confounding one effect which is of no interest or least interest. In practice, one chooses the highest order inter-action. The procedure is very simple. Write the contrast for the effect which is to be confounded. All those treatment combinations with positive sign are randomly assigned in one half replicate (block) and those with negative coefficients in the other half. The same entries of the blocks are repeated in other replications with a fresh randomization within blocks.

We give below the layout of a 2^3 factorial with factors A, B and C each at two levels 0 and 1. The effect ABC is confounded to obtain the blocks of size four units.

$$\begin{aligned} ABC &= (a-1)(b-1)(c-1) \\ &= abc + a + b + c - ab - ac - bc - (1) \end{aligned}$$

Rep. I		*Rep. II*		*Rep. III*	
Bl. 1	*Bl. 2*	*Bl. 3*	*Bl. 4*	*Bl. 5*	*Bl. 6*
a	(1)	*c*	*ab*	*b*	*bc*
abc	*ab*	*b*	*ac*	*c*	(1)
b	*ac*	*abc*	*bc*	*a*	*ac*
c	*bc*	*a*	(1)	*abc*	*ab*

Q. 106 In what manner, can the blocks of two units in 2^3 factorial in a randomized block design be obtained through confounding?

Ans. In a 2^3 factorial, there are eight factorial effects. *First,* we confound one effect to reduce the blocks to the size of 4 units. Again we confound another effect to get the blocks of two units. When we confound second effect, we will always find that the four entries within a block will now have two entries with positive sign and two with negative signs. So each block of four units will further be splitted up into two blocks each containing two units.

Here one should be wary of the fact that when we confound two treatment effects to reduce the blocks into one-fourth size, their generalised interaction is automatically confounded. Hence, one should not choose two treatment effects whose generalised interaction is a main effect or some other effect of greater interest. For instance, if we choose to confound first *ABC* then *AB*, then the generalized interaction, $ABC \times AB = A^2 B^2 C = C$ (powers reduced to modulo 2), which is a main effect, is automatically confounded. An automatically confounded effect means that the effect will be same as the block differences in confounded design. So it is preferable to confound two first order interactions here say, *AB* and *AC* are confounded.

The contrast AB is,

$$\begin{aligned} AB &= (a-1)(b-1)(c+1) \\ &= abc + c + ab + (1) - b - a - bc - ac \end{aligned}$$

Blocks reduced to half in two replications are,

Rep. I

Bl. (i)	*Bl. (ii)*
abc	*a*
c	*b*
(1)	*bc*
ab	*ac*

Rep. II

Bl. (iii)	*Bl. (iv)*
c	*bc*
(1)	*b*
ab	*ac*
abc	*a*

The contrast *AC* is,

$$\begin{aligned} AC &= (a-1)(b+1)(c-1) \\ &= abc + (1) - c - ab + b + ac - a - bc \end{aligned}$$

Rep. I

Bl. 1	*Bl. 2*	*Bl. 3*	*Bl. 4*
abc	*c*	*b*	*bc*
(1)	*ab*	*ac*	*a*

Rep. II

Bl. 5	*Bl. 6*	*Bl. 7*	*Bl. 8*
(1)	*c*	*ac*	*bc*
abc	*ab*	*b*	*a*

Now we check up how *BC* is confounded. The contrast for *BC* is as given below:

$$\begin{aligned} BC &= (a+1)(b-1)(c-1) \\ &= abc + (1) - b - ac + a + bc - c - ab \\ &= Bl.\ 1 - Bl.\ 3 + Bl.\ 4 - Bl.\ 2\ (Rep.\ I) \end{aligned}$$

In this way *BC* is also confounded.

Q. 107 Give general rule for confounding effects in a 2^n factorial to reduce the blocks to size of 2^{n-k} units.

Ans. In a 2^n factorial to have 2^k blocks of 2^{n-k} units each, one has to confounding *k* effects subject to the restriction that none of them is a generalized interaction of the others amongst themselves. In this situation, $(2^k - k - 1)$ effects are automatically confounded.

Q. 108 Give the layout of a partially confounded design in 2^3 factorial in which the effects *ABC*, *AC* and *BC* are confounded in three replicates.

Ans. The layout of a 2^3 factorial experiment in which *ABC*, *AC* and *BC* are confounded in three replicates is as given below.

The contrasts are,

$$\begin{aligned} ABC &= (A-1)(b-1)(c-1) \\ &= (abc + a + b + c) - (ac + bc + ac + (1)) \\ AC &= (a-1)(b+1)(c-1) \\ &= (abc + b + ac + (1)) - (a + c + ab + bc) \\ BC &= (a+1)(b-1)(c-1) \\ &= (abc + a + bc + (1)) - (ab + ac + b + c) \end{aligned}$$

Thus, the layout is,

Rep. I		*Rep. II*		*Rep. III*	
Bl. 1	*Bl. 2*	*Bl. 3*	*Bl. 4*	*Bl. 5*	*Bl. 6*
a	*ac*	*abc*	*a*	(1)	*b*
abc	(1)	*b*	*ab*	*a*	*ab*
b	*ab*	(1)	*bc*	*bc*	*c*
c	*bc*	*ac*	*c*	*abc*	*ac*

Effects confounded

ABC *AC* *BC*

Q. 109 How can we implement confounding in a 3^n factorial design?

Ans. In 3^n factorials, the effects can be confounded with the help of modulo technique. Here we have an additional advantage that the same interaction effect can be subdivided into components each with two degrees of freedom. So by confounding one component in one replication and the other in second replication, etc., the confounded effect can be estimated through its components from other replications barring its own replication.

In 3^n factorial, confounding can be carried through modulo technique easily. Supposing that in 3^2 factorial experiment in randomized block design, the interaction effect $A \times B$ is confounded to form blocks of size 3 units. Interaction $A \times B$ has 4 d.f. which can be subdivided into components AB and AB^2, each with 2 d.f.

Nine treatment combinations 00, 01, 02, 10, 11, 12, 20, 21, 22 can be sorted out to confound AB in one replication such that $i + j = 0$ mod 3, $i + j = 1$ mod 3 and $i + j = 2$ mod 3 where i represents the level of A and j the level of B. Similarly in the second replication, AB^2 is confounded where the blocks contain the treatment combinations satisfying the conditions $i + 2j = 0$ mod 3, $i + 2j = 1$ mod 3 and $i + 2j = 2$ mod 3. Following the technique, the design is as given below:

Rep. I

Bl. 1	*Bl. 2*	*Bl. 3*
00	10	02
12	01	20
21	22	11

AB confounded

Rep. II

Bl. 4	*Bl. 5*	*Bl. 6*
11	10	20
00	21	12
22	02	01

AB^2 confounded

This technique can be extended to 3^n ($n \geq 3$) factorials in general.

Q. 110 How can one carry out the balanced confounding in asymmetrical factorials?

Ans. Let us consider in general an asymmetrical factorial with k factors at levels p_1, p_2, ..., p_k, respectively. Suppose that the total number of treatment combinations $p_1 \times p_2 \times ... \times p_k = P$. We want to obtain a block of size R units. Now determine a quantity Q which is equal to P/R such that Q is either a prime number or a prime power, *i.e.*, $Q = s^m$, where s is a prime number and m is an integer.

All factors in the asymmetrical factorial possessing s levels are called *real factors* and each factor having the levels other than s is called a *factor of asymmetry.*

Now for the construction of a confounded balanced design, the asymmetrical factorial is converted into a symmetrical factorial s^n by adding certain factors each at levels 's' to the factors of asymmetry. These additional factors are called *pseudo factors.* The levels of real and pseudo factors are denoted by the elements of the Galois field s (modulo technique). For distinction we will denote the real factors by A, B, C, ... and factors of asymmetry by X, Y, Z, ... and pseudo factors corresponding to X by X_1, X_2, X_3, ..., and for Y by Y_1, Y_2, Y_3, ..., and so on.

The decision about the number of pseudo factors depends on the value of s and the factors having levels other than s. Suppose the factor F_i has levels p_i less than s. Then convert the asymmetrical factorial to symmetrical factorial as s^m by choosing a single pseudo factor for each X, Y, Z, ... as X_1, Y_1, Z_1,.... For those factors where p_i is greater than s, choose as many pseudo factors for X, Y, etc., as required like X_1, X_2, ...; Y_1, Y_2, ..., etc., to convert the asymmetrical factorial into a symmetrical factorial with factors each having s levels. From the factorial combinations delete all those combinations of pseudo factors which are non-existing in the asymmetrical factorial experiment. Now form incomplete blocks of required size by modulo technique. Also reduce the number of pseudo factors to the number of factors of asymmetry by jointly considering their level combinations to a single level. In this way, we get a confounded design for an asymmetrical factorial.

Q. 111 Give the layout of a balanced confounded design of an asymmetrical factorial $2 \times 2 \times 3$ with blocks of size 4 units and two replications.

Ans. No. of combinations, $P = 2 \times 2 \times 3 = 12$

Block size, $R = 4$

$$Q = \frac{P}{R} = \frac{12}{4} = 3$$

There is one real factor say, A at 3 levels and two factors of asymmetry as X and Y each at 2 levels. Since, the levels of the factors X, Y with levels equal to 2 are less than 3, we convert the asymmetrical

Rep. III

Bl. 5	*Bl. 6*
0000	0001
0011	0010
0101	0100
0110	0111
1010	1000
1001	1011

$$(X_2\,X_2\,AB)_{0 \bmod 2}\,(X_I\,X_2\,AB)_{1 \bmod 2}$$

The above layout involves four factors, two real factors *A*, *B* and two pseudo factors X_1 and X_2 for the factor of asymmetry *X*. Now we amalgamate the levels of X_1 X_2 into a single level to get the confounded design for the factors *X*, *A* and *B*. For this we recode the level combinations of X_1 and X_2 as 00 to 0, 01 to 1 and 10 to 2. In this way, we get the required balance confounded design as given below:

Rep. I		*Rep. II*		*Rep. III*	
Bl. 1	*Bl. 2*	*Bl. 3*	*Bl. 4*	*Bl. 5*	*Bl. 6*
000	001	000	001	000	001
011	010	011	010	011	010
100	101	101	100	101	100
111	110	110	111	110	111
210	200	200	201	210	200
201	211	211	210	201	211

Q. 113 Give the concept of fractional replication.

Ans. The idea of fractional replication was propounded by D.J. Finney in 1945. Some workers later instituted it as *fractional factorials*. A factorial experiment conducted in a single replicate by utilising a selected few treatment combinations with the interest of estimating only lower order effects with a reasonable degree of precision under the assumption of absence of higher order interactions is known as fractional replication.

Q. 114 What do you understand by orthogonal fractional factorial designs?

Ans. Regular fractional factorial plans for symmetrical factorials which make possible the estimation of all relevant effects with zero correlation are called orthogonal fractional factorial designs. The regular fractions are always orthogonal but the converse is not true.

(Box and Hunter defined the regular fraction as a fraction whose treatment combinations form a subgroup.)

Q. 115 Clarify the idea of fractional replication through an example.

Ans. To clarify the idea of fractional replication, we consider a 2^3 factorial experiment for the sake of brevity and simplicity though this is not an ideal example in the present context. Conduct the experiment with ½ replicate having the treatment combinations of positive signs in the contrast for the interaction ABC. From the experimental results, the main effects and interactions can be delineated as given below.

Treatment Combinations	*Main effects and interactions*						
	A	*B*	*C*	*AB*	*AC*	*BC*	*ABC*
a	+	−	−	−	−	+	+
b	−	+	−	−	+	−	+
c	−	−	+	+	−	−	+
abc	+	+	+	+	+	+	+

On the basis of above set of contrasts, we specifically reveal the following points:

(i) The contrast(s) which is/are used to split a replicate into a divisible fraction is/are called *defining contrast(s)*. In this example, *ABC* is the defining contrast. It cannot be estimated at all.

(ii) From the above set of contrast, it is clear that the effect *A* is same as *BC*, *B* is same as *AC* and *C* is same as *AB*. In this way, the effects *A* and *BC*, *B* and *AC*, *C* and *AB* are inseparable. Hence, the effects which are estimated by the same contrast are called *aliases*. Here main effects and first order interactions are

ANOVA Table - 17

Source of Variation	*d.f.*	*Expected M.S.* *Model-I* *(A and B fixed)*	*Expected M.S.* *Model-II* *(A and B random)*
Whole Plot			
Rep.	$r-1$	$\sigma_\varepsilon^2 + q\sigma_\delta^2 + pq\sigma_\rho^2$	$\sigma_\varepsilon^2 + q\sigma_\delta^2 + pq\sigma_\rho^2$
A	$p-1$	$\sigma_\varepsilon^2 + q\sigma_\delta^2 + \frac{rq}{p-1}\sum_{i=1}^{p}\alpha_i^2$	$\sigma_\varepsilon^2 + q\sigma_\delta^2 + rq\ \sigma_\alpha^2$
Error (*a*)	$(r-1)(p-1)$	$\sigma_\varepsilon^2 + q\sigma_\delta^2$	$\sigma_\varepsilon^2 + q\sigma_\delta^2$
Sub-Plot			
B	$q-1$	$\sigma_\varepsilon^2 + \frac{rp}{q-1}\sum_{j=1}^{q}\beta_j^2$	$\sigma_\varepsilon^2 + r\sigma_{\alpha\beta}^2 + rp\ \sigma_\beta^2$
$A \times B$	$(p-1)(q-1)$	$\sigma_\varepsilon^2 + \frac{r}{(p-1)(q-1)}\sum_{i=1}^{p}\sum_{j=1}^{q}(\alpha\beta)_{ij}^2$	$\sigma_\varepsilon^2 + r\sigma_{\alpha\beta}^2$
Sub-plot error (*b*)	$p(r-1)(q-1)$	σ_ε^2	σ_ε^2
Total	$rpq-1$		

Considering an experiment with two sub-units per unit in a split plot design, the error variance of a unit is,

$$E(\varepsilon_{i1u} + \varepsilon_{i2u})^2 = E(\varepsilon_{i1u}^2) + E(\varepsilon_{i2u}^2) + 2E(\varepsilon_{i1u} \cdot \varepsilon_{i2u})$$

$$= \sigma^2 + \sigma^2 + 2\rho\sigma^2$$

$$= 2\,\sigma^2\,(1 + \rho)$$

If there are q sub-units per unit, the corresponding error variance of a unit total works-out to be $\sigma^2\,\{1 + (q-1)\,\rho\}$.

Again, the error variance of the effect of B is equal to the variance of the difference between two sub-units within a unit, *i.e.*,

$$E(\varepsilon_{i1u} - \varepsilon_{i2u})^2 = E(\varepsilon_{i1u}^2) + E(\varepsilon_{i2u}^2) - 2E(\varepsilon_{i1u}\,\varepsilon_{i2u})$$

$$= 2\sigma^2\,(1 - \rho)$$

This expression remains the same even if there are q sub-units per unit.

Q. 122 The layout of a split plot design, in which the irrigations (I) at four levels are assigned to main-plots and fertilizers (F), the combination of N, P, K each at 2 levels are allocated to sub-plots, is given below. Grain yields of maize per plant in gms is also given along with the treatment combinations:

Rep. I				*Rep. II*			
I_0	I_1	I_3	I_2	I_3	I_2	I_0	I_1
(000) 31.0	(001) 33.0	(110) 32.0	(100) 31.3	(110) 26.7	(011) 32.3	(100) 30.7	(110) 31.5
(010) 29.0	(010) 28.7	(111) 28.8	(011) 24.7	(001) 24.7	(000) 33.7	(101) 31.7	(000) 32.7
(011) 23.3	(101) 25.7	(000) 25.0	(110) 30.0	(010) 25.2	(110) 30.0	(111) 29.3	(001) 29.7
(100) 28.6	(110) 30.0	(010) 29.3	(101) 31.0	(101) 27.8	(010) 32.7	(110) 32.7	(010) 32.7
(101) 25.0	(100) 24.6	(001) 25.0	(111) 27.0	(011) 27.5	(111) 33.0	(000) 27.3	(011) 30.2
(110) 29.0	(111) 27.0	(011) 29.0	(001) 26.3	(100) 28.2	(101) 35.3	(001) 26.3	(100) 35.8
(001) 26.7	(011) 28.0	(100) 28.0	(010) 30.0	(111) 24.3	(001) 32.7	(010) 30.0	(101) 30.8
(111) 26.7	(000) 27.3	(101) 27.3	(000) 27.0	(000) 27.3	(100) 35.0	(011) 34.0	(111) 35.7

Rep. III

I_1	I_2	I_3	I_0
(001) 30.0	(000) 29.0	(111) 29.5	(011) 30.0
(010) 29.3	(001) 26.7	(000) 28.0	(100) 29.0
(011) 33.0	(010) 30.7	(110) 31.8	(101) 28.7
(100) 29.7	(011) 29.7	(001) 29.7	(110) 28.3
(101) 26.3	(100) 24.7	(101) 25.5	(111) 29.3
(110) 31.0	(101) 29.3	(010) 30.2	(001) 28.7
(111) 29.7	(110) 33.7	(100) 28.3	(000) 31.3
(000) 38.0	(111) 31.0	(011) 30.2	(010) 27.0

Analyse the experimental data and draw conclusions.

Ans. For main-plot analysis we prepare the following two way table and calculate sum of squares.

	I_0	I_1	I_2	I_3	*Total*
R_1	219.3	224.3	227.3	224.4	895.3
R_2	242.0	259.1	264.7	211.7	977.5
R_3	232.3	247.0	234.8	233.2	947.3
Total	693.6	730.4	726.8	669.3	2820.1

$$\text{C.F.} = (2820.1)^2/96$$
$$= 82843.38$$

$$\text{S.S.(I)} = \frac{1}{24}\left(693.6^2 + 730.4^2 + 726.8^2 + 699.3^2\right) - \text{C.F.}$$

$$= \frac{1990765.85}{24} - 82843.38 = 105.20$$

$$\text{S.S. due to Replications} = \frac{1}{32}(895.3^2 + 977.5^2 + 947.3^2) - \text{C.F.}$$

$$= \frac{265444.63}{32} - 82843.38$$
$$= 108.04$$

$$\text{Main-plot S.S.} = \frac{1}{8}(219.3^2 + 224.3^2 + \ldots + 234.8^2 + 233.2^2) - \text{C.F.}$$

$$= \frac{665488.99}{8} - 82843.38$$
$$= 342.74$$

$$\text{Error (a) S.S.} = 342.74 - 105.20 - 108.04$$
$$= 129.50$$

For sub-plot analysis, we prepare the following table:

S.S. due to sub-plot treats. (F)

$$= \frac{1}{12}\left(357.6^2 + 353.9^2 + \ldots + 351.9^2 + 351.3^2\right) - \text{C.F.}$$

S.S. for factorial effects by Yates' method

		I_0	I_1	I_2	I_3	*Total*	*(i)*	*(ii)*	*(iii)*	*Mean effect*	*S.S.*
(1)	000	89.6	98.0	89.7	80.3	357.6	711.5	1433.0	2820.1	29.38	82843.37
n	100	88.3	90.1	91.0	84.5	353.9	721.5	1387.1	12.5	0.26	1.63
p	010	86.0	90.7	93.4	84.7	354.8	683.9	8.2	29.3	0.61	8.94
np	110	90.0	92.5	93.7	90.5	366.7	703.2	4.3	10.1	0.21	1.06
k	001	81.7	92.7	85.7	79.4	339.5	–3.7	10.0	45.9	0.96	21.94
nk	101	85.4	82.8	95.6	80.6	344.4	11.9	19.3	–3.9	–0.08	0.16
pk	011	87.3	91.2	86.7	86.7	351.9	4.9	15.6	9.3	0.19	0.90
npk	111	85.3	92.4	91.0	82.6	351.3	–0.6	–5.5	–21.1	–0.44	4.64

$$= \frac{994591.81}{12} - 82843.38$$

$$= 39.27$$

$$\text{S.S.}(I \times F) = \frac{1}{3}\left(89.6^2 + 98.0^2 + \ldots + 91.0^2 + 82.6^2\right) - \text{C.F.} - \text{S.S.}(I) - \text{S.S.}(F)$$

$$= \frac{249200.69}{3} - 82843.38 - 105.20 - 39.27$$

$$= 79.05$$

$$\text{Total S.S.} = \left(31.0^2 + 33.0^2 + \ldots + 30.2^2 + 27.0^2\right) - \text{C.F.}$$

$$= 83648.95 - 82843.38$$

$$= 805.57$$

$$\text{Error (b) S.S.} = 805.57 - 79.05 - 39.27 - 342.74$$

$$= 344.51$$

ANOVA Table

Source of variation	*d.f.*	*S.S.*	*M.S.*	*F-value*
Main-plot				
Replications	2	108.04	54.02	2.50
Irrigation (I)	3	105.20	35.07	1.62
Error (a)	6	129.50	21.58	
Sub-plot				
Fertilizer (F)	7	39.27	5.61	0.91
N	1	1.63	1.63	0.26
P	1	8.94	8.94	1.45
NP	1	1.06	1.06	0.17
K	1	21.94	21.94	3.57
NK	1	0.16	0.16	0.03
PK	1	0.90	0.90	0.15
NPK	1	4.64	4.64	0.75
$I \times F$	21	79.05	3.76	0.61
Error (b)	56	344.51	6.15	
Total	95	805.57		

Comparing the calculated F-values given in ANOVA table below for various component factors with the respective F-values as, $F_{.05,(2,6)} = 5.14$; $F_{.05,(3,6)} = 4.76$; $F_{.05,(7,56)} = 2.17$; $F_{.05,(1,56)} = 4.00$; and $F_{.05,(21,56)} = 1.74$,

we find that all treatments and their interactions are non-significant. Also there is no significant difference among the replications as well.

Q. 123 What are the variations usually exploited in split plot design?

Ans. Many variations in split plot design are exploited depending on the number and nature of the treatments. A few are given below:

(i) When there are more than two factors and the factors require different size experimental units, then double, triple or more splits are utilised as per the number of factors.

(ii) In some situations if the number of replicates are as many as the levels of the main-plot factor A and the experimental conditions are conducive, then the levels of A in whole plots can be taken in a Latin square design. This sort of arrangement has certain added advantages.

(iii) Another variation of the split plot design is to arrange the sub-plots in a Latin square design for the same level of the whole plot treatment provided the number of replications is a multiple of the levels of the sub-plot treatment.

(iv) When both the factors require large size experimental units, one has to choose split block (strip plot) design.

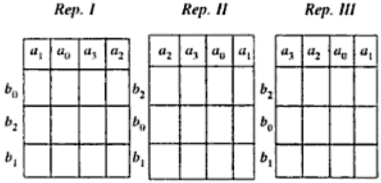
(v) When sub-plot treatments are factorials, it is many times beneficial to confound certain interactions(s) within the main plots. This helps in reducing the size of the main plot which may otherwise be unpalatable.

The break-up of degrees of freedom for the above design will be as presented below.

Source of variation	*d.f.*
Whole-plots	
Replications	2
A	3
Error (a)	6
Sub-plots	
B	2
$A \times B$	6
Orders within plots	8
Error (b)	8
Total	35

Q. 128 When two factorial treatments require large plots, suggest a suitable experimental design and give its layout plan.

Ans. Suppose there are two sets of treatments, say, A at 4 levels and B at 3 levels. Also both the treatments are such that they require large size plots due to operational problems, *e.g.*, depth of ploughing and irrigation, spacing and seed rate, etc. In such cases, a design analogous to split plot design is appropriate named as *split block design* or *strip plot design*. In this design, the field is cut into strips for one set of treatments superposed over the strips for the other set of treatments at right angles. Respective treatments are allocated randomly to the strips. The field layout of split block design with 3 replications is displayed below:



Q. 129 What shall be the break-up of the degrees of freedom of a split block design?

Ans. Let there be two factors A and B at levels p and p respectively. The treatments are allocated in strips at right angles. Also assume that there are r replications. The break-up of degrees of freedom due to various components is as follows:

Source of variation	*d.f.*
Replications	$r - 1$
A	$p - 1$
Error (a)	$(r - 1)(p - 1)$
B	$q - 1$
Error (b)	$(r - 1)(q - 1)$
$A \times B$	$(p - 1)(q - 1)$
Error (c)	$(r - 1)(p - 1)(q - 1)$
Total	$rpq - 1$

Q. 130 What are the merits of a split plot design?

Ans. There are several merits of a split plot design which are delineated below:

(i) Treatments requiring large experimental units are easily accommodated without hampering other treatments.

(ii) Greater precision is attained on sub-plot treatment and interaction as compared to randomized complete block design.

(iii) The precision of split plot design as a whole over randomized block design can be increased by taking whole plot treatment levels in a Latin square arrangement.

(iv) The design is very convenient for agricultural experiments where treatments like ploughing, spacing, irrigation, etc., are involved.

(v) It saves lot of area which would have otherwise been required in any other design.

Q. 131 Mention the demerits of a split plot design.

Ans. Demerits of a split plot design are mentioned below:

(i) The precision of whole plot treatment is less as compared to their precision if they were tried in randomized block design.

(ii) If there is any missing observation, the analysis becomes much more complex as compared to randomized block design.

(iii) The calculation of standard errors for different pairs of treatment means is quite cumbersome. Moreover, there are too many standard errors.

(iv) The increased accuracy obtained on the subplot treatment and interaction is attained at the cost of main plot treatment.

(v) The degrees of freedom for error variance for both the sets of comparisons is less relative to a parallel randomized block design. Hence, randomized block design is superior for all sets of comparisons.

Q. 132 Give the possible layouts of a confounded split plot design in which the sub-unit treatments are 2^3 factorials and also the unit treatment has two levels. Also compare these layouts.

Ans. The layout of a confounded 2×2^3 split plot design with factor A at 2 levels in main units and factors B, C and D in sub-units confounding an effect can possibly be arranged in two ways. Since, the highest order interaction is most preferred to be confounded, we confound BCD in sub-units. This obviously reduces the sub-unit size from 8 units to 4 units. First we give two layouts:

Arrangement (i)				*Arrangement (ii)*			
Block I		*BlockII*		*Block I*		*Block II*	
a_0	a_0	a_1	a_1	a_0	a_1	a_0	a_1
c	bd	(1)	b	c	cd	bd	d
d	(1)	bc	bcd	bcd	bd	(1)	c
bcd	cd	cd	d	b	(1)	cd	bcd
b	bc	bd	c	d	bc	bc	b

If we compare arrangements (i) and (ii) we find that:

(1) in arrangement (i), the factorial combinations with positive and negative signs are at the same level of A in sub-units.

(2) arrangement (i) leads to more accurate estimation of effect A and sub-plot treatments except that of BCD and $ABCD$.

(3) in arrangement (ii), we have gone a step further. The sub-unit treatments which have opposite signs in the contrast BCD are placed under a_0 and a_1 in the same sub-block.

(4) in arrangement (ii), $ABCD$ is completely confounded with sub-blocks whereas BCD is orthogonal with sub-blocks just like A.

(5) arrangement (ii) enables to obtain more precise estimators of A and BCD as compared to (i) at the cost of the information of $ABCD$.

(6) arrangement (ii) is more appropriate than (i) in view of the above points.

Q. 133 What are the commonly used designs to have incomplete blocks when no factorial treatments are at hand.

Ans. There is a series of balanced incomplete block (B.I.B) designs and partially balanced incomplete block (P.B.I.B.) designs which accommodate only a limited number of treatments in a block. These designs have been constructed to estimate the effects and test their significance efficiently. Their construction and analysis part are more complicated than basic experimental designs.

Note. B.I.B. and P.B.I.B. designs are kept out of the scope of this chapter.

SECTION-B

Fill in the blanks

Fill in the suitable word(s)/phrase(s) in the blanks:

1. Design of experiments is a branch of ________.

2. The plan of an experiment which controls all factors as far as possible except the treatments in known as ________.

3. The designs of experiments were originated

38. Random effect model is also called ________ or ________.

39. If the interest of a researcher is confined in the treatments involved in the experiment, the appropriate statistical model for it is a ________ model.

40. If the inferences are to be drawn about the population of treatments of which they are the sample, the type of statistical model is ________.

41. If A is a fixed effect having p levels, then $\sum_{i=1}^{p} \alpha_i$ is equal to ________.

42. A commonly used approach to test the significance of differences between pairs of treatment means is the ________ method.

43. The formula for least significant difference with usual notations is ________.

44. Student-Newman Kuel's test takes care of the ________ between two means in an ordered set of treatment means.

45. The chance of committing Type II error is ________ and of Type I error is ________ in Duncan's multiple range test as compared to least significant difference test.

46. Tukey utilized ________ or the ________.

47. A linear combination of k treatments, in which the sum of the coefficients of the treatments is zero, is called a ________ or ________.

48. The linear combination $T_1 - 3T_2 + \mathrm{T}_3$ of three treatments is ________.

49. The linear combination $-3T_1 - T_2 + T_3 + 3T_4$ of four treatments is a ________.

50. If the sum of the cross product of the corresponding coefficients of two contrasts each involving k treatments is zero, they are said to be ________ contrasts.

51. The idea of orthogonal contrasts emerged from ________.

52. Sum of square due to a contrast $Z_w = T_1 - T_2 - T_3 + T_4$, where each treatment total is a sum of 4 observations is ________.

53. Among k treatments, there can at most be ________ orthogonal contrasts.

54. Each contrast has ________ d.f.

55. Orthogonal contrasts among three treatments enable to estimate ________ and ________ effects.

56. Contrasts provide ________ information about treatments than F-test alone in analysis of variance.

57. There are ________ categories of experimental designs.

58. The experimental designs not involving any randomization process are called ________ designs.

59. The estimates obtained from systematic designs are ________.

60. Systematic designs can cause ________ errors.

61. Personal biases cannot be ruled out in ________ designs.

62. Systematic designs make sampling of area ________.

63. In systematic designs, the chances of drifting of one treatment effect over the other are ________.

64. The experiments in which the treatments are allocated to the units through a random process are called ________ designs.

65. A completely randomized design is used when all experimental units are ________.

66. When all experimental units are homogeneous, the most suitable design for an experiment is ________.

67. Completely randomized design yields ________ degrees of freedom for error.

68. Missing observation in a completely randomized design creates ________.

69. If the experimental units are likely to fail to respond, ________ is a suitable design.

70. Completely randomized design is not suitable for ________ experiments.

71. If an exact F-test for treatments, based on the statistical model of an experiment is available in ANOVA, the model is said to be _________.

72. Analysis of data of a completely randomized design with two treatments and a t-test for their equality are _________.

73. Taking more than one observation on each experimental unit with regard to the same character is known as _________.

74. The analysis of variance under sub-sampling involves two error components in ANOVA table namely _________ and _________ errors.

75. If in ANOVA table, F-value for treatments comes out to be less than unity, it creates doubt on the validity of the _________.

76. F-value in ANOVA table smaller than one may occur if the observations are not distributed _________.

77. If the experimental units are subjected to one way variation, then one may prefer to use _________ design.

78. Each block in a randomised block design is a _________.

79. Each treatment occurs _________ in a block of randomized complete block design.

80. There are as many units in a block as the number of _________ in a randomized block design.

81. When every level of a factor is tried with every level of the other factor, then such an arrangement is called _________ classification.

82. The process of repeated sampling and sub-sampling is known as _________ or _________ classification.

83. If all levels of a factor are tried at each single level of the other factor, the later factor is _________ in the former factor.

84. In a randomized block design, blocks are formed in _________ direction to fertility gradient.

85. Usually randomized block design in field experiments is _________ efficient than completely randomized design.

86. According to Cochran, the error mean square of a randomized block design is _________ per cent to that of an equivalent completely randomized design.

87. For an equal amount of information in completely randomized design _________ replication are equivalent to _________ replications in randomized block design.

88. In a randomized block design, too many treatments _________ accommodated in a block.

89. If there are t treatments and m blocks in a randomized block design, the error degrees of freedom in ANOVA table be _________.

90. Statistical model for a randomized block design under Model-I is a _________ model.

91. Error sum of squares _________ be negative.

92. The formula for one missing value in a randomized block design with k treatments and b blocks following usual notations is _________.

93. If there are two missing values in a randomized block design with 4 blocks and 5 treatments, the error degrees of freedom will be _________.

94. Missing value in a randomized block design causes an _________ bias in the treatment sum of square.

95. In case of sub-sampling in a randomized design, the pooling of sampling and experimental errors depends on the result of _________.

96. In case of sub-sampling in experiments, the analysis of data is safer if the investigator decides for _________ pooling rather than _________ pooling.

97. A Latin square design is a _________ two way classification scheme.

98. A Latin square design controls _________ heterogeneity.

99. A Latin square design is a ________ restrictional design.

100. The number of treatments, rows and columns in a Latin square design are ________.

101. Each treatment occurs ________ in a row or a column in a Latin square design.

102. Each row and each column is a ________ in a Latin square design.

103. The number of rows or columns denote the ________ of a Latin square.

104. Two Latin squares L_1 and L_2 are called ________ if L_1 is superimposed over L_2, then each pair of letters occurs only once in the composite square.

105. If there is a Latin square of order p, then there can at most be ________ orthogonal Latin squares.

106. Error mean square in the analysis of data of a Latin square is ________ than the error mean square in case of RBD and CRD with the same experimental material.

107. A Latin square design of order 4 is as efficient as a randomized block design with ________ replications for the same treatments.

108. A 3 × 3 Latin square design is ________ efficient than a randomized block design with 3 replications.

109. A composite Latin square design using Latin and Greek letters is called ________ design.

110. The main assumption of a Latin square design is that treatment effects ________ with the row and column effects.

111. The additive model for a Latin square design is a ________ model.

112. The formula for estimating a missing value in a Latin square design having k treatments with usual notations is ________.

113. There is an ________ bias in treatment sum of square of a Latin square being analysed with a missing value.

114. Relative efficiency of a Latin square of order k over completely randomized design with usual notation can be calculated by the formula ________.

115. Relative efficiency of a Latin square design with k treatments as compared to randomized block design utilizing the same material can be obtained by the formula ________.

116. To have sufficient degrees of freedom for error from lower order Latin square, the alternative is to consider a ________ Latin squares.

117. If we consider m Latin squares of order k in an experiment, the degrees of freedom for error is ________.

118. Another name of cross-over design is ________ or ________ or ________ design.

119. Cross-over design is suitable for ________ number of treatments.

120. Change over designs is capable of estimating ________ and ________ effects.

121. Cross-over designs are frequently used in experiments on ________.

122. In cross-over design each treatment is followed by the other treatments ________ number of times.

123. An experiment involving two or more factors at various levels is called a ________ experiment.

124. A factorial experiment, with equal number of levels of all factors, is called a ________ factorial experiment.

125. An experiment involving 5 levels of nitrogen, 4 levels of phosphorous and 3 levels of potash is ________ factorial experiment.

126. The factorial experiments were also called ________ experiments.

127. The name factorial experiment is due to ________.

128. In factorial experiments, one estimates ________ and ________ effects.

129. The measure of change in the response due to change of level of a factor averaged over all levels of other factors is called ________.

130. The failure of a factor to give the same response at different levels of the other factor is known as ________.

131. The additional effect due to combined effect of two or more factors is the ________ effect.

132. The interaction effect of the factors N and P is ________ as that of factors P and N.

133. In a 2^n factorial, each main and interaction effect can be expressed as a ________.

134. In a 2^n factorial, the contrasts for main effects and interaction(s) are ________ to each other.

135. In a 2^n factorial, the lower level of a factor is called its ________.

136. In a 2^n factorial, the higher level of a factor is known as ________.

137. If A and B are two factors each at 2 levels, the simple effect of A at the first level of B is ________.

138. Given two factors A and B each at 2 levels, the simple effect B at the second level of A is ________.

139. In a 2^2 factorial with factors A and B, the simple effect of B, $(b) - (1)$ is equivalent to ________.

140. The interaction between two factors A and B each at two levels is the ________ of simple effects of A at second and first levels of B.

141. Sum of squares for main effects and interaction(s) in 2^n factorial can easily be calculated with the help of ________ technique.

142. The sum of squares in a 3^n factorial can easily be calculated with the help of ________ technique.

143. In a 3^n experiment with factors A and B, the interaction $A \times B$ has ________ d.f.

144. In a 3^2 factorial, the interaction $A \times B$ can further be splitted up as ________ and ________.

145. In a 3^3 factorial, the interaction component A^2B^2C is equivalent to the interaction component ________.

146. The statistical model for a two factor factorial is a ________ model.

147. The technique of reducing the block size in factorial experiments by scrificing one or more effects is known as ________.

148. Preferably ________ interaction is chosen for confounding.

149. The method of confounding to reduce block size is applicable only for ________ experiments.

150. If the same effect is confounded in all the replications, it is known as ________.

151. If the same effect is not confounded in all the replicates, it is known as ________.

152. In 2^n factorials, confounding can easily be carried out with the help of ________.

153. If p^n factorial ($p > 2$), confounding can conveniently be carried out through ________.

154. When the effects of the same order are confounded an equal number of times in all the replications of a factorial experiment, it is known as ________.

155. In a 2^4 factorial, the effects ABC and BCD are confounded to obtain the blocks of size 4 units, the effect ________ is automatically confounded with the blocks.

156. The product of any two interactions in a p^n factorial reduced to mod p is known as ________.

157. Confounding in asymmetrical factorials is carried with the help of ________ factors.

158. The additional factors required to convert the asymmetrical factorial into a symmetrical factorial for the purpose of confounding are called ________ factors.

159. When different factors require different size of units, one should go for ________ design.

160. The large size plots used for one factor in split plot design are called ________ plots.

(c) heterogeneity among blocks
(d) none of the above

Q. 10 Randomizations in an experiment provide:
(a) the estimate of experimental error
(b) impetus to the treatments
(c) a check to the variation in soil fertility
(d) none of the above

Q. 11 Number of replications in an experiment is based on:
(a) the precision required
(b) experimental material available
(c) heterogeneity of experimental material
(d) all the above

Q. 12 The factors responsible for deciding the number of replications in an experiment are:
(a) the desire of the experimenter
(b) minimum degrees of freedom required for experimental error
(c) shape of experimental units
(d) all the above

Q. 13 The decision about the number of replications is taken in view of:
(a) size of experimental units
(b) competition among experimental units
(c) fraction to be sampled
(d) all the above

Q. 14 The formula for determining the number of replications 'r' with usual notations is:
(a) $r = 2t_\alpha^2 s^2/d^2$
(b) $r = \sqrt{2}\, t_\alpha s^2/d^2$
(c) $r = t_\alpha^2 s^2/d^2$
(d) $r = 2t_\alpha s/d$

Q. 15 Local control is a device to maintain:
(a) homogeneity among blocks
(b) homogeneity within blocks
(c) both (a) and (b)
(d) neither (a) and (b)

Q. 16 Local control in the field is maintained through:
(a) uniformity trials
(b) randomization
(c) natural factors
(d) all the above

Q. 17 Local control in experimental designs is meant to:
(a) increase the efficiency of the design
(b) reduce experimental error
(c) to form homogeneous blocks
(d) all the above

Q. 18 Experimental error is due to:
(a) experimenter's mistakes
(b) extraneous factors
(c) variation in treatment effects
(d) none of the above

Q. 19 Experimental error is necessarily required for:
(a) testing the significance of treatments effects
(b) comparing treatment effects
(c) calculating the information released from an experiment
(d) all the above

Q. 20 Errors in a statistical model are always taken to be:
(a) independent
(b) distributed as $N\left(0, \sigma_e^2\right)$
(c) both (a) and (b)
(d) neither (a) and (b)

Q. 21 If s^2 is the estimate of error variance in an experiment based on n degrees of freedom, the information gathered from the experiment is equal to:
(a) $\dfrac{n+3}{n+1}\dfrac{1}{s^2}$
(b) $\dfrac{n+1}{n+3}\dfrac{1}{s^2}$
(c) $\dfrac{n+3}{n+1}s^2$
(d) $\left(\dfrac{n+1}{n+3}\cdot\dfrac{1}{s}\right)^2$

Q. 22 The information from an experiment stabilizes when error degrees of freedom is at least:

(a) 6
(b) 8
(c) 10
(d) 12

Q. 23 If σ_1^2 is the error variance of design-1 and σ_2^2 of design-2 utilizing the same experiment material, the efficiency of design 1 over 2 is

(a) $\frac{1}{\sigma_1^2} \Big/ \frac{1}{\sigma_2^2}$

(b) $\frac{1}{\sigma_2^2} \Big/ \frac{1}{\sigma_1^2}$

(c) σ_1^2/σ_2^2

(d) none of the above

Q. 24 If sampling is adopted from plots in an experiment, the loss of information due to sampling as compared to complete harvesting is:

(a) negligible
(b) one-third
(c) two-third
(d) none of the above

Q. 25 Analysis of experimental data means:

(a) estimation of treatment effects
(b) dividing the total variance into component variances
(c) testing of hypothesis about the parameters involved in the experimental model
(d) all the above

Q. 26 A statistical model is a mathematical relationship between the:

(a) response measure and the factors responsible for the response
(b) response measure and the treatment effects
(c) both (a) and (b)
(d) neither (a) and (b)

Q. 27 A statistical model is categorized into:

(a) one type
(b) two types
(c) three types
(d) four types

Q. 28 The categorization of a statistical model is based on:

(a) assumption about the experimental error
(b) the nature of treatments
(c) characteristics of the response variable
(d) all the above

Q. 29 If the conclusions are to be drawn for the treatments in general, then the statistical model shall be known as:

(a) analysis of variance model
(b) component of variance model
(c) mixed effect model
(d) none of the above

Q. 30 If the researcher's interest is confined in the treatments only involved in the experiment, then the statistical model in this situation will be categorized as:

(a) analysis of variance model
(b) component of variance model
(c) mixed effect model
(d) none of the above

Q. 31 If an experiment involves two or more treatments in which some treatments are fixed and the others are of random nature, one should choose:

(a) analysis of variance model
(b) component of variance model
(c) mixed effect model
(d) none of the above

Q. 32 In a fixed effect model, the hypothesis about the treatments under test is:

(a) $\sigma_\tau^2 = 0$

(b) $\tau_i = 0$

(c) $\Sigma \tau_i = 0$

(d) none of the above

Q. 33 In case of a random effect model, the hypothesis which is to be tested with regard to the treatments is:

(a) $\sigma_\tau^2 = 0$

(b) $\tau_i = 0$

(c) $\Sigma \tau_i^2 = 0$

(d) $\Sigma \tau_i = 0$

Q. 34 For a fixed effect model
$y_{ij} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ij}$,
the assumptions are:

(a) $\Sigma \alpha_i = 0, \Sigma \beta_j = 0, \Sigma (\alpha\ \beta)_{ij} = 0$

(b) $\sigma_\alpha^2 = 0, \sigma_\beta^2 = 0$ and $\sigma_{\alpha\beta}^2 = 0$

(c) $\sigma_i \sim N(0, \sigma_\alpha^2), \beta_j \sim N(0, \sigma_\beta^2)$ and $(\alpha\beta)_{ij} \sim N(0, \sigma_{\alpha\beta}^2)$

(d) none of the above

Q. 35 If a model is such that it provides an exact test about the treatments, then it is called:
(a) unspecified model
(b) incompletely specified model
(c) completely specified model
(d) none of the above

Q. 36 If a statistical model fails to provide an exact test about the treatment effects, the model is classified as:
(a) unspecified model
(b) incompletely specified model
(c) completely specified model
(d) none of the above

Q. 37 Which of the pairs of treatment means differ significantly out of a set of treatment means, can be ascertained by:
(a) least significant difference test
(b) student-Newman Kuel's test
(c) Duncan's multiple range test
(d) all the above

Q. 38 Out of many multiple range test, the test which is considered superior is:
(a) student-Newman Kuel's test
(b) Duncan's multiple range test
(c) Tukeys' test
(d) none of the above

Q. 39 A linear combination of treatments is said to be a contrast iff:
(a) the sum of the treatment effects is zero
(b) all the coefficients of the treatments are unity
(c) the sum of the coefficients of the treatments is zero
(d) the number of positive and negative coefficients is same

Q. 40 Two contrasts of the same treatments are said to be orthogonal iff:
(a) they are at right angles
(b) both of them have same coefficients of the treatments
(c) both of them have equal coefficients but opposite in sign
(d) the sum of the cross product of the coefficients of the same treatments is zero

Q. 41 Which of the following is a contrast?
(a) $3T_1 + T_2 - 3T_3 + T_4$
(b) $T_1 + 3T_2 - 3T_3 + T_4$
(c) $-3T_1 - T_2 + T_3 + 3T_4$
(d) $T_1 + T_2 + T_3 - T_4$

Q. 42 Which of the following is not a contrast among three treatments?
(a) $T_1 + 2T_2 - T_3$
(b) $T_1 - T_3$
(c) $T_1 - 2T_2 + T_3$
(d) $-T_1 + 2T_2 - T_3$

Q. 43 The maximum possible number of orthogonal contrasts among four treatments is:
(a) four
(b) three
(c) two
(d) one

Q. 44 With the help of contrasts, one can estimate the:
(a) linear effect
(b) quadratic effect

(c) incompletely specified
(d) none of the above

Q. 58 The process of determining more than one observation on each experimental unit pertaining to the same character is known as:
(a) multiple recording
(b) sub-sampling
(c) repeated observations
(d) none of the above

Q. 59 In case of sub-sampling in a completely randomized design, the treatment mean square is tested against:
(a) experimental error M.S.
(b) sampling error M.S.
(c) pooled error M.S.
(d) none of the above

Q. 60 If *F*-value for treatments comes out to be less than one, it may be due to:
(a) improper randomization
(b) non-normality of response measure
(c) selection of wrong statistical model
(d) all the above

Q. 61 If every level of a factor is taken at every level of the other factor in an experiment, it is known as:
(a) crossed classification
(b) nested classification
(c) hierarchical classification
(d) all the above

Q. 62 When all levels of a factor are woven within each level of some other factor, it is known as:
(a) nested classification
(b) hierarchical classification
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 63 If a plant breeder includes *s* species and *n* strains within each specie, he has to analyse the data according to:
(a) fixed effect model
(b) nested model
(c) mixed model
(d) none of the above

Q. 64 The following layout,

A	*B*	*C*	*D*
A	*C*	*B*	*D*
B	*A*	*C*	*C*
A	*A*	*B*	*C*

meets the requirements of a:
(a) completely randomized design
(b) randomized block design
(c) Latin square design
(d) none of the above

Q. 65 The layout,

A	*C*	*A*	*B*
C	*B*	*C*	*D*
B	*A*	*D*	*A*
D	*D*	*B*	*C*

stands for:
(a) cross-over design
(b) randomized block design
(c) Latin square design
(d) none of the above

Q. 66 In the field layout of a randomized block design, the blocks are formed in the direction:
(a) parallel to fertility gradient
(b) perpendicular to fertility gradient
(c) diagonally to fertility gradient
(d) none of the above

Q. 67 Randomized block design is a:
(a) three restrictional design
(b) two restrictional design
(c) one restrictional design
(d) no restrictional design

Q. 68 A randomized block design has:
(a) two way classification
(b) one way classification
(c) three way classification
(d) no classification

Q. 69 In the analysis of data of a randomized block design with *b* block and *v* treatments, the error degrees of freedom are:
(a) $b(v-1)$
(b) $v(b-1)$
(c) $(b-1)(v-1)$
(d) none of the above

Q. 70 Error sum of squares in RBD as compared to CRD using the same material is:
(a) more
(b) less
(c) equal
(d) not comparable

Q. 71 According to W.G. Cochran, error mean square in case of RBD as compared to CRD utilising same experimental material is:
(a) 60 per cent
(b) 80 per cent
(c) 40 per cent
(d) none of the above

Q. 72 The ratio of the number of replications required in CRD and RBD for the same amount of information is:
(a) 6 : 4
(b) 10 : 6
(c) 10 : 8
(d) 6 : 10

Q. 73 The formula for obtaining a missing value in randomized block design by minimizing the error mean square was given by:
(a) W.G. Cochran
(b) T. Wishart
(c) F. Yates
(d) J.W. Tukey

Q. 74 The formula for estimating one missing value in a randomized block design having *b* blocks and *k* treatments with usual notations is:

(a) $\dfrac{bT' + kB' - G'}{(b-1)(k-1)}$

(b) $\dfrac{bB' + bT' - G'}{(b-1)(k-1)}$

(c) $\dfrac{bT' + kB' - kG'}{(b-1)(k-2)}$

(d) $\dfrac{bB' + kT' - G'}{(b-1)(k-1)}$

Q. 75 In a randomized block design with 4 blocks and 5 treatments having one missing value, the error degrees of freedom will be:
(a) 12
(b) 11
(c) 10
(d) 9

Q. 76 A missing value in an experiment is estimated by the method of:
(a) minimizing the error mean square
(b) analysis of covariance
(c) both (a) and (b)
(d) neither (a) and (b)

Q. 77 When there occurs a missing value in an experiment, treatment sum of square has:
(a) an upward bias
(b) a downward bias
(c) no bias
(d) none of the above

Q. 78 In case of sub-sampling in a randomized block design, if preliminary test reveals a non-significant difference between experimental error and sampling error, one should prefer:
(a) to pool the two errors
(b) not to pool the two errors
(c) may or may not pool the two errors
(d) none of the above

Q. 79 The idea of preliminary test of significance was propounded by:
(a) T.A. Bancroft
(b) A.E. Paull
(c) F. Yates
(d) none of the above

Q. 80 In the want of preliminary test of significance in case of sub-sampling in the analysis of data of an experiment, it is always preferable:
(a) not to pool the two errors
(b) to pool the two error
(c) to use sampling error
(d) to use any one of the errors

Q. 81 In a Latin square design, number of rows, columns and treatments are:
(a) all different
(b) always equal

(b) $[(kT' + kR' + kC' - 2G'\}/(k-1)(k-2)]^2$

(c) $[\{(k-1)T' + R' + C' - G'\}/(k-1)(k-2)]^2$

(d) $[\{(k-1)T' + kR' + kC' - 2G'/(k-1)(k-2)]^2$

Q. 95 For lower order Latin square, adequate degrees of freedom for error can be obtained by taking:

(a) a group of Latin squares at a time
(b) a Latin square at two locations
(c) both (a) and (b)
(d) neither (a) and (b)

Q. 96 Cross-over design is suitable for measuring:

(a) direct treatment effect
(b) treatments' residual effect
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 97 Cross-over design is also named as:

(a) change over design
(b) switch back design
(c) reversal design
(d) all the above

Q. 98 Error degrees of freedom for a change over design are more when the subjects are taken in:

(a) replicates
(b) a group of Latin squares
(c) any order
(d) all the above

Q. 99 The experiments, in which the effects of the levels of a factor are considered at various levels of the other factor are called:

(a) symmetrical experiments
(b) rotational experiments
(c) factorial experiments
(d) all the above

Q. 100 An experiment having several factors with equal number of levels is known as:

(a) complex experiment
(b) symmetrical factorial experiment
(c) asymmetrical experiment
(d) all the above

Q. 101 Two types of effects measured in a factorial experiment are:

(a) main and interaction effects
(b) simple and complex effects
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 102 The additional effect gained due to combined effect of two or more factors is known as:

(a) main effect
(b) interaction effect
(c) either of (a) or (b)
(d) neither of (a) or (b)

Q. 103 The experiments with various factors having unequal number of levels are called:

(a) asymmetrical factorials
(b) symmetrical factorials
(c) typical factorials
(d) none of the above

Q. 104 An experiments, in which all factors have different number of levels, is known as:

(a) asymmetrical factorial
(b) typical factorial
(c) pseudo factorial
(d) real factorial

Q. 105 All contrast, representing the effects of a 2^n factorial, are:

(a) linear contrasts
(b) orthogonal contrasts
(c) both (a) and (b)
(d) neither (a) nor (b)

Q. 106 The main and interaction effects in a 2^n-factorial can easily be estimated with the help of:

(a) simple effects
(b) contrasts
(c) both (a) and (b)
(d) neither (a) and (b)

Q. 107 If the responses for treatments in a factorial experiment with factors *A* and *B* each at two levels from three replications are, $a_0b_0 = 18$, $a_1b_0 = 17$, $a_0b_1 = 25$ and $a_1b_1 = 30$, the sum of square for the interaction *AB* is equal to:

(a) 4

Q. 133 In a split plot design, more precision is attained for:

(a) main plot treatment
(b) sub-plot treatment
(c) block differences
(d) all the above

Q. 134 In a split plot design, more error degrees of freedom is obtained for:

(a) main plots
(b) sub-plots
(c) replicates
(d) none of the above

Q. 135 In a split plot design, smaller error mean square is obtained for:

(a) main plot error
(b) sub-plot error
(c) experimental error
(d) none of the above

Q. 136 If there is a double split plot design, the analysis of variance table will contain:

(a) two error components
(b) three error components
(c) four error components
(d) one error components

Q. 137 In a split plot design with factor A at p levels in main-plots, factor B at q levels in sub-plots and r replications, the degrees of freedom for sub-plot error is equal to:

(a) $(q-1)(r-1)$
(b) $q(p-1)(r-1)$
(c) $(p-1)(q-1)(r-1)$
(d) $p(q-1)(r-1)$

Q. 138 For a split plot design with factor A in main plots at 4 levels, factor B in sub-plots at 3 levels and having 3 replications, sub-plot error degrees of freedom will be:

(a) 24
(b) 27
(c) 16
(d) 12

Q. 139 For a split plot experiment conducted with 5 concentrations of an insecticide in main plots and 4 varieties of gram in sub-plots and having 3 replications, main plot error degrees of freedom will be:

(a) 8
(b) 10
(c) 24
(d) 6

Q. 140 Model-I of a split plot design is:

(a) an incompletely specified model
(b) a completely specified model
(c) an unspecified model
(d) none of the above

Q. 141 For a double split plot design with 4 levels of main plot treatment, 3 levels of sub-plot treatment and 2 levels of sub-subplot treatment and 2 replications, the error sub-sub-plot error degrees of freedom is:

(a) 48
(b) 36
(c) 24
(d) 12

Q. 142 When two treatments at various levels involved in an experiment require large size plots, the appropriate experimental design is:

(a) split plot design
(b) strip plot design
(c) Latin square design
(d) any one of the above

Q. 143 Split plot design for all sets of comparisons as compared to a randomized block design with the same experimental material is:

(a) superior
(b) inferior
(c) equally good
(d) none of the above

Q. 144 In a split-split plot design, maximum precision is attained on:

(a) whole plot treatment
(b) sub-plot treatment
(c) sub-subplot treatment
(d) none of the above

Q. 145 A split block experiment is conducted with 5 level of irrigation (I) and 4 levels of an insecticide (M) for foliar spray. The experiment contained 3 replications. The error

degrees of freedom for the interaction effect will be:
(a) 24
(b) 30
(c) 32
(d) 40

Q. 146 The precision of whole-plot treatment can be increased by assigning the treatments to whole plots:
(a) randomly
(b) in randomized block arrangement
(c) in a Latin square arrangement
(d) all the above

Q. 147 The accuracy of estimates after confounding in sub-plots increases:
(a) for main-plot treatments
(b) for all sub-plot treatments
(c) for all sub-plot treatments except those which are confounded
(d) for no treatments

Q. 148 In a 3^3 factorial with factors A, B and C each at 3 level, the interaction $A^2 BC^2$ is same as the interaction:
(a) ABC
(b) AB^2C
(c) $AB^2 C^2$
(d) $A^2 BC$

Q. 149 The analysis of variance of an experimental data is based on the assumption(s) that:
(a) the response variable is distributed normally
(b) the errors are independent
(c) the errors are homoscedastic
(d) all the above

Q. 150 If in a randomized block design having five treatments and 4 replications, a treatment is added, the increase in error degrees of freedom will be:
(a) 1
(b) 2
(c) 3
(d) 4

Q. 151 If in a Latin square design with five treatments, a treatment is added, the increase in error degrees of freedom will be:
(a) 2
(b) 4
(c) 6
(d) 8

Q. 152 If two levels of a main plot treatment are increased, the degrees of freedom for main-plot error in a split plot design with three replications will be increased by:
(a) 2
(b) 4
(c) 6
(d) 8

Q. 153 If two levels of a sub-plot treatment are increased, the main-plot error in a split plot design with two replications will be:
(a) increased
(b) decreased
(c) same
(d) none of the above

Q. 154 If one level of main plot treatment is increased in a split plot design having its p levels, q levels of sub-plot treatment and r replications, then the sub-plot error degrees of freedom will be increased by:
(a) r
(b) $(r - 1)(p - 1)$
(c) $(r - 1)(q - 1)$
(d) $(p - 1)(q - 1)$

Q. 155 If two levels of a sub-plot treatment are increased in a split plot design, the main plot error degrees of freedom will be increased by:
(a) 0
(b) 1
(c) 2
(d) none of the above

Q. 156 If in a split plot design with two treatments A and B at levels 4 and 3 respectively and three replications, a level of each treatment is dropped, then the total degrees of freedom will be reduced by:
(a) 8
(b) 10

(c) 12
(d) 18

Q. 157 If in a split plot design with treatment A at 4 levels in main-plots, B at 3 levels in sub-plots and 3 replications, one level of both the treatments is reduced, then the sub-plot error degrees of freedom will be decreased by:
(a) 6
(b) 8
(c) 10
(d) 12

Q. 158 If in a split plot design with two factors, A in main-plots and B in sub-plots at levels p and q respectively and r replications, a replication is discarded, then the sub-plot error degrees of freedom will be decreased by:
(a) rpq
(b) $p(q-1)$
(c) $(p-1)q$
(d) $(r-1)(p-1)(q-1)$

Q. 159 If in a split plot design with two factors, A in main plots and B in sub-plots at levels p and q respectively and r replications, a replication is rejected, then the main-plot error degrees of freedom will be decreased by
(a) $r-1$
(b) $(p-1)$
(c) $q-1$
(d) $r(p-1)$

Q. 160 If in a strip plot design with factors A and B at levels p and q respectively and r replications, a level of both the factors is dropped, then the total degrees of freedom will be reduced by
(a) $r(p-1)(q-1)$
(b) $r(p+q-1)$
(c) $r(p+q-2)$
(d) $r(pq-1)$

Q. 161 In randomized block design we always have:
(a) No. of blocks = No. of treatments
(b) No. of blocks > No. of treatments
(c) No. of blocks < No. of treatments
(d) none of the above

Q. 162 Local control helps to:
(a) reduce the no. of treatments
(b) increase the no. of plots
(c) reduce the error variance
(d) increase the error d.f.

Q. 163 In one way classification with more than two treatments, the equality of treatment means is tested by:
(a) t-test
(b) χ^2-test
(c) F-test
(d) none of the above

Q. 164 The degrees of freedom for F-ratio in a 6 × 6 Latin square design is:
(a) (5, 15)
(b) (5, 20)
(c) (6, 15)
(d) (6, 20)

Q. 165 In a 5 × 5 Latin square with one missing value, the totals of row, column and treatment having the missing value are 25, 40, 35 respectively and the total of all the available observations is 100. The estimate of the missing value is:
(a) 15
(b) 20
(c) 25
(d) 30

Q. 166 The concept of fractional replication was first expounded by:
(a) F. Yates
(b) D.J. Finney
(c) C.R. Rao
(d) G.E.P. Box

Q. 167 Fractional replication came into existence for the first time in the year:
(a) 1961
(b) 1950
(c) 1945
(d) 1937

Q. 168 Another name given to fractional replications is:
(a) balanced confounded design
(b) fractional factorial design

(a) Resolution - IV design
(c) Resolution - V design
(b) Resolution - VI design
(d) Resolution - VII design

Q. 182 A fractional factorial design in which mean, main effects and first order interactions are estimable and second and higher order interactions are presumed to be negligible is a design of:
(a) Resolution-III
(b) Resolution-IV
(c) Resolution-V
(d) none of the above

Q. 183 The total of sum of squares (S.S.) due to all orthogonal contrasts in 2^n-factorial experiment is equal to:
(a) replication S.S.
(b) treatment S.S.
(c) total S.S.
(d) error S.S.

Q. 184 In experimental designs, randomization is necessary to make the estimates:
(a) valid
(b) accurate
(c) precise
(d) biased

Q. 185 The name of the design in which main effect is confounded is:
(a) Latin square design
(b) cross-over design
(c) split plot design
(d) none of he above

Q. 186 The pairwise contrast among three treatments is:
(a) $T_1 + T_2 - 2T_3$
(b) $2T_1 + T_2 - 3T_3$
(c) $T_1 + T_3 - 2T_2$
(d) $T_3 - T_1$

Q. 187 A non-pairwise contrast among four treatments is:
(a) $3T_1 + T_2 - T_3 - 3T_4$
(b) $3T_1 + T_2 + T_3 - 3T_4$
(c) $T_1 + T_2 - 2T_3 + T_4$
(d) $T_1 - T_4$

Q. 188 For two treatments there can be in all:
(a) one contrast
(b) two contrasts
(c) three contrasts
(d) no contrast

Q. 189 A contrast constructed while interpreting the results will be categorised as:
(a) posteriori contrast
(b) planned contrast
(c) biased contrast
(d) orthogonal contrast

Q. 190 A decision of testing the quadratic, cubic and quadratic effects of five fertilizers at the beginning of experimentation will lead to construction of:
(a) posteriori contrasts
(b) post hoc contrasts
(c) a priori contrasts
(d) non-orthogonal contrasts

ANSWERS

SECTION-B

(1) statistics (2) design of experiment (3) field experiments (4) experimental unit (5) treatment (6) randomization (7) human bias (8) randomization (9) mathematical (10) randomization (11) randomization (12) replications (13) experimental error (14) better (15) replications (16) local control (17) fertility maps (18) extraneous factors (19) efficient (20) more (21) heterogeneous (22) twelve (23) competition (24) more (25) $\left(\sqrt{2}\, t_\alpha \cdot s/d\right)^2$ (26) Fairfield Smith (27) $\frac{n+1}{n+3} \cdot \frac{1}{s^2}$ (28) R.A. Fisher (29) inverse (30) one-third (31) component variances (32) true mean effect; error term (33) $N\left(o, \sigma_e^2\right)$ (34) three (35) additive (36) J.W. Tukey (37) analysis of variance model; Model-I (38) component of variance model; Model II (39) fixed effect (40) Model-II (41) zero (42) least significant difference (43) $\sqrt{2\frac{s_e^2}{r}} \times t_{.05,v}$ (44) dis-

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