Confounding

Confounding

- What is confounding?
- Why we need it?
- Advantage and Disadvantage.
- How to reduce block size?
- Types of Confounding.
- Examples.
- ANOVA TABLE

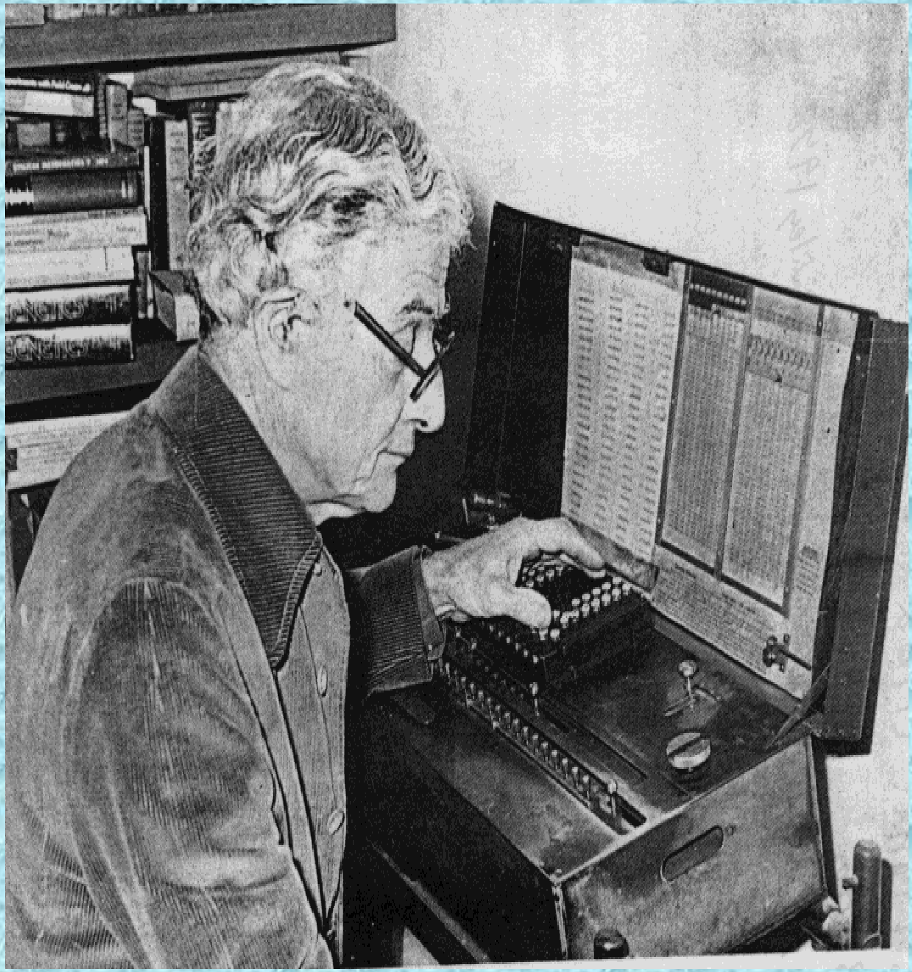
Concept of Confounding.

- In a factorial experiment we have n factors each are at s levels. If the factors and level of the factors are increased, then total no. of treatment combinations will also increase which we call the effect of heterogeneity that is the variability among the treatment combination will increase and consequently the errors variance will increase.

- Because of this result the estimate will be least precise estimate. So one is interested to remove the variability among the treatment combination. which is only possible if the size of the block will reduce to make homogeneity in the block.

- This concept of reducing the block size is call confounding.

Who has developed confounding theory?



- R.A. Fisher
- Yates

Definition

- Confounding of factorial experiment is defined as reduction of block size in such a way that one block is divided into two or more blocks such that treatment comparison of that main or interaction effect is mixed up with block effect.

- Main or Interaction effect which is mixed up with block effect are called Confounded effect and this experiment is called Confounding experiment.

Block
n_0p_0
n_1p_0
n_0p_1
n_1p_1

Block-1
n_1p_0
n_1p_1
$n_1p_1+n_1p_0$

Block-2
n_0p_0
n_0p_1
$n_0p_1+n_0p_0$

- Block Effect :- $(n_1p_1+n_1p_0) – (n_0p_1+n_0p_0)$

- Treatment Effect.:-

  =Main effect N

  $= (n_1-n_0)(p_1+p_0)$

  $=n_1p_1+n_1p_0-n_0p_1-n_0p_0$

  $=(n_1p_1+n_1p_0) – (n_0p_1+n_0p_0)$

Thus,

Treatment comparison – Block effect =0

Here, We can say that main effect N is mixed up with block effect .

So N is confounded.

Example

A soccer coach wanted to improve the team's playing ability, so he had them run two miles a day. At the same time the players decided to take vitamins. In two weeks the team was playing noticeably better, but the coach and players did not know whether it was from the running or the vitamins.

Advantage

- It reduce the block size and make homogeneity in block. And it reduce the heterogeneity of data.

Disadvantage

- In the confounding experiment the drawback is that, the main effect or interaction effect which are confounded can not be estimated. And hence we loose all information of that confounded effect.

- **In confounding experiment, we loose all the information about effect which is confounded.**
- **So we should try to confound only those contrasts which have little or no importance.**
- **therefore one must confound higher order interaction, so we can save main effects and two factor interaction.**

Factorial Experiment	No. of Possible confounding	Possible Treatment Combinations.	
2^2	$2^{2}-1=3$	A,B,AB	
2^3	$2^{3}-1=7$	A,B,C,AB,AC,BC,ABC	
3^2	$3^{2}-1=8$	A,B,AB, A^2B	A^2, B^2 ,A^2B^2 ,AB^2
3^3	$3^{3}-1=26$	A,B,C,AB, BC,AC,ABC ,A^2B,B^2C, A^2C,A^2BC, AB^2C,ABC^2	A^2, B^2,C^2,A^2B^2 B^2C^2 ,A^2C^2, $A^2B^2C^2$,AB^2,BC^2 ,AC^2,AB^2C^2, A^2BC^2,A^2B^2C

How to confound?

- Modular arithmetic is the value of the remainder after dividing the original number by the modulus (devisor).

Ex. 15 mod 5 = 0 OR 15= 0(mod 5)

- When two integers have the same remainder they are said to be congruent. Our interests will focus on sets of congruent integers.

Ex. 13 mod 3 = 1 , 10 mod 3 = 1

Or 13=1(mod 3) , 10=1(mod 3)

2^3 factorial experiment confounded in 2^2 block size.

Block
$a_0b_0c_0$
$a_1b_0c_0$
$a_0b_1c_0$
$a_1b_1c_0$
$a_0b_0c_1$
$a_1b_0c_1$
$a_0b_1c_1$
$a_1b_1c_1$

Replication-1	
B-1	B-2
$a_0b_0c_0$	$a_0b_1c_0$
$a_0b_0c_1$	$a_0b_1c_1$
$a_1b_1c_0$	$a_1b_0c_0$
$a_1b_1c_1$	$a_1b_0c_1$
AB confounded	

Replication-2	
B-3	B-4
$a_0b_0c_0$	$a_0b_0c_1$
$a_0b_1c_0$	$a_0b_1c_1$
$a_1b_0c_1$	$a_1b_0c_0$
$a_1b_1c_1$	$a_1b_1c_0$
AC is confounded	

Coefficient of,

- **a+b=0(mod 2) in block 1**
- **a+b=1(mod 2) in block 2.**

Coefficient of,

- **a+c=0(mod 2) in block 3**
- **a+c=1(mod 2) in block 4.**

Example:- **3^2 Factorial Experiment confounded in 3^1 block size**

Replication-1		
B-1	B-2	B-3
a_0b_0	a_1b_0	a_2b_0
a_0b_1	a_1b_1	a_2b_1
a_0b_2	a_1b_2	a_2b_2
A is confounded		

Replication-2		
B-1	B-2	B-3
a_0b_0	a_1b_1	a_2b_2
a_1b_0	a_2b_1	a_0b_2
a_2b_0	a_0b_1	a_1b_2
B is confounded		

Replication-3		
B-1	B-2	B-3
a_0b_0	a_0b_1	a_0b_2
a_1b_2	a_1b_0	a_2b_0
a_2b_1	a_2b_2	a_1b_1
AB is confounded		

Replication-4		
B-1	B-2	B-3
a_0b_0	a_1b_0	a_0b_1
a_1b_1	a_0b_2	a_1b_2
a_2b_2	a_2b_1	a_2b_0
AB^2 is confounded		

AB^2 confounding.
Coefficient of,

- **$a+2b=0 \pmod 3$ in block 1**
- **$a+2b=1 \pmod 3$ in block 2.**
- **$a+2b=2 \pmod 3$ in block 3.**

Types Of Confounding

- Total Confounding (complete confounding)
- Partial Confounding
- Balanced Confounding

Total Confounding

- In a confounding factorial experiment if all the main effect or interaction effect are same in all the replication , then the experiment is called Total confounding or Complete confounding.
- In complete confounding we could not estimate the value of confounded main or interaction effect.

Example:- 2^3 Factorial Experiment

Replication-1		Replication-2		Replication-3	
B-1	B-2	B-3	B-4	B-5	B-6
$a_0b_0c_0$	$a_0b_1c_0$	$a_0b_0c_0$	$a_0b_1c_1$	$A_1b_1c_1$	$a_0b_1c_0$
$a_0b_0c_1$	$a_0b_1c_1$	$a_1b_1c_1$	$a_0b_1c_0$	$a_0b_0c_0$	$a_1b_0c_1$
$a_1b_1c_0$	$a_1b_0c_0$	$a_0b_0c_1$	$a_1b_0c_1$	$a_1b_1c_0$	$a_1b_0c_0$
$a_1b_1c_1$	$a_1b_0c_1$	$a_1b_1c_0$	$a_1b_0c_0$	$a_0b_0c_1$	$a_0b_1c_1$
AB is confounded		AB is confounded		AB is confounded	

Complete confounding of 2^3 into 2^2 block size with ABC is confounded.

S.V.	d.f.	S.S.	M.S.S.	F_c
Block	b-1	ΣBj^2 /k -c.f.$=S_b^2$	$S_b^2/(b-1)=\sigma_b^2$	σ_b^2/σ_e^2
Replication	r-1	ΣRi^2 /t -c.f.$=S_r^2$	$S_r^2/(r-1)=\sigma_r^2$	σ_r^2/σ_e^2
ABC	1	$[ABC]^2$	$[ABC]^2/1=\sigma_{ABC}^2$	$\sigma_{ABC}^2/\sigma_e^2$
ABC*replication	1*(r-1)	$S_b^2- S_r^2-[ABC]^2=S_p^2$	$S_p^2/1*(r-1)=\sigma_p^2$	σ_p^2/σ_e^2
Treatment	6	ΣTi^2 /r -c.f.$=S_t^2$	$S_t^2/6 =\sigma_t^2$	σ_t^2/σ_e^2
A	1	$[A]^2$	$[A]^2/1=\sigma_A^2$	σ_A^2/σ_e^2
B	1	$[B]^2$	$[B]^2/1=\sigma_B^2$	σ_B^2/σ_e^2
AB	1	$[AB]^2$	$[AB]^2/1=\sigma_{AB}^2$	σ_{AB}^2/σ_e^2
C	1	$[C]^2$	$[C]^2/1=\sigma_C^2$	σ_C^2/σ_e^2
AC	1	$[AC]^2$	$[AC]^2/1=\sigma_{AC}^2$	σ_{AC}^2/σ_e^2
BC	1	$[BC]^2$	$[BC]^2/1=\sigma_{BC}^2$	σ_{BC}^2/σ_e^2
Error	6(r-1)	By sub. $=S_e^2$	$S_e^2/6(r-1)=\sigma_e^2$	
Total	2^3r-1	ΣYij^2-c.f.		

Complete confounding of 3^2 into 3^1 block size with AB is confounded.

S.V.	d.f.	S.S.	M.S.S.	F_c
Block	b-1	$\Sigma Bj^2/k - c.f. = S_b^2$	$S_b^2/(b-1)=\sigma_b^2$	σ_b^2/σ_e^2
Replication	r-1	$\Sigma Ri^2/t - c.f. = S_r^2$	$S_r^2/(r-1)=\sigma_r^2$	σ_r^2/σ_e^2
AB	2	$[AB]^2$	$[AB]^2/2=\sigma_{AB}^2$	σ_{AB}^2/σ_e^2
AB*replication	2*(r-1)	$S_b^2 - S_r^2 - [ABC]^2 = S_p^2$	$S_p^2/2*(r-1)=\sigma_p^2$	σ_p^2/σ_e^2
Treatment	6	$\Sigma Ti^2/r - c.f. = S_t^2$	$S_t^2/6 = \sigma_t^2$	σ_t^2/σ_e^2
A	2	$[A]^2$	$[A]^2/2=\sigma_A^2$	σ_A^2/σ_e^2
B	2	$[B]^2$	$[B]^2/2=\sigma_B^2$	σ_B^2/σ_e^2
AB^2	2	$[AB^2]^2$	$[AB^2]^2/2= \sigma_{(AB^2)}^2$	
Error	6(r-1)	By sub. $=S_e^2$	$S_e^2/6(r-1)=\sigma_e^2$	
Total	3^2r-1	$\Sigma Yij^2 - c.f.$		

Partial Confounding

- A confounded factorial experiment is called partial confounding if different main effects and interaction effects are confounded in different replications.

Example:- **2^3 Factorial Experiment**

Replication-1	
B-1	B-2
$a_0b_0c_0$	$a_0b_1c_0$
$a_0b_0c_1$	$a_0b_1c_1$
$a_1b_1c_0$	$a_1b_0c_0$
$a_1b_1c_1$	$a_1b_0c_1$
AB is confounded	

Replication-2	
B-3	B-4
$a_0b_0c_0$	$a_0b_0c_1$
$a_0b_1c_0$	$a_0b_1c_1$
$a_1b_0c_1$	$a_1b_0c_0$
$a_1b_1c_1$	$a_1b_1c_0$
AC is confounded	

Replication-3	
B-5	B-6
$a_0b_0c_0$	$a_0b_0c_1$
$a_0b_1c_1$	$a_0b_1c_0$
$a_1b_0c_1$	$A_1b_0c_0$
$a_1b_1c_0$	$a_1b_1c_1$
ABC is confounded	

Example:- 3^2 Factorial Experiment

Replication-1

B-1	B-2	B-3
a_0b_0	a_1b_0	a_2b_0
a_0b_1	a_1b_1	a_2b_1
a_0b_2	a_1b_2	a_2b_2

A is confounded

Replication-2

B-1	B-2	B-3
a_1b_0	a_0b_0	a_2b_0
a_1b_1	a_0b_1	a_2b_1
a_1b_2	a_0b_2	a_2b_2

A is confounded

Replication-3

B-1	B-2	B-3
a_2b_0	a_1b_0	a_0b_0
a_2b_1	a_1b_1	a_0b_1
a_2b_2	a_1b_2	a_0b_2

A is confounded

Replication-4

B-1	B-2	B-3
a_0b_0	a_1b_1	a_2b_2
a_0b_2	a_1b_0	a_2b_1
a_0b_1	a_1b_2	a_2b_0

A is confounded

Replication-5

B-1	B-2	B-3
a_0b_0	a_1b_1	a_2b_2
a_1b_0	a_2b_1	a_0b_2
a_2b_0	a_0b_1	a_1b_2

B is confounded

Advantage

- In partial confounding the advantage is that we do not loose the entire information on that confounded main effect or interaction effect.

- Main effect or interaction effect which is confounded in one replication is not confounded in another replication. So we can estimate fraction of information of that main or interaction effect from those replication in which they are not confounded.

Partial confounding of 2^3 into 2^2 block size. With AB,AC,BC,ABC confounded in r replication.

S.V.	d.f.	S.S.	M.S.S.	F_c
Block	b-1	$\Sigma Bj^2/k$ -c.f.$=S_b^2$	$S_b^2/(b-1)=\sigma_b^2$	σ_b^2/σ_e^2
Replication	r-1	$\Sigma Ri^2/t$ -c.f.$=S_r^2$	$S_r^2/(r-1)=\sigma_r^2$	σ_r^2/σ_e^2
Block within replication	(b-1)-(r-1) =b-r	$S_b^2-S_r^2=S_p^2$	$S_p^2/(b-r)=\sigma_p^2$	σ_p^2/σ_e^2
Treatment	7	$\Sigma Ti^2/r$ -c.f.$=S_t^2$	$S_t^2/6=\sigma_t^2$	σ_t^2/σ_e^2
A	1	$[A]^2$	$[A]^2/1=\sigma_A^2$	σ_A^2/σ_e^2
B	1	$[B]^2$	$[B]^2/1=\sigma_B^2$	σ_B^2/σ_e^2
AB’	1	$[AB']^2$	$[AB]^2/1=\sigma_{AB}^2$	σ_{AB}^2/σ_e^2
C	1	$[C]^2$	$[C]^2/1=\sigma_C^2$	σ_C^2/σ_e^2
AC’	1	$[AC']^2$	$[AC]^2/1=\sigma_{AC}^2$	σ_{AC}^2/σ_e^2
BC’	1	$[BC']^2$	$[BC]^2/1=\sigma_{BC}^2$	σ_{BC}^2/σ_e^2
ABC’	1	$[ABC']^2$	$[ABC]^2/1=\sigma_{ABC}^2$	$\sigma_{ABC}^2/\sigma_e^2$
Error	8r-b-7	By sub. $=S_e^2$	$S_e^2/6(r-1)=\sigma_e^2$	
Total	2^3r-1	ΣYij^2-c.f.		

Partial confounding of 3^2 into 3^1 block size with AB(2 times), AB^2(2 times) is confounded. In r replication.

S.V.	d.f.	S.S.	M.S.S.	F_c
Block	b-1	ΣBj^2 /k -c.f.=S_b^2	$S_b^2/(b-1)=\sigma_b^2$	σ_b^2/σ_e^2
Replication	r-1	ΣRi^2 /t -c.f.=S_r^2	$S_r^2/(r-1)=\sigma_r^2$	σ_r^2/σ_e^2
Block within replication	(b-1)-(r-1) =b-r	$S_b^2 - S_r^2 = S_p^2$	$S_p^2/(b-r)=\sigma_p^2$	σ_p^2/σ_e^2
Treatment	8	ΣTi^2 /r -c.f.=S_t^2	$S_t^2/6 = \sigma_t^2$	σ_t^2/σ_e^2
A	2	$[A]^2$	$[A]^2/2=\sigma_A^2$	σ_A^2/σ_e^2
B	2	$[B]^2$	$[B]^2/2=\sigma_B^2$	σ_B^2/σ_e^2
AB’	2	$[AB']^2$	$[AB]^2/2=\sigma_{AB}^2$	σ_{AB}^2/σ_e^2
$AB^{2'}$	2	$[AB^{2'}]^2$	$[AB^2]^2/2= \sigma_{(AB^2)}^2$	
Error	9r-b-8	By sub. =S_e^2	$S_e^2/6(r-1)=\sigma_e^2$	
Total	3^2r-1	ΣYij^2-c.f.		

- Note:-

we can estimate the value of confounded main or interaction effect as below.

Ex.

Suppose AB is confounded in r replication. Then it is not confounded in (r-2) replications.

Information on AB can be estimated with 2 / r.

s.s. of AB = $S_{AB}^2 * 2/r$

Balanced Confounding

- When all the effect of same order are confounded with equal no. of times, the confounding is said to be balanced confounding.

  or balanced partial confounding.

- In the effects of certain order are confounded an unequal no. of times in the replications ,this system of confounding is known as unbalanced partial confounding.

Advantage

- **In balanced confounding we can estimate the confounded interaction with same amount of information**
- **So it is best confounding.**

Disadvantage

- **It required large no. of replication as the no. of factor and level of factor increases. So experiment becomes costly and we may not recommended balanced confounding.**

Example:- 2^3 Factorial Experiment

Replication-1	
B-1	B-2
$a_0b_0c_0$	$a_0b_1c_0$
$a_0b_0c_1$	$a_0b_1c_1$
$a_1b_1c_0$	$a_1b_0c_0$
$a_1b_1c_1$	$a_1b_0c_1$
AB is confounded	

Replication-2	
B-3	B-4
$a_0b_0c_0$	$a_0b_0c_1$
$a_0b_1c_0$	$a_0b_1c_1$
$a_1b_0c_1$	$a_1b_0c_0$
$a_1b_1c_1$	$a_1b_1c_0$
AC is confounded	

Replication-3	
B-5	B-6
$a_0b_0c_0$	$a_0b_0c_1$
$a_0b_1c_1$	$a_0b_1c_0$
$a_1b_0c_0$	$a_1b_0c_1$
$a_1b_1c_1$	$a_1b_1c_0$
BC is confounded	

- It is a balanced partial confounding.
- Because AB, AC, BC have same order 1 and also they are equal no. of time replicated.

Example:- 2^3 Factorial Experiment

Replication-1	
B-1	B-2
$a_0b_0c_0$	$a_0b_1c_0$
$a_0b_0c_1$	$a_0b_1c_1$
$a_1b_1c_0$	$a_1b_0c_0$
$a_1b_1c_1$	$a_1b_0c_1$
AB is confounded	

Replication-2	
B-3	B-4
$a_0b_0c_0$	$a_0b_0c_1$
$a_0b_1c_0$	$a_0b_1c_1$
$a_1b_0c_1$	$a_1b_0c_0$
$a_1b_1c_1$	$a_1b_1c_0$
AC is confounded	

Replication-3	
B-5	B-6
$a_0b_0c_0$	$a_0b_0c_1$
$a_0b_1c_1$	$a_0b_1c_0$
$a_1b_0c_0$	$a_1b_0c_1$
$a_1b_1c_1$	$a_1b_1c_0$
BC is confounded	

Replication-4	
B-1	B-2
$a_0b_0c_0$	$a_0b_1c_0$
$a_0b_0c_1$	$a_0b_1c_1$
$a_1b_1c_0$	$a_1b_0c_0$
$a_1b_1c_1$	$a_1b_0c_1$
AB is confounded	

- It is a unbalanced partial confounding.
- Because AB, AC, BC have same order 1 but they are not replicated equal no. of times.

Example:- 3^2 Factorial Experiment

Replication-1		
B-1	B-2	B-3
a_0b_0 a_0b_1 a_0b_2	a_1b_0 a_1b_1 a_1b_2	a_2b_0 a_2b_1 a_2b_2
A is confounded		

Replication-2		
B-1	B-2	B-3
a_0b_0 a_1b_0 a_2b_0	a_1b_1 a_2b_1 a_0b_1	a_2b_2 a_0b_2 a_1b_2
B is confounded		

Replication-3		
B-1	B-2	B-3
a_0b_0 a_1b_2 a_2b_1	a_0b_1 a_1b_0 a_2b_2	a_0b_2 a_2b_0 a_1b_1
AB is confounded		

- It is a Partial confounding but not balanced confounding .

  Because A and B have 0 order but AB have 1 order.
- So it is a unbalanced partial confounding.

Balanced confounding of 2^3 into 2^2 block size. With AB,AC,BC confounded in r replication.

S.V.	d.f.	S.S.	M.S.S.	F_c
Block	b-1	$\Sigma Bj^2/k$ -c.f.$=S_b^2$	$S_b^2/(b-1)=\sigma_b^2$	σ_b^2/σ_e^2
Replication	r-1	$\Sigma Ri^2/t$ -c.f.$=S_r^2$	$S_r^2/(r-1)=\sigma_r^2$	σ_r^2/σ_e^2
Block within replication	(b-1)-(r-1) =b-r	$S_b^2-S_r^2=S_p^2$	$S_p^2/(b-r)=\sigma_p^2$	σ_p^2/σ_e^2
Treatment	7	$\Sigma Ti^2/r$ -c.f.$=S_t^2$	$S_t^2/6=\sigma_t^2$	σ_t^2/σ_e^2
A	1	$[A]^2$	$[A]^2/1=\sigma_A^2$	σ_A^2/σ_e^2
B	1	$[B]^2$	$[B]^2/1=\sigma_B^2$	σ_B^2/σ_e^2
AB'	1	$[AB']^2$	$[AB]^2/1=\sigma_{AB}^2$	σ_{AB}^2/σ_e^2
C	1	$[C]^2$	$[C]^2/1=\sigma_C^2$	σ_C^2/σ_e^2
AC'	1	$[AC']^2$	$[AC]^2/1=\sigma_{AC}^2$	σ_{AC}^2/σ_e^2
BC'	1	$[BC']^2$	$[BC]^2/1=\sigma_{BC}^2$	σ_{BC}^2/σ_e^2
ABC	1	$[ABC]^2$	$[ABC]^2/1=\sigma_{ABC}^2$	$\sigma_{ABC}^2/\sigma_e^2$
Error	8r-b-7	By sub. $=S_e^2$	$S_e^2/6(r-1)=\sigma_e^2$	
Total	2^3r-1	ΣYij^2-c.f.		