वियोज्य *DETACHABLE*

GENERAL ENGLISH

Time Allowed : Three Hours	Maximum Marks : 100

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions.

There are **SIX** questions and all are to be attempted.

The number of marks carried by a question / part is indicated against it.

Answers must be written in **ENGLISH** only.

Candidates are required to write clear, legible and concise answers and to adhere to word limits, wherever indicated. Failure to adhere to word limits may be penalized.

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

1. Write an essay on any *one* of the following topics in about 800 words : 30

(a) Accountability, not mere stability, essential for good governance

(b) Pros and Cons of online education and examinations

(c) Rural Sector's potential to contribute to the Indian economy

(d) Social Media attains credibility with objectivity and humanity

(e) Effective Centralised Enforcement Agency necessary to prevent harassment and violence against women

2. Make a précis of the following passage in about one-third of the original length, using your own words : 30

Education has not been of the kind that can help you towards happiness. It has not been of the kind that can make you capable of freedom. The aim of education in a democracy must be the same for all the people. Education must aim at making them all capable of freedom, for they are all to be free. Whatever education will achieve, this aim must be given to every person in proportion to his capacity to receive it. If education is necessary for one, it must be necessary for all. Only in this way can we hope to prevent the exploitation of one group by another. Only in this way can we develop a true community. The citizens must be able to understand one another. They must have a common culture. They must have the intellectual training needed to comprehend it and to communicate with those who share it.

Education for freedom consists in transmitting to the rising generation the civilization they have inherited, together with the techniques by which it may be understood. No man or woman is equipped to be a ruler without this education. As liberal education helps us to establish a free community, so does it to preserve it. As Thomas F. Woodlock has said in the *Wall Street Journal*, "Democracy rests ultimately upon 'public opinion' as its base. Public opinion follows upon free speech, free interchange of ideas, of judgements, of opinions; it is generated by these things. Men interchange these things by words..... Sound logic it was that put grammar, logic, and rhetoric as preliminary to geometry, arithmetic, music, and astronomy in the scheme of the seven liberal arts in an age which was notable for the clearness of its thinking and the exactitude of its expression. Both these things are absolutely necessary for the functioning of democracy as the best form of government. If we are going to 'educate for democracy', we had better find the right way to teach them." And the ability to read and write is the best defence against anti-democratic propaganda. Under the name of 'indoctrination of democracy' a host of lies is being propagated. When a person equipped with the liberal arts has critically studied the basic theoretical questions, he is proof against the seductions of the New Order. The reason why we may justly fear such propaganda today is that we are uneducated.

In addition to all the above merits, liberal education has yet another : it is the best vocational training. Vocational training of the usual sort trains hands. The ideal education for freedom is a liberal education in the liberal arts and the cultural heritage of the country. Since it is the ideal, we must aim at it for every citizen. Unless the citizen can master the liberal arts and the fundamental principles of a free society he cannot function as a member of a free community. He cannot help to keep the community as a free community.

If we cannot give liberal education to every citizen in proportion to his capacity to receive it we might as well give up our hopes achieving a democratic community. We cannot insist on free men and at the same time say that we cannot educate our population for freedom. We cannot insist on a community and at the same time say that it is impossible to strengthen the basis of communication among our people. Individual happiness, good citizenship, and the improvement of society all depend on our success in helping the people to achieve liberal education. These objectives do not depend on our success in training people to earn a living or in adjusting them to their environment. What the world needs, what the country must have, is free minds. This does not mean minds loaded with archaic information or equipped with obsolescent techniques. It means minds committed to the general good by good moral and intellectual habits. It means minds informed by principles derived from human experience through the ages, minds that will operate well no matter what waves of change beat upon them.

If we are to admit everybody to citizenship we must try to give everybody this kind of education. In so far as men are men they must have this education. Since in our society all men are politically equal, since all men are either rulers or the ruled, they must all be educated to be rulers and the ruled. If we cannot give them all this education, we may as well drop the pretence of democracy. We may as well admit that, though it was a good idea, it would not work. (770 words)

3. Write a paragraph in about 200 words on any *one* of the following expressions / statements : 10

(a) Every cloud has a silver lining.

(b) Handsome is as handsome does.

(c) Good fences make good neighbours.

(d) That's the way the cookie crumbles.

(e) If gold rusts, what shall the iron do.

4. Use the following words in sentences so as to bring out their meaning clearly. Do not change the form of the word. No credit will be given for a vague or ambiguous sentence. 2×5=10

(a) recondite

(b) largesse

(c) macabre

(d) zealot

(e) disingenuous

5. Use the following idioms and phrasal verbs in sentences so as to bring out their meaning clearly : 2×5=10

(a) infringe upon

(b) hear on the grapevine

(c) fend for

(d) caught in the crossfire

(e) go off at a tangent

6. Correct the following sentences without changing their meaning. Do not make unnecessary changes in the original sentence : 1×10=10

(a) My friend reached the airport on time to have a coffee before boarding.

(b) He was found guilty of stealing his employer.

(c) Sonu got a present from his father but he hasn't opened it.

(d) I am obliged to serve him, isn't it ?

(e) I would watch a football match than a cricket series.

(f) They were annoyed at not to be able to see the puppet show properly.

(g) The economics of the plan is worrying the investors.

(h) He got up at 5 O'clock not to be late for the interview.

(i) If they offered me the position, I shall decline it.

(j) I go to the work in afternoon.

GENERAL STUDIES

Time Allowed : Three Hours	Maximum Marks : 100

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **FOUR** questions and **all** are to be attempted.*

Candidate should attempt questions / parts as per the instructions given in the questions.

The number of marks carried by a question / part is indicated against it.

Candidates are required to write clear, legible and concise answers and to adhere to word limits, wherever indicated. Failure to adhere to word limits may be penalized. The answers must be written within the space provided in the Question-cum-Answer Booklet.

*Answers must be written in **ENGLISH** only.*

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Q1. Answer all of the following (in not more than 200 words each) : 5×5=25

(a) Do you think Kautilya's 'Arthashastra' provides guidelines to modern Indian polity or governance ? Discuss.

(b) Do you consider that the rise of British power in India, primarily owes to the decline of Mughal power or British naval and trade supremacy ? Give reasons.

(c) During 1940s Gandhiji said, "We do not seek independence out of British ruin." Explain the circumstances and outcome of the Gandhian policy in this context.

(d) Nehru preferred "non-linguistic states in the interests of a strong centre," which could not be enforced. Elucidate.

(e) Was 'Rama Setu', a 'man made' or 'nature made' monument ? Explain with reference to the recent researches and findings of the various commissions.

Q2. Answer all of the following (in not more than 200 words each) : ***5×5=25***

(a) Assess the climatic impact of the existence of the Himalayas in the north along with oceans and seas in three other directions of India.

(b) What is the significance of biodiversity conservation in the forests and water bodies of India ?

(c) Comment on the relative performance of the Indian states in terms of population growth and human development.

(d) "High pace of urbanisation in India, though transforming landscape, is a necessary process of development." Give your arguments.

(e) Account for the recent emphasis on rainwater harvesting and small check dams on rivulets in India in preference to big dams on the main rivers.

Q3. Answer all of the following (in not more than 200 words each) : ***5×5=25***

(a) "India is a fast transforming knowledge economy." Comment.

(b) Give an account on the growth of Information Technology and Biotechnology sectors and their impact on GDP and employment in India.

(c) Explain the term 'Demonetisation'. Assess the effects of demonetisation on the economy and on black money.

(d) Farmers' suicides in India are the result of agrarian crisis. Give reasons for it and suggest remedial measures.

(e) Critically examine the 'Universal Healthcare Scheme' as pronounced in the Union Budget, 2018.

Q4. Answer all of the following (in not more than 200 words each) : ***5×5=25***

(a) What is the significance of inner party democracy in political parties ? How could it be ensured in our political system ?

(b) What are the salient features of a welfare state ? Is India a welfare state ? Give reasons for your answer.

(c) Do you agree with the view that the elections to the Lok Sabha and the State Legislative Assemblies be held simultaneously ? Give reasons for your answer.

(d) Khap Panchayats have been in the news for functioning as conscience keepers of society, often delivering pronouncements on the legality of marriage. Discuss the stand taken by the Supreme Court in this regard.

(e) What are the economic constraints which hinder the success of the South Asian Association for Regional Cooperation (SAARC) ?

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T.B.C. : YLO-U-DTSS

Test Booklet Series

C

Serial

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TEST BOOKLET

STATISTICS

Paper I

Time Allowed : Two Hours *Maximum Marks : 200*

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET ***DOES NOT*** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. **Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series Code A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the Answer Sheet liable for rejection.**
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. ***DO NOT*** write ***anything else*** on the Test Booklet.
4. This Test Booklet contains **80** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ***ONLY ONE*** response for each item.
5. You have to mark all your responses ***ONLY*** on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. All items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator ***only the Answer Sheet***. You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**

    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.

    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third (0·33)** of the marks assigned to that question will be deducted as penalty.

    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.

    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

1. Consider the following statements :

   1. The weight of strawberry box is measured in interval scale.
   2. Number of students admitted in each IIM in the year 2007 is cross-sectional data.
   3. Bar code for items in departmental store is measured in nominal scale.

   Which of the statements given above are correct ?

   (a) 1 and 2 only
   (b) 1 and 3 only
   (c) 2 and 3 only
   (d) 1, 2 and 3

2. The minimum value of $Var(Y - aX)$, for all the values of 'a' is given by

   (a) $\rho^2 \frac{Var(Y)}{Var(X)}$
   (b) $\rho^2 \frac{Var(X)}{Var(Y)}$
   (c) $\rho^2 Var(Y)$
   (d) $(1 - \rho^2) Var(Y)$

3. For a certain frequency distribution, the numerical computation yields the following :

   Mean = 62, Median = 65, Coefficient of skewness = – 0·3

   The standard deviation is equal to

   (a) 10
   (b) 30
   (c) 90
   (d) 300

4. The data relating to smoking habits of father and son are given below :

   1. Father is a smoker and so is son : 50
   2. Father is a smoker, but not the son : 10
   3. Father is not a smoker, but son is a smoker : 10
   4. Father is not a smoker, son is not a smoker : 30

   The coefficient of association between smoking habits of father and son is

   (a) 0
   (b) $\frac{7}{8}$
   (c) $\frac{-7}{8}$
   (d) $\frac{1}{4}$

5. Consider the following statements :

   1. Two attributes A and B are said to be positively associated if the presence of one attribute A is accompanied by the presence of the other attribute B.
   2. Two attributes A and B are said to be negatively associated if the presence of an attribute A ensures the absence of the other attribute B or vice versa.
   3. Two attributes A and B are called independent in case the presence or absence of one attribute has linear relationship with the presence of the other attribute.

   Which of the statements given above are correct ?

   (a) 1, 2 and 3
   (b) 1 and 2 only
   (c) 1 and 3 only
   (d) 2 and 3 only

6. If the joint pdf of (X, Y) is

$$f(x, y) = \begin{cases} \frac{1}{3} x^2 e^{-y(1+x)}; & x \geq 0,\ y \geq 0 \\ 0; & \text{otherwise} \end{cases}$$

then the regression equation of Y on X is

(a) $y = \frac{1}{1+x}$

(b) $y = \frac{x^2}{1+x}$

(c) $y = 1 + x$

(d) $y = \frac{x^2}{2(1+x)^2}$

7. Let $(X, Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$.

Consider the following statements :

1. X and Y are independent if and only if $\rho = 0$.
2. Every linear combination of X and Y is a normal variate.
3. The regression equation of Y on X and the regression equation of X on Y are linear and homoscedastic.

Which of the statements given above are correct ?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

8. The correlation coefficient between the marks obtained in Mathematics and Statistics in First Semester Examination for a given set of 60 students is 0·65. After re-evaluation the marks in Mathematics of all the 60 students increased by 3 marks. The coefficient of correlation between the original marks in Statistics and the revised marks in Mathematics will be

(a) 0·60

(b) 0·65

(c) 0·70

(d) Almost 1

9. If the correlation coefficient of zero order in a set of 3 variates were equal to ρ each, then the multiple correlation $R^2_{1\cdot 23}$ is equal to

(a) ρ

(b) $\frac{2\rho^2}{1+\rho}$

(c) $\frac{\rho^2}{1+\rho}$

(d) + 1

10. If r is the coefficient of correlation for a sample of N independent observations from a bivariate normal population with population coefficient of correlation zero, then $E(1 - r^2)^{-1}$ is

(a) $\frac{(N-3)}{(N-4)}$

(b) $\frac{2(N-3)}{(N-4)}$

(c) 0

(d) 1

11. Let A_1, A_2, A_3 and A_4 be independent events such that $P(A_1) = \frac{1}{2}$, $P(A_2) = \frac{1}{4}$, $P(A_3) = \frac{1}{8}$ and $P(A_4) = \frac{1}{16}$.
What is $P(A_1 \cup A_2 \cup A_3 \cup A_4)$ equal to ?

(a) $\frac{15}{16}$

(b) $\frac{1023}{1024}$

(c) $\frac{709}{1024}$

(d) $\frac{315}{1024}$

12. A lot of 15 mobiles contains 4 defective mobiles. The mobiles are taken out one by one at random and examined. The examined ones are not put back. The probability that the ninth mobile examined is the last defective is

(a) $\frac{56}{195}$

(b) $\frac{8}{195}$

(c) $\frac{1}{7}$

(d) $\frac{24}{65}$

13. Let X be a random variable having probability density function

$$f(x) = \begin{cases} \frac{x}{2}, & 0 < x \leq 1 \\ \frac{1}{2}, & 1 < x \leq 2 \\ \frac{3-x}{3}, & 2 < x \leq 3 \end{cases}$$

What is $P(1{\cdot}5 < X \leq 2{\cdot}5 \mid X > 1)$ equal to ?

(a) $\frac{3}{8}$

(b) $\frac{1}{2}$

(c) $\frac{5}{8}$

(d) $\frac{1}{4}$

14. The first of three urns contains 7 white and 10 black balls; the second urn contains 5 white and 12 black balls; and the third urn contains 17 white balls only. A person chooses an urn at random and draws a ball from it. The ball is white. What are the probabilities that the ball drawn is from the first, second and third urns respectively ?

(a) $\frac{7}{29}, \frac{6}{29}, \frac{18}{29}$

(b) $\frac{7}{29}, \frac{5}{29}, \frac{20}{29}$

(c) $\frac{7}{29}, \frac{5}{29}, \frac{17}{29}$

(d) None of the above

15. Let X and Y be jointly distributed with pdf

$$f(x, y) = \begin{cases} \frac{1+xy}{y}, & |x|<1, |y|<1 \\ 0, & \text{otherwise} \end{cases}$$

Which of the following statements is/are correct ?

1. X and Y are independent.
2. X^2 and Y^2 are independent.

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

16. Let X be a random variable with pdf

$$f(x) = \begin{cases} \frac{2}{x^3}, & x \geq 1 \\ 0, & x < 1 \end{cases}$$

Which one of the following statements is correct ?

(a) Both mean and variance exist.

(b) Mean exists but variance does not exist.

(c) Both mean and variance do not exist.

(d) Variance exists but mean does not exist.

17. Let X be a random variable following Binomial distribution with parameters n = 13 and p = 0·3. Consider the following statements :

1. Mode of the distribution of X is 3.
2. Distribution is negatively skewed.

Which of the above statement(s) is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

18. If X follows Normal distribution with mean 4 and variance 100, then the distribution of $Y = \frac{1}{2}\left(\frac{X-4}{10}\right)^2$ is

(a) Exponential with mean 4

(b) Chi square with one degree of freedom

(c) Gamma distribution with parameter 1/2

(d) Normal with mean 4 and variance 10

19. Let X and Y be independent gamma $G(\alpha_1, \beta)$ and $G(\alpha_2, \beta)$ random variables respectively. Then $X|(X+Y)$ is distributed as

(a) $G(\alpha_1 + \alpha_2, \beta)$

(b) $\beta_1(\alpha_1, \alpha_2)$

(c) $U(0, 1)$

(d) $G(\alpha_1, \alpha_2)$

20. For exponential distribution $f(x) = \theta e^{-\theta x}$; $x > 0$, consider the following statements :

1. Variance is greater than mean if $0 < \theta < 1$.
2. Variance is equal to mean if $\theta = 1$.
3. Variance is less than mean if $\theta > 1$.

Which of the above statements are correct ?

(a) 1 and 3 only

(b) 2 and 3 only

(c) 1 and 2 only

(d) 1, 2 and 3

21. If the relation $ax_1 + bx_2 + cx_3 = 0$ holds for all sets of values of x_1, x_2 and x_3, then the partial correlation coefficient $r_{12\cdot3}$ is

(a) 0

(b) 1

(c) -1

(d) 0·5

22. If X_1, X_2, X_3 and X_4 are four uncorrelated random variables each with variance σ^2, then what is the value of the correlation coefficient between U and V where $U = X_1 + X_2 + X_3$ and $V = X_1 + X_2 + X_4$?

(a) 0

(b) 1

(c) $\frac{1}{3}$

(d) $\frac{2}{3}$

23. Let $(X_1, X_2, X_3, ..., X_n)$ be a random sample of size n from a uniform distribution U[– 3, 3]. The large sample distribution of $\sqrt{n}\,\bar{X}$ is

(a) N(0, 1)

(b) N(0, 3)

(c) $U\left[-\frac{3}{\sqrt{n}}, \frac{3}{\sqrt{n}}\right]$

(d) $U\left[-\frac{3}{n}, \frac{3}{n}\right]$

24. A normal population has a mean 0·1 and standard deviation 2·1. What is the probability that the mean of a sample of size 900 will be negative ?
[Given $P(0 < Z < 1{\cdot}43) = 0{\cdot}4236$]

(a) 0·0764

(b) 0·6408

(c) 0·6074

(d) 0·7046

25. Suppose that five random variables $(X_1, X_2, X_3, X_4, X_5)$ are independent and each has standard normal distribution. The random variable $\frac{C(X_1 + X_2)}{(X_3^2 + X_4^2 + X_5^2)^{1/2}}$ will have a t-distribution if the constant C is equal to

(a) $\frac{\sqrt{3}}{2}$

(b) $\sqrt{\frac{3}{2}}$

(c) $\frac{3}{2}$

(d) $\sqrt{\frac{2}{3}}$

26. If X is an F(m, n) random variable, where $m > 2$ and $n > 2$, then $E(X)E\left(\frac{1}{X}\right)$ equals

(a) $\frac{n(n-2)}{m(m-2)}$

(b) $\frac{m(m-2)}{n(n-2)}$

(c) $\frac{mn}{(m-2)(n-2)}$

(d) $\frac{m(n-2)}{n(m-2)}$

27. Which one of the following non-parametric tests is analogous to chi-square test of goodness of fit ?

(a) Mann-Whitney test

(b) Kolmogorov-Smirnov test

(c) Wilcoxon test

(d) Median test

28. Consider the following statements :

1. Normal distribution is a particular case of chi-square distribution with 1 degree of freedom.
2. All moments of order less than n of t-distribution with n degrees of freedom exist.
3. If a statistic t follows Student's t-distribution with n degrees of freedom, then t^2 follows F-distribution with (1, n) degrees of freedom.

Which of the statement(s) given above is/are correct ?

(a) 1 only
(b) 1 and 2 only
(c) 2 and 3 only
(d) 1, 2 and 3

29. For a random sample of size 2 from $N(0, \sigma^2)$ population, what is the value of $E[X_{(1)}]$, where $X_{(1)}$ is first order statistic ?

(a) $\frac{2\sigma}{\sqrt{\pi}}$
(b) $\frac{\sigma}{\pi}$
(c) $-\frac{\sigma}{\sqrt{\pi}}$
(d) $-\frac{\sigma}{\sqrt{2\pi}}$

30. If X_i, i = 1, 2, 3, ..., n are independent and identically distributed random variables with cdf $F(x) = x$, $0 \le x \le 1$, then the pdf g(r) of the range $R = X_{(n)} - X_{(1)}$ is

(a) $n(n-1)r^{n-2}(1-r)$
(b) $n(n-1)r(1-r)^{n-2}$
(c) nr^{n-1}
(d) $(n-1)r^{n-2}$

31. A rod of length $2l$ is broken into two pieces at random. The mean and variance of the length of the shorter of the two pieces are respectively

(a) $\frac{l}{2}, \frac{l^2}{3}$
(b) $l, \frac{l^2}{3}$
(c) $\frac{l}{2}, \frac{l^2}{12}$
(d) $l, \frac{l^2}{12}$

32. The joint distribution function of X and Y is given by

$$F(x, y) = \begin{cases} (1-e^{-x})(1-e^{-y}); & \text{for } x > 0,\ y > 0 \\ 0; & \text{otherwise} \end{cases}$$

What is the value of $P(|X - Y| \le 10)$?

(a) $(1-e^{-10})$
(b) $\frac{1}{2}(1-e^{-10})$
(c) $\left(1 - \frac{e^{-10}}{2}\right)$
(d) None of the above

33. The joint density function of X and Y is given by

$$f(x, y) = \begin{cases} 2e^{-x}e^{-2y}; & 0 < x < \infty, 0 < y < \infty \\ 0; & \text{otherwise} \end{cases}$$

What is the value of $P(X < Y)$?

(a) 1/3
(b) 1/2
(c) 1/4
(d) 1/6

34. The joint pdf of X and Y is given by

$$f(x, y) = \begin{cases} \frac{1}{3}(x + y); & 0 < x < 1,\ 0 < y < 2 \\ 0; & \text{elsewhere} \end{cases}$$

What is the value of E [3X(X + 2Y)] ?

(a) $\frac{9}{2}$

(b) $\frac{29}{6}$

(c) $\frac{31}{6}$

(d) $\frac{11}{2}$

35. The joint probability distribution of X and Y is given by

Y \ X	– 1	0	1
– 1	1/6	1/3	1/6
0	0	0	0
1	1/6	0	1/6

Consider the following statements :

1. X and Y are independent random variables.
2. Cov(X, Y) = 0.

Which of the statement(s) given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

36. If the pdf of X is given by

$$f(x) = \begin{cases} 1 + x, & -1 < x \le 0 \\ 1 - x, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

The random variables are defined as $U = X$ and $V = X^2$.

Consider the following statements :

1. Cov(U, V) = 0.
2. U and V are independent random variables.

Which of the above statement(s) is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

37. Suppose X and Y are two independent normal random variables with means 10, 5 and variances 9, 4 respectively. Define $Z = 4X - 5Y$. The moment generating function of Z [i.e. $M_Z(t)$] is given by

(a) $\exp[15t + 22t^2]$

(b) $\exp[15t - 22t^2]$

(c) $\exp[15t + 122t^2]$

(d) $\exp[15t - 122t^2]$

38. A fair die is thrown repeatedly until 3 or 4 appears. Let X denote the number of throws required. The probability generating function of X when $|s| < 3/2$ is given by the expression

(a) $P(s) = \frac{2s}{3-s}$

(b) $P(s) = \frac{s}{3-2s}$

(c) $P(s) = \frac{3-s}{2s}$

(d) $P(s) = \frac{3-2s}{s}$

39. The number of automobiles sold weekly at a certain dealership is a random variable with expected value 16 and variance 9. The probability that next week's sales are between 10 and 22 is at least

(a) 0·50

(b) 0·56

(c) 0·75

(d) 0·95

40. The pdf of random variable X is given by

$$f(x) = \begin{cases} \frac{kx^3}{(1+2x)^6}; & x > 0 \\ 0; & \text{otherwise} \end{cases}$$

The distribution of random variable $Y = \frac{2X}{1+2X}$ is β distribution of first kind with parameters :

(a) $m = 4, n = 2$

(b) $m = 5, n = 1$

(c) $m = 3, n = 3$

(d) $m = 5, n = 2$

41. Consider the following statements :

1. ALU is a component of Control Unit.
2. CPU is a component of Control Unit.
3. Control Unit is not a component of ALU.
4. ALU is a component of CPU.

Which of the above statement(s) is/are correct ?

(a) 4 only

(b) 3 and 4 only

(c) 1 and 3 only

(d) 2, 3 and 4

42. Conversion of octal number 325·12 into its decimal number will be

(a) 532·123

(b) 213·15625

(c) 213·56121

(d) 124·123

43. A system consists of 1024 memory locations and each location can store 8 bits. How many number of address lines are required to address ?

(a) 8

(b) 10

(c) 16

(d) 1024

44. The core component of UNIX is

(a) Command shell

(b) Kernel

(c) Directories and programs

(d) None of the above

45. Consider signed magnitude, 1's complement and 2's complement representation of negative integers in computers. Generally 2's complement is used because

1. More numbers can be stored in 2's complement
2. Arithmetic operations can be performed faster
3. It is easy to represent negative numbers in 2's complement

Which of the above are correct ?

(a) 1 and 3 only

(b) 2 and 3 only

(c) 1 and 2 only

(d) 1, 2 and 3

46. Which one of the following is the correct normalised form for 11·234 ?

(a) 11·234

(b) ·11234E + 2

(c) ·011234E + 1

(d) 1·1234E × 10

47. The maximum decimal integer number that can be stored in memory of 8 bit word processor computer is

(a) $(128)_{10}$

(b) $(127)_{10}$

(c) $(129)_{10}$

(d) $(130)_{10}$

48. Which of the following are ***not*** bitwise operators ?

1. &&
2. &
3. >>
4. !
5. %

Select the correct answer using the code given below :

(a) 2 and 3 only

(b) 1, 3, 4 and 5

(c) 1, 2 and 5

(d) 1, 4 and 5

49. Which type of memory is the auxiliary memory ?

(a) SRAM

(b) Magnetic tape

(c) Flash ROM

(d) Cache memory

50. Which one of the following statements is correct ?

(a) Compiler translates and executes instructions simultaneously.

(b) Interpreter translates and executes the instructions simultaneously.

(c) Interpreter can only translate machine language programs.

(d) None of the above

51. Given the data f(1) = 12, f(2) = 40, f(3) = 90, f(4) = 168, f(5) = 280, f(6) = 432. The degree of the polynomial f(x) is at least

(a) 2

(b) 3

(c) 4

(d) 5

52. The polynomial function $f(x) = x^3$ can be represented as factorial polynomial $f(x) = Ax^{(3)} + Bx^{(2)} + Cx + D$ when interval of differencing is h = 2. The values of A, B, C and D are respectively

(a) 1, 2, 6 and 0

(b) 1, 6, 4 and 0

(c) 2, 4, 6 and 0

(d) 2, 2, 6 and 1

53. The n^{th} divided difference of an n^{th} degree polynomial is

(a) A variable

(b) A constant

(c) Zero

(d) None of the above

54. The Runge-Kutta method of order four is used to solve the initial value problem $\frac{dy}{dx} = f(x)$, $y(0) = 0$ with step size h. Then the solution at $x = h$ is given by

(a) $y(h) = \frac{h}{6}\left[3f(0) + f\left(\frac{h}{2}\right) + 3f(h)\right]$

(b) $y(h) = \frac{h}{6}[f(0) + 5f(h)]$

(c) $y(h) = \frac{h}{6}\left[f(0) + 3f\left(\frac{h}{2}\right) + 2f(h)\right]$

(d) $y(h) = \frac{h}{6}\left[f(0) + 4f\left(\frac{h}{2}\right) + f(h)\right]$

55. The images f(n) of $n \in \{1, 2, 3, \ldots, 99\} \setminus \{50\}$ are given. If you have to estimate the image of 50 using forward difference table, then your hypothesis is

(a) f is a polynomial of degree 50

(b) f is a polynomial of degree 99

(c) f is a polynomial of degree 98

(d) f is a polynomial of degree not more than 97

56. Consider the following table :

x	1	2	3	4	5
f(x)	2	5	7	α	32

The missing term α in the above table is

(a) 14

(b) 15

(c) 16

(d) 17

57. If $u_{10} = 9$, $u_{20} = 39$, $u_{30} = 74$, $u_{40} = 116$ and $u_{50} = 167$, then the value of $E^{3/2}(u_{10})$ is

(a) 35·75

(b) 45·75

(c) 55·75

(d) 65·75

58. The value of k that satisfies the equation $\Delta^2(1 + k\Delta^3)0^5 = 630$ is

(a) 4

(b) 5

(c) 6

(d) 7

59. Consider the following statements (Given that interval of differencing is 1) :

1. $(E - 2)^2 (x2^x) = 0$.
2. $(E - 3)^3 (x3^x) \neq 0$.

Which of the statement(s) given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

60. The following table gives the population (in lakhs) of a town in certain years.

Year	*Population (in lakhs)*
1971	0·9
1981	1·2
1991	1·5
2001	1·9
2011	2·5

The population P_{2006} (in lakhs) in 2006 is estimated by Newton's forward difference formula. Which one of the following is correct ?

(a) $P_{2006} < 2$

(b) $2 < P_{2006} < 2{\cdot}1$

(c) $2{\cdot}1 < P_{2006} < 2{\cdot}2$

(d) $2{\cdot}2 < P_{2006} < 2{\cdot}3$

61. Consider the following code :

```
While (x! = y)
{
        if (x > y)
        x = x – y
        else
        y = y – x
}
```

What does the above code do ?

(a) It computes x mod y using repeated subtraction

(b) It computes x ÷ y using repeated subtraction

(c) It computes the greatest common divisor of x and y

(d) It computes the least common multiple of x and y

62. Consider the following statements :

1. Array is a collection of data elements of same type.
2. Strings are character arrays in which each character is stored using one byte in memory.
3. Array can store different types of elements.
4. Strings can be data array also.

Which of the above statements are correct ?

(a) 1 and 3 only

(b) 1 and 2 only

(c) 3 and 4 only

(d) 1, 2, 3 and 4

63. Consider the following statements :

1. Weighted 4-bit BCD code is known as 1246 weighted code.
2. Octal system is also known as base-8 system.
3. Octal system is known as base-2 system.
4. Weighted 4-bit BCD code is known as 8421 weighted code.

Which of the above statement(s) is/are correct ?

(a) 2 only

(b) 3 and 4 only

(c) 1 and 2 only

(d) 2 and 4 only

64. The maximum permissible integer in a computer with n-bit word processor and one word per integer is equal to

(a) $2^n - 1$

(b) $2^{n-1} - 1$

(c) $2^{n-1} + 1$

(d) $2^n + 1$

65. Which operators follow right to left order of precedence ?

1. Unary operator
2. Assignment operator
3. Logical operator
4. Bit manipulation operator

Select the correct answer using the code given below :

(a) 1 and 2 only

(b) 1, 2 and 4

(c) 3 and 4 only

(d) 1 only

66. Compiler can diagnose

(a) Grammatical errors only

(b) Logical errors only

(c) Both grammatical and logical errors

(d) Neither grammatical errors nor logical errors

67. A software program that cannot be changed easily and is stored in a ROM is known as

(a) Hardware

(b) Firmware

(c) Linker

(d) Editor

68. A computer program can run successfully even if there is

(a) Syntax error

(b) Logical error

(c) Run-time error

(d) None of the above

69. A disc storage has 16 data-recording surfaces. Number of tracks per surface is 256 and number of sectors per track is 16. Bytes per sector are 512. What is storage capacity of the disc system ?

(a) 3325652 bytes

(b) 3256662 bytes

(c) 33554432 bytes

(d) 335444332 bytes

70. Which one of the following is a low-level programming language ?

(a) FORTRAN

(b) Machine language

(c) COBOL

(d) C

71. Consider the following table :

x	2	3	α	5
f(x)	4	5	16	25

The missing value α of the argument in the above table is estimated by Lagrange interpolation formula. Which one of the following is correct ?

(a) $3{\cdot}5 < \alpha < 4$

(b) $\alpha = 4$

(c) $4 < \alpha < 4{\cdot}2$

(d) $4{\cdot}2 < \alpha < 4{\cdot}5$

72. If δ and μ are the central difference and average operator respectively, then $\delta[f(x)g(x)]$is equal to

(a) $\mu f(x)\mu g(x) + \delta f(x)\delta g(x)$

(b) $\delta f(x)\delta g(x) - \mu f(x)\mu g(x)$

(c) $\mu f(x)\delta g(x) - \mu g(x)\delta f(x)$

(d) $\mu f(x)\delta g(x) + \mu g(x)\delta f(x)$

73. If $f(x) = \frac{1}{x}$, then the third divided difference $\underset{b,c,d}{\overset{3}{\Delta}} f(a)$ is equal to

(a) $\frac{1}{abcd}$

(b) $-\frac{1}{abcd}$

(c) $\frac{a-d}{ad}$

(d) $\frac{1}{ad} - \frac{1}{bc}$

74. The equality $f(n^2) = n^3$ holds for n = 1, 2, 3. Then f(3) is given by Lagrange's interpolation as

(a) 4·9

(b) 5·1

(c) 5·3

(d) 5·5

75. The central difference δ is equivalent to which of the following ?

1. $2\sinh\left(\frac{hD}{2}\right)$

2. $E^{-1/2}\Delta$

3. $\Delta(1+\Delta)^{-1/2}$

Select the correct answer using the code given below :

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

76. Consider the following statements regarding quadrature formulae :

1. Geometrically Simpson's one-third rule implies that we have replaced the graph of the given function by n/2 arcs of second degree polynomials.
2. Weddle's rule gives an exact result when f(x) is a polynomial of degree 6 or less.

Which of the statement(s) given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

77. If an integral is computed using Simpson's three-eighth rule dividing the range into three equal parts, then the value of the integral $\int_0^{\frac{1}{2}} \sin \pi x \, dx$ is

(a) $\frac{3(1+\sqrt{2})}{32}$

(b) $\frac{5+3\sqrt{3}}{32}$

(c) $\frac{3+5\sqrt{3}}{32}$

(d) $\frac{3\sqrt{3}}{8}$

78. The formula $\alpha f\left(-\frac{1}{2}\right) + \beta f(0) + \gamma f\left(\frac{1}{2}\right)$ which approximates the integral $\int_{-1}^{1} f(x)\,dx$ is exact for every polynomial of degree less than 3. Then which one of the following is correct ?

(a) $\alpha = \gamma = \frac{4}{3}, \beta = -\frac{2}{3}$

(b) $\alpha = \gamma = \frac{1}{3}, \beta = 0$

(c) $\alpha = \gamma = \frac{4}{3}, \beta = 1$

(d) $\alpha = \beta = \gamma$

79. If the integral is computed using Trapezoidal rule by dividing the range into ten equal parts, then the value of $\int_0^{10} 2^x \, dx$ belongs to

(a) (1400, 1450)

(b) (1450, 1500)

(c) (1500, 1550)

(d) (1550, 1600)

80. For the initial value problem $\frac{dy}{dx} = \frac{y}{x^2}$, $y(1) = 1$, approximations y_1 and y_2 are computed using Picard's method with $y_0 = y(1)$. Then the constant terms of $y_1(x)$ and $y_2(x)$ are respectively

(a) 1, 1

(b) 2, 2

(c) $2, \frac{3}{2}$

(d) None of the above

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TEST BOOKLET

C

STATISTICS

PAPER—II

Time Allowed : Two Hours **Maximum Marks : 200**

INSTRUCTIONS

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2. **Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the Answer Sheet liable for rejection.**
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9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**

    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.

    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.

    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.

    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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1. A random sample $(X_1, X_2, X_3, ..., X_n)$ is drawn from a Poisson distribution with parameter λ. Which one of the following is an unbiased estimator of λ^2?

(a) $\frac{1}{n}\sum_{i=1}^{n} x_i$

(b) $\frac{1}{n}\sum_{i=1}^{n} x_i^2$

(c) $\frac{1}{n}\sum_{i=1}^{n} x_i(x_i - 1)$

(d) $\frac{1}{n}\sum_{i=1}^{n} x_i(x_i + 1)$

2. Let (X_1, X_2, X_3) be a random sample from binomial $(1, p)$. Which one of the following statistics is ***not*** sufficient for p?

(a) $X_1 + X_2 + X_3$

(b) $(X_1, X_2 + X_3)$

(c) $(X_1 + X_2, X_3)$

(d) $X_1 - X_2 + X_3$

3. Suppose a random sample of siz was selected from a population the p.d.f.

$$f_\theta(x) = \frac{2}{\theta^2}(\theta - x);\ 0 < x < \theta$$

The maximum likelihood estimator will be

(a) $\frac{(x_1 + x_2) + \sqrt{(x_1 - x_2)^2 + 4x_1x_2}}{4}$

(b) $\frac{(x_1 + x_2) + \sqrt{9(x_1 - x_2)^2 + 4x_1x_2}}{4}$

(c) $\frac{3(x_1 + x_2) + \sqrt{(x_1 - x_2)^2 + 4x_1x_2}}{4}$

(d) $\frac{3(x_1 + x_2) + \sqrt{9(x_1 - x_2)^2 + 4x_1x_2}}{4}$

4. If T is unbiased for θ and $\text{var}(T) \to 0$ the sample size tends to ∞, then T consistent for θ. This result follows fr

(a) central limit theorem

(b) Cramer-Rao inequality

(c) Rao-Blackwell theorem

(d) Tchebychev's inequality

5. If one observation from Bernoulli distribution with parameter $p \in \left[\frac{1}{3}, \frac{2}{3}\right]$ is obtained, then the MLE of p is

(a) x

(b) $(2x + 1)/3$

(c) $(x - 1)/3$

(d) $(x + 1)/3$

6. Let $X_i \sim N(\mu, \sigma^2)$, where $i = 1, 2, 3, ..., n$ and $\Theta = \{(\mu, \sigma^2) : -\infty < \mu < \infty, \sigma^2 > 0\}$ such that μ is known. Then the MLE of σ^2 is

(a) $\dfrac{\sum_{i=1}^{n}(X_i - \mu)^2}{n}$

(b) $\dfrac{\sum_{i=1}^{n-1}(X_i - \mu)^2}{n-1}$

(c) $\dfrac{n-1}{n^2}\sum_{i=1}^{n}(X_i - \mu)^2$

(d) $\dfrac{n-1}{n}\sum_{i=1}^{n}(X_i - \mu)^2$

7. Let $(X_1, X_2, X_3, ..., X_n)$ be a random sample of size n from uniform distribution with the p.d.f.

$$f(x) = \begin{cases} \dfrac{1}{\theta_2 - \theta_1}, & \theta_1 < x < \theta_2 \\ 0, & \text{otherwise} \end{cases}$$

Let T_1 and T_2 be the estimates of θ_1 and θ_2 respectively obtained by method of moments. Which one of the following is correct?

(a) $T_1 = \bar{X} + \sqrt{\dfrac{3}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2}$ and

$$T_2 = \bar{X} - \sqrt{\frac{3}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2}$$

(b) $T_1 = \bar{X} - \sqrt{\dfrac{3}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2}$ and

$$T_2 = \bar{X} + \sqrt{\frac{3}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2}$$

(c) $T_1 = \bar{X} + \sqrt{\dfrac{3}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2}$ and

$$T_2 = \bar{X} + \sqrt{\frac{3}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2}$$

(d) $T_1 = \bar{X} - \sqrt{\dfrac{3}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2}$ and

$$T_2 = \bar{X} - \sqrt{\frac{3}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2}$$

8. Let $(X_1, X_2, X_3, ..., X_n)$ be i.i.d. random variables with the density function

$$f(x, \mu) = e^{-(x-\mu)}, \; x \geq \mu$$

The MLE of μ is given by

(a) $X_{(1)}$

(b) $X_{(n)}$

(c) $\dfrac{X_{(1)} + X_{(n)}}{2}$

(d) None of the above

9. If T is an MLE of θ and $\phi(\theta)$ is one-to-one function of θ, then $\phi(t)$ is the MLE of $\phi(\theta)$. This property is known as

(a) invariance property of MLE

(b) asymptotic property of MLE

(c) consistency property of MLE

(d) regularity conditions of MLE

10. Which of the following statements are correct?

1. MLEs are consistent.
2. MLEs are always unbiased.
3. MLEs follow asymptotically normal distribution.
4. MLE need not be a unique estimator.

Select the correct answer using the code given below.

(a) 1 and 3 only

(b) 1, 3 and 4

(c) 1, 2 and 3

(d) 2, 3 and 4

11. Consider the model

$$Y_1 = \beta_1 + \beta_2 + e_1$$
$$Y_2 = \beta_1 + \beta_3 + e_2$$
$$Y_3 = \beta_1 + \beta_2 + e_3$$

Then $\lambda_1\beta_1 + \lambda_2\beta_2 + \lambda_3\beta_3$ is estimable if and only if

(a) $\lambda_1 + \lambda_3 = \lambda_2$

(b) $\lambda_1 + \lambda_2 = \lambda_3$

(c) $\lambda_2 + \lambda_3 = \lambda_1$

(d) $\lambda_1 + \lambda_2 + \lambda_3 = C$

12. Consider the model

$$y_i = \beta_0 + \beta_1 x_i + e_i \; (i = 1, 2, 3)$$

where $x_1 = -1$, $x_2 = 0$ and $x_3 = 1$. The best linear unbiased estimate of β_0 and β_1 is

(a) $\left(\overline{y}, \dfrac{y_3 - y_1}{2}\right)$

(b) $\left(\dfrac{y_2 - y_1}{2}, \dfrac{y_3 - y_1}{2}\right)$

(c) $\left(\overline{y}, \dfrac{y_2 - y_1}{2}\right)$

(d) $\left(\dfrac{y_1 + y_2}{2}, \overline{y}\right)$

where $\overline{y} = \dfrac{y_1 + y_2 + y_3}{3}$

13. Consider the following statements :

1. If multicollinearity exists, $(X'X)^{-1}$ cannot be computed.
2. The method of least squares is a statistical method.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

14. Consider the following statements :

1. The g-inverse of a matrix, if it exists, is unique.
2. The problem of multicollinearity can be tackled by increasing sample size.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

15. In the context of the linear model $\mathbf{Y = X\beta + \varepsilon}$, consider the following statements :

1. If $\mathbf{X}$ is a full rank matrix, then all linear parametric functions are estimable.
2. Each estimable linear parametric function is of the form $\mathbf{c'X\beta}$ for some vector $\mathbf{c}$.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

16. For a two-way ANOVA with 6 rows, 4 columns and 3 observations per cell, the degrees of freedom for interaction and residual are respectively

(a) 15 and 45

(b) 15 and 48

(c) 24 and 45

(d) 24 and 48

17. As the variability due to chance in an analysis of variance decreases, the F-statistic in the ANOVA

(a) will always increase

(b) will always decrease

(c) will remain constant

(d) requires additional information to claim any change

18. When k population means in an ANOVA are truly different from each other, it is likely that the average error deviation, for a significant F-statistic,

(a) is relatively large compared to the average treatment deviations

(b) is relatively small compared to the average treatment deviations

(c) is equal to the average treatment deviations

(d) differs significantly between at least two of the populations

19. Independent observations are drawn from the model

$$y_1 = \beta_0 + \beta_1 x_i + \mu_i;\ i = 1, 2, 3, \ldots, 10$$

It is given that the errors in the model are independent with mean 0 and variance unity. It is also given that the sum of regressors in the model equals 50 and their sum of squares is 260. The possible covariance between the least squares estimates of β_0 and β_1 would be

(a) 0

(b) 5

(c) $-0{\cdot}5$

(d) $0{\cdot}5$

20. Let

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 0 & 1 \\ 4 & 2 & 3 \end{pmatrix}$$

Consider the following matrices :

$$P = \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & -\frac{3}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & \frac{1}{2} & 0 \end{pmatrix}$$

and $$R = \begin{pmatrix} 0 & \frac{1}{2} & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Which one of the following is correct?

(a) P only does not give generalized inverse of A.

(b) Q only does not give generalized inverse of A.

(c) P and R only do not give generalized inverse of A.

(d) None of the options is correct

21. In sampling from a normal population $N(\theta, 1)$ with $\bar{X}$ being sample mean, which of the following classes describes all MVB estimators of θ?

(a) All estimators of the type $A\bar{X} + B$, where A and B are given constants

(b) $\bar{X}$ only

(c) All the estimators of $f(\bar{X})$, where f is a real-valued function

(d) No estimator attains MVB in this case

22. Let 5, 2, 3, 4, 3, 2, 4, 5, 3, 2 be a sample taken from a normal population with mean 0 and variance σ^2. The minimum variance bound estimator for σ^2 is

(a) 33

(b) 3·3

(c) 1·21

(d) 12·1

23. If $(X_1, X_2, X_3, ..., X_n)$ is a random sample from $N(1, \sigma^2)$, then which one of the following is unbiased for σ^2 and has variance equal to Cramer-Rao bound?

(a) $\frac{1}{n}\sum_{i=1}^{n}(X_i - 1)^2$

(b) $\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$

(c) $\frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$

(d) $\frac{1}{n-1}\sum_{i=1}^{n}(X_i - 1)^2$

24. If $(X_1, X_2, X_3, ..., X_n)$ be i.i.d. $N(\mu, \sigma^2)$ with μ known, then C-R bound for estimation of $\sigma^2 = \theta$ will be

(a) $\frac{2\theta}{n}$

(b) $\frac{\theta}{n}$

(c) $\frac{2\theta^2}{n}$

(d) $\frac{\theta^2}{n}$

25. Suppose the p.d.f. is

$$f(x, \theta) = \begin{cases} \frac{1}{\theta}, & 0 \le x \le \theta \\ 0, & \text{otherwise} \end{cases}$$

The sufficient statistic for θ based on a random sample of size n is

(a) $\sum_{i=1}^{n} X_i$

(b) $\overline{X}$

(c) $X_{(1)}$

(d) $X_{(n)}$

26. Let $(X_1, X_2, X_3, \ldots, X_n)$ be a random sample from a population with the p.d.f.

$$f(x, \theta) = \theta x^{\theta-1};\ 0 < x < 1,\ \theta > 0$$

Which one of the following is correct?

(a) $t_1 = \sum_{i=1}^{n} X_i$ is sufficient for θ.

(b) $t_1 = \prod_{i=1}^{n} X_i$ is sufficient for θ.

(c) No sufficient statistic exists for θ.

(d) None of the above

27. Let 5, 3, 3, 6, 2, 3, 5, 4, 3, 6 be a sample of size 10 from a population following Poisson (λ) distribution. The UMVUE of λ is

(a) 40

(b) 8

(c) 4

(d) 3

28. A random sample of 60 schoolchildren gives 40 spectacle users. The 95% confidence limits of the spectacle users of the population of schoolchildren are

(a) 66%, 89%

(b) 66%, 79%

(c) 55%, 66%

(d) 55%, 79%

29. A sample of size one is drawn from Poisson distribution with parameter θ. To test $H_0 : \lambda = 1$ against $H_1 : \lambda = 2$, consider the new randomized test with critical region $\omega = \{x : x > 3\}$. The probability of Type–I error is

(a) $(3e^2 - 19)/(3e^2)$

(b) $(3e - 5)/3$

(c) $(3e - 8)/(3e)$

(d) $(3e^2 - 16)/(3e^2)$

30. Let $(X_1, X_2, X_3, ..., X_n)$ be a random sample from $N(\mu, \sigma^2)$ distribution, where both μ and σ^2 are unknown. The shortest expected length confidence interval for μ is obtained using

(a) standard normal distribution

(b) *t*-distribution

(c) *F*-distribution

(d) Chi-square distribution

31. Consider the following statements :

1. Scheffe's test of multiple comparisons tests all simple and complex contrasts.
2. Bartlett's test of multiple comparisons tests simple contrasts.
3. Tukey's test of multiple comparisons reduces the Type–I error in the test.

Which of the above statements are correct?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

32. Consider the following statements for multiple comparison tests conducted after a significant result is obtained in ANOVA :

1. The *t*-test on all pairwise comparisons inflates Type–I error.
2. Scheffe's test requires equal group sizes for multiple comparisons.
3. Newman-Keuls test reduces the risk of Type–II error.

Which of the above statements are correct?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

33. Consider the model

$$E(Y_{ij}) = \mu_i,\ \text{var}(Y_{ij}) = \sigma^2$$

$$j = 1, 2, 3, \ldots, n_i;\ i = 1, 2, 3, \ldots, p$$

Which of the following is/are correct?

1. Any linear parametric function is estimable.
2. $\hat{\mu}_i = \dfrac{\sum_{j=1}^{n_i} Y_{ij}}{n_i}$

Select the correct answer using the code given below.

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

34. Let A be an $n \times p$ matrix and A^- be the generalized inverse of A. Then which of the following statements are correct?

1. The order of A^- is $p \times n$.
2. (A^-A) and (AA^-) are symmetric matrices.
3. If the rank of the matrix A is n, then $A^- = (A'A)^{-1}A'$.
4. If A is non-singular, then $A^{-1} = A^-$.

Select the correct answer using the code given below.

(a) 1, 2 and 3

(b) 1, 3 and 4

(c) 1, 2 and 4

(d) 2, 3 and 4

35. Consider a two-way classification with r observations per cell and interaction present

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + e_{ijk}$$

$$i = 1, 2, 3, \ldots, p;\ j = 1, 2, 3, \ldots, q;$$

$$k = 1, 2, 3, \ldots, r$$

Then the total number of parameters in this model is

(a) $1 + p + q + pq$

(b) $p + q + pq$

(c) pq

(d) $p + q + 1$

36. For a sample of unit size from a population with the p.d.f.

$$f(x, \alpha) = \frac{2}{\alpha^2}(\alpha - x);\quad 0 < x < \alpha$$

consider the following statements :

1. $2x$ is the maximum likelihood estimator of α.
2. $2x$ is the unbiased estimator of α.

Which of the above statements is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

37. Let $X_1, X_2, X_3, ..., X_n$ be i.i.d. Bernoulli (p) random variables and $S_n = \sum_{i=1}^{n} X_i$. Then the unbiased estimator of p^3 is

(a) $\frac{S_n^3}{n^3}$

(b) $\frac{S_n (S_n - 1)(S_n - 2)}{n(n-1)(n-2)}$

(c) $\frac{(S_n - 1)(S_n - 2)(S_n - 3)}{(n-1)(n-2)(n-3)}$

(d) None of the above

38. A sample of n independent observations $X_1, X_2, X_3, ..., X_n$ is taken from $N(\mu, \sigma^2)$. For what value of k will

$$T = \frac{k}{(n-1)} \sum_{i=1}^{n-1} (X_{i+1} - X_i)^2$$

be an unbiased estimator of σ^2?

(a) 1

(b) $\frac{1}{2}$

(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

39. Let $(X_1, X_2, X_3, ..., X_n)$ be a random sample from an exponential population with mean $\frac{1}{\theta}$. If $Z = nX_{(1)}$, where $X_{(1)} = \min(X_1, X_2, X_3, ..., X_n)$, then which of the following is/are correct?

1. Z is an unbiased estimator of $\frac{1}{\theta}$.
2. Z is consistent estimator of $\frac{1}{\theta}$.

Select the correct answer using the code given below.

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

40. Let t_1 be a most efficient estimator and t_2 be a less efficient estimator with efficiency e and let ρ be the correlation coefficient between the two estimators t_1 and t_2. Which one of the following is correct?

(a) $\rho = e$

(b) $\rho = 2e - 1$

(c) $\rho = \sqrt{e}$

(d) $\rho = e^2$

41. Which of the following are true for Sustainable Development Goals (SDGs)?

1. It has 8 goals.
2. It has 17 goals.
3. It is recommended by UNICEF.
4. It is recommended by UNDP.
5. Life below water, Life on land, No Poverty and Climate Action are included in SDGs.

Select the correct answer using the code given below.

(a) 1, 3 and 5

(b) 2 and 4 only

(c) 2, 4 and 5

(d) None of the options is correct

42. Which one of the following is ***not*** a desirable property of official statistics?

(a) It should be relevant.

(b) It should be brought out in a timely manner.

(c) It should be reliable and credible.

(d) It should be able to disclose an individual unit's data.

43. The reliability of data collected through sample surveys may be measured by

(a) average

(b) standard deviation

(c) coefficient of variation

(d) standard error

44. The Ultimate Stage Unit for data collection in socio-economic survey of NSSO (National Sample Survey Office) is

(a) individual

(b) family

(c) household

(d) village/UFS (Urban Frame Survey) block

45. The most common sampling design used by NSSO in its socio-economic surveys is

(a) simple random sampling

(b) circular systematic sampling

(c) probability proportional to size (PPS) sampling

(d) two-stage stratified sampling

46. Metadata is

(a) unit level data

(b) data in tabular format

(c) raw data (i.e., no data validation has been done)

(d) data about data/information about data

47. The subject of Statistics comes under which of the following Lists of the Constitution of India?

(a) Union List

(b) State List

(c) Concurrent List

(d) All of the above

48. Which of the following are the main activities of CSO (Central Statistics Office)?

1. Coordination of statistical activities in the country
2. Evolving statistical standards
3. Collecting data through sample surveys
4. Providing guidance and training in the field of official statistics

Select the correct answer using the code given below.

(a) 1, 2 and 3

(b) 2, 3 and 4

(c) 1, 2 and 4

(d) 1, 3 and 4

49. The Collection of Statistics Act, 2008 is applicable on

1. industrial and business concerns
2. individual and household
3. government and private agencies/enterprises

Which of the above are correct?

(a) 1 and 2 only

(b) 1 and 3 only

(c) 2 and 3 only

(d) 1, 2 and 3

50. The Dearness Allowance (DA) of Central and State Government employees is linked with

(a) WPI (Wholesale Price Index)

(b) CPI (Consumer Price Index)—U&R (Urban and Rural)

(c) CPI (Consumer Price Index)—IW (Industrial Workers)

(d) CPI (Consumer Price Index)—AL (Agricultural Labour)/RL (Rural Labour)

51. Let X be a random variable having exponential distribution with mean $\frac{1}{\theta}$. Consider the critical region $\omega = \{x : x > 2\}$ to test $H_0 : \theta = 1$ against $H_1 : \theta = 2$. The power of the test at $\theta = 2$ is

(a) $(e^2 - 1)/e^2$

(b) $1/e^4$

(c) $(e^4 - 1)/e^4$

(d) $1/e^2$

52. The Neyman-Pearson fundamental lemma gives the method of construction of

(a) uniformly most powerful test

(b) most powerful test

(c) unbiased test

(d) randomized test

53. For the problem of testing $\mu = \mu_0$ against $\mu \neq \mu_0$ in sampling from $N(\mu, \sigma^2)$ distribution, where both μ and σ^2 are unknown, the likelihood ratio test is equivalent to

(a) uniformly most powerful test

(b) uniformly most powerful similar test

(c) uniformly most powerful invariant test

(d) uniformly most powerful unbiased test

54. Let $X_i (i = 1, 2, 3, \ldots, n)$ be distributed as $N(\mu_i, 1)$ distribution. The likelihood ratio test for testing $H_0 : \mu_1 = \mu_2 = \ldots = \mu_n = 0$ against $H_1 : \mu_i \neq 0$, for some i, is based on the statistic

(a) $\sum_{i=1}^{n} \left(\frac{X_i^2}{\mu_i} \right)$

(b) $\sum_{i=1}^{n} X_i^2$

(c) $\sum_{i=1}^{n} (X_i - \bar{X})^2$

(d) $\sum_{i=1}^{n} X_i$

55. Under the regularity conditions, if λ_n is the likelihood ratio, then the asymptotic distribution of $-2\log\lambda_n$, as $n \to \infty$, is

(a) normal distribution

(b) Chi-square distribution

(c) F-distribution

(d) t-distribution

56. In a Sequential Probability Ratio Test (SPRT), λ_m denotes the likelihood ratio at mth stage of the experiment; α and β are the probabilities of Type–I and Type–II errors respectively. Which of the following pairs is/are correctly matched?

1. $\lambda_m \geq \frac{1-\beta}{\alpha}$: Terminate the process with the rejection of null hypothesis H_0
2. $\frac{\beta}{1-\alpha} < \lambda_m < \frac{1-\beta}{\alpha}$: Terminate the process with the acceptance of H_0
3. $\lambda_m \leq \frac{\beta}{1-\alpha}$: Continue sampling by taking an additional observation

Select the correct answer using the code given below.

(a) 1 only

(b) 1 and 2

(c) 1 and 3

(d) 2 and 3

57. If N is stopping time for an SPRT, then which one of the following is correct?

(a) $P[N < \infty] = \frac{1}{4}$

(b) $P[N < \infty] = \frac{1}{2}$

(c) $P[N < \infty] = 1$

(d) $P[N < \infty] = \frac{3}{4}$

58. Let $\delta(x)$ be an estimator of θ and $L[\theta, \delta(x)]$ be the loss function associated with $\delta(x)$ at θ. Then the risk $R(\theta, \delta) = E[L(\theta, \delta(x))]$ is mean squared error if the associated loss function is

(a) $L[\theta, \delta(x)] = (\theta - a)^3$

(b) $L[\theta, \delta(x)] = (\theta - a)^2$

(c) $L[\theta, \delta(x)] = |\theta - a|$

(d) $L[\theta, \delta(x)] = \theta(\theta - a)^2$

59. Let $(X_1, X_2, X_3, \ldots, X_n) \sim N(\mu, 1)$ and let prior distribution of μ be $N(0, 1)$. Let $L(\mu, \delta) = [\mu - \delta(x)]^2$. Then the Bayes' risk associated with δ is

(a) $\dfrac{\sum_{i=1}^{n} x_i}{n+1}$

(b) $\dfrac{2}{n+2}$

(c) $\dfrac{1}{n+1}$

(d) $\dfrac{\sum_{i=1}^{n} x_i^2}{n}$

60. The Bayes' estimator of a parameter under squared error loss function is

(a) posterior mean

(b) posterior median

(c) posterior mode

(d) posterior variance

61. The retail inflation in India is being measured on which one of the following indices?

(a) CPI (Consumer Price Index)—IW (Industrial Workers)

(b) WPI (Wholesale Price Index)

(c) CPI (Consumer Price Index)—U&R (Urban and Rural)

(d) CPI (Consumer Price Index)—AL (Agricultural Labour)/RL (Rural Labour)

62. Annual inflation rate can be calculated as

(a) $\dfrac{\text{CPI (current year)} - \text{CPI (last year)}}{\text{CPI (last year)}} \times 100$

(b) CPI (current year) – CPI (last year)

(c) $\dfrac{\text{CPI (current year)} - \text{CPI (base year)}}{\text{CPI (base year)}} \times 100$

(d) CPI (current year) – CPI (base year)

63. Price deflators are used to obtain

(a) GDP at constant prices

(b) GDP at current prices

(c) GDP at basic prices

(d) GVA at producer prices

64. Which of the following are the sources of industrial data in India?

1. Annual Survey of Industries (ASI)
2. Economic Census (EC)
3. Employment, Unemployment Survey of NSSO
4. Micro, Small and Medium Enterprise (MSME) Survey

Select the correct answer using the code given below.

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1, 2 and 4 only

(d) 1, 2, 3 and 4

65. Which one of the following is ***not*** correct for Minimum Support Price (MSP)?

(a) MSP is the price at which government purchases food grains from the farmers.

(b) MSP ensures adequate food grains production in the country.

(c) MSP is a direct benefit transfer.

(d) MSP is recommended by CACP (Commission for Agricultural Costs and Prices).

66. Which among the following is a data source from where Infant Mortality Rate can be calculated for India?

(a) NSSO Health Survey

(b) Economic Census

(c) NSSO Disability Survey

(d) Population Census

67. Which one of the following organizations releases Wholesale Price Index (WPI)?

(a) Labour Bureau

(b) NSSO

(c) CSO

(d) Office of Economic Adviser, the Ministry of Commerce and Industry

68. The periodicity of Agriculture Census in India is

(a) 5 years

(b) 7 years

(c) 10 years

(d) 15 years

69. Effective literacy rate is the literacy rate

(a) taking the population from age 7 years and above into account

(b) taking the entire population into account

(c) taking the population from age 7 years to 60 years into account

(d) None of the above

70. The Human Development Index (HDI) developed by UNDP does ***not*** include

(a) life expectancy at birth

(b) gender equality

(c) expected and mean years of schooling

(d) GNI per capita

71. Which one of the following statements is ***not*** correct?

(a) If any Bayes' solution is unique, it must be admissible.

(b) A uniformly best decision rule always exists.

(c) Posterior median is the Bayes' estimator under the absolute error loss function.

(d) A Bayes' decision rule, whose risk is constant, is also a minimax decision rule.

72. Let $(X_1, X_2, X_3, ..., X_n)$ denote a random sample from a population with the density $f(x, \theta) = \sqrt{\theta} x^{\sqrt{\theta}-1}$, $0 \le x \le 1$, $\theta > 0$. The MLE of θ is given by

(a) $\dfrac{\sum_{i=1}^{n} \log x_i}{n}$

(b) $\left(\dfrac{\sum_{i=1}^{n} \log x_i}{n}\right)^2$

(c) $\dfrac{n}{\sum_{i=1}^{n} \log x_i}$

(d) $\dfrac{n^2}{\left(\sum_{i=1}^{n} \log x_i\right)^2}$

73. Let $(X_1, X_2, X_3, ..., X_n)$ be an i.i.d. sequence of random variables $(n > 2)$ from the distribution whose density function is $f(x, \theta) = \theta x^{\theta-1}$, $0 < x < 1$, $\theta > 0$. If $Z = -\left(\sum_{i=1}^{n} \log X_i\right)$, then an unbiased estimator of θ is

(a) $\frac{n}{Z}$

(b) $\frac{n-1}{Z}$

(c) nZ

(d) $(n-1)Z$

74. A statistic $t = t(x_1, x_2, x_3, ..., x_n)$ is sufficient for θ. Consider the following statements :

1. It contains all the information about θ.
2. The likelihood function can be expressed as

   $L(x, \theta) = g(t, \theta)h(x_1, x_2, x_3, ..., x_n)$
3. The conditional distribution of $x_1, x_2, x_3, ..., x_n$ given t is independent of θ.

Which of the above statements are correct?

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

75. Let $(X_1, X_2, X_3, ..., X_n)$ be a random sample from Rayleigh distribution with the p.d.f. $f(x, \theta) = \frac{x}{\theta}\exp\left\{-\frac{x^2}{2\theta}\right\}$; $x > 0$, $\theta > 0$. The complete sufficient statistic for θ is

(a) $\sum_{i=1}^{n} x_i$

(b) $\bar{x}$

(c) $\sum_{i=1}^{n} x_i^2$

(d) $\max(x_1, x_2, x_3, ..., x_n)$

76. A single observation X is drawn from $B(10, p)$ distribution. Reject $H_0 : p = \frac{1}{2}$ in favour of $H_1 : p = \frac{1}{4}$ if $X \leq 3$. Then the power of the test is

(a) $369\left(\frac{3^7}{4^{10}}\right)$

(b) $366\left(\frac{3^7}{4^{10}}\right)$

(c) $372\left(\frac{3^7}{4^{10}}\right)$

(d) $375\left(\frac{3^7}{4^{10}}\right)$

77. For testing H_0 against H_1, if $\phi(x)$ is a UMP, then

(a) $\phi(x)$ is always unbiased

(b) $\phi(x)$ is never unbiased

(c) $\phi(x)$ has power less than its size

(d) None of the above

78. Let X_1 and X_2 be i.i.d. random variables with Poisson. Then $(X_1 + 2X_2)$ is not sufficient because

(a) $P(X_1 = 1, X_2 = 1 \mid T = 3)$ depends on λ

(b) $(X_1 + 2X_2)$ is Poisson

(c) $(X_1 + 2X_2)$ is not Poisson

(d) $P(X_1 = 1, X_2 = 1 \mid T = 3)$ is Poisson with parameter one

where $T = (X_1 + 2X_2)$.

79. Let X be the random variable from the p.m.f.

$f(x, \theta) =$

$$\begin{cases} \left(\frac{\theta}{2}\right)^{|x|}(1-\theta)^{1-|x|}; & x = -1, 0, 1 \text{ and } 0 < \theta < 1 \\ 0 \quad ; & \text{otherwise} \end{cases}$$

The complete sufficient statistic for θ

(a) is X

(b) is $X - 1$

(c) is $|X|$

(d) does not exist

80. Let $(X_1, X_2, X_3, \ldots, X_n)$ be i.i.d. random variables with $N(\theta, 1)$. Then C-R lower bound for the variance of an unbiased estimator of θ^r is

(a) $\frac{\theta^r}{n}$

(b) $\frac{r\theta^{r-1}}{n}$

(c) $\frac{r^2\theta^{2r-2}}{n}$

(d) $\frac{\theta^r}{n^2}$

STATISTICS
Paper – III

Time Allowed : Three Hours | *Maximum Marks : 200*

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions divided under **TWO** sections.*

*Candidate has to attempt **FIVE** questions in all.*

*Both the **TWO** questions in Section A are **compulsory**.*

*Out of the **SIX** questions in Section B, any **THREE** questions are to be attempted.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

The number of marks carried by a question / part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

*Answers must be written in **ENGLISH** only.*

SECTION A

All questions of this section are compulsory.

Q1. (a) Explain sampling and non-sampling errors. State the sources of non-sampling errors. *10*

(b) Consider the three-variable regression model

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i, \quad i = 1, 2, ..., n$$

where X_1 and X_2 are two independent variables, Y is the dependent variable, β's are the coefficients and ε is the random component, $\varepsilon \sim N(0, \sigma^2)$. Obtain the estimates of parameters using OLS method of estimation. *15*

(c) The following figures give the production of a commodity of a country from 2001 to 2007 :

Year	*Production (in thousand metric tonnes)*
2001	4·7
2002	5·9
2003	7·2
2004	7·9
2005	7·6
2006	7·0
2007	7·9

Obtain the trend values by a four-year moving average method. *15*

Q2. (a) The following data show the cost of living indices for different groups along with their weights (% of expenditure), for middle-class people of some city in 2010. Obtain the general cost of living index number. Mr. X was getting a salary of ₹ 25,000 in 2010 and ₹ 82,900 in 2016. Calculate how much he ought to have received as extra allowance in 2016 to maintain his same standard of living. *10*

Group	*Base year group index*	*Weight*
Food	396	71·3
Clothing	550	2·7
Fuel and Light	360	9·3
House rent	110	6·7
Miscellaneous	290	10

(b) A simple random sample of 25 households was drawn from a city area containing 13745 households. The number of persons per household in the sample were as follows :

5, 6, 3, 4, 2, 3, 3, 3, 4, 4, 3, 2, 7, 4, 7, 6, 4, 4, 1, 4, 3, 4, 5, 6, 3.

Estimate the total number of people in the area and also compute the variance of the estimate. *15*

(c) Discuss the concept of heteroscedasticity. Explain this by graphical representation. Show that OLS estimators are unbiased even under the condition of heteroscedasticity. Obtain its variance. *15*

SECTION B

Answer any *three* questions out of the six questions given below.

Q3. (a) A finite population of size 98 is divided into three strata. It is given that $N_1 = 2N_2 = 4N_3$ and $S_1 = 2S_2 = 4S_3$. If a sample of 21 is to be selected from the population, obtain the number of units to be selected from each stratum under Neyman allocation. *10*

(b) Define linear systematic sampling scheme with N = nk. Obtain the expression for the variance of the sample mean in this case. When will this be more advantageous than SRSWOR scheme ? *15*

(c) Define Horvitz-Thompson unordered estimator $\hat{Y}_{HT}$ for the population total Y. Obtain an expression for $V(\hat{Y}_{HT})$. Comment on the estimator. *15*

Q4. (a) Explain generalized least squares method of estimation. Write its model along with assumptions. Obtain the estimates of parameters and their variances. *15*

(b) Discuss the meaning of Autocorrelation. Give a comparison between autocorrelation and serial correlation with an example. Give example of variance-covariance matrix of size 3 to show

(i) heteroscedasticity with no autocorrelation

(ii) homoscedasticity with autocorrelation

(iii) heteroscedasticity with autocorrelation.

Prove that mean of autocorrelated U's turns out to be zero. *15*

(c) For general linear model, show that least squares estimator $\hat{\beta}$ is the best linear unbiased estimate of β. *10*

Q5. (a) Two stationary time series u_t and u_t' are added together and a new series is formed by $v_t = u_t + u_t'$. If u_t and u_t' are independent, show that the autocorrelation of lag κ of v_t is given by

$$\frac{\rho_\kappa \operatorname{Var}(u_t) + \rho_\kappa' \operatorname{Var}(u_t')}{\operatorname{Var}(u_t) + \operatorname{Var}(u_t')}$$

where ρ_κ and ρ_κ' refer to the autocorrelation of u_t and u_t' respectively. *15*

(b) State the law of demand.

Consider the following econometric model :

q_i (quantity demanded) = $a_0 + b_0 p_i + u_i$ $(b_0 < 0)$

q_i (quantity supplied) = $a_1 + b_1 p_i + v_i$ $(b_1 > 0)$

the errors u_i and v_i being uncorrelated.

Find the slope of the regression line q_i on p_i. 15

(c) What is periodogram ?

A time series u_t is composed of one periodic with period λ and amplitude a and the other term an irregular component (b_t)

$$u_t = a \sin \frac{2\pi t}{\lambda} + b_t$$

Describe a method for finding the true cyclical period λ.

Indicate how you will modify the method if the cyclical component is composed of several periodic terms $\lambda_1, \lambda_2, ..., \lambda_k$. 10

Q6. (a) Explain double sampling scheme. Give an example. Obtain the ratio estimator for the population mean of the study variate when the mean of the auxiliary variate is unknown. 10

(b) In a two-stage sampling, the first and the second stage units are drawn by adopting SRSWOR. Suggest an unbiased estimator for the population mean. Derive an expression for its variance.

(You may assume variance of sample mean under SRSWOR). 15

(c) State the linear regression estimate for population mean. Obtain the estimate of the regression coefficient by least square method. Show that this estimate is unbiased. Under what condition is the estimate unbiased ? 15

Q7. (a) What is the need of simultaneous equation system in economic theory ? Discuss Keynesian model as a structural equation model of a national economy. Obtain its reduced form equations and hence reduced form disturbance term. 15

(b) Let Y_1 and Y_2 be two endogenous variables. Let further X_1, X_2 and X_3 be three predetermined variables. A simultaneous equation is expressed as

$$Y_1 = \gamma_{21} Y_2 + \beta_{11} X_1 + \varepsilon_1$$

$$Y_1 = \gamma_{22} Y_2 + \beta_{22} X_2 + \beta_{32} X_3 + \varepsilon_2$$

Discuss a method to identify the model. 15

(c) Discuss full information maximum likelihood method of estimation. Discuss only how to find the normal equations. 10

Q8. (a) Define a stationary time series.

For the stationary AR(2) process

$$X_t = \frac{5}{6} X_{t-1} - \frac{1}{6} X_{t-2} + e_t$$

where $e_t \sim N(0, \sigma_e^2)$ denotes the noise.

Find the autocorrelation function of lags 1 and 2. *10*

(b) Describe the different schemes for explaining the oscillations in a stationary time series. Explain the use of correlograms for discriminating between the above schemes. *15*

(c) Define Gini coefficient.

The following table presents data on distribution of personal income by decile groups of households for each group separately for both rural and urban sectors of a country for the year 2010.

On a graph paper, draw the Lorenz curve for each sector.

Also, compute the Gini coefficient for each sector.

Hence compare the income inequality of two sectors. *15*

Household	*Percentage share in total income*	
	Rural	*Urban*
0 – 10	3·0	2·3
10 – 20	4·4	3·3
20 – 30	5·4	4·1
30 – 40	6·4	4·8
40 – 50	7·4	5·8
50 – 60	8·3	7·0
60 – 70	9·6	8·7
70 – 80	10·9	10·0
80 – 90	13·8	14·0
90 – 100	30·8	40·0

STATISTICS

PAPER—IV

Time Allowed : Three Hours

Maximum Marks : 200

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are FOURTEEN questions divided under SEVEN Sections.

Candidate has to choose any TWO Sections and attempt the questions therein. All the Sections carry equal marks.

The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Normal Distribution Table is given at the end.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the QCA Booklet must be clearly struck off.

Answers must be written in ENGLISH only.

SECTION—A

(Operations Research and Reliability)

1. *(a)* The costs per year (in ₹) for running a truck and its resale value are given below :

Year	:	1	2	3	4	5	6	7
Running cost	:	1,000	1,200	1,400	1,800	2,300	2,800	3,400
Resale value	:	3,000	1,500	750	375	200	200	200

Determine at what year the replacement is due if the purchase price is ₹ 6,000. 10

(b) Five jobs have to be processed by machines *A* and *B* in the order 'first *A*, then *B*'. The processing times of the jobs in each machine are given below :

Jobs	:	1	2	3	4	5
Machine A	:	6	2	10	4	11
Machine B	:	3	7	8	9	5

Determine the sequence of jobs that will minimize the total elapsed time. What are the idle times for the machines? 10

(c) Explain the criteria—*(i)* optimistic, *(ii)* pessimistic and *(iii)* minimax regret in decision making under conditions of uncertainty. 10

(d) The cumulative distribution function of times to failure (measured in months) for a system is

$$F(t) = 1 - \frac{100}{(t+10)^2}, \quad t \geq 0$$

(i) What is the reliability function?

(ii) What is the failure rate as a function of time?

(iii) Does the failure rate increase or decrease?

(iv) What is MTTF? 10

(e) Reduce the game (whose payoff matrix is given below) by principle of dominance and solve it : 10

		Player B			
		I	*II*	*III*	*IV*
Player A	*1*	3	2	4	0
	2	3	4	2	4
	3	4	2	4	0
	4	0	4	0	8

2. Answer any *two* of the following :

(a) (i) The following table gives the profit earned by assigning jobs *1*, *2*, *3* and *4* to contractors *A*, *B*, *C* and *D* :

		Contractors			
		A	*B*	*C*	*D*
Jobs	*1*	13	11	7	12
	2	10	13	9	12
	3	14	11	8	11
	4	8	14	10	15

Find the assignment rule that will yield the maximum profit. 10

(ii) Given the simplex tableau for a maximization-type linear programming problem :

X_B	X_1	X_2	X_3	X_4	X_5	X_6	RHS
X_2	0	1	−1	*a*	0	−1	*b*
X_5	0	0	*c*	−3	1	3	18
X_1	1	0	2	−1	0	2	16
$Z_j - C_j$	0	0	*d*	*e*	0	1	

Find the set of values for *a*, *b*, *c*, *d* and *e*, so that each of the following cases is true : 15

The current tableau

(1) represents an optimal solution

(2) represents an infeasible solution

(3) is feasible but the problem has no finite optimum

(b) (i) Consider a system of n components connected in series. The failure time, T_i, of ith component has Weibull distribution $W(\alpha, \lambda_i)$, $i = 1, 2, ..., n$. Assuming T_i are independent, find the MTTF of the system. 10

(ii) The recorded lifetimes (in hours) for identical items are as follows :

10·2, 89·6, 54·0, 96·0, 23·3, 30·4, 41·2, 0·8, 73·2, 3·6, 28·0 and 31·6

Assuming lifetime has exponential distribution with parameter λ, find the failure rate λ and estimate the reliability at $t = 10$ hours. 15

(c) There are totally nine different routes from plants P, Q and R (each with respective capacity 100, 80 and 120) to markets A, B and C (each with respective demand 150, 50 and 100). The costs associated with the transportation of one unit of commodity from any plant to any market are given below :

		Markets		
		A	B	C
Plants	P	4	8	10
	Q	2	12	4
	R	8	14	2

Find the optimal set of routes and the number of items to be transported in the optimal route. 25

(d) (i) For a single-period inventory model with probabilistic demand and no setup cost, let c denote unit cost, h holding cost per item and unit time and p shortage cost per item and unit time.

Derive the optimal order quantity by assuming continuous density function for the demand. 10

(ii) A servicing facility has three channels to receive customers. On the average, 48 customers arrive for service in an 8-hour day. Each channel spends 15 minutes on an average to serve.

Find the average number of customers in the system, average number of customers waiting to be served and average time a customer spends in the system and in the queue. 15

SECTION—B

(Demography and Vital Statistics)

3. *(a)* Discuss the direct and indirect method of standardizing death rates. 10

(b) Fill in the blanks of the following table which are marked with question marks : 10

Age x	l_x	d_x	q_x	p_x	L_x	T_x	e_x^0
20	6,93,435	?	?	?	?	35,081,126	?
21	6,90,673	—	—	—	—	?	?

(c) Compute *(i)* GFR, *(ii)* SFR and *(iii)* TFR for the following data : 10

Age group of child-bearing females	*Number of women ('000)*	*Total births*
15–19	16·0	260
20–24	16·4	2244
25–29	15·8	1894
30–34	15·2	1320
35–39	14·8	916
40–44	15·0	280
45–49	14·5	145

(d) The number of persons dying at age 75 is 476 and the complete expectation of life at 75 and 76 years are 3·92 and 3·66 years respectively. Find the number of persons living at ages 75 and 76. 10

(e) Explain E. C. Rhodes method of fitting logistic curve for population forecasting. 10

4. Answer any *two* of the following :

(a) What is Vital Statistics? State the uses of vital statistics. Explain registration and census method of obtaining vital statistics in brief. What are the shortcomings of these methods? 25

(b) Write the different measures of mortality. Discuss the merits and demerits of each. 25

(c) For the following values of $l(x)$ of a life table, fit a Makeham curve : 25

Age x :	30	35	40	45	50	55
$l(x)$:	89685	86137	82277	77918	72795	66566

(d) How would you decide that the population has a tendency to increase, decrease or remain stable? Hence, define appropriate measures for measuring growth in population. Establish the relationship between gross reproduction rate and net reproduction rate. 25

SECTION—C

(Survival Analysis and Clinical Trials)

5. *(a)* Define cumulative hazard function, survival function and mean residual life function. Obtain the same for a Pareto distribution and describe its IFR/DFR property. 10

(b) In the usual notations, show that

$$ {}_n q_x = \frac{n({}_n m_x)}{\left[1+\frac{n}{2}({}_n m_x)\right]} $$ 10

(c) Explain Gehan test. The survival times from the time of randomization of Treatment *A* patients and Treatment *B* patients are as follows :

Treatment A : 2, 3, 5, 5+, 5+, 7, 8+, 8+, 10

Treatment B : 2+, 4, 4, 7, 7+, 8, 9, 9, 10+, 11

where + denotes censored observation. Compute Gehan large sample statistic.

[Normal distribution table has been provided at the end] 10

(d) Explain the steps for planning and conduct of multicentre trials. 10

(e) Explain the aspects of data management in clinical trials. 10

6. Answer any *two* of the following :

(a) *(i)* Explain Cox proportional hazards model with several covariates. Discuss the maximum likelihood estimation of the parameter vector involved in the model. 10

(ii) 43 female breast cancer patients with negative auxiliary lymph nodes and a minimum 10-year follow-up were selected from Ohio State University Hospital Cancer Registry. Of the 43 patients, 9 were immuno- peroxidase positive and the remaining 34 remained negative. Survival times (in months) for both groups of patients are as follows :

Immunoperoxidase Negative :

19, 25, 30, 34, 37, 46, 47, 51, 56, 57, 61, 66, 67, 74, 78, 86, 122+, 123+, 130+, 134+, 136+, 141+, 143+, 148+, 151+, 152+, 153+, 154+, 156+, 162+, 164+, 165+, 182+, 189+

Immunoperoxidase Positive :

22, 23, 38, 42, 73, 77, 89, 115, 144+

where + denotes censored observation.

Perform a proportional hazards regression (two iterations only) with immunoperoxidase status as the single covariate in the model. Start with an initial guess equal to zero.

[Normal distribution table has been provided at the end] 15

(b) *(i)* Derive the proper survival distribution for cause C_α, $\alpha = 1, 2, 3, \ldots, k$ in competing risks. Prove the properties of various survival distributions when crude hazard rates are proportional. 15

(ii) Suppose 20 patients are followed for a period of 1 year and to the nearest tenth of a month, deaths were observed at the following times :

0·5, 1·5, 1·5, 3·0, 4·8, 6·2, 10·5 months

In addition, losses to follow up were recorded at :

0·6, 2·0, 3·5, 4·0, 8·5, 9·0 months

Construct Kaplan-Meier life table for 20 patients and variances. 10

(c) Discuss the types of endpoints in clinical trials and their advantages and limitations. 25

(d) Explain crossover trials and their limitations. What are their advantages over parallel trials? Discuss various aspects in designing and monitoring phase III sequential clinical trials. 10+15

SECTION—D

(Quality Control)

7. *(a)* *(i)* Explain the use of p-charts and c-charts. What is the key difference between these two?

(ii) What is meant by process capability? How can it be measured? Why is it important? 10

(b) An $\bar{X}$-chart with 3-sigma control limits has the following parameters :

$$UCL = 104, \quad CL = 100, \quad LCL = 96$$

Suppose that the measured characteristic is normally distributed and has true mean of 98 and standard deviation of 8. What is the probability that the control chart will raise an out-of-control alarm at the fourth sample point? [Normal distribution table has been provided at the end] 10

(c) Compute measures of capability C_p and C_{pk} for the data given below and comment on it : 10

$$USL = 80, \; LSL = 50, \; \text{Process } \mu = 60, \; \text{Process } \sigma = 5$$

(d) The control charts for $\bar{X}$ and R are to be set up. The sample size is 5, and $\bar{X}$ and R values are computed for each of 35 samples. The summary data are

$$\sum_{i=1}^{35} \bar{X}_i = 7805, \qquad \sum_{i=1}^{35} R_i = 1200$$

(i) Find the trial control limits for $\bar{X}$ and R.

(ii) Assuming that the process is in control, estimate the process mean and standard deviation.

Use the following values :

n	4	5	6	7
A_2	0·729	0·577	0·483	0·4419
d_2	2·059	2·326	2·534	2·704
D_3	0	0	0	0·076
D_4	2·782	2·115	2·004	0·1924

10

(e) Explain the following terms : 10

(i) AQL

(ii) LTPD

(iii) Consumer's risk

(iv) Producer's risk

(v) OC curve

8. Answer any *two* of the following :

(a) (i) Describe a CUSUM control chart. Compare this with a Shewhart chart with respect to performance. How is V-mask useful in CUSUM charts? 15

(ii) What are Average Sample Number (ASN) and Average Total Inspection (ATI)? 10

(b) (i) Three bagging machines of a company have the following characteristics :

Machines	:	*A*	*B*	*C*
Standard deviation	:	0·2	0·3	0·05

If specifications are set as LSL = 12·35 and USL = 12·65, determine which of the machines are capable of producing within specifications. 5

(ii) The number of defective bulbs in 10 random samples with 30 observations each are found to be

1, 3, 3, 1, 0, 5, 1, 1, 1, 1

Construct a suitable 3-sigma control chart and offer your comments. 20

(c) (i) Discuss the key differences between chance causes and assignable causes. 5

(ii) Describe single and double sampling plans used in acceptance sampling. Define the OC function of a sampling plan. 10

(iii) Explain the moving average charts and exponentially weighted moving average charts. 10

(d) A quality control inspector of a company has taken 10 samples with four observations each. The data are given below :

1	2	3	4	5	6	7	8	9	10
12·5	12·8	12·1	12·2	12·4	12·3	12·6	12·4	12·6	12·1
12·3	12·4	12·6	12·6	12·5	12·4	12·7	12·3	12·5	12·7
12·6	12·4	12·5	12·5	12·5	12·6	12·5	12·6	12·3	12·5
12·7	12·8	12·4	12·3	12·5	12·6	12·8	12·5	12·6	12·8

Draw the control charts—

(i) if the process standard deviation is given to be 0·2;

(ii) by using sample ranges to find an estimate of process variability. 25

SECTION—E

(Multivariate Analysis)

9. (a) Let $\mathbf{X}_1$, $\mathbf{X}_2$, $\mathbf{X}_3$ and $\mathbf{X}_4$ be independent $N_p(\boldsymbol{\mu}, \Sigma)$ random vectors.

(i) Find the marginal distribution for each of the random vectors

$$\mathbf{V}_1 = \frac{1}{4}\mathbf{X}_1 - \frac{1}{4}\mathbf{X}_2 + \frac{1}{4}\mathbf{X}_3 - \frac{1}{4}\mathbf{X}_4$$

and $$\mathbf{V}_2 = \frac{1}{4}\mathbf{X}_1 + \frac{1}{4}\mathbf{X}_2 - \frac{1}{4}\mathbf{X}_3 - \frac{1}{4}\mathbf{X}_4$$

(ii) Find the joint density of the random vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ defined in (i). 10

(b) Let $\mathbf{X} \sim N_3(\boldsymbol{\mu}, \Sigma)$ with $\boldsymbol{\mu}' = (2, 3, -1)$ and

$$\Sigma = \begin{pmatrix} 7 & 3 & 2 \\ 3 & 4 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

(i) Obtain the conditional distribution of x_1 and x_2, given $x_3 = 2$.

(ii) Obtain the partial correlation coefficient between x_1 and x_2, given x_3, namely $\rho_{12.3}$. 10

(c) Explain the technique of principal component for handling a problem in multivariate analysis with too many variables. If $\sum = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$, where $\rho > 0$, then find the principal components associated with matrix Σ and find the percentage of total variance explained by the first principal component. 10

(d) Let $\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_n$ be n independent observations from $N_p(\boldsymbol{\mu}, \Sigma)\,(n > p)$ population. Obtain maximum likelihood estimators of the parameters. Are they independently distributed? Write the distribution of the estimators you have obtained. 10

(e) Define sample correlation coefficient matrix $R = (r_{ij})$ and obtain its null distribution. 10

10. Answer any *two* of the following :

(a) Discuss the procedure for testing the null hypothesis $H_0 : \boldsymbol{\mu} = \boldsymbol{\mu}_0$ against $H_1 : \boldsymbol{\mu} \neq \boldsymbol{\mu}_0$ regarding the specified mean vector of the multivariate normal population $N_p(\boldsymbol{\mu}, \Sigma)$.

Let the data matrix for a random sample of size $n = 3$ from a bivariate normal population be

$$X = \begin{pmatrix} 6 & 10 & 8 \\ 9 & 6 & 3 \end{pmatrix}$$

Evaluate the observed value of T^2 for $\boldsymbol{\mu}_0' = (9, 5)$. What is the sampling distribution of T^2 in this case? 25

(b) A researcher wants to determine a procedure for discriminating between two multivariate populations. The researcher has enough data available to estimate the density functions $f_1(\mathbf{X})$ and $f_2(\mathbf{X})$ associated with populations Π_1 and Π_2 respectively. Let $c(2/1) = 50$ and $c(1/2) = 100$.

In addition, it is known that about 20% of all possible items (for which the measurements $\mathbf{X}$ can be recorded) belong to Π_2.

(i) Give the minimum ECM rule (in general form) for assigning a new item to one of the two populations.

(ii) Measurements recorded on a new item yield the density values $f_1(\mathbf{X}) = 0{\cdot}3$ and $f_2(\mathbf{X}) = 0{\cdot}5$. Given the preceding information, assign this item to population Π_1 or population Π_2. 25

(c) Define canonical correlation coefficients and canonical variates. In the usual notations, show that the canonical correlations are solution of the determinant equation

$$\begin{vmatrix} -\lambda\Sigma_{11} & \Sigma_{12} \\ \Sigma_{12} & -\lambda\Sigma_{22} \end{vmatrix} = 0$$

Hence or otherwise, show that multiple correlation and simple correlation are special cases of canonical correlation. 25

(d) Define Wishart matrix and the distribution of the Wishart matrix D. Write the applications of Wishart distribution. If $D \sim W_p(D \mid n \mid \Sigma)$, then show that $\frac{|D|}{|\Sigma|}$ is distributed as the product of p independent Chi-square variates with d.f. n, $(n-1)$, $(n-2)$, $\cdots$, $(n-p+1)$ respectively. Also obtain $E(|D|^h)$, $h = 1, 2, 3, \ldots$. If $\mathbf{X}_1$, $\mathbf{X}_2$ and $\mathbf{X}_3$ are identically independently distributed as $N_2(\boldsymbol{\mu}, \Sigma)$ with $\boldsymbol{\mu} = \mathbf{0}$ and $\sum = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$, obtain $E|\mathbf{X}_1\mathbf{X}_1' + \mathbf{X}_2\mathbf{X}_2' + \mathbf{X}_3\mathbf{X}_3'|$. 25

SECTION—F

(Design and Analysis of Experiments)

11. (a) In the usual notations, considering the following linear model, obtain the best estimates of the parameters : 10

$$y_1 = 2\beta_1 + 3\beta_2 + \varepsilon_1$$
$$y_2 = 3\beta_1 + 4\beta_2 + \varepsilon_2$$
$$y_3 = 4\beta_1 + 5\beta_2 + \varepsilon_3$$

(b) If S_L and S_M are orthogonal sum of squares with n_1 and n_2 degrees of freedom, also $E(\text{error}) = 0$, then establish that $\frac{S_L / n_1}{S_M / n_2}$ follows non-central F-distribution. 10

(c) In the usual notations, assuming that the linear model of CRD is $Y_{ij} = \mu + \tau_i + \varepsilon_{ij}$ $(i = 1, 2, \cdots, m$ and $j = i, i+1, \cdots, n_i)$ and if $a\mu + \Sigma t_i \tau_i$ is estimable, then prove that the best estimate is $\Sigma t_i \bar{Y}_i$. 10

(d) Mentioning the need for RBD and its advantages over CRD and specifying its statistical model, write the detailed statistical analysis with one missing observation. 10

(e) Derive the efficiency of LSD in comparison with the corresponding RBD in which either rows or columns are omitted. 10

12. Answer any *two* of the following :

(a) Write a detailed note on Kruskal-Wallis test to carry out ANOVA in one-way classification. 25

(b) Discussing the layout of an RBD to conduct 3^3 factorial experiment in 3 blocks, explain its ANOVA along with detailed computational procedure. 25

(c) Considering a 2^3 design, describe the layout of the design by confounding second-order interaction in each replicate. Explain ANOVA with 3 replications. 25

(d) Distinguish between ANOVA and ANCOVA. Explain detailed analysis of covariance for RBD. 25

SECTION—G

(Computing with C and R)

13. (a) Write a C-program to evaluate and print

$$Q = \sum_{j=1}^{m}\left[\sum_{i=1}^{j}(3j+ji)^{i+1}\right]^2$$ 10

(b) Write a C-program to verify and output whether a given square matrix is orthogonal or not. 10

(c) Write a C-program to verify whether a given string is a PALINDROME. 10

(d) Write an R-program to find 'SADDLE POINT' in a two-person zero-sum game and print its position in the matrix and also its value. 10

(e) Given $X_{n\times k}$ and $Y_{n\times l}$, write an R-program to print the vector $\hat{\beta} = (X'X)^{-1}X'Y$. 10

14. Answer any *two* of the following :

(a) Write a suitable function in C and main program to invoke the same to print the values of the following : 15+10

$$P = \frac{x + \sin(x)}{\sqrt{1 + \cos\left(\frac{x}{x^2 + y^2}\right)}} \quad \text{and} \quad Q = \frac{\tan(x)}{x^2 + 5\sqrt{1 + \cos\left(\frac{x^3}{x^4 + 1}\right)}}$$

(b) Write a C-function to compute

$$\sin^{-1}(x) = x + \frac{1}{2}\frac{x^3}{3} + \frac{1}{2}\frac{3}{4}\frac{x^5}{5} + \frac{1}{2}\frac{3}{4}\frac{5}{6}\frac{x^7}{7} + \cdots \infty$$

and main program to print the value of $\cos^{-1}(x)$. 15+10

(c) Given the 3 vectors X, Y and Z, write a program to compute—

(i) multiple correlation coefficient;

(ii) partial correlation coefficient. 12+13

(d) Write a program to—

(i) create stack on n elements;

(ii) insert an element after rth element;

(iii) delete rth element. 9+8+8

★ ★ ★